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NOTE

ON THE ADOPTION OF INNOVATIONS WITH 'NETWORK' EXTERNALITIES

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We present a simple dynamic model of adoption of an innovation when there are 'network' externalities, that is, when one agent's benefit from adoption increases with the number of other adopters. We assume there is a continuum of differentiated potential adopters who are perfectly informed rational agents. Our main conclusion is that if network externalities are strong, then the equilibrium adoption path is discontinuous, even when there is no coordination between potential adopters. We also argue that a steep S-shaped adoption path can be interpreted as the approximation to a discontinuous point (catastrophe) of the equilibrium adoption path.

Key words: Network externalities; adoption and diffusion of innovations.

1. Introduction

Positive externalities often occur in the diffusion of innovations or standards: the value of adopting an innovation or standard depends positively on how many others adopt the same innovation or standard. Typical examples include communication networks (e.g. fax machines) and technology standards (e.g. the R.I.S.C. design for computer architecture).

There is a vast literature on diffusion of innovations. Some authors justify the process of diffusion as the result of incomplete information about the value of an innovation (cf. Jensen, 1982). For others, differences in the time of adoption follow from differences between potential adopters (cf. David, 1969). Finally, in the case when there are adoption externalities, diffusion can be understood as the equilibrium of a game played by potential adopters (cf. Reinganum, 1981).

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There is also a vast literature of the issue of network externalities, which includes Rohlfs (1974), Schelling (1978), Dybvig and Spatt (1983), Farrell and Saloner (1985), and Granovetter and Soong (1986) (additional references are given below). Most of these authors consider static models, or assume somewhat ad hoc dynamics. One common theme that seems to emerge from this literature is that network externalities likely lead to multiple equilibria, some of which Pareto dominate others.

In this paper we present a model similar to Ireland and Stoneman's (1986) and David and Olsen's (1984, 1986). We assume there is a continuum of heterogeneous potential adopters and that benefits from adoption depend positively on the measure of adoption (network externalities). We further assume that agents are rational and perfectly informed. As in Ireland and Stoneman (1986) and David and Olsen (1984, 1986), we look for equilibrium adoption paths. The main innovation of this paper is the qualitative characterization of the equilibrium adoption path, based on the same tools as the study of regular economies and catastrophe theory (e.g. Debreu, 1976; Balasko, 1978). We argue that in the presence of network externalities there may exist multiple (in fact, a continuum of) equilibrium adoption paths. Assuming there is no exogenous coordination between adopters, we can restrict ourselves to a single equilibrium path, i.e. the one corresponding to minimum adoption. We show that if network externalities are strong, then the equilibrium adoption path is discontinuous (i.e. includes a catastrophe point). This is in sharp contrast with the case of diffusion with no network externalities. In the latter, assuming the primitives of the model (distribution of potential adopters, benefits) are smooth functions, the equilibrium adoption path is also a smooth (and thus continuous) function.

We finally argue that a steep S-shaped adoption path can be interpreted as the approximation to a discontinuous point (catastrophe) of the equilibrium adoption path. We thus believe our model to be consistent with the empirical evidence of diffusion of innovations.

2. Basic model

Consider a given innovation, with positive 'network' externalities, and a measure one of potential adopters.[1] Each potential adopter is characterized by a preference parameter, υ - the higher υ, the higher the net 'benefit' of using the technology, other things equal.[2] Specifically, upon adoption, each agent receives a benefit flow given by $B(\upsilon, x, t)$, where x is the measure of adopters (at time t) and t is time. It

[1] For convenience of exposition, we will use the term 'network' externalities to represent any kind of positive adoption externalities, which may be due to other factors, e.g. 'learning-by-doing' economies. See Cabral (1987).

[2] Different explanations for the heterogeneity of adopters are provided by the literature on the diffusion of innovations. See David and Olsen (1986).

is assumed that B is smooth, all first-order partial derivatives are positive, and υ has a smooth c.d.f. $F(\upsilon)$.

The assumption that $B_\upsilon > 0$ is a matter of convention. $B_x > 0$ corresponds to the idea of 'network' externalities. The crucial assumption of the model is that $B_t > 0$. It is agreed among some theorists that the driving force for the diffusion of innovations is an exogenous trend by which adoption benefits increase over time. B_t is supposed to capture such an exogenous trend. One must note, however, that B_t also includes the effects of changes in the conditions of supply of the innovation. Therefore, we are assuming that the conditions of supply are sufficiently stable that the increasing trend in benefits is not reversed.[3]

3. Existence and uniqueness of equilibrium

Since by assumption benefits at time t depend only on the measure of adoption at time t, we can begin by looking at the static problem of finding the equilibrium values of x for each value of t.

