

# Bertrand Supertraps

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We study oligopoly price competition between multiproduct firms—firms whose products interact in the profit function. Specifically, we focus on the impact of intrafirm product interactions on the level of equilibrium profits. This impact may be decomposed in two different ways: (a) a direct effect (keeping the competitors' actions fixed) plus a strategic effect (i.e., through the competitors' actions); or, alternatively, (b) a competitive advantage effect (change in firm  $i$  only) plus an imitation effect (change in all other firms). We derive conditions such that (a) the strategic effect more than outweighs the direct effect, and conditions such that (b) the imitation effect more than outweighs the competitive advantage effect: *Bertrand supertraps*. For example, an increase in the degree of economies of scope would increase profits if prices were fixed or if the change were limited to firm  $i$ 's cost function. However, if all firms increase the degree of economies of scope then all firms receive lower profits. A variety of other applications is considered, including learning curves, core competencies, demand synergies, systems competition, compatibility, bundling, network effects, switching costs, durable goods, long-term contracts.

*Key words:* competition; strategic complementarity; economies of scope; learning curves; core competencies; demand synergies; systems competition; compatibility; bundling; network effects; switching costs; durable goods; long-term contracts

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## 1. Introduction

Most firms produce more than one product: Ford produces cars and trucks; Kodak sells cameras and film; American Airlines offers air travel services along various routes; and so forth. Not only do these firms sell different products, they sell products that “interact” with each other in the firm’s profit function. For example, if some Ford-loyal consumers are undecided between buying a car and buying a truck, selling more trucks may imply selling fewer cars. For Kodak, by contrast, increasing sales of cameras is likely to imply an increase in the sales of film. As for American Airlines, increasing output or capacity in the Chicago–St. Louis and St. Louis–New York routes is likely to decrease the cost of offering air travel from New York to Chicago, another example of intrafirm products interaction.

Similarly, in a dynamic context, we can interpret a firm selling a given product in different periods as a multiproduct firm. Specifically, we can interpret the output of a given product in different periods as different outputs. In this framework, interactions across products within a firm result from dynamic effects on the firm’s demand or cost function. For example, increasing the output of aircraft sold today lowers Boeing’s cost of selling aircraft next period. We thus

have an additional class of examples of multiproduct oligopoly competition with interactions across products. Switching costs and dynamic network effects would provide additional examples within the same class of dynamic product interactions within a firm.

In this paper we look at oligopoly price competition between multiproduct firms, firms whose products interact in their profit function (as in the above examples). We are interested in the impact of intrafirm product interactions on the equilibrium profit level. We consider two possible approaches to this comparative statics exercise: (a) a common change in the degree of intrafirm product interactions; and (b) a firm-specific change. The first approach may correspond to an industrywide shock that affects all firms’ costs and/or demand. For example, technology change may imply greater economies of scope. The second approach is more appropriate when considering endogenous changes in the firms’ cost and demand functions. For example, by changing its product design firm  $i$  may induce a demand curve with greater intrafirm product interactions.

Our results provide conditions such that “positive” changes in the profit function (e.g., greater economies of scope or demand complementarity), if common to all firms, lead to a negative change in equilibrium profits, a situation we refer to as a *Bertrand supertrap*.

The term *Bertrand trap* has been used by various authors (e.g., Hermalin 1993) as a reference to the situation in which equilibrium profits under some form of single-product competition (e.g., Hotelling) remain constant despite seemingly favorable exogenous changes. For example, if all firms' (constant) marginal cost declines by the same amount then equilibrium profits remain at zero. Our results show that things may go further than that (thus the term "supertrap"): when intrafirm product interactions are present, there are cases when otherwise positive shocks (e.g., greater economies of scope) lead to lower equilibrium profits.<sup>1</sup>

More specifically, we will say there is a Bertrand supertrap if a common change in some exogenous parameter (e.g., economies of scope) leads to a decrease in firms' profits, even though (a) holding prices constant profits would increase; or (b) were the change exclusive to firm  $i$  its profits would increase.<sup>2</sup>

The key to a Bertrand supertrap is the intensity of price competition. Under approach (a) above, the direct effect of a common exogenous change (i.e., holding prices constant) is positive, whereas the total effect is negative. This implies that the *strategic* effect is negative and greater (in absolute terms) than the direct effect. Under approach (b) above, the effect of a common shift is negative, whereas the effect of a firm  $i$  exclusive shift is positive. This implies that the *imitation* effect is negative and greater in absolute value than the *competitive advantage* effect, where the *competitive advantage* effect is the profit effect of a change exclusive to firm  $i$ , whereas the *imitation* effect is the effect on firm  $i$ 's profits of a matching change by all of its rivals.

We present several instances of Bertrand supertraps. For example, increasing the degree of economies of scope has a positive direct effect (that is, lower costs while keeping the competitors' prices fixed). Moreover, increasing the degree of firm  $i$ 's economies of scope may increase firm  $i$ 's equilibrium profits. However, when all firms benefit from increasing economies of scope, the lower marginal costs induce more aggressive price competition to the point that equilibrium profits are lower than in the initial situation. Likewise, under certain conditions, an increase in the degree of intrafirm demand complementarity (that is, demand complementarity between the products offered by a given firm) implies positive direct and competitive advantage effects but a negative total effect (and so very negative strategic and imitation effects). Similar patterns take place in the

context of learning curves, network effects, systems competition, bundling, switching costs, and Internet cross-referencing.<sup>3</sup>

Our contribution is threefold. First, we present a framework that unifies a series of contributions to the economics, marketing, and strategy literatures. In other words, we show that a series of apparently different results share the same basic intuition. Second, our framework suggests additional applications hitherto not considered where Bertrand supertraps may arise. Third, our general results provide a tool for comparative statics in situations where analytical solutions are not feasible.

The focus of our analysis is on multiproduct competition, for two reasons. First, we would like to provide a unifying framework to understand a series of previous papers that feature Bertrand supertraps, and most of these papers feature multiproduct competition. Second, we believe that most real-world applications of interest involve multiproduct competition. That said, we should add that Bertrand supertraps are also possible in the context of single-product competition. In the appendix, we present a simple example where Bertrand supertraps arise in a one-product Hotelling-like duopoly.<sup>4</sup>

The rest of the paper is organized as follows. In the next section we present an extensive example, competition in wide-body aircraft, arguing that it satisfies the assumptions and implications of our results. Section 3 presents formal definitions of Bertrand traps and supertraps. Section 4 introduces the general model. Sections 5 through 7 present the main results in the paper: conditions under which an industry features a Bertrand supertrap. Section 8 develops the complete analytical solution of a particular example (economies of scope), illustrating how the general results can be applied. Section 9 considers a series of additional applications, several of which have been previously developed in the literature, that follow from our general results. Finally, §10 contains a discussion of the results, including implications for strategy.

## 2. Example: Wide-Body Aircraft

One important characteristic of aircraft manufacturing is the learning curve: the cost of developing the first few aircraft is greater by a significant factor than the cost of producing the 100th or 200th unit.

<sup>1</sup> Bulow et al. (1985) also point out that otherwise positive exogenous changes in one market may have negative effects in another market because of the strategic interaction between firms.

<sup>2</sup> See §3 for a more formal definition.

<sup>3</sup> The competitive price discrimination literature that addresses the issue of product choice (Katz 1984, Champsaur and Rochet 1989, Stole 1995) is related to our work. See also Borenstein (1985), Corts (1998), and Armstrong and Vickers (1999).

<sup>4</sup> See also Seade (1987), Moorthy (1988), Corts (1998), Iyer and Soberman (2000), Anderson et al. (2001), Anderson (2002), Kuksov (2004).

Such steep learning curves imply that sellers have a lot to gain from making the initial sales. Failure to do so implies falling behind in the learning curve race and thus becoming less competitive. In fact, as Newhouse (1988) argues, “the business of making and selling commercial airliners is not for the diffident or faint of heart. It is remarkably difficult and, by anyone’s standard, intensely competitive” (p. 3).

The 1960s were a time of rapid technological progress, especially in the area of engine design. Wide-body aircraft, a quantum leap in terms of commercial aircraft size, became a technological possibility. Economically, such planes promised a much lower capital and operating cost per passenger mile. However, the cost of developing and producing the first aircraft was also much greater, even when calculated on a per-passenger-mile basis. In sum, the advent of wide-body aircraft can be seen as an industry shock whereby learning curves become steeper and lower (higher initial level, lower asymptote, lower integral over a high number of units). For a given price and (reasonably large) number of units, wide-body aircraft implied a greater profit per-passenger-mile capacity sold to the airlines. (See, for example, Newhouse 1988.)

