R&D cooperation and product market competition

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Abstract

I derive equilibria (that is, self-enforcing agreements) for firms that cooperate in improving their product and compete in marketing it. I am particularly interested in the interaction between R&D and product market decisions along optimal equilibria (that is, equilibria that maximize firm value). I assume that (i) the product is homogeneous and any product improvement is a public good (in other words, spillovers are complete); (ii) product improvement is a function of the effort by each firm; (iii) firms cannot observe each other’s effort; (iv) firms market the product by simultaneously setting prices.

The analysis reveals some surprising results. I find that, in some cases, although monopoly prices can be sustained in equilibrium, optimal equilibria call for firms to set lower-than-monopoly prices in the short run, thus providing the right incentives for investment in R&D. Under a different set of assumptions, I find that it is optimal to reduce R&D effort below the efficient level as a means to sustain short-run collusion in prices, even though efficient R&D effort could be achieved in equilibrium. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The number of formal alliances between firms has grown dramatically in recent
years. In the US, for example, this number has increased from 750 in the 1970s to 20,000 in 1987–1992 (The Economist, September 2nd, 1995). Although the scope of these alliances cannot always be easily ascertained, R&D cooperation seems to play an important role in most of them.\(^1\) The actual mechanism underlying each agreement can take various forms, from strategic alliances without equity commitments to mergers or joint ventures that involve a separate shared entity.

The increase in cooperative R&D activity results, to a large extent, from the tolerance — and even encouragement — that competition policy authorities have increasingly shown towards this kind of agreement.\(^2\) However, despite this freedom from regulatory constraint, it remains a fact that formal, complete interfirm contracts pertaining to R&D activities are extremely difficult to write down and enforce.\(^3\) Firstly, R&D programs normally progress over a long period, and it is difficult to contract on future actions. Secondly, the contribution that each firm brings into the alliance is frequently not observable, or at least it is only imperfectly observable.\(^4\)

It is well known from Game Theory that non-contractibility does not preclude agreement: when interaction is repeated, firms may establish self-enforcing agreements that are as good as, or almost as good as, enforceable contracts (Fudenberg et al., 1994). In the particular case of R&D alliances, the issue is complicated — or enriched — by the fact that R&D partners are frequently competitors in the product market. The set of instruments for establishing cooperative R&D agreements is thus expanded to include product market decisions as well.\(^5\)

In this paper, I derive equilibria (that is, self-enforcing agreements) for firms that cooperate in improving their product and compete in marketing it. I am particularly interested in the interaction between R&D and product market decisions along optimal equilibria (that is, equilibria that maximize firm value). I assume that (i) the product is homogeneous and any product improvement is a public good (in other words, spillovers are complete); (ii) product improvement is

\(^1\)According to Veugelers (1993), R&D was the motivation for more than 20% of the alliances involving Belgian firms.


\(^3\)This may explain why agreements are frequently very simple (e.g. cross licensing of newly developed technologies); and why formal joint ventures are so unstable. For discussions on the issue of stability of joint ventures, see Harrigan (1985), Kogut (1989) and Minehart (1993).

\(^4\)For example, Shapiro and Willig (1990, p. 114) argue that RJV participants may ‘contribute its less able personnel or withhold its most advanced technology from the venture’, both of which are instances of imperfect observability.

\(^5\)The supergames I will consider in this paper are strictly speaking not repeated games, since R&D introduces stock variables and nonstationary processes. However, the insights from the repeated-game literature apply.
a function of the effort by each firm; (iii) firms cannot observe each other’s effort; (iv) firms market the product by simultaneously setting prices.

The analysis of a simple game with this structure reveals some surprising results. I find that, in some cases, although monopoly prices can be sustained in equilibrium, optimal equilibria call for firms to set lower-than-monopoly prices in the short run, thus providing the right incentives for investment in R&D. Under a different set of assumptions, I find that it is optimal to reduce R&D effort below the efficient level as a means to sustain short-run collusion in prices, even though efficient R&D effort could be achieved in equilibrium.