In equilibrium, all types υ for whom B is non-negative must adopt the innovation. Denote by $g(x,t)$ the indifferent adopter's υ level, i.e. $B(g(x,t),x,t)=0$. Define $H(x,t) \equiv 1 - F(g(x,t))$. A static equilibrium for a given time t is a value x such that $x = H(x,t)$. Denote by $\phi(t)$ the set of (static) equilibrium measures of adoption for each time t.

Our first result characterizes the graph of the equilibrium correspondence, ϕ, which we denote by E. Some additional definitions are needed, however. Points in E where the tangent is vertical are called singular points (e.g. points A and B in Fig. 1). Denote by π the natural projection of E onto $\mathbf{R}$ (the domain of t). A regular value t is a value such that $\pi^{-1}(t)$ includes no singular points. Finally, denote by $\mathscr{R}$ the set of regular values.

Proposition 1. (i) *E is a one-dimensional smooth manifold.* (ii) *$\mathscr{R}$ is an open dense subset of $\mathbf{R}$.* (iii) *The restriction of E to a connected component of $\mathscr{R}$ consists of an odd number of functions $f_i(t)$ such that $f_i(t) > f_{i+1}(t)$ and $f_i'(t) > 0$ iff i is odd.*

These results are well known from the study of regular economies, catastrophe theory, and standard differential topology (see Balasko, 1978, and references there-

[3] If, for example, the innovation were supplied by a monopolist, one would expect him or her to act strategically and – possibly – set a non-time-stationary price schedule. If this were the case, $B_t > 0$ would be too narrow an assumption. There are two basic situations in which $B_t > 0$ is a reasonable assumption to make. The first one is when supply is competitive, so that changes in price only reflect changes in cost. The second one is when the innovation is an 'unsponsored' innovation, i.e. an innovation which is not the property of any firm in particular, so that one cannot talk about 'supply' in the usual sense of the word. Examples of 'unsponsored' innovations are the QWERTY keyboard standard and the R.I.S.C. design in computer architecture.

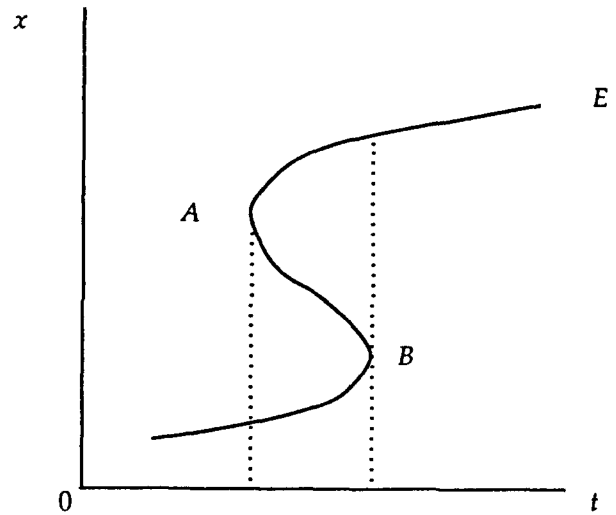

Fig. 1. Equilibrium manifold.

in). An example of what E may look like is given by Fig. 1. The idea of Proposition 1 is that E will in general look like Fig. 1.

Our next step is now to find an equilibrium adoption path (EAP). This is a function $X(t)$ such that $X(t) \in \phi(t)$ for all t. Inspection of E reveals that generically there exist multiple equilibrium adoption paths; in fact, a continuum of equilibrium adoption paths. One natural way of selecting among these is to assume there is no coordination among potential adopters, which seems to be consistent with the assumption that there is a continuum of them. Suppose that each agent makes his or her adoption decision at time t based on the extent of adoption at time $t-\delta$, where δ is arbitrarily small, i.e. each agent assumes that the extent of adoption at time t is very close to what it was at time $t-\delta$. The next result shows that under this assumption there exists a unique equilibrium adoption path, given by the lower envelope of E.

