Vertical Integration
and Right of First Refusal

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Abstract

We consider a partially integrated industry comprising a vertically integrated firm, independent downstream sellers, and an independent upstream supplier. We examine the effects of contracts with a right of first refusal, whereby the vertically integrated firm has the option to match a quote from the independent supplier to supply an independent downstream firm. We show that such contracts, also known as matching contracts, while reducing total input costs, also lead to higher prices, mainly because they soften the integrated firm’s downstream pricing incentives.

We apply our results to the Portuguese gasoline industry and show that the net effect of matching contracts on consumer surplus is negative but small.

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1 Introduction and summary

Many industries are characterized by partial vertical integration: a few firms are vertically integrated while others are not. For example, a common structure of the Iberian (Portugal and Spain) refining and retailing gasoline industry is the co-existence of vertically integrated national firms (which refine gasoline and sell it in the retail market) and independent firms which operate at the retail level only.

An additional characteristic of the Portuguese gasoline industry is the prevalence of contracts stipulating a right of first refusal, also known in this context as matching contracts. A matching contract gives the vertically integrated firm the right to supply the non-integrated independent firms’ input at a price equal to the best alternative price they can find elsewhere. Specifically, in the Portuguese industry the non-integrated downstream firms (Repsol and BP) can either (a) import gasoline from third parties located abroad, (b) import from their own upstream division abroad (e.g. Repsol in Spain), or (c) purchase gasoline from Galp (the vertically integrated firm). The matching contract establishes that Galp has the option to match the price that Repsol or BP would pay if they were to directly import the fuel to be sold at their retail outlets (Autoridade da Concorrência, 2009, page 180). Because Galp produces at a lower price than the import price (including transportation cost), the matching contract has “bite”: fuel imports are minimal, that is, Repsol and BP purchase most of their input from the vertically integrated firm (Autoridade da Concorrência, 2009, page 119).

Our goal is to analyze the competitive effects of matching contracts in a partially vertically integrated industry. We show that matching contracts have three different effects, two on price and one on costs. First, there is a competition softening effect: since the vertically integrated firm supplies the downstream non-integrated firms, it becomes less aggressive in the downstream market. In fact, losing market share is not so bad to the extent that it leads to increased input sales to independent downstream firms at a positive margin.

Second, since firms must commit ex-ante to a matching contract, there are cases when the independent downstream sellers purchase from the vertically integrated firms at a higher price than they would get if they were to ask the vertically integrated firm for a bid before revealing their best alternative. In other words, a matching contract eliminates the independent downstream suppliers’ information rents. Similarly to the first effect, this leads to higher
downstream prices.

Third, there is also an import substitution effect. To the extent that the vertically integrated firm is more efficient than the independent supplier, matching implies more efficient input supply, and this efficiency is captured by the (domestic) vertically integrated firm. That is, under matching, the independent firm observes no change in the price it pays for the input, but there is an additional rent that is transferred to the more efficient domestic producer.

After showing the above results under fairly general conditions, we specialize our framework to a Salop circular-city model. We calibrate the model and thus estimate the magnitude of the above effects in the Portuguese gasoline refining and retailing industry. Our results suggest that the import substitution (positive) effect more than compensates for the (negative) competition softening effect. Both effects are however relatively small (less than 1% of total sales).

**Related literature.** As mentioned above, a matching contract essentially entitles the vertically integrated firm to a Right of First Refusal (ROFR). Contracts with such a clause are common in the real estate and entertainment industries (see Lee, 2008, for additional examples). Burguet and Perry (2005, 2009) identify conditions under which a ROFR clause takes place in equilibrium, either because the seller offers it to a buyer or because the seller and a buyer jointly negotiate it (the idea is similar to, but different from, Aghion and Bolton, 1990). Lee (2008) and Grosskopf and Roth (2009) also examine variations of the ROFR clause. However, none of these papers looks at the effect of ROFR in the context of a vertically integrated firm selling to an independent downstream competitor.