The era of wide-body aircraft started in 1965, when Boeing and PanAm signed an agreement whereby PanAm would secure the first order of a new large aircraft to be developed by Boeing—the 747. A few days after Boeing and PanAm’s contract was signed, F. Kolk from American Airlines sent Boeing and the other manufacturers a proposal for a new aircraft of larger size than the existing ones but smaller than the proposed 747.<sup>5</sup> Responding to this appeal, Lockheed and McDonnell Douglas entered the market at approximately the same time, with the first proposals submitted in September 1967. A new duopoly (or triopoly, if we include the 747 in the same market) had just emerged.

Competition for the first wide-body orders was quite intense. The first set of bids (after the PanAm-Boeing agreement) was submitted in February 1968 to American Airlines. The bids were reportedly within \$200,000 of each other, a trifle in comparison with the overall size of the order in question. While it is difficult to determine the exact price, it is agreed that it fell in the \$15–\$17 million range, less than 20% of cost. On February 19, American Airlines announced the first order—to McDonnell Douglas.

Despite this initial setback, Lockheed was able to secure some of the subsequent orders, effectively cementing the duopoly situation (or triopoly, if we include Boeing). To do so, Lockheed had to maintain prices at very low levels, a move that was matched by

its rivals. In fact, sale prices were consistently below \$20 million.

Given these pricing levels, it is not surprising that the sellers found themselves in financial trouble. Boeing’s viability was questionable in the late 1960s, having cut its workforce from 105,000 in 1968 to 38,000 in 1971 (Bluestone et al. 1981). Rolls-Royce, Lockheed’s main engine supplier, was nearly bankrupt in the early 1970s, calling on Lockheed and the British government for help.

Eventually, it was the Boeing 747 that won the race, whereas the DC10 and L1011 programs amounted to huge losses. While Boeing did go past the break-even level of orders for the B747, it is generally agreed that this gain less than compensated for its rivals’ losses. In fact, over the period in question, the commercial aircraft industry performed significantly below comparable manufacturing industries, proving once again that high tech does not necessarily imply high profits. Specifically, compared to 1960, each company’s share-price index peaked at between 2 and 3 in the mid-1960s, then fell to as low as 0.3 to 0.5 in the mid-1970s. By 1980, share-price levels were back to the 1960 levels approximately.

The wide-body aircraft industry, particularly in its first generation, shares many of the features that we focus on in this paper: (a) a small number of firms; (b) price competition; (c) multiproduct profit functions with intrafirm interactions. The last assertion warrants some additional explanation. As we mentioned in the introduction, we can interpret intertemporal competition as competition over several products (aircraft sales today, aircraft sales tomorrow). In this context, the learning curve implies intrafirm interactions insofar as sales today affect marginal profitability tomorrow (through lower marginal costs). And an increase in the steepness of the learning curve implies an increase in the degree of intrafirm profit interactions.

The main point in the paper is that, under certain conditions, intrafirm profit interactions lead to Bertrand supertraps. In the context of our current example, a Bertrand supertrap corresponds to one of the following conditions.

(a) If prices were fixed at a certain level, firms would be better off with steeper learning curves; but steeper learning curves make pricing so much more aggressive that firms are worse off with steeper learning curves.

(b) A firm would be better off if it were the only to have a steeper learning curve. However, if all firms’ learning curves become steeper then all firms become worse off.

In the next section, we generalize the definitions of Bertrand supertraps presented above and make them analytically precise. We then provide necessary

<sup>5</sup> Although the initiative came from American Airlines, several other airlines agreed that the 747 was too big for their needs.

and sufficient conditions for an industry to exhibit a Bertrand supertrap.

### 3. Bertrand Traps and Bertrand Supertraps

The purpose of this paper is to characterize situations in which Bertrand supertraps arise. In this section, we formally define this concept, which we do in reference to an existing concept—Bertrand traps. Both Bertrand traps and Bertrand supertraps correspond to comparative statics exercises where we measure changes in profits that result from changes in exogenous parameters  $s = (s_i)$ , where  $i$  refers to firm  $i$ . Specifically, we assume that firm  $i$  has profit  $\Pi_i(p, s)$ , where  $p$  is a vector of prices and  $s$  a vector of exogenous parameters.<sup>6</sup>

We consider two types of exogenous changes: a common change in all  $s_i$ , which (with some abuse of notation) we will denote as a change in  $s$ , and a change in  $s_i$  alone. Our framework is fairly general, and  $s_i$  can be interpreted either as a cost function or a demand function parameter. So, for example, a change in  $s$  may arise from an industrywide decrease in cost; a change in  $s_i$  may correspond to an increase in firm  $i$ 's product quality, or a decrease in its cost.

In this section, we first define Bertrand traps and then the main concept in the paper, Bertrand supertraps.

#### Bertrand Traps

The Bertrand model is the prototypical example of the hazards of price competition. It has the striking feature that, under the assumptions of product homogeneity and constant marginal cost, equilibrium profits are zero even if only two competitors are present.

The Bertrand model also implies drastic comparative statics. A common decrease in marginal cost implies no change in equilibrium profits; whereas, if prices remain constant, or if the decrease in marginal cost is exclusive to firm  $i$ , then the cost decrease implies an increase in profits (for firm  $i$  in the latter case). Following previous authors, we use the term "Bertrand trap" to denote these striking comparative statics.

**DEFINITION 1 (BERTRAND TRAP).** The following cases correspond to a Bertrand trap:

- (a)  $\partial\Pi_i/\partial s > 0$  and  $d\Pi_i/ds = 0$ .
- (b)  $d\Pi_i/ds_i > 0$  and  $d\Pi_i/ds = 0$ .

The two types of Bertrand trap correspond to two different perspectives. Part (a) highlights the effects of price competition:  $\partial\Pi_i/\partial s > 0$ , that is, a change  $s_i$  that is common to all firms implies an increase in profits

keeping prices constant (thus the use of partial derivatives); however,  $d\Pi_i/ds = 0$ , that is, the total effect of the common change in  $s$  is zero. This implies that price competition completely wipes out the potential benefits from the change in  $s$ .

Part (b) of the definition highlights the element of competitive advantage. Whereas  $d\Pi_i/ds_i > 0$ , that is, a unilateral change in  $s_i$  implies an increase in firm  $i$ 's profits,  $d\Pi_i/ds = 0$ , that is, a common change in  $s$  leaves profits unchanged.

An alternative way of presenting Definition 1 is to decompose the total effect of a common variation of  $s$  in two alternative ways that correspond to the two perspectives:

- (a) Total effect of  $ds$   
= Direct effect of  $ds$  (holding prices fixed)  
+ Strategic effect of  $ds$  (through price changes);
- (b) Total effect of  $ds$   
= Effect of  $ds_i$  (competitive advantage)  
+ Effect of  $ds_k$  ( $k \neq i$ ) (imitation).

So Definition 1a states that the direct effect is positive, but the strategic effect is negative and equal, in absolute value, to the direct effect. Definition 1(b), in turn, states that the competitive advantage is positive but the imitation effect is negative and equal, in absolute value, to the competitive advantage effect.

Finally, it is worth noting that, while the Bertrand model provides the primary example for the concept of a Bertrand trap, the concept is actually broader. For example, the Hotelling model of product differentiation features similar Bertrand traps.

#### Bertrand Supertraps

We now turn to the central concept of our paper, Bertrand supertraps. As the name suggests, this corresponds to the case when the positive effect of an exogenous change is *more than wiped out* by the effects of imitation and price competition.

**DEFINITION 2 (BERTRAND SUPERTRAP).** The following cases correspond to a Bertrand supertrap.

- (a)  $\partial\Pi_i/\partial s > 0$  and  $d\Pi_i/ds < 0$ .
- (b)  $d\Pi_i/ds_i > 0$  and  $d\Pi_i/ds < 0$ .

By analogy with Bertrand traps, we may rephrase Definition 2 by decomposing the total effect of a change in  $s$ . Definition 2a then states that the direct effect is positive, but the strategic effect is negative and greater, in absolute value, than the direct effect. Definition 2b, in turn, states that the competitive advantage is positive but the imitation effect is negative and greater, in absolute value, than the competitive advantage effect.

<sup>6</sup> See the next section for a more complete characterization of the basic model.

Definition 2 is fairly broad. In this paper, we focus on situations in which firms produce several products and the exogenous changes in  $s$  correspond to changes in the degree of intrafirm product interaction. Examples include economies of scope and demand complementarity across two or more of the firm's products.

Finally, we note that in most of the applications we consider, both parts of Definition 2 hold. It is conceivable, however, that one holds true and the other one does not.

