

Umbrella Branding with Imperfect Observability and Moral Hazard

Luís M B Cabral*

New York University

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Abstract

In a framework of repeated-purchase experience goods with seller's moral hazard and imperfect monitoring, umbrella branding may improve the terms of the implicit contract between seller and buyers, whereby the seller invests in quality and buyers pay a high price. In some cases, umbrella branding leads to a softer punishment of product failure, which increases the seller's value. In other cases, umbrella branding leads to a harsher punishment of product failure, which allows for a reputational equilibrium that would otherwise be impossible. On the negative side, under umbrella branding one bad signal may kill two revenue streams, not one. Combining costs and benefits, I determine the set of parameter values such that umbrella branding is an optimal strategy.

Keywords: branding, repeated games.

JEL Code Nos.: C7, L0, M3.

*Stern School of Business, 44 West 4th Street, New York, NY 10016, lcabral@stern.nyu.edu. I am grateful to Co-Editor Jay-Pil Choi, two referees, and seminar participants at Berkeley and Chicago for useful comments and suggestions. The useful disclaimer applies. This paper has had a long history: it first started as "Multimarket Contact Under Imperfect Observability, With an Application to Umbrella Branding" (1998) then moved on to "Optimal Brand Umbrella Size" (2007), before converging to the current title.

1 Introduction

Umbrella branding, the practice of labeling more than one product with a single brand name, is common practice among multiproduct firms in a variety of markets.¹ Examples include *Canon* cameras and photocopiers, *Colgate* toothpaste and toothbrushes, *Levi's* jeans and sneakers. To underscore the importance of the strategy of umbrella branding, Aaker and Keller (1990) quote a Nielsen report according to which “from 1977 to 1984, approximately 40% of the 120 to 175 new brands that were introduced into supermarkets annually were extensions” (p. 27).

In this paper, I propose a model that highlights the costs and benefits of umbrella branding. I consider a framework of repeated-purchase experience goods with seller's moral hazard. Firms have a short-run incentive to reduce quality and save costs, as consumers can only observe quality ex post. However, there exist equilibria whereby firms refrain from cheating consumers. In these equilibria, when consumers infer that the firm has cheated them, the firm's reputation breaks down, whereby consumers no longer pay a high price for the firm's product and the firm no longer produces quality products.²

I show that umbrella branding may improve the terms of the “implicit contract” between firm and consumers. This may happen for two reasons. First, umbrella branding may provide for a harsher punishment on cheating sellers. For example, suppose that a low-quality product fails with probability $1 - \beta$. If a firm sells two products under different names and consumers punish each product failure separately, then the probability that cheating is detected in a given product is given by $1 - \beta$. If however the firm sells both products under the same name and the firm is punished in both products following any product failure, then the probability of punishment given that the firm shirks in both products is given by $1 - \beta^2$, which is greater than $1 - \beta$.

A second reason why umbrella branding may improve equilibrium payoff is that, if there is slack in the no-deviation constraint, consumers may be more lenient with a firm that sells two products under the same name. Specifically, there exist parameter values such that the threat of punishing a firm only

1. The terms “brand extension” or “brand stretching” are also used. Some authors distinguish between “line extension” (when the new product is in the same class) and “brand extension,” when the new product is in a different class. Given the level of generality of the model considered in this paper, this distinction is not crucial.
2. See Klein and Leffler (1981), Shapiro (1983).

when two simultaneous product failures take place is sufficient to keep the firm from shirking; and lowering the probability of punishment increases equilibrium payoff.

Generically, if the value of the discount factor is very high then the optimal equilibrium is for the seller to umbrella brand and for consumers to punish the seller only if two simultaneous product failures take place. For lower values of the discount factor, no umbrella branding is optimal. For still lower values of the discount factor, the optimal equilibrium is for the seller to umbrella branding and for consumers to punish the seller for any product failure. Finally, if the discount factor is very low then the only equilibrium is the repetition of the static Nash equilibrium (where no quality products are sold).

The optimal equilibrium when the discount factor is very high has interesting implications. As mentioned above, consumers only lose trust in a brand if they observe two simultaneous product failures. Let α be the probability that a product works given that the seller put effort into it. In a single-product framework, an increase in α is unambiguously good for the seller: it increases the severity of the punishment for shirking and it decreases the probability of reverting to a punishment phase along the equilibrium path — both good things from the seller’s point of view. Under umbrella branding, however, the seller may be strictly worse off as the value of α increases. The reason for this seemingly unreasonable comparative statics is that a higher α improves the seller’s payoff from shirking in one product only. In the limit when $\alpha = 1$, the seller would have an incentive to always do so, in the certainty that consumers would only punish its brand following two simultaneous product failures.

While there is an extensive economics literature on umbrella branding, which I review next, my paper shows three novel features of umbrella branding. First, in favorable environments (where reputation equilibria exist even without umbrella branding), umbrella branding efficiently softens the punishment resulting from product failure. Second, in unfavorable environments (where reputation equilibria do not exist without umbrella branding), umbrella branding allows for harsher punishments even when the two products are symmetric (that is, even when the no-deviation constraint looks the same for both products individually). Finally, to the best of my knowledge this is the first paper to show that, in the context of umbrella branding, the seller may be strictly worse off when the the buyer’s monitoring technology

improves.³

■ **Related literature.** There is a fairly sizeable literature on brand extension and umbrella branding. One first explanation is that umbrella branding is a form of economies of scope, as it economizes on the costs of creating a new brand (e.g., Tauber, 1988). A related idea is that brands have an intrinsic value (status or otherwise). Brands are therefore like a “public good” in the sense that the more products are sold under the same brand the greater the total value created (see Pepall and Richards, 2002). A different perspective on brand extensions is that, in a world where consumers are uncertain about product characteristics (due to horizontal or vertical differentiation), brands may play an informational role.⁴ Still another possible role for umbrella branding is entry deterrence (see Choi and Scarpa, 1992).

More closely related to my paper, Choi (1998) proposes a moral-hazard theory of brand extensions. In a repeated-game framework, sellers introduce high-quality products and buyers pay a high price for them. Buyers can observe quality *ex post* and punish sellers who cheat by introducing low-quality products; the nature of the punishment is that future introductions are no longer believed to be of high quality as before. My paper differs from Choi (1998), among other things, in that it considers the costs, not only the benefits, from umbrella branding.

Andersson (2002), like myself, considers a repeated game setting. In his framework, umbrella branding improves a seller’s prospects for reasons similar to the benefits of multimarket contact (Bernheim and Whinston, 1990). Specifically, if the two products are asymmetric then we can construct an equilibrium where some of the slack in one no-deviation constraint is used to ease up the overall no-deviation constraint. By contrast, the reasons why umbrella branding improves a seller’s payoff do not depend on this type of asymmetries. In fact, I consider a symmetric setting and still find that umbrella branding improves equilibrium payoffs. The key difference is that, unlike Andersson (2002), I assume imperfect observability.

