The Rise and Fall of Reputation

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Abstract

I present an adverse selection model of seller reputation which, despite its symmetric primitives, produces asymmetric cycles: on average, it takes longer for reputation to increase than to decrease. The reason is that a seller with lower reputation not only sells for a lower price but also sells less frequently.
1 Introduction

It takes 20 years to build a reputation and five minutes to ruin it.
—Warren Buffett.

Conventional wisdom has it that reputation takes a long time to build up but is destroyed rapidly. Economic models of reputation can explain this fact by appropriately choosing the player’s type space or the player’s strategies. For example, Phelan (2006) proposes a model where governments accumulate trust gradually and, when it reaches a high level, destroy it to their gain.

In this note, I present an adverse selection model of seller reputation with two characteristics: First, it is a pure adverse selection model, that is, the seller takes no strategic actions. Second, the type-space primitives are symmetric, that is, the probability that a bad seller offers a good product is equal to the probability that a good seller offers a bad one. Notwithstanding these characteristics, I show that the model produces asymmetric equilibrium reputation cycles. The key intuition is that a seller with lower reputation not only sells for a lower price but also sells less frequently. This implies that, starting from a low level, reputation is updated at a slower rate than starting from a high level.

The closest paper to this note is Veldkamp (2005). She considers a model where there are a number of borrowers and lenders. The element of uncertainty is the state of the economy, which may be good or bad. Veldkamp shows that, the higher the market belief that the economy is in a good state, the greater the number of loans, and a result the faster the Bayesian updating regarding the state of the economy.

2 Model

I consider a model of pure adverse selection with changing types (see Mailath and Samuelson, 1998). A seller’s type, $\tau$, is given by the probability that his products work well. Specifically, I assume that there are two seller types, corresponding to $\tau = \alpha$ and $\tau = 1 - \alpha$; and without further loss of generality, I assume $\alpha > \frac{1}{2}$. I thus have a “good” seller (type $\alpha$) and a “bad” seller (type $1 - \alpha$). The seller only has one unit for sale in each period, which he values at zero. At the end of each period the type changes with probability $\lambda$ (which I assume
is less than \( \frac{1}{2} \)). The values \( \alpha \) and \( \lambda \) are common knowledge. The seller’s type, however, is unknown to consumers.

Consumers have a valuation one for a product that works and zero for a product that does not work. Consumers are risk neutral and hold a belief \( \mu \) that the seller is of a good type. It follows that willingness to pay is given by

\[
v = \mu \alpha + (1 - \mu) (1 - \alpha)
\]

In each period, two consumers are born. They must simultaneously decide (a) whether to bid for the seller’s product and (b) how much to bid. Making a bid costs \( c \). Finally, consumers only live for one period.

The timing within each period \( t \) is as follows. First, buyers form a belief about the seller’s type during that period, \( \mu_t \). Next buyers make purchase decisions. Based on the (publicly observed) outcome of the sale (if any), buyers update the seller’s reputation to \( \mu'_t \). Finally, Nature decides if the seller’s type changes into next period.

A Bayesian equilibrium consists of an optimal bidding decision (whether to bid and, if so, how much to bid) given beliefs \( \mu \); and a belief \( \mu \) that is consistent with history and Bayes rule. A symmetric Bayesian equilibrium is a Bayesian equilibrium such that all consumers play the same strategy. For the remainder of the paper, I will focus on symmetric Bayesian equilibria.

3 Equilibrium dynamics

The state of the dynamic system is given by the seller’s reputation, \( \mu \). My results characterize the evolution of \( \mu_t \).

**Definition 1** Let \( \Delta(\mu; \tau) \) be the expected motion of reputation (expected at the beginning of the period) given that its current level is \( \mu \) and seller type is \( \tau \); that is

\[
\Delta(\mu; \tau) \equiv E(\mu_{t+1} | \mu_t, \tau) - \mu_t
\]

The central result in this note is that the stochastic process of \( \mu_t \) is asymmetric. Specifically, the upwards expected motion of a good type’s reputation is lower, starting from a low level, then the downwards expected motion of a bad type, starting from a high level:

**Proposition 1** If \( \mu < \frac{1}{2} \), then \( \Delta(\mu; \alpha) > 0 \), \( \Delta(1 - \mu; 1 - \alpha) < 0 \), and

\[
|\Delta(1 - \mu; 1 - \alpha)| > |\Delta(\mu; \alpha)| \tag{1}
\]
**Proof:** First notice that

\[
\mu_{t+1} = \lambda \mu'_t + (1 - \lambda) (1 - \mu'_t)
\]

(2)

that is, given the post-trade, interim posterior regarding the seller’s type, consumers update their beliefs based on the possibility of a Nature-induced type change.

Now consider the updating from \(\mu_t\) to \(\mu'_t\). If no sale takes place at \(t\), then no update on \(\mu\) takes place: \(\mu'_t = \mu_t\). It follows that

\[
E(\mu'_t - \mu_t) = P(\mu_t; \tau) Q(\mu_t; \tau)
\]

(3)

where \(P(\mu; \tau)\) is the probability that a sale takes place and \(Q(\mu; \tau)\) is the expected change \(\mu'_t - \mu_t\) given that a sale takes place.

