Mathematical Social Sciences 37 (1999) 97–106

mathematical social sciences

Meaningful cheap talk must improve equilibrium payoffs

Lanny Arvan[a], Luís Cabral[b,*], Vasco Santos[c]

[a]University of Illinois at Urbana-Champaign, Department of Economics, 1206 South Sixth Street, Champaign, Illinois 61820, USA

[b]London Business School, Sussex Place, Regent's Park, London NW1 4SA, UK

[c]Universidade Nova de Lisboa, Faculdade de Economia, Travessa Estêvão Pinto, P-1070 Lisbon, Portugal

Received 2 November 1997; received in revised form 31 December 1997; accepted 31 March 1998

Abstract

We generalize Farrell's (1987) idea of coordination by means of cheap talk. We show that if cheap talk is meaningful (in the sense that babbling equilibria are ruled out) and if there is room for cooperation (namely if there exists at least one pure-strategy equilibrium Pareto superior to the default equilibrium), then cheap talk must increase the equilibrium expected payoff relative to the play of the game without preplay communication. The result is limited to proper equilibria of the communication game and to games with two players. It is shown that

Keywords: Cheap talk; Communication; Equilibrium

1. Introduction

In a seminal paper, Farrell (1987) has shown that nonbinding, costless preplay communication (cheap talk) may improve the equilibrium payoff in a game with the structure of the 'battle of the sexes.' The 'battle of the sexes' is a game that entails both elements of competition and of coordination. It possesses two pure-strategy equilibria which are not Pareto ranked, and a third equilibrium in mixed strategies which is Pareto inferior to the pure-strategy equilibria. The first fact induces the competitive element of the game, while the second one adds the cooperative element. Farrell's (1987) idea is that cheap talk may help players coordinate in playing one of the pure-strategy equilibria: while players' preferences are opposite regarding the pure-strategy equilibria, they both prefer any of these equilibria to the mixed-strategy equilibrium, which, by

*Corresponding author. Tel.: 0044-171-7066963; fax: 0044-171-4020718; e-mail: lcabral@lbs.ac.uk

0165-4896/99/$ – see front matter
PII: S0165-4896(98)00018-3

assumption, is the default equilibrium to be played in case no ‘agreement’ is reached at the communication stage.

Specifically, Farrell considers an extended game whereby, before any action is taken, players simultaneously announce which action they intend to take. Although announcements are not binding (cheap talk), they serve as a means for coordinating on a Nash equilibrium superior to the default equilibrium. In fact, Farrell shows that there exists an equilibrium of the ‘extended’ game which yields a higher payoff than the default equilibrium.[1]

We generalize and strengthen Farrell’s result in two ways. First, we consider games where each player chooses between $n \geq 2$ pure-strategies, whereas Farrell only considered the case $n = 2$.[2] Second, and more importantly, our results show that, provided some conditions are satisfied, cheap talk must imply an improvement in equilibrium payoffs, whereas Farrell only argues that it may imply an improvement in equilibrium payoffs.

A central assumption underlying our result refers to the relation between communication and action. Following Farrell, we assume that, if players’ announcements correspond to a Nash equilibrium of the game to be played, then such equilibrium becomes focal and is indeed played; if, on the other hand, communication does not lead to any particular equilibrium, then a default equilibrium is played. Moreover, we assume that there exist pure-strategy Nash equilibria which are Pareto superior to the default equilibrium, i.e., we assume there is scope for improvement in the equilibrium payoff.

The intuition behind our result is as follows. At the communication stage, players have no incentive to announce a strategy associated with an equilibrium that yields the other player a low payoff. This is so because, in equilibrium, such players will not announce the action corresponding to that equilibrium. In fact, players will only make announcements associated with superior equilibria (equilibria beneficial to both players), which in turn implies that cheap talk must improve expected equilibrium payoffs.

In addition to the assumptions spelled out above, our result requires that the number of players be two and that the equilibrium of the augmented game be proper.[3] In Section 3, we show, by means of examples, that both these requirements are necessary. In fact, the restriction to the case of two players is necessary even when we consider a richer message space whereby players announce action profiles instead of actions (cf. Section 4).

