Standards battles and public policy
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Abstract

We examine the effectiveness of public policy in a context of competing standards with network externalities. We show that, if the policymaker is very impatient, then it is optimal to support the leading standard; whereas, if the policymaker is very patient, then it is optimal to support the lagging standard. We also consider the timing for optimal intervention and provide sufficient conditions under which it is optimal to delay or not to delay intervention.

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1 Introduction

VHS vs. Betamax VCRs; Apple MacIntosh vs. PC DOS microcomputers; discrete vs. matrix quadraphonic systems. These are three of a long list of examples from recent history where two (or more) alternative versions of a new standard battled for market dominance. One aspect common to most of these standards is the importance of network effects: the fact that many users buy a DOS-based microcomputer increases the utility of buying a DOS-based microcomputer (among other reasons because the amount of software, technical support, etc., available for DOS users will be better and more widely available).

Given the importance of network industries, it is surprising that little attention has been paid to the role of public policy in standards battles.1 Consider the cases of high-definition television (HDTV) and mobile

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telecommunications. Public policy toward these industries differed greatly between Europe and the US: Whereas the European Commission (EC) was primarily concerned with early standardization, the US’s Federal Communications Commission (FCC) adopted the more patient approach of letting market forces decide the winning standard. At first sight, the European approach seems preferable in that it takes better advantage of network effects. The US approach, in turn, is more likely to lead to a higher-quality standard.

Our purpose in this paper is to analyze the different trade-offs involved in the policymaker’s decisions with respect to standardization in network industries. Specifically, there are at least two questions that a policymaker should address. First, the decision of which standard to support, if any. Second, the decision of when to intervene. Regarding the first question, we show that, if the policymaker is sufficiently patient, then it is optimal to favor the lagging standard. Conversely, if the policymaker is sufficiently impatient, then it is optimal to favor the leading standard. Regarding the second question, we show that, if the policymaker is sufficiently patient, then it is optimal to delay intervention. Conversely, if the policymaker is sufficiently impatient, then it is optimal not to delay intervention.

In our analysis, we consider the extreme cases of an infinitely patient and an infinitely impatient policymaker. An infinitely patient policymaker is one who cares exclusively about the welfare of future adopters, whereas an infinitely impatient policymaker cares exclusively about current adopters. We consider these extremes for illustrative purposes only; reality is likely to fall somewhere in between. There are two factors that determine the degree of policymaker “patience” in each particular case. One is the policymaker’s preferences: witness, for example, the contrast between Europe and the US in wireless telecommunications, or the contrast between Japan and the US in HDTV.

More importantly, the degree of patience is likely to reflect the nature of the technology in question. Take for example the case of color television in the 1950s. Given the success of monochrome TV and the absence of a likely substitute for TV, a policymaker should take a long-term view of the standardization process. Whatever solution is achieved, it is likely to stay for a long time and be used by a great number of future adopters. Our infinitely patient policymaker assumption tries to capture this feature.
By contrast, digital audio tape (DAT) is a good example of a technology with a relatively short expected life span, considering the rapid advancements in storage and recording devices such as CDs. In such a situation, a policymaker is more likely to concentrate on the existing set of adopters and the standardization problems they face. Our infinitely impatient policymaker assumption tries to capture this situation.

In addition to issues of time horizon and patience, our model treats the policymaker’s actions in a stylized way. Specifically, we assume that the policymaker has the option to “tilt” the system in favor of one standard or the other. In reality, this may come about through a variety of mechanisms such as direct subsidies (e.g., HDTV in Japan), government regulations (e.g., wireless in Europe), or direct adoption decisions by the policymaker when the latter is a “large” user (e.g., nuclear reactors in the US).

Other papers, such as Mitchell and Skrzypacz (2004), develop a model similar to ours and derive outcomes as a function of the agents’ discount factor. However, their policy analysis is limited to comparing the welfare-maximizing to the unregulated solutions. Our approach to modeling public policy, while certainly very stylized, is a useful first step toward a more complete treatment of policymakers’ options under imperfect information about the quality of emerging standards. In recent years, “heavy-handed” regulation which picks one winner from several standards has increasingly been abandoned in favor of “softer” intervention of the form modeled in our paper. Especially in situations where a mistake would carry significant costs, helping the market to make the efficient choice rather than making the decision itself may be the policymaker’s best strategy.