Our paper is naturally related to the literature on vertical integration and vertical relations (see Rey and Tirole, 2006, for a survey). Much of this literature is concerned with the foreclosure or otherwise anti-competitive effects of vertical integration (Salop and Scheffman, 1987; Riordan and Salop, 1995; Nocke and White, 2007). Particularly germane to our paper is Chen (2001). While he does not consider ROFR clauses or matching contracts as we do, he too examines collusion and efficiency effects of partial vertical integration. In addition to the different focus, our analysis also differs in that it includes a simple calibration to evaluate the size of the various effects in a particular case.
Structure of the paper. The remainder of the paper is organized as follows. In Section 2, we lay down our general framework and show the main effects of matching contracts. In Section 3, we specialize this framework to the Salop circular-city model, deriving analytical expressions for the competition softening and import substitution effects. We use these expressions in Section 4, where we calibrate the model to data from the Portuguese gasoline industry. Finally, Section 5 concludes the paper.

2 General results

Consider an industry with two competitors, each with constant marginal cost $c_i$.\(^1\) Suppose firm $i$’s demand is given by $q_i(p)$, where $p$ is the price vector. We make the following assumptions regarding demand.

Assumption 1 (strategic complementarity) For all $i$ and $j \neq i$,

$$\frac{\partial^2 q_i(p)}{\partial p_i \partial p_j} > 0$$

Assumption 2 (market coverage) For all $p$,

$$\sum_{i=1}^{n} q_i(p) = S$$

We assume Firm 1 is vertically integrated: it produces its own input (e.g., gasoline) and sells it in retail outlets (e.g., gasoline stations). Firm 2, by contrast, is only active at the retail level. It must either import its input at a price $v$ or purchase it from Firm 1 at a price $w$. For simplicity we assume no cost of retailing beyond the cost of acquiring the required input.

We will compare two alternative regimes of organization of production and sales: one where there is a matching contract and one where there is no such matching contract. Under a matching contract, Firm 2 (credibly) reveals to Firm 1 the price $v$ at which it can obtain the input from a third party and Firm 1 has the option to match that price and supply Firm 2 its

\(^1\) The qualitative nature of our results extends to $n$ firms. In fact, the empirical application we consider in the next section considers the case when there are three firms (as is the case in Portugal). However, for the purpose of the present section, the analysis is greatly simplified by considering two firms only.
input. Under no matching rule, Firm 1 offers Firm 2 a wholesale price $w$, and Firm 2 then decides whether to purchase from Firm 1 at $w$ or from a third party at $v$.

We assume that, a priori, $v$ is distributed according to a commonly known continuous c.d.f. $F(v)$ with bounded density $f(v)$. Moreover, for simplicity we assume that $v \geq c_1$, that is, the vertically integrated firm is more competitive than the international market in terms of supplying the input (this may be due to transportation costs or other reasons). Finally, we also assume that $q_2(p) < s$ (uniformly). Stated as such, this assumption is without additional loss of generality with respect to Assumption 2. However, our result below considers the limiting case when $s \to 0$, which corresponds to Firm 2 being very small.

The question at hand is, what impact does a matching regime have on equilibrium prices? The next result provides an unequivocal answer to the question.

**Proposition 1** Under a matching contract, with probability $\rho > 0$ all equilibrium prices are uniformly higher than without a matching contract. Moreover, $\rho \to 1$ as $s \to 0$.

The proof of this and subsequent results may be found in the Appendix. Intuitively, the thrust of Proposition 1 is that, under a matching contract, the integrated firm’s downstream first-order condition is not a function of its cost, only of its rival’s. To the extent that the vertically integrated firm is more efficient than its rival, this leads to less aggressive pricing behavior by the integrated firm. Finally, strategic complementarity leads to less aggressive behavior all around.