#### 4. General Model

Consider an oligopoly with  $I$  price-setting firms. Each firm offers a set  $J$  of products. Let  $p_j^i$  be the price of product  $j$  set by firm  $i$ ;  $p^i = (p_j^i)$ , the vector of firm  $i$ 's prices;  $p = (p^i)$ , the vector of all prices;  $p^{-i}$ , the price vector of firm  $i$ 's competitors;  $p_{-j}^i$ , the vector of firm  $i$ 's prices except  $p_j^i$ ; and  $p_j$ , the vector of prices across firms of product  $j$ . Firm  $i$ 's profit function is  $\Pi^i(p, s)$ , where  $s = (s_i)$  is a vector of exogenous parameters that measure the level of intrafirm product interaction ( $s_i$  for firm  $i$ ); the role of  $s$  is at the center of this paper.<sup>7</sup> Firms simultaneously set prices in a one-shot game, the equilibrium of which is given by  $\hat{p}$ .

The profit function can be written as

$$\Pi^i(p, s) = \sum_{j \in J_i} p_j^i D_j^i(p, s) - C^i(D^i(p, s), s),$$

where  $D_j^i$  is the demand for product  $j$  sold by firm  $i$ ;  $D^i(p)$ , the vector of firm  $i$ 's demands; and  $C^i(\cdot)$ , firm  $i$ 's cost of supplying  $D^i$ .

Throughout the paper we maintain the following two assumptions.

**ASSUMPTION 1 (COMPETITIVE MARKETS).** For each firm  $i$ ,

$$\frac{\partial \Pi^i}{\partial p_j^k} \geq 0,$$

for all  $k \neq i, j \in J$ , and  $p$ . Moreover, there is at least a  $j \in J$  for which the inequality is strict.

Assumption 1 states that a firm is never worse off when a competitor raises one of its prices.

**ASSUMPTION 2 (STRATEGIC COMPLEMENTARITY).** For each pair of firms  $i, k$ , for all  $j \in J$

$$\frac{\partial^2 \Pi^i}{\partial p_j^i \partial p_j^k} > 0.$$

<sup>7</sup> We assume throughout that the profit function is differentiable and continuous. Several of our results can be derived without these assumptions, using the methods presented in Milgrom and Roberts (1990) or Villas-Boas (1997).

Furthermore, for any products  $j, \ell \in J$ , the

$$\left. \frac{\partial^2 \Pi^i}{\partial p_j^i \partial p_\ell^k} \right|_{s=0} \geq 0.$$

Assumption 2 corresponds to the traditional assumption of strategic complementarity in prices and an extension to the case of a multiproduct firm.

The market equilibrium  $\hat{p}(s)$  is determined by the first-order conditions for all the firms,

$$\frac{\partial \Pi^i(\hat{p}(s), s)}{\partial p_j^i} = 0, \quad \forall i \in I, j \in J.$$

We assume throughout that the second-order conditions are satisfied, that the best-response functions are unique, and that the equilibrium exists and is unique and differentiable for any  $s$ .

Our main goal is to determine the impact on profits,  $\Pi^i(\hat{p}(s), s)$ , of an increase in the degree of product interactions, either a common increase for all firms,  $ds$ , or an increase for firm  $i$  only,  $ds_i$ .

In the common variation case we will also distinguish between the total effect of a change in  $s$ ,  $d\Pi_i/ds$ , and the direct effect of a change in  $s$ ,  $\partial\Pi_i/\partial s$  (that is, holding prices constant). By the envelope theorem, we know that

$$\frac{d\Pi^i(\hat{p}(s), s)}{ds} = \frac{\partial \Pi^i(\hat{p}(s), s)}{\partial s} + \sum_{k \neq i} \sum_{j \in J} \frac{\partial \Pi^i(\hat{p}(s), s)}{\partial p_j^k} \frac{d\hat{p}_j^k(s)}{ds}.$$

The difference between the total effect and the direct effect (the second term on the right-hand side) is the *strategic effect*, that is, the effect of price competition.

An alternative decomposition of the total effect is as follows:

$$\frac{d\Pi^i(\hat{p}(s), s)}{ds} = \frac{d\Pi^i(\hat{p}(s), s)}{ds_i} + \sum_{k \neq i} \frac{d\Pi^i(\hat{p}(s), s)}{ds_k}.$$

We will refer to the two terms on the right-hand side as the *competitive advantage* effect and the *imitation* effect, respectively.<sup>8</sup>

Bertrand supertraps are situations in which the strategic effect or the imitation effect is so negative that even though  $\partial\Pi_i/\partial s$  and  $d\Pi_i/ds_i$  are positive, the total effect of a common change in  $s$ ,  $d\Pi_i/ds$ , is negative.

In each of the next two sections, we consider two polar cases: the case when product interactions occur only through the cost function, and the case when product interactions occur only through the demand functions.

In the cost interactions case, we assume that the profit function can be written as

$$\Pi^i(p, s) = \sum_{j \in J_i} p_j^i D_j^i(p_j) - C(D^i, s_i).$$

<sup>8</sup> Note that the imitation effect can be written as  $d\Pi_i/ds - d\Pi_i/ds_i$ .

The role of  $s$  in the cost interactions is presented in the following assumption.

ASSUMPTION 3 (COST INTERACTIONS).

$$\frac{\partial^2 C^i}{\partial D_j^i \partial D_\ell^i} \Big|_{s_i=0} = 0, \quad \frac{\partial}{\partial s_i} \left( \frac{\partial^2 C^i}{\partial D_j^i \partial D_\ell^i} \right) < 0,$$

$$\frac{\partial C(0)}{\partial s_i} = 0, \quad \frac{\partial C(D)}{\partial s_i} < 0 \quad (D \neq 0),$$

According to this parameterization, the case  $s_i > 0$  corresponds to cost complementarity (or economies of scope) (that is,  $\partial^2 C_j^i / \partial D_j^i \partial D_k^i < 0$ ), whereas  $s_i < 0$  corresponds to diseconomies of scope. We make also the technical assumption that  $\partial D_j^i / \partial p_j^i$  evaluated at the equilibrium prices does not vary when the equilibrium prices vary with  $s$ . For the symmetric case this means that  $\partial D_j^i / \partial p_j^i$  evaluated when the prices are equal across firms does not vary. This assumption guarantees that the effect of changes in economies of scope is not reversed by significant shifts in the slope of the demand curve.

In the demand interactions case, we assume that the profit function can be written as

$$\Pi^i(p, s) = \sum_{j \in I_i} p_j^i D_j^i(p, s) - \sum_{j \in I_i} C_j^i(D_j^i(p, s)).$$

The role of  $s$  in the demand interactions is presented in the following assumption.

ASSUMPTION 4 (DEMAND INTERACTIONS).

$$\frac{\partial D_j^i}{\partial p_\ell^i} \Big|_{s=0} = 0, \quad \frac{\partial^2 D_j^i}{\partial p_\ell^i \partial s} \leq 0, \quad \frac{\partial^2 D_j^i}{\partial p_\ell^i \partial s_i} \leq 0,$$

for all  $i, j, \ell \neq j$ , with the inequality strict for a least one pair  $(j, \ell)$ . Moreover,

$$\partial D_j^i / \partial s \geq 0, \quad \partial D_j^i / \partial s_i \geq 0,$$

for all  $(i, j)$ . Finally,  $\partial \pi^i / \partial p_j^i$  is decreasing in  $s$  and  $s_i$  for all  $(i, j)$ .

The first part of the assumption simply states that  $s$  measures the degree of demand complementarity. If  $s = 0$ , then firm  $i$ 's profits are additive across products. If  $s > 0$ , then an increase in the price of product  $\ell$  implies a decrease in the demand for product  $j$ , that is,  $\partial D_j^i / \partial p_\ell^i \leq 0$  when  $s > 0$ . The third part of the assumption is needed for technical reasons. It implies that a firm's best response to an increase in  $s$  or  $s_i$ , keeping fixed all the other prices, is to decrease  $p_j^i$ . This assumption captures the idea that the first derivatives of demand, and their change with respect to  $s$ , or  $s_i$ , are the dominant effects to be considered. This will hold, for example, under constant marginal costs when the direct effect  $\partial D_j^i / \partial s$ , or  $\partial D_j^i / \partial s_i$ , is close to zero.

## 5. Price Competition and Bertrand Supertraps

In this section, we consider the case of a common change in the the values of  $s_i$ , which we denote by a change in  $s$ . We provide sufficient conditions that imply a Bertrand supertrap (type a): The direct effect of a change in  $s$  is positive but the strategic effect is so negative that the total effect is negative as well. In other words, the positive effects of the exogenous, industrywide shock  $s$  are more than wiped out by price competition.

We first consider the case of cost interactions.