The paper is organized as follows. In the following section, we extend Arthur’s (1989) model of standard adoption to consider the possibility of public policy intervention. Next we consider the direction of optimal intervention in the case when the policymaker is very impatient (Section 3) or patient (Section 4). In Section 5, we look at the optimal timing for intervention. Section 6 includes a discussion of some of the results in the context of several recent standards battles. Section 7 concludes the paper.

2 In the case of DAT, there were two different standards, DDS and DataDAT.
2 Basic model

Our analysis departs from Arthur's (1989) seminal model of standard adoption. Suppose there are two unsponsored standards, A and B, available to consumers at constant marginal cost (which we normalize to zero). In each period, one new consumer arrives in the market and buys one unit of one of the standards. Some consumers favor standard A, some standard B; all benefit from the size of the network they link into. Specifically, by choosing standard $i$, a consumer receives, at time $t$, utility $v_i + w n_{it}$, where $v_i$ is stand-alone utility, $w$ is a measure of the strength of network effects, and $n_{it}$ standard $i$’s network size at time $t$. We assume that $v_i \in \{0, 1\}$ and that $v_j = 1 - v_i$.

Following Arthur, we assume that, in each period, consumers make adoption decisions based on that period’s utility levels. Under this assumption, it can be shown that, if standard $i$ is chosen sufficiently more often than standard $j$, then all future adoptions are directed to $i$, even by consumers who, absent network effects, would prefer standard $j$ ($v_j > v_i$). Arthur et al. (1983) have shown that the above stochastic process of technology adoption ends up in one of these absorbing barriers in finite time with probability one. The specific condition for lock-in to standard $i$ is $n_i w > n_j w + 1$, or $\Delta_i > N \equiv \frac{1}{w}$, where $\Delta_i = n_i - n_j$ is the difference in installed bases. The values $-N, N$ are called absorbing barriers since, once crossed ($\Delta_i < -N, \Delta_i > N$), they are never crossed again. Arthur et al.’s result can then be rephrased as: lock-in to one technology occurs in finite time with probability one.

Consider now the following extension of Arthur’s model: suppose that consumers are unevenly distributed: a fraction $p > \frac{1}{2}$ prefers one of the standards. Since standards are otherwise symmetric, it follows that the standard with $p > \frac{1}{2}$ “fans” is the better standard. A crucial assumption in our analysis is that the policymaker knows the above information as well as the prior distribution of $p$, which we assume is

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3 Callaner (2003) develops a model similar to ours in a voting context to illustrate the formation of bandwagons.

4 Standardization, i.e., lock-in to one standard, is optimal in our model as it is in Arthur’s (1989), a result that depends on the assumption of a linear utility function. Farrell and Saloner (1986) and Bassanini and Dosi (1998) develop models where this assumption is relaxed and find that standardization need not be optimal.
symmetric around $\frac{1}{2}$. However, the policymaker does not know the exact value of $p$.

Consider now the problem faced by a welfare maximizing policymaker. Since marginal cost is constant and identical for both standards, a sufficient statistic for social welfare is discounted consumer surplus:

$$W = \sum_{t=1}^{\infty} \sum_{\tau=1}^{t} \delta^{\tau} u(\tau, t),$$

where $u(\tau, t)$ is period $t$ utility of the consumer who joined the network at time $\tau$, and $\delta$ the discount factor.

We will consider the following policy instrument: At a given point in time, the policymaker has the option of forcing the next $s_i$ adoptions of standard $i$. These “forced” adoptions can be interpreted in various ways. One is to assume the policymaker subsidizes adoption by private agents. An alternative interpretation is that the policymaker is itself a large adopter (see Section 6 for examples). As we will see, the direction of the optimal policy depends crucially on the policymaker’s discount factor. We will consider two extreme cases: a very impatient, or myopic, policymaker; and a very patient policymaker.