The result is not straightforward because, with positive probability, a matching contract leads to lower equilibrium prices. Consider the case when there is no matching contract and suppose that $v$, the third-party input supplier price, is lower than, but close to, $w$, the price quoted by the vertically integrated firm in case there is no matching contract. As far as its own cost is concerned, Firm 2 is close to indifferent between the two suppliers. However, purchasing from Firm 1 has the advantage of “softening” Firm 1’s downstream pricing. If $v$ is sufficiently close to $w$, then it pays to purchase from Firm 1 at a higher price. Had Firm 2 signed a matching contract, it would have purchased from Firm 1 at price $v$. It follows that the only difference between a matching contract and no matching contract is that, in
the latter, Firm 2 pays a higher input price. By strategic complementarity, it follows that both prices are lower under a matching contract. The assumption that $s$ is close to zero, that is, that Firm 2 is very small, ensures that the probability that this happens is also close to zero.

Specifically, if $s$ is close to zero, then there are two possibilities: either $v < w$ or $v > w$. If $v < w$, then with or without a matching contract Firm 2 purchases its input at $v$. The main difference is that, with a matching contract its supplier is Firm 1, whereas without a matching contract its supplier is the third party. In this case, a matching contract leads to higher prices because it softens Firm 1’s downstream pricing.\(^2\) If $v > w$, then with or without a matching contract Firm 2 purchases its input from Firm 1. The main difference is that, with a matching contract, it pays $v$, whereas without a matching contract it pays only $w$. In this case, a matching contract leads to higher prices because it implies a higher input cost for one of the competitors.

Our next result pertains to the cost efficiency of a matching contract.

**Proposition 2** Total input cost is lower when there is a matching contract.

The proof of this result is fairly straightforward: Firm 1 is more efficient than the third-party supplier, and a matching contract implies that Firm 2 always purchases from Firm 1, whereas, absent such matching contract and with positive probability, Firm 2 purchases from the third-party input supplier. Intuitively, a matching contract eliminates the monopoly inefficiency present in Firm 1’s input pricing: although it is common knowledge that $v > c_1$, Firm 1 optimally sets $w > c_1$, thus creating a monopoly distortion.\(^3\)

### 3 A circular-city model

With a view at obtaining more precise results, we now specialize our general framework with a particular demand structure, namely the Salop circular

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2. This part of the argument is similar to the collusive effect identified in Chen (2001) in the context of vertical endogenous mergers. Our result differs in the particular context it is applied.

3. We should note the inequality of Proposition 2 is weak. There may be parameter values such that $w$ is so low (possibly even lower than $c_1$) such that Firm 2 always purchases from Firm 1. In this case, total input cost remains the same with or without a matching contract. A small value of $s$ is a sufficient condition to make the inequality of Proposition 2 strict.
city model. Consumers are uniformly distributed along the circle and have a linear transportation cost as they “travel” from their home to the seller they purchase from. There are \( n \) sellers, numbered from 1 to \( n \) and located sequentially clockwise in the circle. Seller 1 is located at point zero and the distance from seller \( i \) to seller \( i + 1 \) is \( l_i \). Seller \( i \) has constant marginal cost \( c_i \). As before, we assume \( c_i > c_1 \), \( \forall i > 1 \).

We consider an interior equilibrium where all sellers are active (i.e., prices are not very different) and all buyers make a purchase (i.e., the gross valuation is high). This implies that all buyers between sellers \( i \) and \( i + 1 \) purchase from one of these sellers. Let \( x_i \) be the distance between seller \( i \) and the buyer who is indifferent between sellers \( i \) and \( i + 1 \). We thus have

\[
p_i + t x_i = p_{i+1} + t (l_i - x_i)
\]
or simply

\[
x_i = \frac{l_i}{2} - \frac{p_i - p_{i+1}}{2 t} 
\]

(1)

Firm \( i \)'s demand is given by \( x_i \) consumers located clockwise from its location in addition to \( l_{i-1} - x_{i-1} \) consumers located counter-clockwise from its location:

\[
q_i = x_i + (l_{i-1} - x_{i-1})
\]