PROPOSITION 1 (STRATEGIC EFFECT WITH COST INTERACTIONS). *If total demand elasticity is close to zero and firms are approximately symmetric, then an increase in cost complementarity (increase in  $s$ ) implies (a) a positive direct effect and (b) a negative total effect in equilibrium firm profits.*

We should note that this result (and the ones that will follow) is based on two important assumptions: (a) low market demand elasticity and (b) approximate symmetry. An elastic market demand curve may imply a more favorable total effect than predicted by the proposition. Consider, for example, the case of economies of scope. An increase in  $s$  implies a positive direct effect (cost savings) and a negative strategic effect (lower prices). If market demand is very elastic, then we would expect the strategic effect to be less negative, perhaps to an extent that the direct effect dominates the strategic effect. Likewise, in an asymmetric oligopoly, the strategic effect on a large firm's profit is likely to be smaller. We would then expect the total effect to have the same sign as the direct effect. In fact, in the limit of monopoly (the extreme of an asymmetric oligopoly), the strategic effect is zero and the total effect has the same sign as the direct effect.

A result analogous to Proposition 1 holds for the case of demand interactions.

PROPOSITION 2 (STRATEGIC EFFECT WITH DEMAND INTERACTIONS). *If total demand elasticity is close to zero and firms are approximately symmetric, then an increase in the degree of demand complementarity (increase in  $s$  with  $s$  close to zero) implies (a) a positive direct effect, and (b) a negative total effect in equilibrium firm profits.*

Notice that the result also works the other way around (demand substitutes): a small decrease in  $s$  starting from a negative value close to zero leads to (a) a negative direct effect and (b) a positive total effect.

As in the cost interactions case, Proposition 2 relies on the assumptions of low market demand elasticity and approximate symmetry. As we move from these extremes, the direct effect eventually dominates the strategic effect. For example, although Acrobat Reader and Acrobat Writer exhibit demand complementarity

of the sort considered here, it is likely that the total effect of this demand interaction is positive, considering that Adobe is subject to virtually no competition.

## 6. Imitation and Bertrand Supertraps

In the previous section, we considered the case when the change in  $s$  is common to all firms. This is the relevant comparative statics when firms are subject to an industrywide change in technology or consumer preferences. In many situations, however, changes in  $s_i$  result from firm-level decisions, and it makes more sense to consider a unilateral change in  $s_i$ . In this section, we present Bertrand supertrap results for this case. Specifically, we provide sufficient conditions such that (a) a unilateral change in  $s_i$  increases firm  $i$ 's profits, but (b) a common change in all  $s_i$ 's decreases firm  $i$ 's profits. The benefits from firm  $i$ 's competitive advantage, and more, are wiped out by imitation. To illustrate this idea we restrict attention to linear demands and costs, except for the product interaction terms.

Moreover, for the cost interactions result we assume that the cost function takes the form

$$C^i(D^i(p)) = \sum_j C_j^i(D^i(p))$$

with

$$C_j^i(D^i(p)) = \tilde{C}_j^i(D_j^i(p_j)) - s_i D_j^i(p_j) \sum_{\ell \neq j} D_\ell^i(p_\ell),$$

for all  $i, j$ , and where  $\tilde{C}_j^i(D_j^i(p_j))$  represents the cost for product  $j$  of firm  $i$  if  $s = 0$ .

Under these conditions and under Assumptions 1–3 one can then obtain the following result.

**PROPOSITION 3 (IMITATION EFFECT WITH COST INTERACTIONS).** *If total demand elasticity and  $s$  are close to zero and firms are approximately symmetric, then (a) an increase in firm  $i$ 's cost complementarity (increase in  $s_i$ ) implies an increase in firm  $i$ 's profits and (b) a common increase in cost complementarity (increase in  $s$ ) implies a decrease in firm  $i$ 's profits.*

A result analogous to Proposition 3 holds for the case of demand interactions. We now assume that demand interactions take the form

$$D_j^i(p, s) = \tilde{D}_j^i(p_j) + s_i \sum_{\ell \neq j} \tilde{D}_\ell^i(p_\ell) - \frac{1}{I-1} \sum_{k \neq i} s_k \sum_{\ell \neq j} \tilde{D}_\ell^k(p_\ell)$$

for all  $i, j$ , where  $\tilde{D}_j^i(p_j)$  represents the demand for product  $j$  of firm  $i$  if  $s = 0$ . (The third term in the demand function guarantees that total demand does not change with changes in any  $s_i$ .)

Under these conditions and under Assumptions 1, 2, and 4, one can then obtain the following result.

**PROPOSITION 4 (IMITATION EFFECT WITH DEMAND INTERACTIONS).** *If total demand elasticity and  $s$  are close to zero, and firms are approximately symmetric, then (a) an increase in firm  $i$ 's demand complementarity (increase in  $s_i$ ) implies an increase in firm  $i$ 's profits and (b) a common increase in demand complementarity (increase in  $s$ ) implies a decrease in firm  $i$ 's profits.*

## 7. Dynamic Product Interactions

As suggested in the introduction, we can think of multiperiod competition as a particular case of multiproduct competition. Suppose that each firm offers one product over  $T$  periods. This situation is analogous to that of a firm selling  $T$  products. Intrafirm product interactions then correspond to dynamic interactions in the production or sale of the firm's product. Examples of this are learning curves, network externalities, or switching costs. In these situations, the results of the previous sections still apply if we look at open-loop equilibria, that is, the case when all prices are set at the beginning of time.

Consider for example the case of an oligopoly with one-product firms competing over two periods. Our results indicate that, when second-period demand is increasing in first-period demand (switching costs or network effects), then discounted equilibrium profits are lower the greater the extent of switching costs or network effects. Turning to cost interactions, if each firm's second-period cost is decreasing in its first-period output (learning curve), then the "steeper" the learning curve, the lower the discounted equilibrium profits.

In most cases, firms are able to change their prices at different moments in time. It thus makes sense to focus on Markov perfect equilibria, that is, equilibria such that the firms' strategies in each period are only a function of the payoff-relevant state variables.<sup>9</sup> In all of the applications we have considered, the results of §§5 and 6 can also be extended to the case of Markov equilibria. The general conditions under which this is true are quite messy.<sup>10</sup> They are quite intuitive, however. They state that future profits only depend on today's prices through differences across firms. Moreover, under cost interactions, the conditions state that a firm benefits from having a lower cost even

<sup>9</sup> See Maskin and Tirole (1997) for a definition of Markov perfect equilibria. Under general conditions this set of equilibria is equal to the set of closed-loop equilibria.

<sup>10</sup> For the case of two-period competition between one-product firms, a set of sufficient conditions is given by

$$\pi_2^i(p_2, p_1, s) = \pi_2^i(p_2, s(p_1^i - p_1^i), \dots, s(p_1^i - p_1^i))$$

$$\frac{\partial^2 \pi_2^i}{\partial s \partial p_1^i} + \frac{I-1}{(a-b)(a+b(I-1))} \frac{\partial \pi_2^i}{\partial p_2^k} \left[ a \frac{\partial}{\partial s} \frac{\partial^2 \pi_2^i}{\partial p_2^i \partial p_1^k} - b \frac{\partial}{\partial s} \frac{\partial^2 \pi_2^i}{\partial p_2^i \partial p_1^i} \right] < 0,$$

where  $a \equiv -\partial^2 \pi_2^i / \partial (p_2^i)^2$  and  $b \equiv -\partial^2 \pi_2^i / \partial p_2^i \partial p_2^k$  ( $k \neq i$ ).

though its competitors may behave more aggressively because of the firm’s lower cost. Under demand interactions, the conditions state that the direct impact of the first-period demand on second-period profits dominates any possible effect through the competitors’ actions.

### 8. Application: Economies of Scope

In this section, we completely solve an example of multiproduct oligopoly competition. Specifically, we consider the case of Hotelling competition with economies of scope. We first consider an instance of a Bertrand trap with single-product firms. We then extend this to multiproduct competition and show how supertraps may arise.

#### Bertrand Traps

Consider the following Hotelling game. There is a mass one of consumers uniformly distributed along a unit segment; consumers pay a transportation cost of  $t$  per unit of distance, and firms are located at the extremes of the segment. Firm  $i$ ’s demand is equal to  $D^i(p^i, p^k) = \frac{1}{2} + (p^k - p^i)/(2t)$ . Moreover, firm  $i$ ’s cost is given by  $C^i = (c - s_i)D^i$ , where  $s_i$  measures firm  $i$ ’s efficiency improvement (which may be industrywide or firm specific). It follows that firm  $i$ ’s profits are given by

$$\Pi^i(p^i, p^k) = (p^i - c + s_i) \left( \frac{1}{2} + \frac{p^k - p^i}{2t} \right).$$

What is the impact of a common increase in  $s$ , that is, an equal increase in  $s_1$  and  $s_2$ ? Clearly, the direct effect,  $\partial \Pi^i / \partial s$ , is positive. What about the total effect? Straightforward computation shows that the equilibrium prices and profits are given by

$$\hat{p}^i = c + t - s,$$

and

$$\hat{\Pi}^i = \frac{t}{2}.$$

It follows that the total effect,  $d\hat{\Pi}^i/ds$ , is zero. In words, the strategic effect exactly cancels out the direct effect—a Bertrand trap of type a.