3 The case of an impatient policymaker

We start with the case of a very impatient policymaker. Our main result is that such a policymaker should favor the leading standard.

**Proposition 1**
If $\delta$ is close to zero, and given that policy intervention takes place in state $\Delta_i$, it is optimal to favor standard $i$ if $\Delta_i$ is sufficiently greater than zero.

**Proof**
If $\delta$ is close to zero, then all periods after the next are of second-order importance. The network benefits added to the current base of users are given by $wn_i$. The difference between the two standards is thus $w\Delta_i$. If $\Delta_i$ is sufficiently high, then the benefits on the existing users outweigh the benefits received by the new user, and the result follows. ■
An alternative version of the result is as follows. Suppose that the policymaker has the option of offering a subsidy to the new user at time \( t \). Then the subsidy to standard \( i \) is positive if and only if \( \Delta_i > 0 \).

This result corresponds to the “classical” case of an externality. Since the discount factor is close to zero, there is no informational issue; that is, the policymaker is not concerned with the value of \( p \) and how it will influence the expected pattern of future adoptions (beyond the next period). The main thing the policymaker is concerned with is how the next adopter will affect the previous adopters. If the \( i \) installed base is greater than the \( j \) installed base, then the externality is greater when an \( i \) adoption takes place, and thus the policymaker is better off subsidizing this standard.

4 The case of a patient policymaker

Consider now the opposite case with respect to the previous section, namely that of a very patient policymaker. From an optimization point of view, this is the rather more interesting case. The policymaker’s problem is that, while knowing that one of the standards is superior (higher \( p \)), it does not know which one is which. All that the policymaker knows is the prior distribution on \( p \), which we assume is symmetric around \( \frac{1}{2} \). In other words, the two standards look the same at the start of the process. Naturally, as the adoption process unfolds, the policymaker acquires more information, specifically, the number of adoptions of each standard.

Our main result is that the policymaker’s optimal policy is to favor the lagging standard:

**Proposition 2**

If \( \delta \) is close to one, and given that policy intervention takes place in state \( \Delta_i \), it is optimal to favor standard \( j \) by \( s^* = \frac{1}{2} \Delta_i \).

**Proof**

See the appendix.

This result states that the optimal intervention intensity is to pull the leading standard halfway back to the symmetric state (\( \Delta_i = 0 \)). Intuitively, moving the process halfway back takes into account the trade-off between keeping the process away from the absorbing
barriers for some more time (which implies supporting the lagging standard) and making use of the information gained from the process prior to intervention (which suggests that the leading standard is leading for a reason: it is more likely that it is indeed the right one). Notice that the result does not depend on the particular distribution of \( p \); the only restriction is that the distribution is symmetric, i.e., the two standards have a priori an equal chance of being the optimal standard.

Specifically, consider the extreme case of a binomial distribution and suppose that \( p \) is very close to 1. In other words, suppose that each of the standards is equally likely to be favored by a fraction \( p \) of the population, where \( p \) is close to one. Even then, the optimal policy would be to delay the lock-in process. This may at first seem counterintuitive: if so many adopters have chosen standard \( i \) previously, then it is very likely that this is the right standard. But precisely because \( p \) is close to one and the policymaker is very patient, favoring the lagging standard is an optimal policy: in the (likely) event that the leading standard is the right standard, then favoring the lagging standard won’t do much harm; most likely, the leading standard will eventually win anyway.

Broadly speaking then, Proposition 2 seems consistent with David’s (1987) prescription that “one thing that public policy could do is to try to delay the market from committing.”

5 Optimal timing for public intervention

So far, we have addressed the question: given that the policymaker must make a decision at time \( t \), which standard should the policymaker favor? The natural next step is to ask when the policymaker should intervene. We will address a somewhat more specific question: given that the policymaker must choose a single time at which to intervene, what is the optimal time \( t^* \)?