(2)

The above equations apply to \( 1 < i < n \). For \( i = 1 \), \( i - 1 \) should be replaced by \( n \); for \( i = n \), \( i + 1 \) should be replaced by \( 1 \). With this correction in mind, and substituting (1) for (2) we get

\[
q_i = \left( \frac{l_i}{2} - \frac{p_i - p_{i+1}}{2 t} \right) + \left( l_{i-1} - \frac{l_{i-1}}{2} + \frac{p_{i-1} - p_i}{2 t} \right)
\]

\[
= \frac{l_i + l_{i-1}}{2} + \frac{p_{i+1} - p_i}{2 t} + \frac{p_{i-1} - p_i}{2 t}
\]

We next turn to the input markets. To keep the analysis tractable, we consider the case when Firm 2 purchases from Firm 1 if and only if there is a matching contract; that is, we are always in the case when \( v < v' \) (where \( v' = w \) in the extreme when \( s \) is close to zero). This procedure may be justified in two ways. First, to the extent that \( v < v' \) is the case when a matching contract softens Firm 1’s downstream pricing, our choice provides an upper bound to the anticompetitive effect of a matching contract; and as we will see below this upper bound is fairly low. Second, there are
distributions \( F(v) \) that imply \( v < v' \) almost surely. Specifically, suppose that \( v = \bar{v} \) with probability \( \alpha \) and \( v = \underline{v} \) with probability \( 1 - \alpha \), where \( c_1 < \underline{v} < \bar{v} \). If \( \underline{v} \) is sufficiently close to \( c_1 \), then Firm 1’s optimal \( u \) is such that, under no matching contract, Firm 2 purchases from Firm 1 if and only if \( v = \underline{v} \). And if \( \underline{v} \) is sufficiently close to \( c_1 \) we can make the value of \( \alpha \) arbitrarily close to zero.

We now consider separately the cases of competition under no matching and matching contract and compare equilibrium prices in each case.

\( \square \) **Case 1: no matching contract.** Seller \( i \)'s profit function is given by

\[
\pi_i = (p_i - c_i) q_i = (p_i - c_i) \left( \frac{l_i + l_{i-1}}{2} + \frac{p_{i+1} - p_i}{2t} + \frac{p_{i-1} - p_i}{2t} \right)
\]

The first-order condition for profit maximization is given by

\[
\left( \frac{l_i + l_{i-1}}{2} + \frac{p_{i+1} - p_i}{2t} + \frac{p_{i-1} - p_i}{2t} \right) - (p_i - c_i) \left( \frac{1}{2t} + \frac{1}{2t} \right) = 0
\]

\( \square \) **Case 2: matching contract.** Seller \( i \)'s profit function, for \( i > 1 \), remains the same as before. Regarding firm 1, we have

\[
\pi_1 = (p_1 - c_1) q_1 + \sum_{i \neq 1} (c_i - c_1) q_i
\]

\[
= (p_1 - c_1) \left( \frac{l_1 + l_3}{2} + \frac{p_2 - p_1}{2t} + \frac{p_3 - p_1}{2t} \right) + \sum_{i \neq 1} (c_i - c_1) \left( \frac{l_i + l_{i-1}}{2} + \frac{p_{i+1} - p_i}{2t} + \frac{p_{i-1} - p_i}{2t} \right)
\]

Notice now that the last term of the previous profit function only depends on
when \( i = 2 \) or \( i = n \). We can thus re-write the profit function as follows.