[1] The extended game consists of a communication stage (comprising one or more rounds) followed by play of the original game.

[2] However, we require all pure-strategy Nash equilibria of the original game to be strict. Alternatively, we may assume instead that there is only ‘one pure-strategy Nash equilibrium per row,’ a weaker hypothesis than strictness. Finally, we can also consider a different message space than the one proposed by Farrell; cf. Section 4.

[3] Cf. Myerson (1978). Essentially, properness implies that, for each two player i’s actions, a_i and a_i', if a_i yields a lower payoff than a_i', then, in the play of the perturbed game, a_i should be chosen with a probability that is at least one order of magnitude lower than the probability that a_i' is chosen.

2. Basic results

Consider the following two-player game in normal form, denoted by G: S_i is the space of player i's pure strategies and s_i is a generic element of S_i, $i = 1, 2$; $S \equiv S_1 \times S_2$ and $s \equiv (s_1, s_2)$; finally, $u_i(s)$ denotes player i's utility.

Based on game G, we now define an extended game, denoted by Γ, which involves a communication stage prior to playing G. At the communication stage, players simultaneously announce actions from their own pure-strategy sets S_i (the terms 'action' and 'pure strategy' meaning the same thing). After all messages are sent and received, game G is played.

Games of this sort always admit equilibria in which cheap talk has no influence on the outcome of G ('babbling equilibria').[4] Following Farrell (1987), we concentrate on no-babbling equilibria, that is, equilibria with the following properties:

1. If announcements correspond to a Nash equilibrium of G, then that equilibrium becomes focal and is thus played.
2. If announcements do not correspond to a Nash equilibrium of G, then G is played as if no communication has taken place: a 'default' equilibrium is played independently of which announcements are made. Payoffs in this default equilibrium are given by $\bar{u}_i$.

Notice that properties (i)–(ii) induce a well-defined reduced game in the communication stage, so the set of no-babbling equilibria (Nash, perfect Nash, or proper Nash) is nonempty.

Finally, we define s to be a superior equilibrium of G if and only if s is a pure-strategy equilibrium and $u_i(s) > \bar{u}_i$, $\forall i$. Our main result will be based on the hypothesis that there exists at least one superior equilibrium. This hypothesis implies that G is to some extent a game of coordination, specifically, a game with at least two Nash equilibria which are Pareto ranked.

Theorem 1. *Assume that all pure-strategy Nash equilibria of G are strict. If there exists some superior equilibrium of the original game, then, in a proper no-babbling equilibrium of the extended game, only actions corresponding to superior equilibria are announced, and the expected payoff of both players is strictly greater than in the game with no communication.*

Proof. Consider a 'perturbed' game in the communication stage. In this game, every message is announced with positive probability. Therefore, it is a strictly dominated strategy for player i to announce an action corresponding to a pure-strategy Nash

[4] See, for example, Farrell and Gibbons (1989). Seidmann (1992) shows that the same is not necessarily true in games of incomplete information.

equilibrium, say s^*, yielding him or her a payoff less than $\bar{u}_i$. In fact, the expected payoff of announcing such an action would be lower than $\bar{u}_i$, by the assumption that all pure-strategy Nash equilibria are strict; whereas announcing an action corresponding to a superior equilibrium, say s^{**}, would yield player i an expected payoff greater than $\bar{u}_i$. By the requirement of properness (Myerson, 1978), the weight assigned by player i to the action corresponding to s^* must be at least one order of magnitude lower than the weight assigned to the action corresponding to s^{**}.