Proposition 2. *Suppose there exists a t' such that $\phi(t)$ is single-valued for $t<t'$. Consider an alternative model in which the benefit function is given by $B(v, X(t-\delta), t)$, and denote the (unique) equilibrium adoption path by* EAP(δ). *The limit as $\delta \to 0$ of* EAP(δ) *is the lower envelope of E.*

An example of an equilibrium adoption path corresponding to Proposition 2 is shown in Fig. 2. From this we conclude that a necessary condition for the equilibrium adoption path to be discontinuous is that there exists a singular (or catastrophe) point in E. Note, however, that this is not a sufficient condition. Fig. 3 depicts a case in which the equilibrium adoption path is continuous even though E

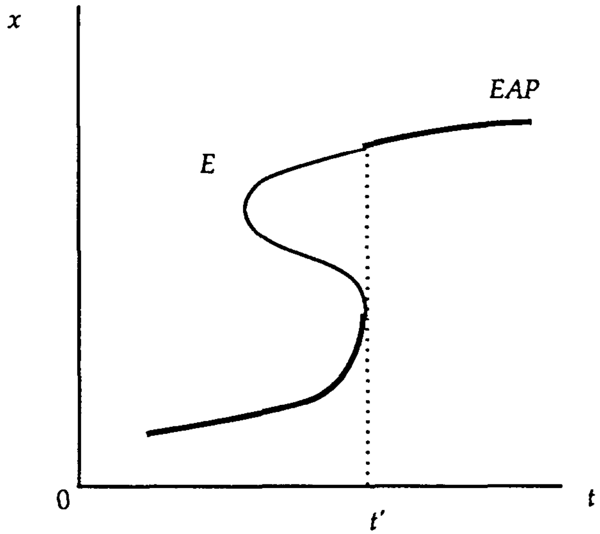

Fig. 2. Equilibrium adoption path.

includes a singular point. Our final result provides sufficient conditions for the existence of a continuous or discontinuous equilibrium adoption path.

Proposition 3. *Suppose that* $H(0,t)=0$ *for all* $t<t'$ *and denote by* x' *the lowest fixed point of* $\lim_{t\to\infty} H(x,t)$. (i) *If for all* $x<x'$, $H_x<1$ *for all* t, *then the equilibrium adoption path is continuous.* (ii) *If there exists an* $x''<x'$ *such that* $H_x(x'',t)>1$ *for all* t, *then the equilibrium adoption path is discontinuous.*

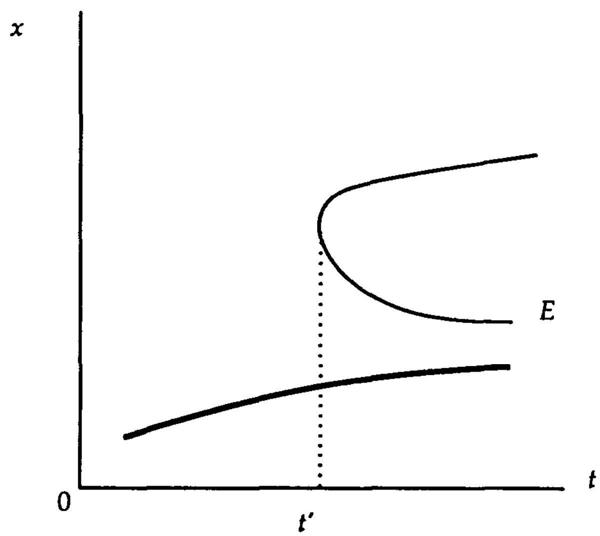

Fig. 3. Continuous equilibrium adoption path.

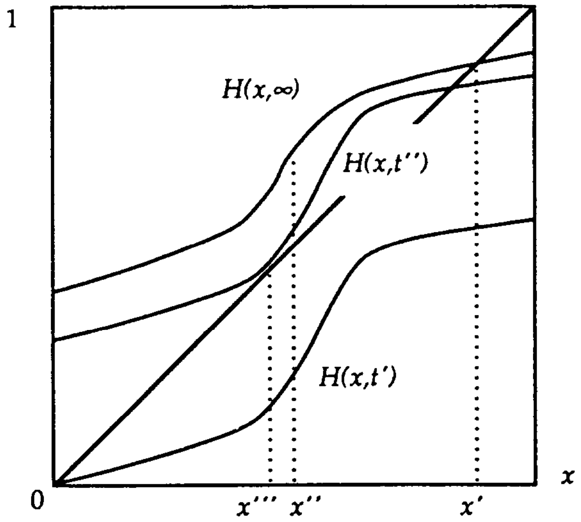

Fig. 4. Discontinuous equilibrium adoption path.