\[
\begin{align*}
\pi_1 &= (p_1 - c_1) q_1 + \sum_{i \neq 1} (c_i - c_1) q_i \\
&= (p_1 - c_1) \left( \frac{l_1 + l_n}{2} + \frac{p_2 - p_1}{2t} + \frac{p_n - p_1}{2t} \right) + \\
&\quad + (c_n - c_1) \left( \frac{l_n + l_2}{2} + \frac{p_1 - p_n}{2t} + \frac{p_2 - p_n}{2t} \right) + \\
&\quad + (c_2 - c_1) \left( \frac{l_2 + l_1}{2} + \frac{p_n - p_2}{2t} + \frac{p_1 - p_2}{2t} \right) + \\
&\quad + \sum_{i \in \{1,2,n\}} (c_i - c_1) \left( \frac{l_i + l_{i-1}}{2} + \frac{p_{i+1} - p_i}{2t} + \frac{p_{i-1} - p_i}{2t} \right)
\end{align*}
\]

where the dependencies on \( p_1 \) are limited to the first three lines of the expression. Intuitively, a change in firm 1’s price only affects its own demand and that of its two neighbors. All other firms’ market shares remain unchanged by a change in \( p_1 \).

The first-order condition for firm 1’s profit maximization is then given by:

\[
\begin{align*}
&\left( \frac{l_1 + l_n}{2} + \frac{p_2 - p_1}{2t} + \frac{p_n - p_1}{2t} \right) - (p_1 - c_1) \frac{1}{t} + \\
&\quad + (c_n - c_1) \left( \frac{1}{2t} \right) + (c_2 - c_1) \left( \frac{1}{2t} \right) = 0,
\end{align*}
\]

Notice that, consistently with (7) in the proof of Proposition 1, firm 1’s best response does not depend on its cost, rather on its rivals’ costs. An additional feature of the above first-order condition — this one specific to the circular city model — is that that firm 1’s best response only depends on distances, costs and prices of its immediate rivals.

### 4 Calibration

We now proceed to calibrate the above Salop model with data from the Portuguese retail gasoline industry. We assume there is one vertically integrated firm and two non-vertically integrated retailers. In reality, there are more
than three retailers, but other than Galp, BP and Repsol they all have very small market shares (Autoridade da Concorrência, 2009, page 250).

In order to obtain tractable expressions, we now consider a special case when there are three equidistant firms: $n = 3$, $l_i = \frac{1}{3}$. As before, let firm 1 be the vertically integrated firm, with cost $c_1$, and let $c_2 = c_3 = c_0$. Substituting these values into the first-order conditions and solving, we get, under no matching,

$$p_1 = \frac{t}{3} + \frac{2}{5} c_0 + \frac{3}{5} c_1$$
$$p_2 = p_3 = \frac{t}{3} + \frac{1}{5} c_0 + \frac{4}{5} c_1$$

Under matching,

$$p_1 = p_2 = p_3 = \frac{t}{3} + c_0 \quad (3)$$

There are two variations that we are interested in. One is the price decrease attained by eliminating matching contracts (which is equivalent to unbundling). The second one is the cost savings (through import substitution) attained by matching contracts. Let $\bar{p}^M$ and $\bar{p}^N$ be average price with and without matching contracts, that is

$$\bar{p}^K \equiv \sum_{i=1}^{3} q_i^K p_i^K$$

$K = M, N$. To simplify notation and calibration, we define

$$\xi \equiv \frac{c_0 - c_1}{c_0}$$

that is, $\xi$ measures the domestic producer’s cost advantage with respect to imports. Computation then establishes that

$$\bar{p}^M = \frac{t}{3} + c_0$$
$$\bar{p}^N = \frac{25 t (t + c_0 (3 - \xi)) - 12 c_0^2 \xi^2}{75 t} \quad (4)$$
The measures we’re interested in are then given by

\[ \Delta P \equiv \frac{\bar{p}^M - \bar{p}^N}{\bar{p}^M} \]

\[ \Delta C \equiv \frac{(q_2 + q_3) (c_0 - c_1)}{\bar{p}^M} \]  \hspace{1cm} (5)

Proposition 1 implies that \( \Delta P \geq 0 \), whereas Proposition 2 implies that \( \Delta C \geq 0 \).