Now, given the above, it is also a dominated strategy for player j to announce the action corresponding to s^*, assuming that payoffs are bounded and that the 'perturbed' game is sufficiently close to the 'unperturbed game.' To see why, notice that player j's expected payoff from announcing the action associated to s^* is

$$A = P(s_i^*)u_j(s^*) + (1 - P(s_i^*))\bar{u}_j \, , \tag{1}$$

where $P(s_i^*)$ is the probability that action s^*_i is announced by player i. On the other hand, announcing the action corresponding to s^{**} yields player j an expected payoff of

$$B = P(s_i^{**})u_j(s^{**}) + (1 - P(s_i^{**}))\bar{u}_j \, . \tag{2}$$

Since $u_j(s^{**}) > \bar{u}_j$ and $P(s_i^*) \leq \epsilon P(s_i^{**})$ (the latter by properness), we have $A < B$.

Finally, a similar argument, if somewhat simpler, applies to actions corresponding to no pure-strategy Nash equilibrium. Likewise, it is straightforward to show, by contradiction, that at least one superior equilibrium must be played with positive probability. ■

It should be remarked that the argument extends to games with $T > 1$ rounds of cheap talk (a case also considered in Farrell (1987)). It suffices to note that the proof applies to any period $t < T$, with the difference that payoffs in case of no agreement at stage t are now given by $(\bar{u}_1^t, \bar{u}_2^t)$, where $u_i^t > \bar{u}_i$.

3. Counterexamples

In this section, we present a series of counterexamples which elucidate the necessity of some of the assumptions and hypotheses underlying the main result of the previous section. All examples involve a communication stage with a single round of cheap talk.

1. The first example shows how the assumption that all pure-strategy Nash equilibria are strict is necessary for the result. In this example, the original game, shown in Fig. 1, has two pure-strategy equilibria, (T, L) and (T, R), neither of which is strict. There also exists a mixed-strategy equilibrium in which both players choose each action with equal probability. The expected payoff in this equilibrium, which we assume to be the default equilibrium, is $2/3$ for each player. Since we restrict our interests to no-babbling equilibria, expected payoffs at the communication stage are summarized by the payoff matrix in Fig. 2. It can be seen that it is a proper equilibrium for the row player to choose M and the column player to choose L with probability α and R with probability

	L		C		R	
T	2	1	0	0	0	1
M	0	1	2	0	0	0
B	1	0	1	2	0	1

Fig. 1. Game G in first example.

	L		C		R	
T	2	1	$2/3$	$2/3$	0	1
M	$2/3$	$2/3$	$2/3$	$2/3$	$2/3$	$2/3$
B	$2/3$	$2/3$	$2/3$	$2/3$	$2/3$	$2/3$

Fig. 2. Game Γ in first example.

$1-\alpha$, with $\alpha < 1/3$. Although there exists a superior equilibrium, namely (T, L), the row player never announces the strategy corresponding to this equilibrium.

The intuition behind the first example is simple. By being indifferent between two equilibria ((T, L) and (T, R)) and announcing more often the one least desired by its opponent ((T, R)), one player may end up preventing coordination.

2. The second example shows that perfection is not a sufficient refinement to produce the main result. The original game in this example is described in Fig. 3. The game has three pure-strategy equilibria, (T, R), (M, C) and (B, L). There also exists a mixed-strategy equilibrium in which players choose the first two actions with equal probability. The expected value in this equilibrium, which we assume to be the default equilibrium, is $3/2$ for each player. The payoff structure at the communication stage is given by the matrix in Fig. 4.

We will now show that it is a perfect equilibrium, although not a proper equilibrium,

	L		C		R	
T	0	0	3	-3	9	1
M	-3	3	6	6	0	0
B	1	9	0	0	0	0

Fig. 3. Game G in second example.

	L		C		R	
T	$3/2$	$3/2$	$3/2$	$3/2$	9	1
M	$3/2$	$3/2$	6	6	$3/2$	$3/2$
B	1	9	$3/2$	$3/2$	$3/2$	$3/2$

Fig. 4. Game Γ in second example.