If only  $s_i$  increases while  $s_k = 0$ , then equilibrium profits are given by

$$\hat{\Pi}_i = \frac{(3t + s_i)^2}{18t}, \quad \hat{\Pi}_k = \frac{(3t - s_i)^2}{18t}.$$

Clearly,  $d\hat{\Pi}_i/ds_i > 0$  (whereas, as shown before,  $d\hat{\Pi}_i/ds = 0$ ). We thus have a Bertrand trap of type b: Whereas a unilateral change in  $s_i$  increases firm  $i$ ’s equilibrium profits, a common increase in  $s$  leaves equilibrium profits unchanged.

### Bertrand Supertraps

Consider now the case when firms compete in two products. Each product is characterized by a Hotelling demand: there is a mass one of consumers uniformly distributed along a unit segment; consumers pay a transportation cost of  $t$  per unit of distance; and firms are located at the extremes of the segment. The demands for the two products are independent and equal to  $D_j^i(p_j^i, p_j^k) = \frac{1}{2} + (p_j^k - p_j^i)/(2t)$  for all  $i, k \neq i, \text{ and } j$ . Suppose also that each firm’s cost function is given by  $C^i = cD_1^i + cD_2^i - s_i D_1^i D_2^i$ , where  $D_j^i$  is firm  $i$ ’s output of product  $j$ . Thus,  $s_i$  measures the degree of firm  $i$ ’s economies of scope.<sup>11</sup>

Firm  $i$ ’s profit function is given by

$$\Pi^i = \frac{1}{2t} \left( \sum_{j=1}^2 (t + p_j^k - p_j^i)(p_j^i - c) + \frac{s_i}{2t} \prod_{j=1}^2 (t + p_j^k - p_j^i) \right),$$

with  $k \neq i$ .

Consider first the case when  $s_1 = s_2 = s$ . Deriving the first-order conditions and solving for a symmetric equilibrium yields

$$\hat{p}_j^i = c + t - \frac{1}{2}s.$$

Substituting in the profit function and simplifying, we get

$$\hat{\Pi}^i = t - \frac{1}{4}s. \tag{1}$$

In other words, the greater the degree of economies of scope, the lower the equilibrium profits. The direct effect of an increase in  $s$  is clearly positive:  $\partial \Pi^i / \partial s = D_1^i D_2^i$ . Equation (1) thus implies that the strategic effect more than outweighs the direct effect—a Bertrand supertrap.

This result could also be derived from Proposition 1. In fact, it is straightforward to show that the example satisfies the proposition’s conditions. Moreover, there are cases that cannot be solved analytically but can nevertheless be studied based on Proposition 1.

Consider now the case when only  $s_i$  changes, while  $s_k = 0$ . Equilibrium profits are given by

$$\hat{\Pi}_i = \frac{9t^2(4t - s_i)}{(6t - s_i)^2}, \quad \hat{\Pi}_k = \frac{4t(3t - s_i)^2}{(6t - s_i)^2}.$$

Straightforward differentiation implies

$$\frac{d\hat{\Pi}_i}{ds_i} = \frac{1}{12}, \quad \frac{d\hat{\Pi}_k}{ds_i} = -\frac{1}{3}.$$

We thus have a Bertrand supertrap of type b: Although a unilateral change in  $s_i$  implies an increase in firm  $i$ ’s

<sup>11</sup> Throughout the paper, we use the term economies (and dis-economies) of scope to mean profit complementarity (substitutability) under cost interactions. This definition is related, but not identical, to the definition of economies of scope used in the literature (see Panzar 1991).



profits,  $d\hat{\Pi}_i/ds_i = 1/12$ , a common increase in  $s$  implies a decrease in firm  $i$ 's profits,  $d\hat{\Pi}_i/ds_i = -1/4$ .

This result could also be derived from Proposition 3. In fact, it is straightforward to show that the example satisfies the proposition's conditions.

In the next section, we show that this type of comparative statics arises in quite a variety of situations.

## 9. Other Applications

As suggested by the examples in the introduction, intrafirm product interactions are a fairly prevalent phenomenon. In this section, we present a series of applications of our general framework. Many of these applications are featured in previous research. Although some of these applications do not add new results, they are useful in two ways. First, our conditions allow for comparative statics even when the models cannot be solved analytically. Second, we provide a general framework that unifies various seemingly independent results.

### Learning by Doing

The case of learning by doing is similar to that of economies of scope. Consider a duopoly selling one product over two periods. The product index  $j$  refers now to time. The learning curve hypothesis is that second-period marginal cost is decreasing in first-period output (e.g., Fudenberg and Tirole 1983). Suppose specifically that  $C_2^i = (c - sD_1^i)D_2^i$ . It follows that total cost (assuming, for simplicity, no discounting) is given by  $C^i = cD_1^i + cD_2^i - sD_1^iD_2^i$ . This is the exact same cost function as before. Moreover, if demand has the same structure as in the previous section, then we conclude that in an open-loop equilibrium, profits are lower than they would be were there no learning effects. We could also consider the subgame-perfect equilibrium of this game. As mentioned in §7, the additional constraints imposed by subgame perfection are satisfied and the result holds again that equilibrium profits are lower the steeper the learning curve is. Cabral and Riordan (1994) present a similar result in a model with two firms, infinite horizon, and discrete demand.

### Core Competencies

One possible implication of the core competencies hypothesis is that profitability is greater when a firm focuses on a small set of products or services—its core competencies (Prahalad and Hamel 1990). For example, it may be that managers cannot pay enough attention to any particular activity when the firm is involved in too many activities. This can be modelled by a cost function that exhibits diseconomies of scope: It is more costly to produce  $q_1$  and  $q_2$  together than it is to produce both separately. If we believe

that these diseconomies of scope are valid at the margin, then we have the reverse of the case considered before: Industry profits can be greater when two firms produce two products each than when there are four firms, each producing one product. Even though the direct effect of multiproduct firms spinning off one of their products is positive, the total effect is negative: A "focused" firm is not only more efficient but can also be more aggressive, to the point that, in equilibrium, price cuts can outweigh cost savings. To put it differently: If one firm unilaterally decides to become more focused, then its profits increase. However, if all firms decide the same, then price competition becomes more aggressive to the point where equilibrium profits drop.

### Demand Synergies

Consider now the case of "demand synergies": Greater sales of firm  $i$ 's product  $j$  increases the demand for firm  $i$ 's product  $\ell$ . These effects result in demand complementarity and, applying Propositions 2 and 4, we know that greater industrywide demand synergies may yield lower equilibrium profits, though profits would increase for firm  $i$  if it were the only to create such synergies (or if prices were to remain constant).

Consider the following specific example (similar to Strauss 1999) of duopoly competition where firms offer two products subject to these cross-market effects. Each firm  $i = 1, 2$  offers a product  $A$  (demanded by type  $A$  buyers) and a product  $B$  (demanded by type  $B$  buyers). Consumers of each type are uniformly distributed along Hotelling segments and firms are located at the extremes of the segments. Each type  $B$ 's valuation (for a  $B$  product) is given by  $v$  minus the cost of "travelling" to the seller, which is equal to the distance travelled. For type  $A$  consumers, however, gross valuation for firm  $i$ 's  $A$  product also includes the term  $s_i D_B^i$ , where  $D_B^i$  is demand for firm  $i$ 's  $B$  product. So, in one example,  $D_B^i$  would be the number of consumers buying firm  $i$ 's plug-in and  $D_A^i$ , the number of website managers buying firm  $i$ 's software.

It can be shown that the demand for product  $B$  is given by  $D_B^i(p_B^i, p_B^k) = (1 + p_B^k - p_B^i)/2$  (as in a standard Hotelling model), whereas demand for product  $A$  is given by

$$D_A^i(p^i, p^k) = \frac{1 + p_A^k - p_A^i}{2} + \frac{s_i - s_k}{4} + \frac{s_i + s_k}{4}(p_B^k - p_B^i).$$

Finally, assuming zero costs (for simplicity), firm  $i$ 's profits are given by  $\Pi^i = p_A^i D_A^i + p_B^i D_B^i$ . Straightforward differentiation yields  $\partial D_A^i/\partial s = (p_B^k - p_B^i)/2$ , which is zero in a symmetric equilibrium,  $\partial D_A^i/\partial p_A^i = -1/2$ , which is independent of  $s$ , and  $\partial D_A^i/\partial p_B^i = -s/2$ , which is zero at  $s = 0$  and decreasing in  $s$ . We thus have demand complementarity. Proposition 2 implies

that the total effect of an increase in  $s$  is negative, that is, profits are lower the greater the degree of demand synergies, a result derived in Strauss (1999).