The main result in this section states that a patient policymaker should wait, whereas an impatient one should act soon.

**Proposition 3**

*Suppose that the policymaker must choose a single time at which to intervene. If the discount factor \( \delta \) is close to one, then it is optimal to wait until \( \Delta_i = N \). If \( \delta \) is close to zero and \( \Delta_i \) is large, then it is optimal to intervene right away.*
Proof
See appendix. ■

The intuition for the impatient policymaker case is similar to that of Proposition 1. Regarding the patient policymaker case, the question might be asked: Why should a patient policymaker wait until $\Delta_i = N$? The answer is, the closer to $N$ we are the more information the policymaker has. Since $N$ is achieved in finite time with probability one, and the policymaker is infinitely patient, there is no cost of waiting. Waiting for longer than $\Delta_i = N$ does the policymaker no good: once we hit an absorbing barrier, no additional information is gained.

6 Examples

The results presented in the previous sections are as tentative as the model they are based on is stylized. Real world examples are far more complicated than simple models. Still, we believe the theoretical analysis allows us to make some qualitative points about public policy. In this section, we present a few examples of public policy in industries with strong network effects. These examples illustrate the structure and assumptions of our model.

6.1 Second generation mobile telephony

Second generation wireless standard setting provides an interesting testing ground for the economic theory of public policy. The US and the EU took very different approaches to the problem. Whereas in the US the Federal Communications Commission (FCC) followed a “hands-off” approach, in Europe the European Commission (EC) mandated a standard from very early on. Standard-setting in Europe was regarded as a success story, especially in the early stages of 2G technology: early diffusion was faster in European countries than in the

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US at roughly comparable prices, and roaming (i.e., using one’s cellphone outside the provider’s coverage area) was clearly better in Europe early on. As the technology matured however, these differences became less relevant. Currently, diffusion is at similar levels and roaming is virtually seamless in both markets. Finally, as third-generation technology enters the picture, it is interesting to note that the competing standards are both based on CDMA, the standard that survived the battle for supremacy in the policy-neutral US ground.

Our theoretical analysis (Propositions 1–3) suggests that a very patient policymaker should wait and favor the lagging technology before the market sets onto a particular standard; whereas a very impatient policymaker should favor the leading technology early on. Moreover, by continuity, Propositions 1 and 2 suggest that, for intermediate values of \( \delta \), the optimal government policy is not to favor any of the technologies. This result is strengthened if we consider additional sources of uncertainty (for example, uncertainty regarding payoff levels), or if we consider more than two types of adopters. In other words, the best policy may in fact be not to have a policy.

The contrasting approaches taken by the US and the EC suggest that either one of them made the wrong decision, or else that they started from different “utility” functions. The latter may be accounted by different perceived time horizons or different weights placed on early adopters.

6.2 Wide-body aircraft: DC-10 vs. B-747

Over a period of time during the mid-1970s, the US Air Force ordered about sixty military cargo and tanker aircraft. It was seen as a “no brainer” that the USAF would select the Boeing proposal on the grounds of the technical specifications of their planes. As it turned out, the McDonnell Douglas KC-10 was selected. The KC-10 is the military version of the DC-10 and shares many features with the latter. The Air Force decision thus had the effect of keeping the DC-10 program alive for a while longer. In the end, the indirect network effect

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created by the learning curve, as well as a series of DC-10 crashes in the late 1970s, led to a sharp decline in orders for the McDonnell Douglas plane; production was discontinued in 1980.\(^7\) This example illustrates how the policymaker can intervene as a “large” adopter.

### 6.3 Nuclear power reactors

By the late 1950s, there were about a dozen relevant technologies for nuclear power reactors. Of these, the main contenders were light water, heavy water, and gas graphite. None of the technologies was perceived as clearly superior, and early adoption figures indicated that consumers were divided in their preferences. Due to strong learning and network effects, experts predicted that one of the technologies would eventually dominate. One important event in the race was the US Navy’s decision to adopt the light water technology in their nuclear submarines. Eventually, when a market for civilian nuclear power emerged, the light water “absorbing barrier” had been crossed and the industry was locked-in to this technology. According to Cowan (1990), “light water is considered inferior to other technologies, yet it dominates the market for nuclear reactors.” This example thus illustrates, among other things, how suboptimal outcomes may take place in the standard setting process with public intervention.