Recent work that is closely related to mine includes Cai and Obara (2006) and Hakenes and Peitz (2008a). Cai and Obara consider a repeated game

3. It is known that, in various Principal-Agent scenarios, the Principal may be worse off when its monitoring improves.

4. See Sappington and Wernerfelt (1985), Wernerfelt (1988), Aaker and Keller (1990), Sullivan (1990) and Erdem (1998), Cabral (2000), Hakenes and Peitz (2008b), Dana and Spier (2006), and Miklós-Thal (2008).

framework similar to mine. However, they assume that sellers make a once-and-for-all investment in product quality. Moreover, the integrated firm case, which is their equivalent to umbrella branding, assumes that quality decisions are the same across all products. They therefore address a different set of issues. In particular, one of the important issues in my model is precisely whether the binding constraint under umbrella branding is to shirk on one product or to shirk in both products. Hakenes and Peitz consider a firm that sells a product in each of two periods and must decide on product quality at the beginning of the first period. Again, my approach differs from theirs in that I consider repeated quality decisions.

2 Model and results

Consider a firm that that sells two products in an infinite number of periods. At the beginning of its life, the firm must decide how to brand its products (more on this below).⁵ Then, at the beginning of each period, the firm must decide whether to spend effort into producing quality products. If the firm puts effort into a given product, which costs ϵ per product and per period, then that product works (in that period) with probability α , whereas low effort (which costs zero) leads to a product that works with probability $\beta < \alpha$. The firm makes two effort decisions, one for each product. The outcome of these decisions (conditional on effort levels) is independent.

Consumers value a product that works at π/α , so expected value conditional on the firm exerting effort is π . The good in question is an experience good. Specifically, consumers observe ex-post whether a product performs well or rather breaks down. Product performance is batch specific, so all consumers observe the same performance outcome. I assume the market is long on the buyers' side, so consumers pay a price equal to their quality expectation.

Regarding parameter values, I make the following assumption:

Assumption 1 (cost of effort) $\epsilon < \frac{\pi}{\alpha - \beta}$.

Assumption 1 implies that the efficient solution is for the firm to produce high-quality products: the added value it creates more than compensates for the extra cost. And since the seller can extract the consumer's surplus,

5. I assume the firm commits forever to its branding strategy.

it also implies that the seller's optimal solution is to produce high-quality products — provided the incentives are right.

The firm's initial choice of how to brand its products, that is, how to name them, is important because consumers cannot observe the ownership of each brand:

Assumption 2 (brand ownership) *Consumers observe brand names but not the identity of each brand owner.*

A consequence of Assumption 2 is that, if a firm does not umbrella brand, then consumers consider the firm's products to be offered by separate firms.

I assume that, in the eyes of consumers, a firm's history is encapsulated in the value of its brand. I moreover assume that a firm's brand can take two values: trustworthy or not trustworthy. This assumption is motivated by the belief that consumers have finite capabilities to process information, and thus summarize a firm's history into a coarse measure consisting of a finite number of states.⁶ This assumption plays an important role in the results that follow. In particular, I will argue that one of the benefits of umbrella branding is to provide consumers with more signals about a firm's performance. However, to the extent that consumers have longer memories than I assume, history can to some extent substitute for multiple contemporaneous signals.

I consider a simple class of consumer strategies: in each period, consumers are willing to pay a price which is a function of the firm's brand value; and upon observation of the firm's quality level consumers update their belief regarding the brand's trustworthiness. Throughout most of the paper, I assume this updating is deterministic. Later in the paper I also consider the possibility of stochastic reputation updating.⁷

Finally, as often is the case when dealing with repeated games, there exists a plethora of Nash equilibrium. In particular, the repetition of the one-shot game Nash equilibrium (seller makes no effort, buyer pays accordingly) is itself an equilibrium of the repeated game. I will focus on optimal equilibria, that is, equilibria that maximize the seller's expected payoff. I will designate

6. See Monte (2006), Wilson and Ekmekci (2006).

7. My assumption that the state space only takes two values (that is, a firm's reputation is either good or bad) implies that consumers cannot play the equivalent of the Green and Porter (1984) type of strategies, whereby a firm or its reputation is punished for a finite number of periods. Such strategies would require a more complex state space.

such equilibrium as a quality provision equilibrium if the seller's payoff is higher than under the repetition of the static Nash equilibrium.

There are several reasons why one may be interested in optimal equilibria. From a positive point of view, we can think of evolutionary process whereby only the best firms survive. If the only difference between firms is the type of equilibrium they play, then only the firms playing the equilibrium I consider survive. Perhaps a better justification, in the spirit of Klein and Leffler (1981), is to think of a bigger game where firms enter and spend some amount on advertising, an expenditure that indicates the equilibrium the firm expects to be played out and serves as a bond that enforces such equilibrium. That is, the up-front advertising expenditure allows consumers to make the forward induction argument that selects the optimal equilibrium; and justifies the assumption that, if a firm's brand dies, then its continuation payoff is zero.⁸

■ **Perfect monitoring.** As a benchmark, consider the extreme case when $\alpha = 1$ and $\beta = 0$, that is, perfect monitoring: by observing quality, consumers can perfectly infer whether a firm has exerted effort. This case is well known from the literature. Two important results are that: (a) if the discount factor is sufficiently large, then there exist quality provision equilibria (Klein and Leffler, 1982); (b) under symmetry (as is the case with my model), there are no gains from umbrella branding that is, the optimal payoff under umbrella branding can be replicated under separate branding. The intuition for the second result (a result that is isomorphic to Proposition 1 in Berhneim and Whinston, 1990) is that, under umbrella branding, the punishment for a deviation is twice as high — but so is the benefit. In other words, the relevant no-deviation constraint under umbrella branding is identical to that under separate brands.

■ **No umbrella branding.** Let us now assume imperfect monitoring, beginning with the case when the firm sells its products under different brands. The optimal equilibrium for a single product has been characterized before. For completeness, I restate the result in the context and notation of my model:

8. In the context of collusion, the choice of the optimal equilibrium can also be justified by the assumption that firms communicate with each other. This type of argument carries less weight in the present context, as typically there is a very large number of consumers.

Proposition 1 (no umbrella branding) *If $\delta \geq \frac{\epsilon}{\pi(\alpha-\beta)+\beta\epsilon}$, then the following is an optimal equilibrium: (a) the seller chooses high-quality if and only if consumers trust its brand; (b) consumers purchase from the seller (at price π) if and only if they consider the brand to be trustworthy; (c) the brand is trustworthy if and only if past products sold under that brand have performed well.*

The proof of this and the following results may be found in the Appendix.