As in the case of economies of scope, an alternative interpretation of the above result is that a series of two mergers between firms producing different products (each merged firm carries a product  $A$  and a product  $B$ ) may generate a fall in industry profits. For example, the above analysis suggests that profits in the video game industry could be higher if software and hardware were produced by different companies (in reality, some of the games played on Sony's PlayStation, for example, are sold by Sony itself).

Finally, consider the case of a unilateral change in  $s_i$ . It can be shown that equilibrium profits are such that

$$\left. \frac{d\Pi_i}{ds_i} \right|_{s_i=0} = \frac{7}{12}.$$

It follows that a unilateral change in  $s_i$  leads to greater profits for firm  $i$ . Thus we have a Bertrand supertrap of type b as well.

### Bundling

Pure bundling may be interpreted as the limit of systems competition when, for a very high  $s$ , consumers only buy "systems" from the same firm. The above analysis suggests that competition with pure bundling may lead to lower equilibrium payoffs than no bundling. As an illustration, consider the case of a double-Hotelling demand system ( $J = 2$ ), whereby consumers are uniformly and independently distributed along two unit segments (each consumer has a location in each of the segments). Each consumer buys one pair of products. Under no bundling, equilibrium profits are the sum of two Hotelling profits. Under pure bundling, it is as if consumers were only buying one product. Because the valuations for the components are independent, it is as if firms were located at the extreme of a segment of length two. The density of consumers along this segment is triangular, with a value of one at the middle (as in the simple Hotelling game). Therefore, the equilibrium price is the same as for each of the components under no bundling, which means that the equilibrium profits under pure bundling are one half of the equilibrium profits under no bundling.

The result that bundling makes firms more aggressive is not novel. Whinston (1990), for example, considers a model of a monopolist in a given market who leverages its power into a second market. Tying sales of the first and second products may allow the monopolist to drive rivals out of the second market (the tied good market).<sup>12</sup> Although the context in

which Whinston looks at bundling is different from ours, the intuition for the result is the same. Similar results are obtained by Nalebuff (2000).<sup>13</sup> For the case of mixed bundling, a related result is presented by Anderson and Leruth (1993).

### Network Externalities

Consider a product such that consumer utility is increasing in the number of past consumer purchases—a network externality.<sup>14</sup> In this setting, lower prices in earlier periods generate greater demand and profits in future periods. We can then apply the results of the previous sections to obtain that firms may compete so much in the earlier periods that they end up with a lower present value of equilibrium profits.

Consider the following specific duopoly example where firms sell a given durable product in two periods. A measure one of consumers buys one unit from one of the firms in the first period. A second set of consumers face the same choice in the second period. Consumers are uniformly distributed along a Hotelling segment, whereas firms are located at the extremes of the segment. In addition to the Hotelling transportation cost, second-period consumers derive an extra utility that is proportional to the number of consumers who bought the same product in the first period—a network externality.<sup>15</sup> Specifically, consumers in the second period derive net utility

$$U^i = sD_1^i - td^i - p_2^i,$$

where  $U^i$  is the utility from buying from firm  $i$ ,  $D_1^i$  is firm  $i$ 's output in the first period,  $p_2^i$  is firm  $i$ 's price in the second period,  $s$  measures the intensity of network effects,  $t$  is the importance of product differentiation (transportation cost), and  $d^i$  is the distance from the consumer to firm  $i$ . For simplicity, assume that costs are zero.

Firms compete by simultaneously setting prices in each period. Second-period profits are given by

$$\pi_2^i = \frac{1}{2t} p_2^i (t - s - (p_2^i - p_2^k) + 2sD_1^i).$$

market, which can drive its rival's profits below the point where remaining in the market is profitable" (p. 840).

<sup>13</sup> See also Bakos and Brynjolfsson (2000).

<sup>14</sup> Network effects may also arise indirectly through the informational role of market shares (Caminal and Vives 1996), and through compatibility issues (Farrell and Saloner 1986, Katz and Shapiro 1986).

<sup>15</sup> Note that we are assuming that network effects only apply to the second-period consumers. The case where the network effects also apply to the first-period consumers generates similar results, as in the switching costs example solved in Cabral and Villas-Boas (2001).

<sup>12</sup> Specifically, Whinston (1990) states that, for the monopolist, "tying represents a commitment to foreclose sales in the tied good

The equilibrium of the second-period pricing game is given by  $p_2^i = t + \frac{2}{3}sD_1^i - \frac{1}{3}s$ , leading to second-period equilibrium profits of

$$\hat{\pi}_2^i = \frac{1}{2t} \left( t + \frac{1}{3}s(2D_1^i - 1) \right)^2.$$

First-period equilibrium profits are given by  $p_1^i D_1^i + \hat{\pi}_2^i$ .

$$D_1^i(p_1^i, p_1^k) = \frac{1}{2} + \frac{p_1^k - p_1^i}{2t},$$

thus, we have an expression of the first-period equilibrium profits that is a function of  $p_1^i, p_1^k$ . Solving the equilibrium pricing game we get  $p_1^i = t - \frac{2}{3}s$  and total equilibrium profits of  $t - \frac{1}{3}s$ , a value that is lower the greater the extent of network effects.

### Switching Costs

In several markets consumers incur costs if they choose to switch sellers between periods—switching costs. This case can then be construed as a case of dynamic market interactions with intertemporal demand complementarity. That is, a lower price in the first period yields a greater profit in the second period. This is analogous to the case of demand synergies. In its simplest version, we consider the open-loop equilibrium of a game where consumers are myopic. In this case, the results from the static demand synergies case apply immediately. We thus conclude that equilibrium profits are lower the greater the extent of switching costs. More generally, we must consider both the restrictions implied by forward-looking consumers and forward-looking firms. In Cabral and Villas-Boas (2001), we consider such a situation and show that the same results hold.<sup>16</sup> Similar results can be obtained in markets with experience goods (Villas-Boas 2004a, b).

### Durable Goods

A firm selling a durable good over  $J$  periods is analogous to a firm that sells  $J$  substitute products. In fact, the more consumers buy a durable in one period, the less consumers buy it in a different period. Our results suggest that, under oligopoly competition, durability may soften price competition, just as intrafirm product substitutability softens price competition—possibly to the extent of increasing total profits, as Proposition 2 suggests. This is in stark contrast with the case of a durable-goods *monopolist*, where durability has a negative impact on seller profits (Stokey

1981, Bulow 1982). In other words, the monopolist’s durability “curse” may be a blessing to duopolists. For papers on durable-goods oligopoly with related results, see Desai and Purohit (1999) and Driskill (2001). See also Ausubel and Deneckere (1987).

### Long-Term Contracts

A firm selling a given product based on long-term contracts is analogous to a firm that sells a series of products in a given period. In other words, long-term contracts are an example of intertemporal bundling. Our results suggest that competition with long-term contracts may lead to lower profits than competition with spot contracts.

A related situation is that of frequent-flyer programs and similar loyalty schemes. Effectively, these programs lead to a more permanent relationship between the firm and its customers, very much like long-term contracts do. Again, our results suggest that while the direct effect of such loyalty schemes may be positive, one must also consider their impact on increased price competition (see also Kim et al. 2001).

## 10. Discussion

We have examined the impact of intrafirm product interactions on the equilibrium strategies and payoffs of competing oligopolists. As illustrated in §9, there are many market situations in which product interactions imply a negative strategic effect (case a) or imitation effect (case b) that can more than outweigh a positive direct effect (case a) or the unilateral effect (case b), situations that we refer to as “Bertrand supertraps.” In these situations, the effect of price competition is so powerful that “more is less”: stronger product interactions, which in a monopoly situation would imply greater profits, turn out to lower equilibrium profits under competition. This fall in equilibrium profits comes about through lower industry prices.<sup>17</sup>

We considered both the cases of a common change in  $s$  and a unilateral change in  $s_i$ . In the case of a common change, the main point is that there may be exogenous changes in industry conditions (technology, consumer preferences) that lead to lower industry profits even though absent price competition, they would lead to greater industry profits. In other words, there are situations where multiproduct price competition more than wipes away the positive benefits of an exogenous industrywide shock.