My paper is not the first to look at the interaction between R&D cooperation and product market competition. Other references include Martin (1993, 1995) and van Wegberg (1995). Martin (1993), in particular, argues that cooperative R&D may reduce welfare, since ‘joint R&D makes it more likely that firms will be able to sustain tacit collusion on output markets’ (pp. 1–2). The idea is reminiscent of Bernheim and Whinston’s (1990) theory of multimarket contact: firms that interact in more than one market are able to sustain collusion more easily than firms interacting in one market only. Although an R&D agreement is not isomorphic to a product market, Martin (1995) shows that Bernheim and Whinston’s (1990) intuition extends to the R&D cooperation-cum-product-market-competition case. He assumes that R&D investments are contractible and shows that the scope for collusion in the product market is extended when the breakdown of the R&D agreement is taken into account as a possible punishment for deviations in the output market.

My assumptions regarding R&D cooperation differ from Martin’s (1995). In particular, I assume that R&D investments cannot be contracted upon — in fact, are not even observable. Not surprisingly, the results I obtain differ from his. First, I show that product market prices may decrease as a result of an R&D agreement. Second, when product market prices increase as a result of an R&D agreement, the mechanism by which this happens is very different from Martin’s (1995).

The paper is organized as follows. In Section 2 I lay out the basic framework that is common throughout the paper. In Sections 3 and 4 I specialize the model to two particular cases and show how optimal equilibria may involve lower prices
and slower R&D progress than equilibria which do not account for the interaction between R&D and product market decisions. Section 5 concludes the paper.

2. General framework

I consider an infinite-period duopoly industry in which firms make R&D decisions as well as product market decisions. On the R&D side, I assume that firms start with an old technology. By investing in R&D, a new, more profitable technology may be discovered. This happens, in each period, with probability \( P(x_1, x_2) \), where \( x \) is the amount invested in that period. R&D effort costs firm \( i \) \( c(x_1) \).\(^7\) The new technology is a public good: both firms benefit equally from its discovery. This and the assumption of non-contractibility imply that the model applies equally to the case of independent R&D efforts, informal alliances, or a formal RJV.\(^8\)

On the product market side, I assume that firms sell the same product and simultaneously set prices. Product market profits are given by \( \pi(p_1, p) \), if the firms are using the old technology, or \( \pi(p_1, p_1) \), if the firms have discovered the new technology. Notice that, given this level of generality, the innovation in question could either consist of an improvement in the product or a reduction in costs. All that I need to assume is that \( \pi(p, p) > \pi(p_1, p) \), that is, profits increase as a result of the innovation.

The specific timing of the model is illustrated in Fig. 1. At the beginning of each period, firms simultaneously choose price \( p \) and R&D effort \( x \). Firms then receive the payoff during that period, \( \pi(p_1, p) - c(x_1) \). At the end of the period, the outcome of R&D is known. Success in R&D is a zero-one, once-and-for-all event. With probability \( P(x_1, x_2) \), a superior product is discovered and firms move to a price-setting subgame with no R&D and a stage payoff function \( \pi(p_1, p_1) \); with probability \( 1 - P(x_1, x_2) \), nothing results from the R&D effort, and firms start the next period in the same way as the previous one.

\(^7\)I could alternatively assume that the probability of discovery is a function of cumulative effort. It is not clear that this alternative formulation would be more realistic than the one I follow. Suppose, for example, that firms are attempting to discover a room-temperature superconductor. One interpretation of the model is that, in each period, a different material is tried out. In this respect, it makes sense to consider independent, non-cumulative probabilities of success. (An additional motivation for my modeling option — arguably the primary motivation — is that cumulative probabilities of success complicate the analysis considerably.)

\(^8\)In the case of independent R&D investments, I implicitly assume that the rate of spillovers is 100%. This assumption is made for simplicity. The results could be extended to the case when the rate of spillover is strictly less than one. In this case, independent R&D efforts and a formal RJV would be different even in my model.
In other words, success in R&D is an absorbing state. Time can thus be divided in before success and after success in R&D. Before R&D success firms receive a product market payoff $\pi(p_i, p_j)$. After R&D success, firms receive $\bar{\pi}(p_i, p_j)$.

Firms are risk neutral and maximize expected discounted payoff, which includes both product market profits and R&D expenditures. Time is discounted according to the discount factor $\delta$.