■ **Umbrella branding.** Consider now the possibility of umbrella branding. Specifically, at time of birth the seller must decide whether to sell both products under the same name, or rather under different names (and then, in each period, it must choose the quality level of each of its products). The consumers' strategy is now a little more complicated than before. In each period, the outcome can be one of the following: no failure, failure of one product, failure of both products. In each case, consumers must decide whether or not to lose trust in the brand. My main result is as follows:

Proposition 2 (optimal equilibrium) *For each set of values $\alpha, \beta, \epsilon, \pi$, there exist threshold values $0 < \delta_1 \leq \delta_2 \leq \delta_3 \leq 1$, such that*

- *If $\delta < \delta_1$, then no quality provision equilibrium exists, that is, the only equilibrium is the repetition of the static game equilibrium.*
- *If $\delta_1 < \delta < \delta_2$, then the optimal equilibrium is for the seller to umbrella brand and consumers to lose trust following any product failure.*
- *If $\delta_2 < \delta < \delta_3$, then the optimal equilibrium is for the seller not to umbrella brand.*
- *If $\delta > \delta_3$, then the optimal equilibrium is for the seller to umbrella brand and consumers to lose trust following two simultaneous product failures only.*

Moreover, there exist values ϵ_1, ϵ_2 such that

- *If $\epsilon < \epsilon_1$, then $\delta_1 < \delta_2 < \delta_3$*
- *If $\epsilon_1 < \epsilon < \epsilon_2$, then $\delta_1 = \delta_2 < \delta_3$*
- *If $\epsilon > \epsilon_2$, then $\delta_1 = \delta_2 = \delta_3$*

Figure 1 illustrates Proposition 2. Consider first the case when ϵ is small, say $\epsilon = \epsilon_0$, so that the gains from offering a quality equilibrium are particularly high. For this value of ϵ (and for the values of π, α and β considered in the figure), we obtain three thresholds for the value of the discount factor δ . If the discount factor is very low, specifically if $\delta < \delta_1$, then no equilibrium exists where the seller offers quality products, either with or without umbrella branding. If $\delta_1 < \delta < \delta_2$, then the optimal equilibrium is for the seller to umbrella brand and for consumers to lose trust following *any* product failure. For higher values of the discount factor, $\delta_2 < \delta < \delta_3$, the optimal equilibrium is for the seller not to umbrella brand. In this case, the equilibrium in each product (and each brand) follows Proposition 1. Finally, for very high values of the discount factor, $\delta > \delta_3$, the optimal equilibrium is for the seller to umbrella brand and for consumers to lose trust following two *simultaneous* product failures only.

More generally, the optimal solution is as follows: in region UB_2 , follow umbrella branding and the simultaneous failure policy; in region UB_{1+} , follow umbrella branding and the policy of punishing any failure; in region “no UB,” sell the two products under separate names. Finally, in the southeast region (low δ or high ϵ) the optimal solution is to play the repeated Nash equilibrium (no quality provision).

Region UB_2 corresponds to the situation when umbrella branding allows for softer punishments without damaging incentives, thus improving the overall equilibrium value. The idea is quite simple: two signals are better than one signal.⁹ Region UB_{1+} corresponds to the situation when umbrella branding allows for harsher punishments. If seller incentives are insufficient under no umbrella branding, then umbrella branding may lead to a better equilibrium. A similar idea appears in Hakenes and Peitz (2008a), though in a quite different setting.

Notice that the thresholds δ_i are only weakly increasing in i . It is quite possible that $\delta_1 = \delta_2$ or/and $\delta_2 = \delta_3$. In fact, if $\epsilon_1 < \epsilon < \epsilon_2$ then $\delta_1 = \delta_2$, so that the optimal equilibrium is either UB_2 or no umbrella branding. If $\epsilon_2 < \epsilon < \epsilon_3$, $\delta_1 = \delta_2 = \delta_3$, so that the optimal policy is either the static Nash equilibrium or UB_2 .

In order to understand the intuition for the main results, it helps to re-

9. Again, I stress the importance of the assumption that consumers have limited capacity to process information. Without this limitation, under no umbrella branding the seller can do better than what is considered above, and the benefits from umbrella branding are likely to be smaller.

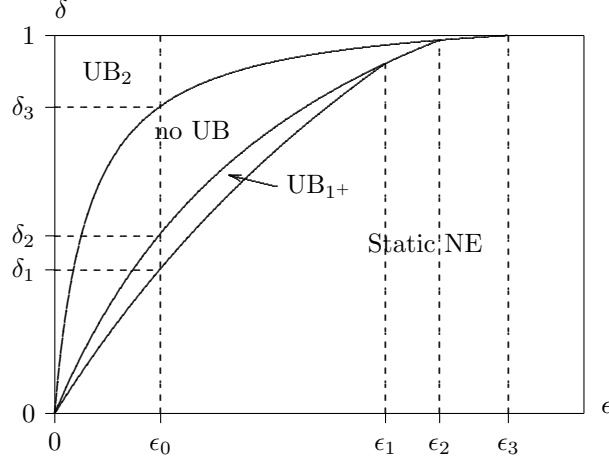


Figure 1: Optimal equilibrium as a function of cost of effort, ϵ , and the discount factor, δ (assuming $\pi = 1$, $\alpha = .9$, $\beta = .6$).

write the no deviation constraint as follows:

$$(\pi - \epsilon) + \delta \rho \frac{(\pi - \epsilon)}{1 - \rho \delta} \geq \pi + \delta \rho' \frac{(\pi - \epsilon)}{1 - \rho' \delta}, \quad (1)$$

where ρ is the continuation probability along the equilibrium path and ρ' the continuation probability following a deviation. Defining $\xi \equiv \frac{\epsilon}{\pi - \epsilon}$, (1) is equivalent to

$$\delta \geq \frac{\xi}{(\rho - \rho') + \rho \xi}$$

This condition shows that there are two factors that determine the lowest value of δ such that a quality equilibrium exists: the continuation probability, ρ , and the difference in continuation probabilities between the equilibrium action and shirking, $\rho - \rho'$. The higher either of these is (everything else constant), the lower the lower bound on the value of δ . Moreover, the higher ξ (that is, the lower the efficiency gains from quality provision), the greater the relative importance of ρ vis-a-vis $\rho - \rho'$.