The case of unilateral change in  $s_i$  has important implications for business strategy. We have identified

<sup>16</sup> We make the additional assumption that each consumer’s “location” in the second period is independent of his location in the first period. This is similar to the assumption in von Weizsacker (1984). Other authors (including, e.g., Beggs and Klemperer 1989), make the opposite extreme assumption, namely that location is the same in all periods.

<sup>17</sup> Throughout the paper, we focus on industry profits. In the cases when profits decrease through lower prices, we would also expect the effect of a greater consumer welfare.

cases when the competitive advantage from a unilateral move are particularly great. In fact, a firm's profit increases if it is the only one to change  $s_i$ ; that firm's profits decrease (with respect to the initial level), however, if all firms proceed with the same change.

A natural extension of our analysis is to consider metagames where firms choose  $s_i$  in addition to prices. Specifically, suppose that firms simultaneously choose  $s_i$  in the first stage and  $p_i^j$  in the second stage. Propositions 3 and 4 suggest that  $d\Pi_i/ds_i > 0$  and  $d\Pi_i/ds_j < 0$ . Although we have not derived general results to this effect, in all cases we considered  $d\Pi_i/ds_j < 0$ . It follows that the first stage has the nature of a prisoner's dilemma. For example, firms may choose technologies with steep learning curves, or selling conditions that imply a high degree of intrafirm demand complementarity (e.g., pure bundling). Even though, unilaterally, firms are better off by doing so, in equilibrium they are worse off than they would be if there were no intrafirm product interactions.

Obviously, this is not a novel idea. For example, Church and Gandal (1992) showed that the decision to merge in a situation of demand synergies has the nature of the prisoner's dilemma. In their paper, the decision to merge may be interpreted as the choice between  $s_i = 0$  (no merger) and  $s_i > 0$  (merger between firms that produce products that interact at the level of demand). Our results suggest that this pattern is quite prevalent.

The situation may be different if the metagame is one of *sequential*, not simultaneous, choice of  $s_i$ . In particular, consider the case where Firm 1, an incumbent, first chooses the value of  $s_1$ , and Firm 2, a potential entrant, then decides whether to enter and which value  $s_2$  to choose.<sup>18</sup> Firm 1's monopoly and duopoly profits are given by  $\Pi^1(s_1)$  and  $\pi^1(s_1, s_2)$ , respectively. Firm 2's duopoly profits are given by  $\pi^2(s_2, s_1; \theta)$ , where  $\partial\pi^2/\partial\theta > 0$ . The variable  $\theta$  is public information at the time Firm 2 decides whether to enter, but unknown at the time Firm 1 chooses  $s_1$ . Suppose the prior on  $\theta$  is given by the cumulative distribution function  $F(\theta)$  and let  $\theta^*(s_1)$  be such that  $\max_{s_2} \pi^2(s_2, s_1; \theta^*(s_1)) = 0$ . Firm 1's expected payoff is then given by

$$F(\theta^*(s_1))\Pi^1(s_1) + \int_{\theta^*(s_1)}^{\infty} \pi^1(s_1, s_2^*(\theta)) dF(\theta),$$

where  $s_2^*(\theta) = \arg \max_{s_2} \pi^2(s_2, s_1; \theta)$ . This analysis suggests an extra reason why a unilateral increase in  $s_1$  may have a positive effect. Not only may an increase

in  $s_1$  increase the value of the duopoly profits (as suggested above) but it may also increase Firm 1's ex ante expected payoff. First, with some probability Firm 1 will be a monopolist, and the total effect of  $s_1$  on  $\Pi^1(s_1)$  is simply the direct effect. Second, if  $\pi^2(s_2^*(\theta), s_1; \theta)$  is decreasing in  $s_1$ , as our results suggest, then an increase in  $s_1$  implies an increase in  $\theta^*$ , which in turn increases Firm 1's expected payoff. In other words, precisely because greater values of  $s$  imply lower duopoly profits, an increase in  $s_1$  may have the strategic effect of deterring entry.

The above extensions of our basic framework suggest a solution to an apparent puzzle raised by our results. Business people and business analysts are wont to stress the positive effect of strategies that lead to demand synergies, cost synergies, greater switching costs, and so forth—the very same strategies that, according to our analysis, lead to lower industry profitability. If the two-stage game has the structure of a prisoner's dilemma (simultaneous choice of  $s_i$ ), then we may interpret the business advice as reflecting the fact that choosing a high  $s_i$  is a dominant strategy, a fact that is consistent with our result that higher values of  $s_i$  by all firms lead to lower profits. If, on the other hand, we consider the sequential choice of  $s_i$ , then our results point to the benefits that early entrants may reap from intrafirm product interactions. In markets with high degree of within-firm product interaction (high  $s$ ), timing is of the essence.

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### Appendix

**Example of a Single-Product Bertrand Supertrap.** The focus of our paper is on multiproduct firms. However, Bertrand supertraps can also occur in a single-product oligopoly context. In this appendix, we present a simple example of a Bertrand supertrap in a one-product Hotelling duopoly.

Consider a unit segment with two firms located at the extremes. Consumers must pay a "transportation" cost  $t$  per unit of distance whereas firm  $i$ 's cost of producing quantity  $q$  is given by  $(c - s)q + \alpha s$ , where  $s, \alpha \geq 0$ . Suppose  $s$  small and  $\alpha < \frac{1}{2}$ . It can be shown that the equilibrium is symmetric and in pure strategies; equilibrium price is given by  $\hat{p} = t + c - s$ . It follows that an increase in  $s$ , keeping prices constant, implies an increase in each firm's profits (because each firm has lower overall costs). However, equilibrium profit for each firm is given by  $\frac{1}{2} - \alpha s$ , which is decreasing in  $s$ . That is, an increase in  $s$  implies a positive direct effect,

<sup>18</sup> In the particular case when  $s_i$  represents the bundling decision, this structure is similar to the one analyzed by Whinston (1990) and Church and Gandal (2000). This general structure is also related to Fudenberg and Tirole (1984).

but the strategic effect is so negative that it outweighs the direct effect—a Bertrand supertrap (of type a).

PROOF OF PROPOSITION 1. Firm  $i$ 's equilibrium profits are given by

$$\hat{\Pi}^i(s) = \sum_{j=1}^J \hat{p}_j^i(s) D_j^i(\hat{p}(s)) - C(D^i(\hat{p}(s)), s).$$

This may be rewritten as

$$\begin{aligned} \hat{\Pi}^i(s) = & \sum_{j=1}^J \left( \hat{p}_j^i(s) - \frac{\partial C}{\partial D_j^i}(D^i(\hat{p}(s)), s) \right) D_j^i(\hat{p}(s)) \\ & + \sum_{j=1}^J \frac{\partial C}{\partial D_j^i}(D^i(\hat{p}(s)), s) D_j^i(\hat{p}(s)) - C(D^i(\hat{p}(s)), s). \quad (A1) \end{aligned}$$

Firm  $i$ 's  $j$ th first-order condition for profit maximization can be written as

$$\left( p_j^i - \frac{\partial C}{\partial D_j^i} \right) \frac{\partial D_j^i}{\partial p_j^i} = -D_j^i.$$

The hypotheses that total demand is close to fixed and the equilibrium is symmetric imply that the right-hand side is invariant with respect to  $s$ . Because  $\partial D_j^i / \partial p_j^i$  is assumed constant across vectors of equilibrium prices, it follows that margins are also approximately invariant with respect to  $s$ . We conclude that an increase in  $s$  implies a decrease in equilibrium profits if and only if the last two terms in (A1) decrease in  $s$ , that is, if

$$\sum_{j=1}^J \frac{\partial^2 C}{\partial D_j^i \partial s} D_j^i - \frac{\partial C}{\partial s} \leq 0. \quad (A2)$$

(We only need to take partial derivatives with respect to  $s$  because the equilibrium demands do not change too much with  $s$ , by the proposition's hypothesis.)

Given the definition of cost interactions the fixed costs are independent of  $s$  because for  $D^i = 0$   $C(\cdot)$  is not a function of  $s$ . We then have

$$\frac{\partial C^i}{\partial s}(D^i, s) = \sum_{j=1}^J \int_0^{D_j^i} \frac{\partial^2 C^i}{\partial D_j^i \partial s}(D_{j-}^i, D_j^i = t, D_{j+}^i = 0) dt,$$

where  $D_{j-}^i$  is the vector with the elements of  $D_k^i$  for  $k < j$  and  $D_{j+}^i$  is the vector with elements  $D_k^i$  for  $k > j$ .

$$(\partial / \partial s)(\partial^2 C^i / \partial D_j^i \partial D_k^i) < 0, \quad \forall i, j \in J_i, k \in J_i$$

(Assumption 3), thus we have

$$\frac{\partial^2 C^i}{\partial s \partial D_j^i} D_j^i \leq \int_0^{D_j^i} \frac{\partial^2 C^i}{\partial D_j^i \partial s}(D_{j-}^i, D_j^i = t, D_{j+}^i = 0) dt,$$

because

$$\frac{\partial^2 C^i}{\partial s \partial D_j^i} \leq \frac{\partial^2 C^i}{\partial D_j^i \partial s}(D_{j-}^i, D_j^i = t, D_{j+}^i = 0)$$

for  $0 \leq t \leq D_j^i$ . Adding up for all products  $j$ , we conclude that (A2) holds, which in turn implies the result.  $\square$

PROOF OF PROPOSITION 2. First notice that by assumption,  $\partial D_j^i / \partial s \geq 0$ , which implies that the direct effect is positive.