Combining (2) and (3), we have

\[
\Delta(\mu; \tau) = \lambda (1 - 2 \mu) + (1 - 2 \lambda) P(\mu; \tau) Q(\mu; \tau)
\]

The first term on the right-hand side, \(\lambda (1 - 2 \mu)\), is symmetric in \(\mu\), that is, \(\lambda (1 - 2 \mu) = -\left(\lambda (1 - 2 (1 - \mu))\right)\). So asymmetry of \(\Delta(\mu; \tau)\) will boil down to the term \(P(\mu; \tau) Q(\mu; \tau)\). In the remainder of the proof, I will show that the values of \(Q(\mu; \tau)\) are symmetric, that is, \(Q(\mu; \alpha) = -Q(1 - \mu; 1 - \alpha)\). However, the values of \(P(\mu; \tau)\) are not symmetric, leading to the phenomenon of slow increase and rapid decrease in reputation.

\[\square\] **Step 1.** Suppose that a sale takes place when reputation is \(\mu < \frac{1}{2}\) and seller type is \(\alpha\). Two things may happen: the product works (probability \(\alpha\)) or the product does not work (probability \(1 - \alpha\)). In the first case, the new value of reputation, \(\mu^+\), is given by

\[
\mu^+ = \frac{\alpha \mu}{\alpha \mu + (1 - \alpha) (1 - \mu)}
\]

In the second case, we have

\[
\mu^- = \frac{(1 - \alpha) \mu}{(1 - \alpha) \mu + \alpha (1 - \mu)}
\]

It follows that the expected change in \(\mu\) is given by

\[
Q(\mu; \alpha) = \alpha \mu^+ + (1 - \alpha) \mu^- - \mu
\]
Suppose now that reputation is $1 - \mu$ and seller type is $1 - \alpha$. We now have

$$
\mu^+ = \frac{\alpha (1 - \mu)}{\alpha (1 - \mu) + (1 - \alpha) \mu}
$$

$$
\mu^- = \frac{(1 - \alpha) (1 - \mu)}{(1 - \alpha) (1 - \mu) + \alpha \mu}
$$

It follows that the expected change in $\mu$ is given by

$$
Q(1 - \mu; 1 - \alpha) = (1 - \alpha) \mu^+ + \alpha \mu^- - (1 - \mu)
$$

Straightforward if tedious computation establishes that

$$
Q(\mu; \alpha) = -Q(1 - \mu; 1 - \alpha).
$$

**Step 2.** The seller has one unit available in each period and will sell it to the highest (positive) bid, if any. Since consumers only live for one period, I can analyze the bidding game as a static game. The symmetric equilibrium is for each consumer to make a bid with probability $\beta$ and, conditional on making a bid, to chose a value in the interval $[0, \bar{b}]$ according to the c.d.f. $F(b)$.

Expected value conditional on making a bid $b$ is given by

$$
((1 - \beta) + \beta F(b)) (v - b)
$$

Indifference between bidding and not bidding implies that this must equal the cost of bidding, $c$. Solving for $F(b)$ we get

$$
F(b) = \frac{1}{\beta} \left( \frac{c}{v - b} - (1 - \beta) \right)
$$

and $\bar{b}$ is given by $F(\bar{b}) = 1$, or simply $\bar{b} = v - c$.

Conditionally on making a bid, a consumer expects a surplus of $(1 - \beta) v$ (to see this, consider a bid of zero). The equilibrium value of $\beta$ is then given by the indifference condition between bidding and not bidding:

$$
(1 - \beta) v = c,
$$

which implies

$$
\beta = 1 - \frac{c}{v}.
$$

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Finally, the probability that a sale takes place is given by

$$\phi = 1 - (1 - \beta)^2 = 1 - \left( \frac{\gamma}{v} \right)^2.$$ 

Recalling that $v = \mu \alpha + (1 - \mu)(1 - \alpha)$ and that $\alpha > \frac{1}{2}$, the above implies that $\phi$ is increasing in $\mu$. Since $\mu < \frac{1}{2}$, we conclude that $P(\mu; \tau) < P(1 - \mu; 1 - \tau)$, and the result follows.

Figure 1 shows a sample paths for the level of reputation ($\mu_t$) assuming that $\alpha = .7$ and $\lambda = .004$ and considering the case when the seller’s type changes four times in 1,000 periods (which corresponds to the probability $\lambda = .004$. The dashed line represents the seller’s type, that is, the probability that the seller’s products work. Since $\alpha = .7$, it switches back and forth between $.3$ and $.7$: the seller starts off as a “bad” type and switches types at time 250, 500 and 750. The thin solid line represents the value of $\mu$ for a particular run of the model. As can be seen the path of $\mu$ is fairly noisy, partly because the signal observed by consumers (0 or 1) is also noisy.

Finally, the thick solid line represents the average of the $\mu_t$ paths over a series of 1000 simulations (all with the same evolution of seller type). This line best illustrates the thrust of Proposition 1: in expected terms, reputation increases more slowly than it decreases.

References


Figure 1: Reputation dynamics. The dashed line illustrates a possible evolution of seller type. The thin line illustrates a possible evolution of $\mu$ given the evolution of seller type. Finally, the thick line shows the average evolution of $\mu$ given the evolution of seller type.