Under no umbrella branding, $\rho = \alpha$ and $\rho' = \beta$. We thus have

$$\delta \geq \frac{\xi}{(\alpha - \beta) + \alpha \xi}$$

Consider now the case of umbrella branding and the equilibrium whereby loss of trust follows any product failure occurrence. In the appendix I show that the binding constraint corresponds to offering low quality in both products. The no-deviation constraint is then given by

$$2(\pi - \epsilon) + \delta \rho \frac{2(\pi - \epsilon)}{1 - \rho \delta} \geq 2\pi + \delta \rho' \frac{2(\pi - \epsilon)}{1 - \rho \delta},$$

Dividing through by two, we get an expression identical to (1). The difference between no umbrella branding and the form of umbrella branding we are considering resides in the values of the continuation probabilities ρ , ρ' . Specifically, under the umbrella branding equilibrium we are considering, we have $\rho = \alpha^2$ and $\rho' = \beta^2$. The condition on δ is therefore

$$\delta \geq \frac{\xi}{(\alpha^2 - \beta^2) + \alpha^2 \xi}$$

Notice that the continuation probability, ρ , under no umbrella branding, $\rho = \alpha$, is higher than the continuation probability under umbrella branding, $\rho = \alpha^2$. Moreover, the differential in continuation probabilities, $\rho' - \rho$, is given by $\rho' - \rho = \alpha - \beta$ under no umbrella branding and $\rho' - \rho = \alpha^2 - \beta^2$ under umbrella branding. If $\alpha + \beta < 1$, then $\alpha - \beta > \alpha^2 - \beta^2$. We thus conclude that, if $\alpha + \beta < 1$, then the quality provision set is greater under no-umbrella branding (with respect to umbrella branding and punishing any deviation). Intuitively, punishing any deviation lowers the equilibrium value (lower continuation probability) with respect to no-umbrella branding. Formally, we have $\rho = \alpha^2$ instead of $\rho = \alpha$. Moreover, if $\alpha + \beta < 1$, then punishing only one deviation increases the incentives for shirking.

If $\alpha + \beta > 1$, then we have two counteracting effects: equilibrium value is greater under no umbrella branding; but the decline in continuation probabilities is greater under umbrella branding (harsher punishment). In this case, no umbrella branding is better only if ξ is high enough, specifically, if

$$\xi \equiv \frac{\epsilon}{\pi - \epsilon} > \xi_1 = \frac{(\alpha - \beta)(\alpha + \beta - 1)}{(1 - \alpha)\alpha}$$

A similar intuition applies to the comparison between no umbrella branding and the policy of umbrella branding and the equilibrium where only two simultaneous product failures lead to the loss of reputation. In this case, the continuation probability favors the umbrella branding equilibrium. In

fact, $\rho = 1 - (1 - \alpha)^2$, which is greater than α . However, the differential in continuation probabilities favors no umbrella branding. As before, if ξ is sufficiently high then the first effect dominates, and umbrella branding leads to a higher set of values of the discount factor.

So far I have only considered the intuition for the relative size of the sets of the discount factor such that a quality equilibrium exists. Going from here to optimal equilibria is easy. In fact, the continuation probability is highest under umbrella branding and punishment of two simultaneous product failures, and lowest under umbrella branding and punishment of any product failure. It follows that, whenever more than one solution is feasible, umbrella branding and punishment of two simultaneous product failures is preferable to both umbrella branding and punishment of any product failure and no umbrella branding; and no umbrella branding is preferable to umbrella branding and punishment of any product failure.

■ **Type I and type II errors.**¹⁰ A different way of presenting the above results is in terms of the hazard rates α and β , which are related to type I and type II errors. (If we think of shirking as the event in question, then the probability of a type I error is given by $1 - \alpha$, whereas the probability of a type II error is given by β .)

Figure 2 slices the parameter space in terms of the values of α and β . As in Figure 1, we obtain four regions: static Nash equilibrium, reputation equilibria without umbrella branding (no UB), umbrella branding with punishment of any product failure (UB₁₊), and umbrella branding with punishment of two simultaneous product failures (UB₂).

Regarding the value of β , the comparative statics yield the expected results. As the value of β decreases (lower type II error), the equilibrium moves from UB₁₊ to no UB, and from no UB to UB₂. In both cases, the decrease in β is associated with a higher equilibrium payoff.

Surprisingly, the same is not necessarily true with respect to α . As the value of α increases (lower type I error), equilibrium payoff *may* decrease. This is illustrated in Figure 2. Suppose $\beta = \beta_1$ and consider two possible values of α , α_1 and α_2 . The first point falls in the UB₂ region, whereas the second one falls in the no UB region. If the values of α_1 and α_2 are sufficiently close to each other, then equilibrium payoff in the second point

¹⁰. I am grateful to Jay-Pil Choi for suggesting this alternative approach to the results on optimal umbrella branding.

is strictly lower than in the first point. Recall that, under UB_2 , punishment only occurs under *simultaneous* product failures, which is clearly better than no umbrella branding.

In order to understand this counterintuitive comparative statics, notice that, under UB_2 , the binding deviation constraint is shirking in one product only (see the proof of Proposition 2). But if α is very close to 1, then there is virtually no punishment from shirking in one product, so that, as much as equilibrium payoff under UB_2 increases, UB_2 is no longer an equilibrium. More generally, I can show the following non-monotonicity result:

Proposition 3 (non-monotonicity) *Suppose there exist values $\alpha, \beta, \epsilon, \delta$ such that UB_2 is the optimal equilibrium. Then there exist values $\alpha_1 < \alpha_2$ such that, under the same β, ϵ, δ values, optimal payoff is lower when the value of α is higher.*

Proposition 3 has the interesting implication that the seller’s payoff may increase by moving to a noisier process of quality monitoring. The reason for this apparently inconsistent result is related to the topsy-turvy principle of repeated games: the lower payoff that we can impose on players who deviate from the equilibrium path, the greater the attainable equilibrium payoff. A decrease in α has the effect of increasing the probability of a switch to punishment conditional on shirking in one product (the relevant deviation). This effect is so great that it more than compensates for the lower probability of staying on the favorable portion of the equilibrium path.

■ **Stochastic reputation updating.** Throughout the paper, I have assumed that consumers follow a deterministic reputation procedure. Alternatively, I could consider the possibility of a random transition process. Specifically, suppose that, following one single product failure, consumers maintain trust in the brand (i.e., “forgive” the product failure) with probability θ_1 ; whereas two simultaneous product failures lead to reputation maintenance of trust with probability θ_2 . The situation I considered in the previous sections thus corresponds to the extreme cases when $\theta_1 = 0$ and $\theta_2 = 0$ (what I called UB_{1+}), or $\theta_1 = 1$ and $\theta_2 = 0$ (what I called UB_2). What if we allow for any values $\theta_i \in [0, 1]$? The following result provides the answer:

Proposition 4 (stochastic reputation update) *For each set of values α, β, π , there exist threshold values ϵ_1 and $0 < \delta_1 \leq \delta_2 \leq 1$, such that*

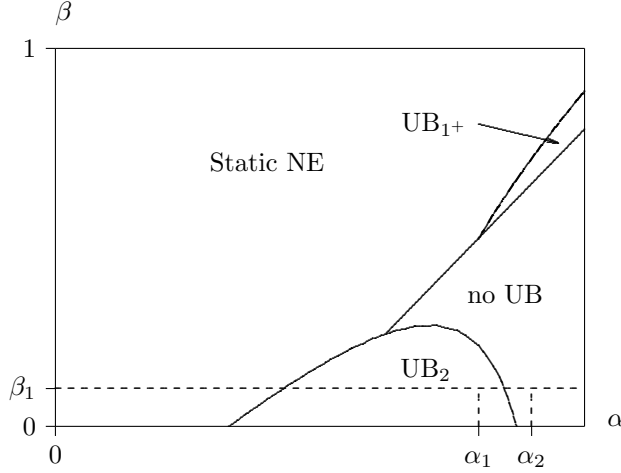


Figure 2: Optimal equilibrium as a function of the success probabilities α , β (assuming $\pi = 1$, $\epsilon = .5$, $\delta = .7$).