Suppose that total demand is fixed and that the equilibrium is symmetric. Firm  $i$ 's first-order conditions are given by  $\partial \Pi^i(\hat{p}) / \partial p_j^i = 0$ ,  $j \in J$  and can be rewritten as

$$p_j^i = f_j^i(p_{-j}^i, p^{-i}, s), \quad j \in J. \quad (A3)$$

Assumptions 2 and 4 imply that, for  $s = 0$ , the right-hand side of (A3) is weakly increasing in all arguments and strictly increasing in at least one. Moreover, by assumption  $\partial \Pi^i / \partial p_j^i$  is decreasing in  $s$ , that is,  $f_j^i$  is decreasing in  $s$ . Standard supermodularity results (Milgrom and Roberts 1990, Villas-Boas 1997) imply that all equilibrium prices are decreasing in  $s$ . Because total demand is close to constant, the equilibrium is symmetric, and costs do not directly depend on  $s$ , it follows that equilibrium prices decrease. Finally, the result follows by continuity.  $\square$

PROOF OF PROPOSITION 3. Consider  $s = 0$ , and the effect of  $s_i$  increasing. The first-order condition for a product  $j$  sold by firm  $i$  is

$$D_j^i + (p_j^i - \tilde{C}_j^i) \frac{\partial D_j^i}{\partial p_j^i} + 2s_i \sum_{\ell \neq j} D_\ell^i \frac{\partial D_j^i}{\partial p_j^i} = 0,$$

and for product  $j$  sold by a firm  $k \neq i$  is

$$D_j^k + (p_j^k - \tilde{C}_j^k) \frac{\partial D_j^k}{\partial p_j^k} = 0.$$

Stacking up these first-order conditions we can denote the system of first-order conditions for all the firms as  $g(p, s_i) = 0$ . To find the effect of  $s_i$  on the prices charged by firm  $k \neq i$ , we can totally differentiate this system of equations with respect to  $p$  and  $s_i$  and evaluate it at  $s = 0$ . Differentiation of  $g(p, s_i)$  with respect to  $p$  yields a matrix  $g_p$  of order  $IJ \times IJ$ . Stacking up matrix  $g$  and vector  $p$  by market, evaluating  $g_p$  at  $s = 0$ , and dropping the second-order derivatives, the matrix  $g_p$  becomes block diagonal with  $I$  blocks, each block  $(J \times J)$  corresponding to each market, having  $2a$  in the diagonal with  $a \equiv \partial \tilde{D}_j^i / \partial p_j^i$  and  $b$  in the off-diagonal elements, with  $b \equiv \tilde{D}_j^i / \partial p_j^k$  for all  $i$  and  $k \neq i$ . Differentiation of  $g(p, s_i)$  with respect to  $s_i$  yields a vector  $g_{s_i}$  where the elements associated with firm  $i$  and product  $j$  are equal to  $2 \sum_{\ell \neq j} D_\ell^i (\partial \tilde{D}_j^i / \partial p_j^i)$  and the elements associated with firm  $k \neq i$  and product  $j$  are equal to zero. The effect of  $s_i$  on  $p$  can be obtained as  $dp / ds_i = -g_p^{-1} g_{s_i}$ . Under the assumptions of symmetry across firms and products (for  $s = 0$ ), and full market coverage we have  $a + (I - 1)b = 0$ , and denoting by  $d$  the total market for each product, we have  $p_j^k - \tilde{C}_j^k = -d / (aI)$  for  $s = 0$  for all  $k, j$ . Then we can obtain for firm  $k \neq i$  and product  $j$  at  $s = 0$ ,  $dp_j^k / ds_i = -2(J - 1)d / (I(2I - 1))$ .

To compute the total effect on firm  $i$  note that this effect is

$$\frac{d\pi^i}{ds_i} = \sum_j D_j^i \sum_{\ell \neq j} D_\ell^i + \sum_j \sum_{k \neq i} (p_j^i - \tilde{C}_j^i) \frac{\partial \tilde{D}_j^i}{\partial p_j^k} \frac{dp_j^k}{ds_i},$$

which yields

$$\frac{d\pi^i}{ds_i} = J(J - 1) \left( \frac{d}{I} \right)^2 \frac{2I - 3}{2I - 1},$$

which is always positive. Part (b) follows immediately from Proposition 1.  $\square$

PROOF OF PROPOSITION 4. Consider  $s = 0$ , and the effect of increasing  $s_i$ . The first-order condition for a product  $j$  sold by firm  $i$  is

$$D_j^i + \left( p_j^i - C_j^i + s_i \sum_{\ell \neq j} (p_\ell^i - C_\ell^i) \right) \frac{\partial \tilde{D}_j^i}{\partial p_j^i} = 0,$$

and for product  $j$  sold by firm  $k \neq i$  is

$$D_j^k + \left( p_j^k - C_j^k - \frac{s_i}{I-1} \sum_{\ell \neq j} (p_\ell^k - C_\ell^k) \right) \frac{\partial \tilde{D}_j^k}{\partial p_j^k} = 0.$$

Stacking up these first-order conditions we can denote the system of first-order conditions for all the firms as  $g(p, s_i) = 0$ . To find the effect of  $s_i$  on the prices charges by a firm  $k \neq i$ , we can totally differentiate this system of equations with respect to  $p$  and  $s_i$  and evaluate it at  $s = 0$ . Differentiation of  $g(p, s_i)$  with respect to  $p$  yields a matrix  $g_p$  of order  $IJ \times IJ$ . Stacking up matrix  $g$  and vector  $p$  by market, evaluating  $g_p$  at  $s = 0$ , and dropping the second-order derivatives, matrix  $g_p$  becomes block diagonal with  $I$  blocks, each block  $(J \times J)$  corresponding to each market, having  $2a$  in the diagonal with  $a \equiv \partial \tilde{D}_j^i / \partial p_j^i$  and  $b$  in the off-diagonal elements, with  $b \equiv \tilde{D}_j^i / \partial p_j^k$  for all  $i, k, j$  with  $k \neq i$ . Differentiation of  $g(p, s_i)$  with respect to  $s_i$  yields a vector  $g_{s_i}$  where the elements associated with firm  $i$  and product  $j$  are equal to  $\sum_{\ell \neq j} (p_\ell^i - C_\ell^i) \partial \tilde{D}_j^i / \partial p_j^i$  and the elements associated with firm  $k \neq i$  and product  $j$  are equal to  $-1/(I-1) \sum_{\ell \neq j} (p_\ell^k - C_\ell^k) \partial \tilde{D}_j^k / \partial p_j^k$ . The effect of  $s_i$  on  $p$  can be obtained as  $dp/ds_i = -g_p^{-1} g_{s_i}$ . Under the assumptions of symmetry across firms and products (for  $s = 0$ ), and full market coverage we have  $a + (I-1)b = 0$ , and denoting by  $d$  the total market for each product, we have  $p_j^k - C_j^k = -d/aI$  for  $s = 0$  for all  $(k, j)$ . Then we can obtain for  $k \neq i$  and a product  $j$  at  $s = 0$ ,

$$\frac{dp_j^k}{ds_i} = -\frac{d(J-1)}{b(I-1)^2(2I-1)} \frac{I+1}{I},$$

To compute the total effect on firm  $i$ , note that this effect is

$$\frac{d\pi^i}{ds_i} = \sum_j (p_j^i - C_j^i) \sum_{\ell \neq j} \tilde{D}_\ell^i(p_\ell) + \sum_j \sum_{k \neq i} (p_j^i - C_j^i) \frac{\partial \tilde{D}_j^i}{\partial p_j^k} \frac{dp_j^k}{ds_i},$$

which yields

$$\frac{d\pi^i}{ds_i} = \frac{J(J-1)}{b(I-1)} \left( \frac{d}{I} \right)^2 \frac{2I(I-2)}{(I-1)(2I-1)},$$

which is equal to zero for  $I = 2$  and strictly positive for  $I > 2$ . Part (b) follows directly from Proposition 2.  $\square$

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