The proof of the second part of the proposition can be seen with reference to Fig. 4. (The proof of the first part is analogous.) The conditions of the proposition imply that $H(x,t')$ and $H(x,\infty)\equiv\lim_{t\to\infty}H(x,t)$ must be like in Fig. 4. If $H_x(x'',t)>1$ for all t, then there must exist a t'' such that (t'',x''') is a singular point, and the equilibrium adoption path is discontinuous at t''.

What are the factors affecting the value of H_x? Straightforward derivation yields:

$$H_x = F'(g(x,t))\frac{B_x}{B_v} = kfB_x, \tag{1}$$

where $k\equiv B_v^{-1}$ and $f\equiv g(x,t)$. This finally brings us to the main point of the paper. Even if we assume there is no coordination among potential adopters, we should expect the equilibrium adoption path to be discontinuous in situations of strong network externalities (large B_x) and relative homogeneity among potential adopters (large f).

4. Example

In this section we consider a simple example in which F is linear, B is a linear function of v,t, and a concave function of x. Specifically:

$$F(v) = v, \quad 0\le v\le 1, \tag{2}$$

and

$$B(v,x,t) = v+x^{\alpha}+t-\beta, \tag{3}$$

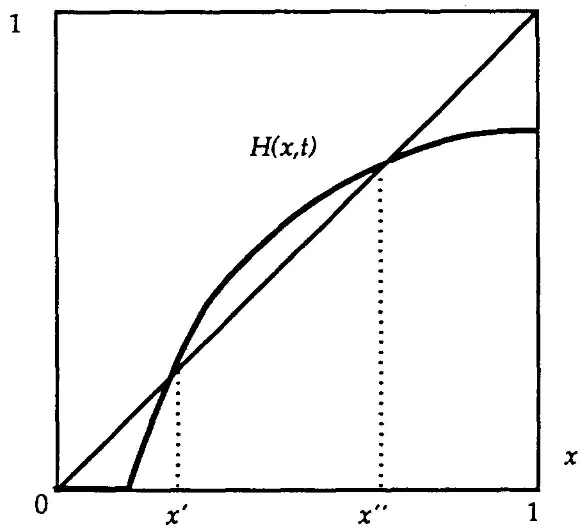

Fig. 5. Static equilibria for a given t.

where $\alpha<1$, $\beta>1$, and $t<1$. From (2)–(3) we get, for $t<1$:

$$H(x,t)=\max\{0,1-\beta+x^{\alpha}+t\}. \tag{4}$$

The restriction of $H(x,t)$ to a particular value of t is depicted in Fig. 5. From this, we can find the static equilibrium values $\phi(t)$ (in this case three values). By varying t, we obtain the equilibrium manifold E (Fig. 6), and taking the lower envelope we

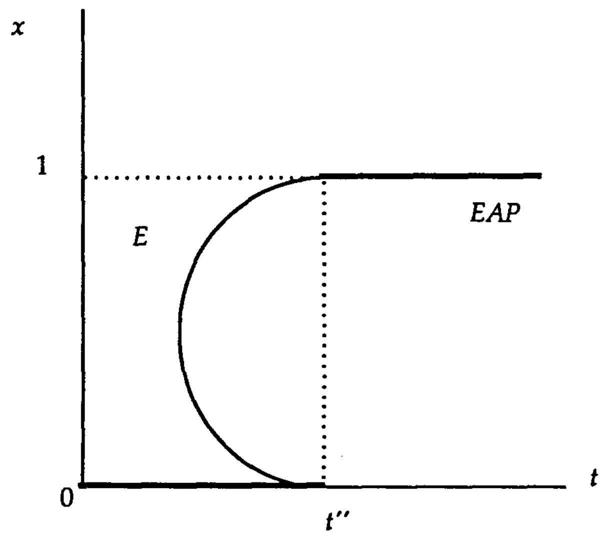

Fig. 6. Equilibrium adoption path.

get the equilibrium adoption path. As can be seen, the equilibrium adoption path is discontinuous at t'', where $t''=\beta-1-(1-\alpha)\alpha^{\alpha/(1-\alpha)}$.