### 6.4 Pest control technology

For a limited period of time, the US Department of Agriculture sponsored one of the alternative technologies for pest control: Integrated Pest Management (IPM). Individual farmers have little incentive to deviate from the common practice in the vicinity, which implies a network effect similar to the one we consider in our theoretical model. For this reason, while the government intervention was temporary, its effects were permanent: the industry got locked-in to IPM, which, according to Cowan and Gunby (1996), was the welfare maximizing outcome. This example illustrates that early public intervention may have a determinant effect even if limited in its extent and duration.

\(^7\) For more on McDonnell Douglas, see http://www.angelfire.com/dc/douglasjets.
6.5 Linux vs. Windows

Recently, several government agencies in the US, Europe and Asia have decided to adopt the Linux operating system at various levels. Acting as large and influential customers, governments may support the lagging technology with the aim of delaying the outcome of the standards battle, or simply to lower the dependency on single software vendors.

7 Conclusion

The above examples illustrate the variety of situations where standards battles take place and government intervention is a possibility. Sometimes the policymaker acts by law, sometimes by offering adoption incentives, sometimes by acting as a lead adopter. Notwithstanding the specificities of each situation, one thing is common to all cases: the policymaker faces the dilemma of which standard to favor, if any, and when. We thus think that our model, stylized as it is, addresses an important set of public policy issues.

Appendix

Proof of Proposition 2: We begin by assuming that the prior distribution of $p$ takes two values, and later generalize to the case of a symmetric distribution. In other words, the policymaker knows that one of the standards is preferred by a majority $p > \frac{1}{2}$ of the population, knows the value of $p$, but does not know which standard is which.

Suppose that at time $t$ the system is in state $\Delta_t > 0$. Let $P(\Delta_t)$ be the probability that the system will eventually get locked-in to $i$. Let $\Pi(\Delta_t)$ be the probability that standard $i$ is the right standard, that is, the standard associated to $p > \frac{1}{2}$. Then the unconditional probability that the system gets locked-in to the right standard is simply

$$\pi(\Delta_t) = \Pi(\Delta_t)P(\Delta_t) + \Pi(-\Delta_t)P(-\Delta_t). \quad (1)$$

8 The list includes: in the US, the Air Force, the Federal Aviation Administration, the Postal Service, and the Departments of Defense, Agriculture, and Energy; in Europe, the European Commission, various government offices in Germany, and France’s Ministries of Culture, Defense, and Education; and China’s Post Office. Sources: http://www.ZDNet.com on 04/06/2002; http://www.usatoday.com on 30/03/2002.
The policymaker’s goal is to maximize the probability that the right absorbing barrier is hit. Absent any intervention, that probability is given by \( p(D_i) \). We assume that the policymaker has the option of starting at time \( t \) and over a period of time forcing adoptions in favor of one of the standards. Define \( \hat{p}(D_i, s) \) as the probability that the right standard is chosen given that, starting in state \( D_i \), the next \( s \) adopters are forced to adopt standard \( i \).

**Derivation of \( P(D_i) \) and \( \Pi(D_i) \)**

Given the stationarity of the process, the probability that the system in state \( D_i \) will get locked-in to \( A \) satisfies the difference equation

\[
P(D_i) = pP(D_i + 1) + (1 - p)P(D_i - 1).
\]

Let \( N \) be the necessary lead for one of the standards to lock in (so that the distance between barriers is \( 2N \)). The boundary conditions are then given by \( P(0) = 1 \) and \( P(2N) = 0 \). We thus get

\[
P(D_i) = p^{2N} - p^{N-D_i}(1 - p)^{N+D_i}.
\]