Substituting (4) into (5), we get

\[ \Delta P = \frac{\xi c_0 (25 t + 12 \xi c_0)}{25 t (t + 3 c_0)} \]

\[ \Delta C = \frac{2 \xi c_0}{t + 3 c_0} \]  \hspace{1cm} (6)

**Application to the Portuguese retail gasoline market.** In what follows, we consider values corresponding to regular gasoline.\(^4\) During August 2008, we estimate average consumer price to be 1.47 Euros/liter (the standard deviation across weeks and firms is approximately .01 Euros). Under a matching contract, the value of \( c_0 \) is given by the wholesale gasoline price paid by independents to the vertically integrated firm. In August of 2008, we estimate this price to be .50 Euros per liter (the standard deviation is less than .01 Euros) (Autoridade da Concorrência, 2009, page 184).

Equation (3) implies that

\[ t = 3 (p - c_0) = 2.91, \]

where we use the above estimates for \( p \) and \( c_0 \). Finally, the value of \( c_0 - c_1 \) corresponds to the vertically integrated firm’s refining margin. Galp Energia reports a refining margin of \$5.4/barrel (third quarter of 2008).\(^5\) A barrel is equivalent to 159 liters. Finally, we consider an exchange rate of \$US 1.47/Eur to get

\[ \xi = \frac{c_0 - c_1}{c_0} = \frac{5.4/159/1.47}{.5} = .0231 \]

\(^4\) We performed similar calculations for diesel oil and obtained very similar results.

\(^5\) Galp Energia’s Report for the first three quarter’s of 2008 lists a refining margin of \$5.4 per oil barrel (see p. 25).
Finally, substituting these values into (6), we get

\[ \Delta P = 0.26\% \]
\[ \Delta C = 0.52\% \]

**Discussion.** Our calibration is subject to a series of approximation errors. For example, we assumed symmetry in the locations of the three competitors, which implies equal market shares. Given the distribution of market shares in the Portuguese gasoline retail industry, this assumption overestimates the market share of the vertically separated retailers, and so overestimates the cost savings due to import substitution. A second approximation error results from assuming gasoline has the same value as other products derived from crude. To the extent that gasoline has a greater value, our estimate of the refining margin underestimates the true margin, which in turn underestimates the price and import substitution effects.

A simple “sanity” test of our calibration exercise is to derive the implied firm-level value of demand elasticity. This is given by

\[ \epsilon \equiv \frac{d q_i}{d p_i} \frac{p_i}{q_i} \approx 1.52 \]

which seems reasonable, though we have not been able to find estimates of firm-level demand elasticity for gasoline. Finally, we estimate \( t = 2.91 \). Since we normalize distance units so that the Salop circle has length 1, and since firms are equidistant from each other, it follows that the average distance traveled by a consumer is \( 1/12 \). We conclude that the average transportation effectively paid is \( 2.91/12 \approx 0.24 \) Euros, or 16.5% of retail price. This seems like a rather large number. One possible explanation is that we assume competitive behavior on the part of retailers. To the extent that there is some degree of tacit or explicit collusion, then this is picked up by the value of \( t \) (the only parameter that “explains” high price-cost margins).

5 Conclusion

The bottom line of our analysis is that matching contracts (a type of ROFR clause) have a negative unilateral effect (softening downstream competition) and a positive efficiency effect (welfare-increasing import substitution). Our
calibration exercise — approximate as it may be — suggests that the absolute size of these effects is modest: in terms of unilateral effects, matching contracts don’t seem to raise competition policy issues.