One may wonder how realistic the idea of a discontinuous equilibrium adoption path is. There is a vast body of empirical research giving evidence of an S-shaped pattern in the diffusion of innovations. We argue that a steep S-shaped adoption path can be interpreted as the approximation to a discontinuous point (catastrophe) of the equilibrium adoption path. To do so, we return to the alternative model of adoption with a short observation lag introduced in the previous section. How does the equilibrium adoption path EAP(δ) behave in the neighborhood of a discontinuous point of EAP(0)? We argue that very likely EAP(δ) looks like an S-shaped diffusion process.

Fig. 7 shows how the equilibrium values $X(t)$ can be obtained in the neighborhood of a catastrophe point of the underlying process. The thing to notice is that in the interval $[x',x'']$ the function $H(x,t)-x$ is quasi-concave. This in turn implies an S-shaped pattern for EAP(δ) as x goes from x' to x''. For example, Fig. 8 depicts equilibrium paths for given parameter values (three values of α, $\beta=1$, and $\delta=0.001$). When network externalities are significant (high values of α) and the observation lag is short (low values of δ), a steep S-shaped continuous path obtains.

With a few exceptions (e.g. Jensen, 1982), the literature on S-shaped diffusion processes is based on rather ad hoc assumptions regarding the adoption process. We believe that one advantage of our model is that it takes an explicit equilibrium approach, while at the same time being consistent with the empirical evidence on diffusion of innovations.

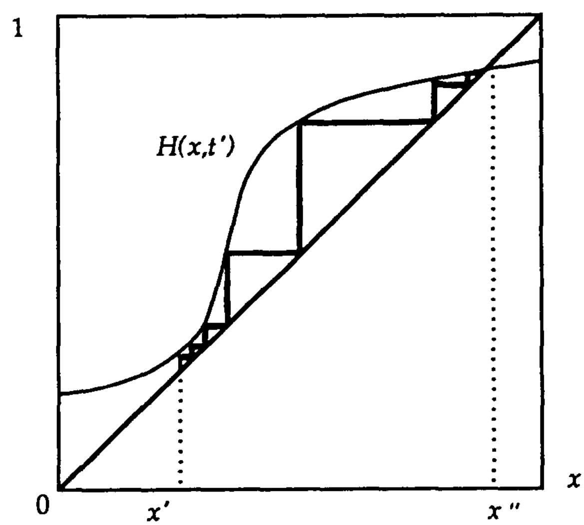

Fig. 7. Approximate equilibrium adoption path in the neighborhood of a singular point.

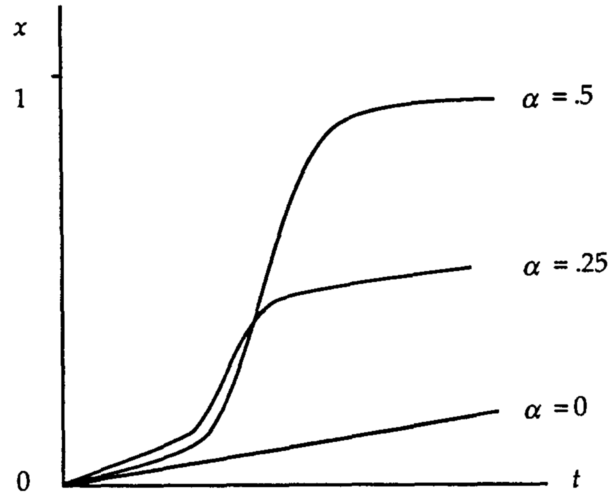

Fig. 8. Approximate equilibrium adoption path as a function of α.

5. Conclusion

The adoption and diffusion of innovations is a far more complex issue than has been modeled in this paper. For example, agents often behave strategically (e.g. Reinganum, 1981; Fudenberg and Tirole, 1983; Quirmbach, 1986), whereas our assumption of a continuum of adopters abstracts from this possibility. On the other hand, patterns of diffusion are often due to uncertainty regarding the benefits of adoption (cf. Jensen, 1982). Finally, there may be several competing innovations (Arthur, 1989; Katz and Shapiro, 1985, 1986a, 1986b), another issue which was not considered in this paper.

The main point of the paper is that even with a simple model like ours - perfect information, no exogenous coordination, no strategic behavior, smooth supply conditions - the mere existence of network externalities leads to a discontinuous equilibrium adoption path.

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