The probability that \( A \) is the right standard, \( \Pi(D_i) \), is defined by the probability that the current state is reached given that \( A \) is associated with \( p > \frac{1}{2} \). If there have been \( t \) adoptions and the probability of adopting \( A \) is \( p \), then the likelihood that state \( D_i \) is reached is

\[
mp^{\frac{t-D_i}{2}}(1 - p)^{\frac{t+D_i}{2}},
\]

where \( m \) is the number of possible combinations of “ups” and “downs” which lead to state \( D_i \). On the other hand, the probability of reaching state \( D_i \), given that \( A \) is associated with \( (1 - p) \) is given by

\[
mp^{\frac{t-A_i}{2}}(1 - p)^{\frac{t+A_i}{2}}.
\]

The posterior probability that \( A \) is associated with \( p \) is therefore given by

\[
\frac{mp^{\frac{t-A_i}{2}}(1 - p)^{\frac{t+D_i}{2}}}{mp^{\frac{t-A_i}{2}}(1 - p)^{\frac{t+D_i}{2}} + mp^{\frac{t+A_i}{2}}(1 - p)^{\frac{t+D_i}{2}}} = \frac{p^{A_i}}{p^{A_i} + (1 - p)^{A_i}}.
\]

Finally, substituting for \( P \) and \( \Pi \) in (1), we get the unconditional probability that the system, \( Y(t) \), will hit the right barrier:
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\[
\pi(\Delta) = \frac{p^\Delta (p^{2N} - p^{N-\Delta}(1-p)^{N+\Delta})}{(p^\Delta + (1-p)^\Delta)(p^{2N} - (1-p)^{2N})} + \frac{p^{-\Delta}(p^{2N} - p^{N+\Delta}(1-p)^{N-\Delta})}{(p^{-\Delta} + (1-p)^{-\Delta})(p^{2N} - (1-p)^{2N})}.
\]

**Optimal intervention for specific \( p \)**

A policymaker will maximize the probability that the right barrier is hit by forcing \( s \) adopters to adopt either the leading or the lagging standard. Let \( s > 0 \) denote forced adoptions of the lagging standard and \( s < 0 \) adoptions of the leading one. Note that \( s \neq 0 \) influences only \( P(\Delta_i) \). That is, the probability that a given barrier is the right one is not affected. We can now see that the new probability of achieving a desired outcome is

\[
\hat{\pi}(\Delta) = \frac{p^\Delta (p^{2N} - p^{N-\Delta+s}(1-p)^{N+\Delta-s})}{(p^\Delta + (1-p)^\Delta)(p^{2N} - (1-p)^{2N})} + \frac{p^{-\Delta}(p^{2N} - p^{N+\Delta-s}(1-p)^{N-\Delta+s})}{(p^{-\Delta} + (1-p)^{-\Delta})(p^{2N} - (1-p)^{2N})}.
\]

We now maximize \( \hat{\pi} \) with respect to \( s \):

\[
\frac{\partial \hat{\pi}(\Delta)}{\partial s} = -\frac{p^\Delta(-p^{N-\Delta+s} \ln(p)(1-p)^{N+\Delta-s} + p^{N-\Delta+s}(1-p)^{N+\Delta-s} \ln(1-p))}{(p^\Delta + (1-p)^\Delta)(p^{2N} - (1-p)^{2N})} + \frac{p^{-\Delta}(p^{N-\Delta+s} \ln(p)(1-p)^{N-\Delta+s} - p^{N+\Delta-s}(1-p)^{N-\Delta+s} \ln(1-p))}{(p^{-\Delta} + (1-p)^{-\Delta})(p^{2N} - (1-p)^{2N})},
\]

or simply

\[
\frac{\partial \hat{\pi}(\Delta)}{\partial s} = -\left(\psi(-\Delta)(p^\Delta + (1-p)^\Delta) + \psi(\Delta)(p^{-\Delta} + (1-p)^{-\Delta})\right) \cdot \frac{(\ln(1-p) - \ln(p))}{(p^{-\Delta} + (1-p)^{-\Delta})(p^\Delta + (1-p)^\Delta)(p^{2N} - (1-p)^{2N})},
\]

where

\( \psi(x) \) is the digamma function.
\[ \psi(\Delta_i) = p^{\Delta_i} p^{N-\Delta_i + s} (1 - p)^{N+s}. \]