- If $\delta < \delta_1$, then no quality provision equilibrium exists, that is, the only equilibrium is the repetition of the static game equilibrium.
- If $\delta_1 < \delta < \delta_2$, then the optimal equilibrium corresponds to $0 < \theta_1 < 1$ and $\theta_2 = 0$ if $\epsilon < \epsilon_1$; and no umbrella branding if $\epsilon > \epsilon_1$.
- If $\delta > \delta_2$, then the optimal equilibrium corresponds to $\theta_1 = 1$ and $0 < \theta_2 < 1$.

Figure 3 illustrates Proposition 4. It can be shown that the optimal solution is to choose the highest values of θ consistent with the no deviation constraints. Specifically, for very low values of δ , even if $\theta_1 = \theta_2 = 1$ no equilibrium exists other than the repetition of the static Nash equilibrium. If $\delta = \delta_1$ (where δ_1 is derived in Proposition 2 and illustrated in Figure 1), setting $\theta_1 = \theta_2 = 0$ exactly satisfies the no-deviation constraint. As the value of δ increases, the optimal value of θ_1 is decreased, keeping $\theta_2 = 0$. For $\delta = \delta_2$ (where δ_2 corresponds to the δ_3 derived in Proposition 2 and illustrated in Figure 1), $\theta_1 = 1$ and $\theta_2 = 0$ exactly satisfies the no-deviation constraint. Finally, for higher values of δ , that is, $\delta > \delta_2$, the optimal solution consist of setting $\theta_1 = 1$ and θ_2 to the highest value consistent with the no-deviation constraint (of shirking in one product only).

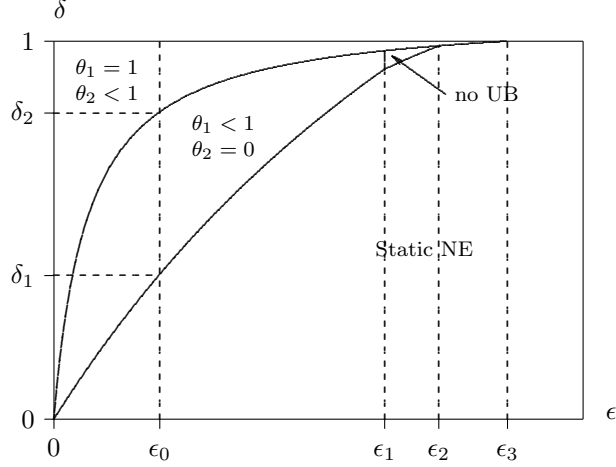


Figure 3: Optimal equilibrium as a function of cost of effort, ϵ , and the discount factor, δ (assuming $\pi = 1$, $\alpha = .9$, $\beta = .6$).

Compared to the situation when consumers play pure strategies, the stochastic reputation updating case implies higher payoffs for the firm. As a result, umbrella branding becomes relatively more attractive. This is reflected in the fact that the no-umbrella-branding region is much smaller in Figure 3 than in Figure 1. The reason why umbrella branding becomes a more attractive option is that it allows consumers to fine-tune the punishment to the lowest punishment required to keep the firm's incentives in check.

The nature of the stochastic punishment is itself interesting. The idea is to concentrate punishment as much as possible in the event of two simultaneous product failures. So, for very high δ , consumers entirely forgive firms for isolated product failures; only when $\theta_2 = 0$ is insufficient to keep the seller from deviating do consumers switch to punish the seller with positive probability in case of a single failure. Intuitively, the idea is that, if $1 - \alpha$ is relatively small, then conditional on the seller offering quality in both products, the probability of two simultaneous product failures is very small, specifically of order $(1 - \alpha)^2$. However, condition on one single deviation, the probability of two product failures becomes $(1 - \beta)(1 - \alpha)$. If β is considerably lower than α then this is a very large number. In the limit when α is close to 1 and β close to 0 (good monitoring) then we go from a second-order probability to a first-order probability.

If the cost of effort, ϵ is very high (or, equivalently, if the difference $\alpha - \beta$

is very small), then no umbrella branding may be a better strategy. The idea is that, because of the unfavorable monitoring technology, punishment does take place along the equilibrium path with a relatively high probability. And when that happens in the region when $\theta_1 < 1$, reputation breakdown extends to two products, whereas no umbrella branding allows the seller to protect the reputation of the product that did not fail.

Finally, I note that in many applications it may not be realistic to assume consumers play a random strategy. However, an alternative interpretation of consumer random strategies is that *each* consumer loses trust with probability θ_i . To the extent that firms are risk neutral and there is a continuum of consumers, the firm's value is the same when all consumers leave the firm with probability θ or a fraction θ leaves with probability 1.¹¹

3 Concluding remarks

Throughout the paper, I assumed that, conditional on effort, the quality outcomes of a firm's products are independent. Notice that all of the results I have developed are continuous and based on strict inequalities. For this reason, the results are not knife-edged: a small measure of correlation among the products would not change the qualitative nature of the results. Now consider the opposite extreme to what I have considered, that is, suppose that quality outcomes are perfectly correlated. Then umbrella branding is trivially irrelevant. More generally, for intermediate patterns of positive or negative correlation, more complicated patterns of optimal equilibria may emerge.

Although I do not develop a full theory of optimal brand size, my results provide some pointers towards the costs and the benefits from umbrella branding.¹² The benefits from umbrella branding may come from one of two sources: if the discount factor is sufficiently high, then umbrella branding allows for an equilibrium with a softer punishment (specifically, the equilibrium where consumers only lose trust in a brand when they observe two

11. In order for this alternative interpretation to work, we would need the cost of effort to be proportional to the number of consumers. Then we can have an equilibrium where the seller always produces quality products and product failure results in a smaller customer base.

12. By contrast, results based on a perfect monitoring model, such as Andersson (2002), have a harder time explaining why firms would *not* choose umbrella branding, as the latter is weakly better than no umbrella branding.

simultaneous product failures). If, by contrast, the discount factor is very small, then umbrella branding may allow for an equilibrium where quality is supplied at all (specifically, the equilibrium where consumers lose trust following any product failure).

Against these benefits, we must also consider the potential costs arising from umbrella branding. Under the equilibrium where any product failure is punished by loss of trust in the brand, the seller is subject to an unfortunate “domino” effect: a bad signal in one product kills two streams of revenue and profit. If the no-deviation constraint is satisfied under no umbrella branding, then the latter is the better choice. In fact, under no umbrella branding, the effect of a product failure is limited to the profit stream from the product that failed.