	L	R
T	0 0 1	0 0 0
B	1 3 3	1 0 0

LM

	L	R
T	4 4 0	0 0 2
B	0 0 2	4 4 4

RM

Fig. 5. Game *G* in third example.

for the row player to announce *T* and the column player to announce *L*. For this purpose, consider the following strategies in an ϵ-perturbed game: each player chooses the first strategy with probability $1-2\epsilon$ and the two remaining strategies with probability ϵ each. Simple inspection reveals that this constitutes a Nash equilibrium given the constraint that all actions be chosen with probability greater than ϵ. Hence, the designated equilibrium is indeed perfect. However, the strategies in the perturbed game do not satisfy the requirement for properness, namely that the third strategy be chosen by each player with a probability which is one order of magnitude lower than the second one. In fact, our main result states that the only equilibrium at the communication stage consists of players announcing the middle strategy with probability one.

The intuition behind the second example is also simple. Perfectness allows actions associated with superior and nonsuperior equilibria to be announced with probabilities of the same order of magnitude. This, in turn, may lead players to try to coordinate on nonsuperior but, to them, particularly favorable equilibria, a behavior that may end up precluding coordination. Properness rules this out by ensuring that the probability of coordinating on a nonsuperior equilibrium is at least one order of magnitude lower than on a superior equilibrium.

3. Finally, the third example shows that the result does not extend to games with more than two players. Fig. 5 depicts the original game in this example. As before, players 1 and 2 choose rows and columns, respectively. Now we add a third player who picks one of the two matrices, *LM* or *RM*. The game has two pure-strategy equilibria, (*B*, *L*, *LM*) and (*B*, *R*, *RM*), both of which are strict. In addition, there exists a mixed-strategy equilibrium in which the first two players mix with equal probability each of their actions and the third player chooses *RM*. Expected payoff under this equilibrium, which we assume to be the default equilibrium, is given by 2 for all players. The payoff matrix at the communication stage is thus the one described in Fig. 6.

	L	R
T	2 2 2	2 2 2
B	1 3 3	2 2 2

LM

	L	R
T	2 2 2	2 2 2
B	2 2 2	4 4 4

RM

Fig. 6. Game Γ in third example.

We will now argue that (T, L, LM) constitutes a proper equilibrium of Γ, even though one of the players, player 1, chooses an action not associated with any Nash equilibrium. Consider the following equilibrium of the perturbed game. Player 1 chooses T with probability $1-\epsilon$ and B with probability ϵ. Player 2 chooses L with probability $1-\epsilon$ and R with probability ϵ. Finally, player 3 chooses LM with probability $1-\epsilon$ and RM with probability ϵ. Clearly this equilibrium converges to the designated Nash equilibrium. We will now show that it constitutes an equilibrium of the properly perturbed game.[5] If player 1 deviates, it gets $1(1-\epsilon)^2+4\epsilon^2+2\cdot 2\epsilon(1-\epsilon)$, which is lower than 2, its equilibrium payoff. If player 2 deviates, it gets $4\epsilon^2+2(1-\epsilon^2)$, which is lower than $3\epsilon(1-\epsilon)+2(1-\epsilon+\epsilon^2)$, its equilibrium payoff. Finally, if player 3 deviates, it gets $4\epsilon^2+2(1-\epsilon^2)$, which is lower than $3\epsilon(1-\epsilon)+2(1-\epsilon+\epsilon^2)$, its equilibrium payoff.

The idea of this example is that, with three or more players, there appear coordination problems which are absent in the case of two players only. Players 2 and 3 would prefer to switch from playing (T, L, LM) to playing (B, R, LM). Unilateral moves, however, can only reduce expected payoff. At (T, L, LM), both player 2 and player 3 look forward to a mistake by player 1, a mistake that will induce a payoff of 3 instead of 2. If player 2 or player 3 unilaterally deviate, then it would require two simultaneous mistakes to increase payoff, a possibility infinitely less likely.

4. A different message space

In the previous sections, we have considered a game with the structure proposed in Farrell (1987): starting from a normal-form game, we augment this by adding a prior stage of communication. The set of messages sent by player i in the communication stage consists of the names of his or her actions in the initial normal-form game. In other words, each player announces what action he or she intends to play.