The denominator is different from zero, and so is \( \ln(1 - p) - \ln(p) \) (for \( p \neq \frac{1}{2} \)). A necessary condition for maximizing \( \pi(\Delta_i, s) \) is therefore that
\[
\psi(-\Delta_i) (p^{\Delta_i} + (1 - p)^{\Delta_i}) + \psi(\Delta_i) (p^{-\Delta_i} + (1 - p)^{-\Delta_i}) = 0.
\]
which implies \( s^* = -\Delta_i/2 \). We take the second derivative to determine whether \( s^* \) is a maximum:
\[
\frac{\partial^2 \hat{\pi}(\Delta_i)}{\partial s^2} = -\left( \psi(-\Delta_i) (p^{\Delta_i} + (1 - p)^{\Delta_i}) + \psi(\Delta_i) (p^{-\Delta_i} + (1 - p)^{-\Delta_i}) \right) \cdot \frac{(\ln(1 - p) - \ln(p))^2}{(p^{-\Delta_i} + (1 - p)^{-\Delta_i})(p^{\Delta_i} + (1 - p)^{\Delta_i})(p^{2N} - (1 - p)^{2N})}.
\]
Observe that all of the bracketed expressions on the right-hand side are positive. It follows then that the second derivative is negative and \( s^* \) is a global maximum.

\section*{Generalization to any symmetric distribution}

The above results readily generalize to any symmetric distribution of \( p \). The idea is that any distribution symmetric about \( \frac{1}{2} \) is the integral of a series of binomial distributions like the one we considered above. Since the optimal solution does not depend on \( p \), it follows that the same solution holds for any distribution of \( p \) that is symmetric around \( \frac{1}{2} \).

\section*{Proof of Proposition 3}

The case when \( \delta \) is close to zero is straightforward: anything that takes place after the current period is of second-order importance; and an intervention in the current period has a positive effect on welfare.

When \( \delta \) is close to one, discounting is irrelevant (or close to irrelevant). We need to find out in which period the impact of public policy is greatest. Define \( \rho \equiv \frac{1-p}{p} \). Notice that, given our assumption that \( p > \frac{1}{2} \), it follows that \( 0 < \rho < 1 \). With this change in variables, we can simplify various previous expressions as follows:
\[
\Pi(\Delta_i) = \frac{1}{1 + \rho^{\Delta_i}}.
\]
\[
P(i) = \frac{1 - \rho^{N+\Delta_i}}{1 - \rho^{2N}}.
\]

Substituting in the expression for \(\pi(\Delta_i)\) and simplifying, we get
\[
\Phi \equiv (1 - \rho^{2N})\pi(\Delta_i) = 1 - \rho^N.
\] (2)

Moreover, since by Proposition 2 we have \(s^* = \frac{1}{2}\Delta_i\), it follows that
\[
P(\Delta_i + s^*) = \frac{1 - \rho^{N+\frac{1}{2}\Delta_i}}{1 - \rho^{2N}}.
\]

Substituting in the expression of \(\hat{\pi}(\Delta, s)\) and simplifying, we get
\[
\Psi \equiv (1 - \rho^{2N})\hat{\pi}(\Delta_i) = 1 + \rho^{N-\frac{1}{2}\Delta_i} \left(\frac{\rho^{N-\Delta_i} - \rho^N}{\rho^{N-\Delta_i} + \rho^N} - 1\right).
\] (3)

Comparing (2) and (3), we conclude that: (a) expected value without policy is independent of \(\Delta_i\); (b) expected value with policy is increasing in \(\Delta_i\). We conclude that the expected incremental value from implementing the optimal policy is increasing in \(\Delta_i\).

References
David, Paul 1986. “Narrow windows, blind giants, and angry orphans: The dynamics of systems rivalries and dilemmas of technology policy,”