Appendix

Proof of Proposition 1: Given Assumption 1, an optimal equilibrium is one that leads the firm to invest in quality. In order for the solution to be an equilibrium, we must impose an incentive compatibility constraint on the firm, namely that it is better off by offering high quality. Given the simple nature of the consumers' strategies, the only hope for an efficient equilibrium is that, just as in the case of perfect monitoring, consumers “punish” the firm by losing trust in the brand whenever low quality is observed. In fact, were consumers not to punish the firm, then it would be the latter's dominant strategy not to offer quality.

Under the equilibrium assumption that consumers punish the firm for a product that breaks down, the firm's no-deviation constraint is derived as follows. If the firm invests in quality, then its expected value is $\pi - \epsilon$, net revenues during the current period, plus $\delta \alpha v$, where v is the firm's value. By shirking today, the firm receives profits π today but only $\delta \beta v$ in the future. The no-deviation constraint is therefore

$$\pi - \epsilon + \delta \alpha v \geq \pi + \delta \beta v \quad (2)$$

Under the equilibrium hypothesis that the seller chooses quality in every period until trust is broken, firm value is given by

$$v = \pi - \epsilon + \delta \alpha v,$$

or simply

$$v = \frac{\pi - \epsilon}{1 - \delta \alpha}. \quad (3)$$

Substituting (3) for v in (2), and solving for δ , we get

$$\delta \geq \frac{\epsilon}{\pi(\alpha - \beta) + \beta \epsilon}, \quad (4)$$

and the result follows. ■

Lemma 1 *Under umbrella branding, the no-deviation constraint corresponding to shirking in n products ($n = 1, 2$) is given by*

$$\delta \geq \frac{\xi}{(3 - n)(\rho - \rho') + \rho \xi}$$

where ρ is the equilibrium continuation probability, ρ' the continuation probability conditional on shirking, and $\xi \equiv \frac{\epsilon}{\pi - \epsilon}$. Moreover, under no umbrella branding, the no-deviation constraint has the same form as under umbrella branding and $n = 2$.

Proof of Lemma 1: Consider first the case of no umbrella branding. The no-deviation constraint is given by

$$(\pi - \epsilon) + \delta \rho \frac{(\pi - \epsilon)}{1 - \rho \delta} \geq \pi + \delta \rho' \frac{(\pi - \epsilon)}{1 - \rho \delta}, \quad (5)$$

This is equivalent to

$$\begin{aligned} -\epsilon + \delta \rho \frac{(\pi - \epsilon)}{1 - \rho \delta} &\geq \delta \rho' \frac{(\pi - \epsilon)}{1 - \rho \delta} \\ -\xi + \delta \rho \frac{1}{1 - \rho \delta} &\geq \delta \rho' \frac{1}{1 - \rho \delta} \\ \delta(\rho - \rho') &\geq \xi(1 - \rho \delta) \end{aligned}$$

Finally, solving with respect to δ we get

$$\delta \geq \frac{\xi}{(\rho - \rho') + \rho \xi} \quad (6)$$

Consider now the case of umbrella branding and the no-deviation constraint corresponding to shirking on two products, $n = 2$. The no-deviation constraint is given by

$$2(\pi - \epsilon) + \delta \rho \frac{2(\pi - \epsilon)}{1 - \rho \delta} \geq 2\pi + \delta \rho' \frac{2(\pi - \epsilon)}{1 - \rho \delta},$$

Dividing through by 2, we get an expression identical to (5).

Consider now the case of shirking on one product only. The no-deviation constraint is now given by

$$2(\pi - \epsilon) + \delta \rho \frac{2(\pi - \epsilon)}{1 - \rho \delta} \geq 2\pi - \epsilon + \delta \rho' \frac{2(\pi - \epsilon)}{1 - \rho \delta},$$

This is equivalent to

$$\begin{aligned} -\epsilon + \delta \rho \frac{2(\pi - \epsilon)}{1 - \rho \delta} &\geq \delta \rho' \frac{2(\pi - \epsilon)}{1 - \rho \delta} \\ -\xi + \delta \rho \frac{2}{1 - \rho \delta} &\geq \delta \rho' \frac{2}{1 - \rho \delta} \\ 2\delta(\rho - \rho') &\geq \xi(1 - \rho \delta) \end{aligned}$$

Finally, solving with respect to ξ we get

$$\delta \geq \frac{\xi}{2(\rho - \rho') + \rho \xi} \quad (7)$$

Putting together (6) and (7), Lemma 1 follows. ■

Lemma 2 *Under umbrella branding, (a) if consumers lose trust following any product failure, then the binding no-deviation constraint is to shirk on both products; (b) if consumers lose trust following two simultaneous product failures, then the binding no-deviation constraint is to shirk on one product only.*

Proof of Lemma 2: Consider the case of umbrella branding with punishment of any product failure. In this case, and using the notation of Lemma 1, we have $\rho = \alpha^2$. If the seller shirks in both products, then $\rho' = \rho'_2 = \beta^2$. If the seller shirks in one product then $\rho' = \rho'_1 = \alpha\beta$. Applying Lemma 1, we see that the lowest value of δ under two deviations is higher than the lowest value of δ under one deviation only if and only if

$$(\rho - \rho'_2) < 2(\rho - \rho'_1)$$

which is equivalent to

$$(\alpha^2 - \beta^2) < 2(\alpha^2 - \alpha\beta) \quad (8)$$

or simply $\beta < \alpha$.

Consider now the case of umbrella branding with punishment of two simultaneous product failures. In this case, $\rho = 1 - (1 - \alpha)^2$. If the seller shirks in both products, then $\rho' = \rho'_2 = 1 - (1 - \beta)^2$. If the seller shirks in

one product then $\rho' = \rho'_1 = 1 - (1 - \alpha)(1 - \beta)$. Applying Lemma 1, we see that the lowest value of δ under two deviations is lower than the lowest value of δ under one deviation only if and only if

$$(\rho - \rho'_2) > 2(\rho - \rho'_1)$$

which is equivalent to

$$\begin{aligned} (1 - (1 - \alpha)^2) - (1 - (1 - \beta)^2) &> 2 \left((1 - (1 - \alpha)^2) - (1 - (1 - \alpha)(1 - \beta)) \right) \\ (2\alpha - \alpha^2) - (2\beta - \beta^2) &> 2 \left((2\alpha - \alpha^2) - (\alpha + \beta - 2\alpha\beta) \right) \\ (\beta^2 - \alpha^2) &> 2(\alpha\beta - \alpha^2), \end{aligned}$$

which is equivalent to (8). ■

Proof of Proposition 2: Generally speaking, equilibrium value is given by

$$v = \frac{2(\pi - \epsilon)}{1 - \rho\delta},$$

where ρ is the continuation probability. Under no umbrella branding, $\rho = \alpha$. Under umbrella branding and punishment of any product failure, $\rho = \alpha^2$. Finally, under umbrella branding and punishment of two simultaneous product failures, $\rho = 1 - (1 - \alpha)^2$. It follows that, if the respective no-deviation constraints are satisfied, equilibrium value is highest under umbrella branding and punishment of two simultaneous product failures, next under no umbrella branding, and finally under umbrella branding and punishment of any product failure. I next show when each proposed solution satisfies the no-deviation constraints.