For part of the analysis that will follow, I will be interested in looking at stationary equilibria. A stationary equilibrium is a subgame perfect Nash equilibrium such that, along the equilibrium path, $x_i$ and $p_i$ are constant both before and after the innovation.\(^9\)

\(^9\)However, the values before the innovation may differ from those after. In particular, after the innovation $x_i = 0$, as there is no additional innovation to be achieved.
3. Low prices to sustain R&D investment

In this section, I assume that \( x_i \in \{0, 1\} \) and that the success function is as follows: \( P(1, 1) = \alpha, \ P(1, 0) = P(0, 1) = \beta, \ P(0, 0) = 0 \). Moreover, I assume that \( \beta \) is small.\(^{10}\) Finally, I assume that the cost of effort is simply \( c(x_i) = x_i. \(^{11}\)

My first result establishes that, under some conditions, firms find it optimal not to collude in prices, even though full collusion can be achieved as an equilibrium. Define \( \pi^a = \pi(p^a) \) and \( \pi^m = \pi(p^m) \), where \( p^a \) and \( p^m \) are the monopoly prices under the old and the new technology, respectively.

**Proposition 1.** Suppose that

\[
\delta \geq \frac{1}{2},
\]

\[
\frac{1 - \delta}{\delta(\alpha - \beta)} < \frac{\pi^m - \pi^a}{\pi^m - \pi^a} < \frac{1 - \delta(1 - \beta)}{\delta(\alpha - \beta)}.
\]

Then (i) there exist stationary equilibria such that firms set monopoly prices in every period; but (ii) optimal stationary equilibria imply prices below the monopoly level.

By *optimal* stationary equilibria I mean equilibria that maximize joint profits. It can be shown that, in an optimal stationary equilibrium, firms set prices below monopoly level until a new technology is discovered. From then on, firms revert to monopoly prices. Finally, in every period before R&D success is achieved, both firms invest \( x_i = 1 \) in R&D, the efficient level.

The proof of this and the following result may be found in Appendix A. The intuition underlying Proposition 1 is akin to Arrow’s (1962) replacement effect: a firm’s incentive to invest in R&D is inversely related to its current profit.\(^{12}\) By setting low prices, the firms create for themselves an additional incentive to invest in R&D. In some cases, this may be what is necessary to turn an unstable R&D

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\(^{10}\) Specifically, the assumption that is required is that

\[
\beta < \frac{1}{\delta(1 + \delta((1 - \alpha) - \delta(2 - \alpha)) - (1 - \delta))}.
\]

This assumption guarantees that there always exists a stationary equilibrium in pure strategies. When the assumption does not hold, the analysis becomes more complicated with no significant gain in insight.

\(^{11}\) I could alternatively assume a continuous set \( X \) for the R&D variable. I conjecture that there exist success and R&D-cost functions that would lead to a similar result as the discrete-variable function I am considering here.

\(^{12}\) I am grateful to Vincenzo Denicolo for pointing this out to me.
agreement into a stable one; and, if the value of the innovation is sufficiently high, then the short-run sacrifice in profits is worthwhile.\footnote{Lin (1995) presents a model in which the replacement effect implies that firms behave less aggressively. His model differs from mine in that he assumes firms commit to future prices (or quantities) before engaging in an R&D race (not an agreement). The intuition for his result is that, by behaving less aggressively, firms are able to soften up their rivals in the R&D race.}

Specifically, the second and third inequalities in Proposition 1 imply that the gain from innovation, $\pi^m - \pi^n$, is sufficiently high that engaging in R&D is efficient, but not so high that such an agreement is sustainable by itself.\footnote{To be precise, ‘sustainable by itself’ means with R&D strategies that are only a function of the history of observable R&D levels and R&D outcomes.} For this intermediate range of the gains from innovation, it is optimal to use the extra instrument of product market prices to sustain the efficient level of R&D investment. Finally, the first condition in the proposition implies that it is possible to sustain any level of prices (including monopoly prices).

3.1. Non-stationary equilibria

So far, I have only considered stationary equilibria, that is, equilibria where firms’ choices of $x_j$, $p_j$ are constant in time (except for the event of R&D success). The set of optimal equilibria is, however, much broader. In particular, there is an alternative simple symmetric equilibrium strategy which yields the same optimal payoff: \footnote{In fact, the equilibrium that follows yields a higher payoff than the low-price stationary equilibrium considered in Proposition 1. However, if we amend the low-price equilibrium to have firms charge monopoly prices in the first period and play the stationary equilibrium thereafter, then the two equilibria yield the same payoff.}

- set $x_j = 1$ in each period while the agreement is still on and success has not been reached;
- set monopoly prices in every period;
- abandon the R&D agreement with probability $y$ conditional on no success having been reached.