However, this is not the end of the story. First, we only considered unilateral effects. A tantalizing possibility, which we did not consider, is the role of matching contracts in increasing the degree of downstream collusion (see Mendi, 2005). Related to this, we might also consider the multimarket nature of the relations between firms. For example, in Portugal Repsol purchases gasoline from Galp, whereas in Spain Galp purchases gasoline from Repsol. Finally, as mentioned in the introduction, vertical integration may have a foreclosure effect, and it’s possible that matching contracts enhance that effect.
Appendix

**Proof of Proposition 1:** Since Firm 2’s payoff is decreasing in the price it pays Firm 1 for its input, under no matching rule there is a threshold value \(v'\) such that Firm 2 selects the third party if and only if \(v < v'\). In general, \(v'\) is different from \(w\): Firm 2 internalizes that, by purchasing from its downstream rival, market outcomes are different. In fact, as we will see below, strategic complementarity implies that purchasing from Firm 1 softens the latter as a price competitor, which is good for Firm 2. This implies that \(v' < w\). However, as \(s \to 0\), the strategic effect of purchasing from Firm 1 instead of a third party supplier becomes arbitrarily small, in which case \(v' \to w\). Moreover, given the properties of \(F(v)\), the probability that \(v \in [v', w]\) converges to zero as \(s \to 0\). In what follows, we assume that \(s = 0\) and \(v' = w\), the results then following by continuity.

If \(v < w\) (Case A), then Firm 2 purchases from the third party supplier if and only if no matching contracts exist. However, in both cases firm 2’s effective cost is given by \(v\). If \(v > w\) (Case B), however, then Firm 2 purchases from Firm 1 regardless of whether there is a matching contract or not. However, Firm 2’s effective cost is given by \(v\) if there is a matching contract and \(w\) otherwise. We next examine each case in turn.

**Case A:** \(v < w\). In this case, \(c_2 = v\) regardless of whether there is or there isn’t a matching contract. If there is a matching contract, and since \(v \geq c_1\), Firm 1 always exercises the option of matching the third party’s price. Firm 1’s profit function is therefore given by

\[
\pi_1 = (p_1 - c_1) q_1 + (c_2 - c_1) q_2 \\
= (p_1 - c_1) q_1 - (S - q_1) c_1 + c_2 q_2 \\
= p_1 q_1 - S c_1 + c_2 q_2
\]

where Assumption 2 is used in the second equality. The first-order condition for profit maximization is now given by

\[
q_1 + p_1 \frac{\partial q_1}{\partial p_1} + c_2 \frac{\partial q_2}{\partial p_1} = 0 \tag{7}
\]

Under no matching, by contrast, firm i’s profit function is given by

\[
\pi_i = (p_i - c_i) q_i
\]
The first-order condition for profit maximization is given by

\[ q_i + (p_i - c_i) \frac{\partial q_i}{\partial p_i} = 0 \]  \hspace{1cm} (8)

By Assumption 2, \( q_1 = S - q_2 \). Therefore,

\[ \frac{\partial q_1}{\partial p_1} = -\frac{\partial q_2}{\partial p_1} \]

It follows that the first-order condition (8) may be re-written for Firm 1 as

\[ q_1 + p_1 \frac{\partial q_1}{\partial p_1} + c_1 \frac{\partial q_2}{\partial p_1} = 0 \]  \hspace{1cm} (9)

Let \( f^z(p) = 0, \ z = N, M \) be the first-order condition under no matching \((z = N)\) and matching \((z = M)\). Comparing (7) to (9) and noting that \( c_1 < c_2 \), we conclude that \( f^M(p) > f^N(p) \). Assumption 1 and standard supermodularity results (e.g., Theorem 2.3 in Vives, 2000) then imply equilibrium prices are uniformly higher under matching.

\[ \square \text{ Case B: } v > w. \] In this case, first-order conditions are given by (8). The difference between no matching and matching contracts is that firm 2’s cost increases from \( w \) to \( v \). This implies an increase in firm 2’s best-response mapping (whereas firm 1’s remains unchanged). By the same argument as above, we conclude this leads to higher prices. \( \blacksquare \)

**Proof of Proposition 2:** Under a matching contract, Firm 2 always purchases from Firm 1. Under no matching contract, Firm 2 purchases from Firm 1 if and only if \( v > v' \). Finally, Firm 1’s cost is lower than the third party input supplier’s. \( \blacksquare \)
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