An alternative message space consists of players announcing action profiles instead of actions.[6] This message space is perhaps as natural as the one assumed in Farrell (1987) and by ourselves in the previous sections. It allows for a somewhat sharper result:

Theorem 2. *If there exists some superior equilibrium of the original game, then, in a proper no-babbling equilibrium of the extended game, only actions corresponding to superior equilibria are announced, and the expected payoff of both players is strictly greater than in the game with no communication.*

Proof. Consider a 'perturbed' game in the communication stage. In this game, every action profile is announced with positive probability. Therefore, it is a strictly dominated strategy for player i to announce an action profile corresponding to a pure-strategy Nash

[5] Since each player has only two pure strategies, properness corresponds to trembling-hand perfection.

[6] This extension follows a referee's suggestion that we consider a message space of the type $\{m_i(s)\}$, where i denotes the player and s an action profile, together with the assumption that $m_i(s)\neq m_i(s')$ for $s\neq s'$. The message space we propose is a natural particular case that satisfies this assumption.

equilibrium, say s^*, yielding him or her a payoff less or equal to $\bar{u}_i$. (For simplicity, we will refer to this move in the communication stage as 'proposing equilibrium s^*.') In fact, the expected payoff from proposing s^* would be strictly lower than $\bar{u}_i$, whereas proposing a superior equilibrium, say s^{**}, would yield player i an expected payoff strictly greater than $\bar{u}_i$.

By the requirement of properness (Myerson, 1978), the weight assigned by player i to s^* must be at least one order of magnitude lower than the weight assigned to s^{**}.

Now, given the above, it is also a dominated strategy for player j to announce s^*, assuming that payoffs are bounded and that the 'perturbed' game is sufficiently close to the 'unperturbed game.' To see why, notice that player j's expected payoff from announcing s^* is

$$A = P(s_i^*)u_j(s^*) + (1 - P(s_i^*))\bar{u}_j\,, \tag{3}$$

where $P(s_i^*)$ is the probability that s_i^* is announced by player i. On the other hand, announcing s^{**} yields player j an expected payoff of

$$B = P(s_i^{**})u_j(s^{**}) + (1 - P(s_i^{**}))\bar{u}_j\,. \tag{4}$$

Since $u_j(s^{**}) > \bar{u}_j$ and $P(s_i^*) \leq \epsilon P(s_i^{**})$ (the latter by properness), we have $A < B$.

Finally, a similar argument, if somewhat simpler, applies to actions profiles corresponding to no pure-strategy Nash equilibrium. Likewise, it is straightforward to show, by contradiction, that at least one superior equilibrium must be played with positive probability. ■

The main difference with respect to Theorem 1 is that Theorem 2 dispenses with the assumption that all pure-strategy Nash equilibria of G are strict. However, the assumption that there are only two players remains a necessary condition, as the following example shows.

The example features a three-player game with payoffs as in Fig. 7. As before, players 1, 2 and 3 chose rows, columns and matrices, respectively. This game has two pure-strategy Nash equilibria, (T, L, LM) and (B, R, RM), and one mixed-strategy equilibrium where players 1 and 2 mix with equal probability and player 3 plays RM. The latter equilibrium yields all players a payoff of 2 and is assumed to be the default equilibrium.

As in Theorem 2, we consider an extended game, Γ, in which players announce action profiles. If the action profile announced by all three players coincides and if this action

	LM: L	LM: R	RM: L	RM: R
T	1 3 3	0 0 0	4 4 0	0 0 2
B	0 0 1	1 0 0	0 0 2	4 4 4

Fig. 7. Game G in fourth example.

	L			R		
T	1	3	3	2	2	2
B	2	2	2	2	2	2

LM

	L			R		
T	2	2	2	2	2	2
B	2	2	2	4	4	4

RM

Fig. 8. Summary of payoffs of game Γ in fourth example.

profile constitutes a Nash equilibrium, then that equilibrium is played; otherwise the default equilibrium is played.

The normal form of the extended game is quite complex: it comprises eight matrices of eight by eight. Instead of writing out the complete normal form, Fig. 8 presents a summary of the payoffs of the extended game when all players propose the same action profile. Whenever announcements differ, payoffs are (2, 2, 2).