If $y$ is set at the highest value such that the no-deviation constraint for $x_j = 1$ is just binding, then the resulting equilibrium is optimal. In this alternative optimal equilibrium, it is the ‘threat’ of abandonment of the R&D agreement, not low prices, that sustains the efficient R&D investment.\footnote{In a non-strategic context, Dutta (1992) presents necessary and sufficient conditions for the optimality of a ‘bold play’ — spending all the remaining budget on the current stage of a sequential R&D project. This would be similar to the choice of $y = 0$.} Interestingly, Kogut (1989) finds that the hazard rate of R&D agreements is extremely high. For example, a large proportion of R&D agreements are dissolved during the first year. This is
consistent with the above equilibrium, although, admittedly, it is also consistent with several other theories.

More generally, two equilibria that yield the same payoff in the first period and the same continuation payoff as a function of the outcome of R&D in the first period are equally optimal. Low prices and project abandonment are two ways of achieving the same continuation payoff; but not necessarily the only ones.  

3.2. Public policy

Previous literature on public policy towards cooperative R&D has emphasized different trade-offs: if spillovers are important, then allowing for R&D cooperation may lead to a more efficient solution. However, if spillovers are relatively less important, then R&D cooperation may imply an R&D level that is less than efficient and less than the level that would prevail in the absence of a cooperative agreement. Moreover, as stated above, R&D agreements may help firms sustain a greater degree of collusion in the product market.

Given the very abstract level at which I am modelling R&D cooperation, it is difficult to discern immediately what the impact of public policy might be. If the effect of public policy is to encourage firms to reach a (self-reinforcing) agreement on R&D investment, then Proposition 1 suggests that such policy might (a) increase the level of R&D expenditures to the efficient level, and (b) decrease the price level (prior to innovation); that is, such policy might unequivocally improve welfare.

However, if the effect of public policy is to allow for contractually based R&D agreements when only self-reinforcing agreements were possible, then welfare may decrease as a result of a policy that favors formal cooperative R&D. In fact, Proposition 1 implies that moving to a contractually based R&D agreement maintains the level of R&D (at the efficient level) but increases (pre-innovation) prices, since firms no longer need the price incentive to comply with the R&D agreement.

4. Project delays to sustain high prices

In the previous section, I have assumed that, upon success in product improvement, the firms carry on playing a repeated price-setting game as before,

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17 This multiplicity is also a feature of models of collusion with imperfect observability. See Green and Porter (1984).
18 In this case, the implication of favoring R&D agreements is similar to what Martin (1995) has found, but for very different reasons.
only with a different payoff function. I now make the assumption that, in the subgame following R&D success, each firm’s minimax value is strictly positive. Specifically, I assume that the difference between the highest payoff, denoted by $\pi^m$, and the minimax payoff, denoted by $\pi^*$, is low. One justification for this is that firms are subject to capacity constraints which initially are not binding but become so once the improved product is introduced in the market. An alternative assumption is that innovation introduces product differentiation.

Moreover, I now assume that $x_i \in [0, 1]$, $P(x_i, x_j) = \min(x_i, x_j)$ and that $c(x_i) = 0$. These somewhat extreme assumptions imply that the efficient level of effort is $x_i = x_j = 1$, as R&D effort is costless. However, as I show below, it may be optimal not to invest the efficient level in R&D.

**Proposition 2.** Suppose that

\[ \delta \geq \frac{1}{2}, \]

\[ \pi^m > \delta (\pi^m - \pi^*) \]

Then (i) there exist equilibria such that firms choose efficient R&D effort levels; however, (ii) optimal equilibria imply that firms choose less than efficient R&D effort levels.

The intuition for the result is quite simple: in order to sustain collusive prices, there must be a credible punishment for deviations. If, as a result of technological progress, that punishment becomes less severe, then firms may prefer to delay innovation with the sole purpose of maintaining a sufficiently credible punishment for cheating on the price agreement.

As in the previous proposition, the condition $\delta \geq 1/2$ is necessary for some price collusion to be sustainable. If $\delta < 1/2$, then prices are equal to zero (prior to innovation), while R&D expenditures are at the efficient level. If, on the other hand, the second condition of Proposition 2 is violated, then firms collude in prices and invest in R&D at the efficient level.