From Lemmas 1 and 2, the lower bounds on the value of the discount factor as follows. Under no umbrella branding, the equilibrium continuation probability is α , whereas the continuation probability is given by β . It follows that the no-deviation constraint is given by

$$\delta > \frac{\xi}{(\alpha - \beta) + \alpha\xi}.$$

Under umbrella branding and the strategy of punishing any product failure, the continuation probability, ρ , is equal to α^2 . By Lemma 2, the binding

deviation is shirking in both products, so $\rho' = \beta^2$. From Lemma 1, it follows that

$$\delta > \frac{\xi}{(\alpha^2 - \beta^2) + \alpha^2 \xi}.$$

Finally, under umbrella branding and the strategy of only punishing two simultaneous occurrences of product failure, the continuation probability, ρ , is equal to $1 - (1 - \alpha)^2$. By Lemma 2, the binding deviation is shirking in one product only, so $\rho' = 1 - (1 - \alpha)(1 - \beta)$. From Lemma 1, it follows that

$$\delta > \frac{\xi}{(1 - (1 - \alpha)^2) - (1 - (1 - \alpha)(1 - \beta)) + (1 - (1 - \alpha)^2) \xi}.$$

which is equivalent to

$$\delta > \frac{\xi}{2(1 - \alpha)(\alpha - \beta) + \alpha(2 - \alpha)\xi}.$$

It follows that the lower bound on the discount factor under umbrella branding with punishment of any product failure is lower than that under no umbrella branding if and only if

$$(\alpha^2 - \beta^2) + \alpha^2 \xi > (\alpha - \beta) + \alpha \xi,$$

which is equivalent to

$$\xi < \xi_1 = \frac{(\alpha - \beta)(\alpha + \beta - 1)}{(1 - \alpha)\alpha}$$

Moreover, the lower bound on the discount factor under umbrella branding with punishment of two product failures is lower than that under no umbrella branding if and only if

$$2(1 - \alpha)(\alpha - \beta) + \alpha(2 - \alpha)\xi > (\alpha - \beta) + \rho\xi,$$

which is equivalent to

$$\xi > \xi_2 = \frac{(\alpha - \beta)(2\alpha - 1)}{(\alpha - \beta)^2 + \beta(1 - \beta)}$$

Finally, $\alpha > \beta$ implies that $\xi_2 > \xi_1$, which leads to the characterization in the proposition. ■

Proof of Proposition 3: From the proof of Proposition 2, we know that the solution UB_2 is feasible if and only if

$$\delta > \frac{\xi}{2(1-\alpha)(\alpha-\beta) + \alpha(2-\alpha)\xi}.$$

As $\alpha \rightarrow 1$, the right-hand side converges to 1, which, for a given $\delta \in (0, 1)$, implies the solution becomes infeasible. At that point, the optimal solution switches to no umbrella branding, which, from the proof of Proposition 2, yields a strictly lower expected payoff. ■

Proof of Proposition 4: Consider the following stochastic reputation update equilibrium:

- The seller invests in quality in both products while consumers trust the seller's brand (used for both products);
- Consumers purchase each product at a price π while they trust the brand;
- Each time one of the products breaks down, with probability $1 - \theta_1$ consumers lose trust in the brand (or, with probability θ_1 trust is maintained);
- If the two products break down in the same period, then with probability $1 - \theta_2$ consumers lose trust in the brand (or, with probability θ_2 trust is maintained).

The firm's value is given by

$$V = 2(\pi - \epsilon) + \left((1 - \alpha)^2 + 2\alpha(1 - \alpha)\theta_1 + \alpha^2\theta_2 \right) \delta V.$$

Each term within brackets corresponds to each of three possible events: no product failure, one product failure, and two product failures, respectively. Solving for V , we get

$$V = \frac{2(\pi - \epsilon)}{1 - \left((1 - \alpha)^2 + 2\alpha(1 - \alpha)\theta_1 + \alpha^2\theta_2 \right) \delta}. \quad (9)$$

A seller that cheats consumers by lowering the quality of one of its products has an expected payoff

$$V'_1 = 2\pi - \epsilon + \left((1-\alpha)(1-\beta) + (\alpha(1-\beta) + \beta(1-\alpha))\theta_1 + \alpha\beta\theta_2 \right) \delta V. \quad (10)$$

Finally, a seller that cheats consumers by lowering the quality of both of its products has an expected payoff

$$V'_2 = 2\pi + \left((1-\beta)^2 + 2\beta(1-\beta)\theta_1 + \beta^2\theta_2 \right) \delta V. \quad (11)$$

An optimal equilibrium is obtained by

$$\begin{aligned} \max_{\theta_1, \theta_2} \quad & V \\ \text{s.t.} \quad & V \geq V'_1 \\ & V \geq V'_2 \end{aligned}$$

where V, V'_1, V'_2 are given by (9)–(11). Differentiating (9)–(11) with respect to V_1, V_2 , we get

$$\begin{aligned} -\frac{\partial \theta_1}{\partial \theta_2} \Big|_V &= \frac{\alpha}{2(1-\alpha)} \\ -\frac{\partial \theta_1}{\partial \theta_2} \Big|_{V'_1} &= \frac{\alpha\beta}{\alpha(1-\beta) + \beta(1-\alpha)} \\ -\frac{\partial \theta_1}{\partial \theta_2} \Big|_{V'_2} &= \frac{\beta}{2(1-\beta)} \end{aligned}$$

where $|_x$ means “maintaining x constant.” Moreover,

$$\begin{aligned} -\left(\frac{\partial \theta_1}{\partial \theta_2} \Big|_V - \frac{\partial \theta_1}{\partial \theta_2} \Big|_{V'_1} \right) &= \frac{\alpha(\alpha - \beta)}{2(1-\alpha)(\alpha(1-\beta) + \beta(1-\alpha))} > 0 \\ -\left(\frac{\partial \theta_1}{\partial \theta_2} \Big|_{V'_1} - \frac{\partial \theta_1}{\partial \theta_2} \Big|_{V'_2} \right) &= \frac{\beta(\alpha - \beta)}{2(1-\beta)(\alpha(1-\beta) + \beta(1-\alpha))} > 0 \end{aligned}$$

Consider an equilibrium such that $V_1 < V$ and $V_2 > 0$. Consider a candidate alternative equilibrium obtained by slightly decreasing θ_2 and increasing θ_1 by

$$d\theta_1 = \frac{\alpha\beta}{\alpha(1-\beta) + \beta(1-\alpha)}$$

This implies that: (a) the slack of the first constraint (cheating in one product) is maintained; (b) the slack of the second constraint (cheating in both products) is increased; (c) V increases. (a) and (b) imply that the new equilibrium is indeed an equilibrium; (c) implies that it is better.

