Problem Set 10

Foundations of Financial Markets

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1. Suppose that the risk-free rate is 5%, that Lego's stock is currently at \$100, and that, the next two years, the stock price movements are well approximated by the following tree:

$t = 0$	$t = 1$	$t = 2$
		144
	120	
100		108
	90	
		81

We see that every year Lego will either increase by 20% or decrease by 10% (with equal probability). Suppose that a 2-year (European-style) "binary" option is traded on Lego. This option pays the option-holder \$10 if the stock price is greater than \$100 at maturity, $t = 2$.

To compute the initial value of the option, you should make a tree for the development of the option price. Fill out the tree, starting from the back, by answering the following questions:

(a) Compute the value of the option at maturity ($t = 2$) at each of the 3 scenarios.

(b) Suppose at time 1, the stock price is 120. Create a portfolio of stocks and risk-free securities that replicates the option's payoff at time 2.

(*Hint:* Recall that the number of stocks in the replicating portfolio is always

$$\Delta = \frac{C^+ - C^-}{S^+ - S^-}$$

where S^+ and S^- are the two possible stock prices in the next period, and C^+ and C^- are the corresponding option values next period. The amount invested in the riskfree security should make the total value of the portfolio equal to the value of the option in both scenarios next period.)

What is the price of this replicating portfolio? This is the value of the option at time 1 in this upper-branch scenario. Put this value in your option tree.

(c) Suppose at time 1, the stock price is 90. Create a portfolio of stocks and risk-free securities that replicates the option's payoff at time 2. What is the price of this portfolio? This is the value of the option at time 1 in this lower-branch scenario. Put this value in your option tree. (Use hint from (1b).)

(d) At time 0, create a portfolio of stocks and risk-free securities that replicates the option values at time 1. (The time-1 option values were computed in (1b)–(1c).) What is the price of this portfolio? This is the value of the option at time 0. Put this value in your option tree. (Use hint from (1b).)

2. Suppose a European call option has an exercise price of \$100 and the underlying stock has a price of \$100. The stock will pay no dividends over the next year. The option expires in 1 year and the continuously compounded interest rate is 6%.

   (a) What is the intrinsic value of this option?

   (b) What will the option be worth on expiration if the stock price in 1 year is \$110? What if the stock price is \$90?

   (c) What is the lower bound on the price of this option today?

   (d) Will the value of the option be larger or smaller if the volatility of the underlying asset is higher than otherwise?

   (e) Will the value be larger or smaller if the option has 3 months rather than 6 months to expiration?

   (f) Will the value be larger or smaller if the interest rate is larger or smaller?

   (g) Would the value be different for an American option? Why or why not?