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19 Consider the following example: demand is initially $D = 2 - P$ and expands to $D = 3(2 - P)$ following the innovation; each firm’s capacity is 1; production costs are zero. In this extreme example, the difference between monopoly profits and minimax profits is zero; in fact, there exists a unique equilibrium. As capacity is increased from 1 up, the difference between monopoly profits and minimax profits increases as well.

20 In the limit when product differentiation is very high, firms earn monopoly profits. The difference between maximum profit and minimax profit is zero.
5. Concluding remarks

Bernheim and Whinston (1990) have shown that, under some conditions, the scope for product market collusion is enhanced by multimarket contact. In other words, the set of attainable equilibria is increased when each firm \( i \) plays strategies \( p_i^k(h^k, h^{k'}) \), where price in market \( k \) is a function of the history of prices in markets \( k \) and \( k' \); compared with the case when strategies \( p_i^k(h^k) \) only depend on the price history in market \( k \).

My results relate to those by Bernheim and Whinston (1990). There is an analogy between competing in several markets \((k \text{ and } k')\) and competing in different strategic variables (price and R&D effort). There are, however, two important differences. First, my model features imperfect observability in one of the strategic variables (R&D), whereas Bernheim and Whinston (1990) consider the case of quasi-perfect information. Second, payoffs in my model are not additive across ‘markets’ as in Bernheim and Whinston (1990). Still, my framework is suggestive of a more general analysis of self-enforcing agreements when firms choose several related strategic variables.\(^{21}\)

There are at least two questions and/or criticisms that may be levelled against the results I present here. First, the fact that I considered a somewhat specialized model and a specific set of parameter values. With respect to the model, I have argued that several generalizations are possible that would maintain the qualitative nature of the results (although at a significant cost in terms of the complexity of the analysis). With respect to the set of parameter values, the implication is that Propositions 1 and 2 are possibility results, not necessity results; I accept that.

A second, perhaps more profound, question relates to the nature of the analysis and its derived claims. Is this positive analysis, that is, a claim of how firms actually behave; or is it rather normative analysis, that is, a prescription of how firms ought to behave if they seek to maximize value. (In fact, this question is relevant for every paper dealing with optimal cartel equilibria, beginning with Porter (1983) and Abreu (1986) and down to the present day.)

A devil’s advocate might create a ‘catch 22’ situation out of this, as follows: if it’s positive analysis we’re talking about, then its validity must be based on supporting empirical evidence (of which I have little, in fact, either supporting or otherwise). If, instead, this is normative analysis, then it is of limited interest to an IO audience.

A more benevolent view — one that I, not surprisingly, advocate — views optimal equilibria, in the first place, as normative analysis; and, secondly, as working propositions for positive analysis. If the empirical evidence is consistent with optimal equilibria, then the validity of the theory as positive analysis is

\(^{21}\)In Cabral (1998), I consider the case of multimarket contact with imperfect observability, an intermediate case between this paper and Bernheim and Whinston (1990).
established (or, at least, strengthened). If the opposite is true and the model primitives are a fair image of reality, then it follows that firms do not play optimal equilibria. This, by itself, is an interesting result, one might argue.

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Appendix

Proof of Proposition 1

First, I show that monopoly prices are sustainable in equilibrium. Suppose, for example, that \( x = 0 \) in every period. Then, we have a standard Bertrand repeated game, and the condition for monopoly prices to be sustainable is \( \delta > 1/2 \), one of the hypotheses in the Proposition.

Next, I show that setting monopoly prices and \( x_i = 1 \) cannot be an equilibrium. Discounted payoff along the proposed equilibrium would be

\[
V = \pi^m + \delta \left( \alpha \frac{\pi^m}{1 - \delta} + (1 - \alpha)V \right) - 1. \tag{A.1}
\]

A one-shot deviation to \( x = 0 \) would imply an expected payoff of

\[
V' = \pi^m + \delta \left( \beta \frac{\pi^m}{1 - \delta} + (1 - \beta)V \right). \tag{22}
\]

The no-deviation constraint with respect to \( x \) is therefore \( V \geq V' \), or simply

\[
\delta(\alpha - \beta) \left( \frac{\pi^m}{1 - \delta} - V \right) \geq 1. \tag{A.2}
\]

From (A.1), we get

\[22\text{We are considering here a one-time deviation, so the value } V \text{ is used on the right-hand side.}

Inexistence of incentive for a one-time deviation is a sufficient condition for equilibrium.
\[ V = (1 - \delta(1 - \alpha))^{-1} \left( \pi^m + \delta \alpha \frac{\pi^m}{1 - \delta} - 1 \right). \]  
(A.3)

Substituting in (A.2), and simplifying, we get

\[ \bar{\pi} - \pi^m \geq \frac{1 - \delta(1 - \beta)}{\delta(\alpha - \beta)}. \]

which contradicts one of the conditions for the Proposition.