Player 1 prefers to propose (*B*, *R*, *RM*) (expected payoff greater than 2) then to propose an action profile that is not an equilibrium (expected payoff of 2); and this, in turn, player 1 prefers to announcing (*T*, *L*, *LM*) (expected payoff lower than 2). Assume that player 1 proposes (*B*, *R*, *RM*) with probability $1-6\epsilon-\epsilon^2$, (*T*, *L*, *LM*) with probability ϵ^2, and all other action profiles with probability ϵ.

Players 2 and 3 prefer to propose (*B*, *R*, *RM*) and (*T*, *L*, *LM*) (expected payoff greater than 2) then to propose any other action profile (expected payoff of 2). Assume that players 2 and 3 propose (*T*, *L*, *LM*) with probability $1-\epsilon^3-6\epsilon^4$, (*B*, *R*, *RM*) with probability ϵ^3 and all other action profiles with probability ϵ^4.

To show that this constitutes an equilibrium of the perturbed game, notice that, by proposing (*T*, *L*, *LM*), players 2 and 3 receive a payoff that exceeds 2 by a factor of order ϵ^2 (the probability of a mistake by player 1). However, by unilaterally deviating to proposing (*B*, *R*, *RM*), expected payoff exceeds 2 by a factor of order ϵ^3 (the probability of a mistake by player 2 or 3, whichever is not deviating).

5. Final remarks

As Farrell (1988) noted, 'cheap talk is notoriously hard to model: there are no obviously 'right' rules about who speaks when, what he may say, and when discussion ends.' Not surprisingly, different structures of preplay communication have been attempted in the literature, some of which present results related to ours.

Farrell (1988) and Watson (1991) consider the case in which one of the players unilaterally suggests to the other which set of strategies to play. Watson (1991) shows that if the original game has a single Pareto-efficient outcome, then this is the only sensible outcome of the extended game.

Matsui (1991) assumes a structure of preplay communication similar to ours, with players sending messages simultaneously. However, the equilibrium concept he considers is quite different from ours. He considers a large population matched to play a game of common interest with cheap talk. He shows that a unique cyclically stable set exists and this contains only Pareto optimal outcomes.

The paper which is closest to ours is Rabin (1994). As in our paper, he considers a two-player game and simultaneous message exchange. However, he considers a wider set of communication possibilities than our paper (and Farrell, 1987). Rabin shows that with enough rounds of cheap talk, each player's expected payoff is at least as great as the payoff from his worst Pareto-efficient Nash equilibrium. Our paper partly confirms the communication-yields-efficiency hypothesis, to borrow Rabin's (1994) expression, although we show that some important qualifications need to be made.

Finally, although our results imply that, under some conditions, equilibrium payoffs must improve through cheap talk, it can be shown that, generically, there exists no lower bound to the size (or the probability) of the improvement in payoff resulting from preplay communication (cf. Farrell, 1987). It thus seems that our result is 'tight' both in terms of its extent and in terms of the required conditions.

Acknowledgements

We are grateful to a referee for comments and suggestions that improved the paper considerably.

References

Farrell, J., 1987. Cheap talk, coordination, and entry. Rand Journal of Economics 18, 34–39.

Farrell, J., 1988. Communication, coordination and Nash equilibrium. Economics Letters 27, 209–214.

Farrell, J., Gibbons, R., 1989. Cheap talk can matter in bargaining. Journal of Economic Theory 48, 221–237.

Matsui, A., 1991. Cheap talk and cooperation in a society. Journal of Economic Theory 54, 245–258.

Myerson, R., 1978. Refinements of the Nash equilibrium concept. International Journal of Game Theory 7, 73–80.

Rabin, M., 1994. A model of pregame communication. Journal of Economic Theory 63, 370–391.

Seidmann, D., 1992. Cheap talk games may have unique informative equilibrium outcomes. Games and Economic Behavior 4, 422–425.

Watson, J., 1991. Communication and superior cooperation in two-player normal form games. Economics Letters 35, 267–271.