It follows that, in an optimal equilibrium, one of the following must be true:

- Failure of *one isolated product* does not affect trust ($\theta_1 = 1$);
- Failure of *both products* implies a loss of trust with probability one ($\theta_2 = 0$).

Specifically, if δ is sufficiently high, then the optimal solution is characterized by $\theta_1 = 1$ and $0 < \theta_2 < 1$. For lower values of δ , we have $0 < \theta_1 < 1$ and $\theta_2 = 0$. Finally, for very low values of δ , no equilibrium exists with provision of high quality.

The critical value of δ when the optimal stochastic reputation equilibrium implies $\theta_1 = 1$ and $\theta_2 = 0$ corresponds to the value δ_3 determined in Proposition 2. The critical value of δ when the optimal stochastic reputation equilibrium implies $\theta_1 = 0$ and $\theta_2 = 0$ corresponds to the value δ_1 also determined in Proposition 2. Computation establishes that, in the $0 < \theta_1 < 1$ case, the binding constraint is deviation in two products if $\delta < \delta_2$ and deviation in one product if $\delta > \delta_2$, where δ_2 corresponds to the minimum δ such that a reputation equilibrium exists under no umbrella branding (see Proposition 2).

Next I need to compare the stochastic reputation umbrella equilibrium to no umbrella branding. First notice that, in the region where $\theta_1 = 1$, umbrella branding is superior to no umbrella branding. In fact, under no umbrella branding both reputations are destroyed when both products fail, just like under umbrella branding. However, if one product only fails, then under no umbrella branding one reputation is destroyed, whereas under umbrella branding the common reputation is maintained.

The case when umbrella branding leads to $\theta_1 > 0$ is a little trickier. In fact, one disadvantage of umbrella branding is that, once reputation is destroyed, it is destroyed for both products. Computation shows that there exists a critical value of ϵ , called it ϵ_1 , such that for $\epsilon > \epsilon_1$ no umbrella branding is superior to umbrella branding whenever $\delta < \delta_3$. ■

References

- AAKER, DAVID, AND KEVIN KELLER (1990), "Consumer Evaluations of Brand Extensions," *Journal of Marketing* **54**, 27–41.
- ANDERSSON, FREDRIK (2002), "Pooling Reputations," *International Journal of Industrial Organization* **20**, 715–730.
- BERNHEIN, B. DOUGLAS AND MICHAEL D. WHINSTON (1990), "Multimarket Contact and Collusive Behavior," *Rand Journal of Economics* **21**, 1–26.
- CABRAL, LUÍS (2000), "Stretching Firm and Brand Reputation," *Rand Journal of Economics* **31**, 658–673.
- CHOI, JAY PIL (1997), "Brand Extension and Informational Leverage," *Review of Economic Studies* **65**, 655–69.
- CHOI, CHONG JU, AND CARLO SCARPA (1992), "Credible Spatial Preemption Through Reputation Extension," *International Journal of Industrial Organization* **10**, 439–447.
- DANA, JAMES, AND KATHRYN SPIER (2006), "Bundling and Product Reputation," Northwestern University.
- ERDEM, TÜLIN (1998), "An Empirical Analysis of Umbrella Branding," *Journal of Marketing Research* **65**, 655–669.
- GREEN, ED AND ROBERT PORTER (1984), "Noncooperative Collusion Under Imperfect Price Information," *Econometrica* **52**, 87–100.
- HAKENES, HENDRIK, AND MARTIN PEITZ (2008a), "Umbrella Branding and the Provision of Quality," *International Journal of Industrial Organization* **26**, 546–556.
- HAKENES, HENDRIK, AND MARTIN PEITZ (2008b), "Umbrella Branding and External Certification," forthcoming in *European Economic Review*.
- CAI, HONGBIN, AND ICHIRO OBARA (2006), "Firm Reputation and Horizontal Integration," U.C.L.A..
- HARSANYI, JOHN C. (1973), "Games with Randomly Disturbed Payoffs: A New Rationale for Mixed-Strategy Equilibrium Payoffs," *International Journal of Game Theory* **2**, 1–23.

- KLEIN, B, AND K LEFFLER (1981), "The Role of Market Forces in Assuring Contractual Performance," *Journal of Political Economy* **89**, 615–641.
- KOTLER, PHILIP, GARY ARMSTRONG, JOHN SAUNDERS AND VERONICA WONG (1996), *Principles of Marketing*, London: Prentice Hall.
- KREPS, DAVID (1990), "Corporate Culture and Economic Theory," in J Alt and K Shepsle (Eds), *Perspectives on Positive Political Economy*, Cambridge: Cambridge University Press, 90–143.
- MIKLÓS-THAL, JEANINE (2008), "Linking Reputations: The Signaling and Feedback Effects of Umbrella Branding," University of Manheim, May.
- MONTE, DANIEL (2007), "Reputation and Bounded Memory," Yale University.
- MONTGOMERY, CYNTHIA, AND BIRGER WERNERFELT (1992), "Risk Reduction and Umbrella Branding," *Journal of Business* **65**, 31–50.
- PEPALL, LYNNE, AND DAN J RICHARDS (2002), "The Simple Economics of Brand-Stretching," *Journal of Business* **75**, 535–552.
- PORTER, ROBERT H. (1983), "Optimal Cartel Trigger Price Strategies," *Journal of Economic Theory* **29**, 313–338.
- SAPPINGTON, DAVID, AND BIRGER WERNERFELT (1985), "To Brand or Not to Brand? A Theoretical and Empirical Question," *Journal of Business* **58**, 279–293.
- SHAPIRO, CARL (1983), "Premiums for High Quality Products as Rents to Reputation," *Quarterly Journal of Economics* **98**, 659–680.
- SULLIVAN, MARY (1990), "Measuring Image Spillovers in Umbrella-Branded Products," *Journal of Business* **63**, 309–329.
- TAUBER, EDWARD M (1988), "Brand Leverage: Strategy for Growth in a Cost-Control World," *Journal of Advertising Research* **28**, 26–30.
- TELSER, L G (1980), "A Theory of Self-enforcing Agreements," *Journal of Business* **53**, 27–44.
- TIROLE, JEAN (1988), *The Theory of Industrial Organization*, Cambridge, Mass.: MIT Press.

WERNERFELT, BIRGER (1988), “Umbrella Branding as a Signal of New Product Quality: An Example of Signalling by Posting a Bond,” *Rand Journal of Economics* **19**, 458–466.

WILSON, ANDREA, AND MEHMET EKMEKCI (2006), “A Note on Reputation Effects with Finite Memory,” Harvard University.