From the above analysis, we conclude that the maximum payoff in an equilibrium where firms set monopoly prices in every period is

\[ \bar{V} = \frac{\pi^m}{1 - \delta}. \]

The final step in the proof is to show that a stationary equilibrium exists with prices below monopoly prices and payoff greater than the monopoly pricing equilibrium. Suppose that firms set \( x_i = 1 \) in each period. Suppose moreover that, in each period before R&D success, firms set \( p, \hat{p}^m \) such that

\[ \bar{\pi} - \pi(p) = \frac{1 - \delta(1 - \beta)}{\delta(\alpha - \beta)}, \]

or simply

\[ \pi(p) = \bar{\pi} - \frac{1 - \delta(1 - \beta)}{\delta(\alpha - \beta)}. \]  
(A.4)

From the analysis above, such price implies that the no-deviation constraint regarding \( x \) is (just) satisfied. Substituting (A.4) for \( \pi^m \) into (A.3), and simplifying, we get

\[ V = \frac{\pi}{1 - \delta} - \frac{1}{\delta(\alpha - \beta)} \equiv V^*. \]

The claim that \( V^* > \bar{V} \) is then equivalent to

\[ \bar{\pi} - \pi^m > \frac{1 - \delta}{\delta(\alpha - \beta)}, \]

which follows from one of the conditions of the Proposition. \( \square \)

\^Notice that, for some parameter values, this may actually imply pricing below marginal cost.
Proof of Proposition 2

I first prove that there exists an equilibrium where firms set \( x_i = 1 \) in each period. Suppose that firms do so. Moreover, suppose they set price at marginal cost in every period. Finally, suppose past deviations from prescribed equilibrium are ignored. Then, neither changes in \( x_i \) nor changes in \( p_i \) would affect either current or future payoff.

I next show that the maximum equilibrium payoff is increasing when \( \alpha \equiv \min(x_1, x_2) \) is decreased from \( \alpha = 1 \). Suppose that firms set price \( p \) before attaining success in R&D and monopoly prices thereafter. Discounted equilibrium payoff is then given by

\[
V = \pi(p) + \delta \left( \frac{\pi^m}{1 - \delta} + (1 - \alpha)V \right). \tag{A.5}
\]

Assuming the most severe punishment for price deviations \( (p_i = 0, x_i = 0) \), payoff from deviation is given by

\[
V' = 2\pi(p) + \delta\alpha\frac{\pi^m - \pi^*}{1 - \delta}. \tag{A.6}
\]

If \( \alpha \approx 1 \), then the no-deviation condition \( V \geq V' \) can be simplified into

\[
\pi(p) \leq \alpha\delta\frac{2\pi^m - \pi^*}{(1 - \delta)(1 - 2\delta(1 - \alpha))}. \tag{A.6}
\]

The value that maximizes discounted payoff corresponds to the maximum \( p \) consistent with the no-deviation constraint, unless the no-deviation constraint is satisfied for monopoly price. Straightforward computation shows that the second condition in the Proposition implies that the no-deviation constraint is binding for \( \alpha = 1 \) and \( p = p^m \).

Substituting the right-hand side of (A.6) for \( \pi(p) \) in (A.5), and simplifying, we get

\[
V = \alpha\delta\frac{2\pi^m - \pi^*}{(1 - \delta)(1 - 2\delta(1 - \alpha))} = V^*. \tag{A.7}
\]

Finally, the derivative of \( V^* \) with respect to \( \alpha \), at \( \alpha = 1 \), is given by

\[
\frac{\partial V^*}{\partial \alpha} \bigg|_{\alpha=1} = \frac{\delta(1 - 2\delta)(2\pi^m - \pi^*)}{1 - \delta},
\]

which is negative for \( \delta > 1/2 \), implying that payoff increases as \( \alpha \) decreases. □

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\(^{24}\)Inequality (A.6) requires that \( 1 - 2\delta(1 - \alpha) > 0 \), which is true for \( \alpha = 1 \).
References


