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# *Betting Against Beta*

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Preliminary  
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# Motivation

## ➤ Background:

- Security Market Line for U.S. stocks too flat relative to CAPM (Black, Jensen, and Scholes (1972))
- Could be related to borrowing constraints (Black (1972, 1993))
- Surprisingly little research on factors based on the flatness of the SML

## ➤ Research questions:

1. Is the SML flat in other markets?
2. Betting-Against-Beta (BAB):
  - How to capture this effect with a factor?
  - BAB returns relative to size/ value/ momentum effects?
3. Additional predictions of a theory of funding constraints?
  - In the cross section?
  - In the time series?

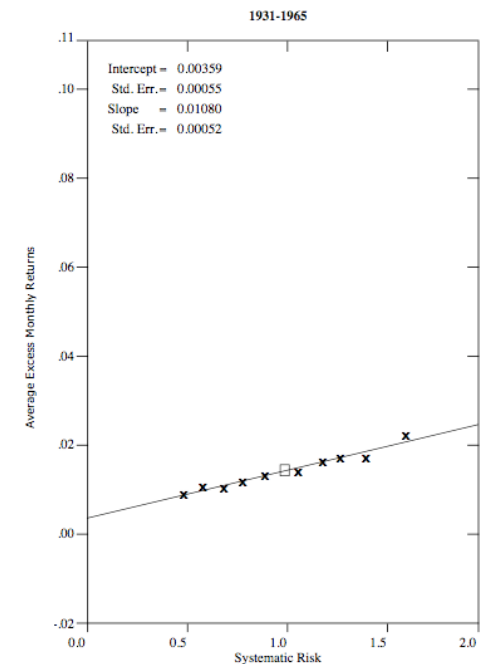


Figure 1 Average excess monthly returns versus systematic risk for the 35-year period 1931-65 for each of ten portfolios (denoted by x) and the market portfolio (denoted by □).

## What We Do

### Theory:

- Predictions of a dynamic model with constrained investors:
  - No leverage: some investors cannot (or will not) use leverage (e.g. pension funds, mutual funds, etc.)
  - Margin requirements: investors who are willing to use leverage are constrained by their margin requirements and may sometimes need to de-lever (e.g. hedge funds, proprietary traders, etc.)

### Evidence:

- Beta-sorted portfolios in numerous major markets and asset classes
  - US stocks
  - Global stocks in 19 developed markets (other than US)
  - Treasuries
  - Credit markets
  - Futures: stock indices, bond futures, currencies, and commodities
- Market neutral Betting-Against-Beta (BAB) factors:
  - Long *levered* low-beta securities, short *de-levered* high-beta securities
- Test cross-sectional and time-series predictions of the theory

## Summary of Results

### Theory:

- Investors accept lower risk-adjusted returns of high beta stocks
  - because they alleviate no-leverage constraint
- BAB factors have positive average excess returns
  - Expected returns increase in “beta spread” and in ex ante tightness of constraints
  - A shock to funding constraints leads to losses to BAB factors
- Funding shocks compress betas towards 1

### Consistent evidence:

- Within each major market: High beta = low alpha and Sharpe Ratio
- Beta-factor is large and present in many asset classes:
  - US stocks: SR of 0.75. Compare: Value Effect:  $SR^{HML}=0.39$  ; Momentum:  $SR^{UMD}=0.50$  ; Size:  $SR^{SMB}=0.25$
  - Global stocks: Positive return in 18 of 19 markets; Overall SR of 0.79
  - Treasuries: SR of 0.85 (long short term bonds, short fewer long-term bonds)
  - Credit: SR of 0.88
  - Futures: positive, but lower returns
- Beta compression
- BAB factors load on measures of funding constraints

## *Related Literature*

- Low return of high-beta stocks in the U.S., borrowing constraints:
  - Black (1972, 1993), Black, Jensen, and Scholes (1972), Gibbons (1982), Kandel (1984), Shanken (1985), Polk, Thompson, and Vuolteenaho (2006)
  
- Stocks with high idiosyncratic volatility have realized low returns:
  - Ang, Hodrick, Xing, Zhang (2006, 2009), Bali, Cakici, and Whitelaw (2010)
  
- Benchmarked managers:
  - Brennan (1993), Baker, Bradley, and Wurgler (2010)
  
- Treasury term premia
  - Fama (1986), Duffee (2010)
  
- Margin requirements and funding constraints can also help explain:
  - Deviations from the Law of One Price (Garleanu and Pedersen (2009))
  - The impact of central banks' lending facilities (Ashcraft, Garleanu, and Pedersen (2010))
  - Variation in market liquidity and liquidity crises (Brunnermeier and Pedersen (2009))

## *Road Map*

- Theory and predictions
- Evidence: testing the main predictions of the model
- Conclusion

## Model

- OLG economy where agents maximize their utility:

$$\max x'(E_t(P_{t+1}) - (1+r^f)P_t) - \frac{\gamma^i}{2} x' \Omega_t x$$

subject to a portfolio constraint:

$$m_t^i \sum_s x^s P_t^s \leq W_t^i$$

which can capture

- No leverage,  $m^i=1$  (as in Black (1972))
  - No leverage and cash constraint,  $m^i>1$
  - Margin constraints,  $m^i<1$  (Garleanu and Pedersen (2009) consider assets with different margins)
- Competitive equilibrium:

$$\sum_i x^i = x^*$$

where  $x^*$  are shares outstanding

## Equilibrium Required Returns

### Proposition 1.

(i) The equilibrium required return for security  $s$  is:

$$E_t \left( r_{t+1}^s \right) = r^f + \psi_t + \beta_t^s \lambda_t$$

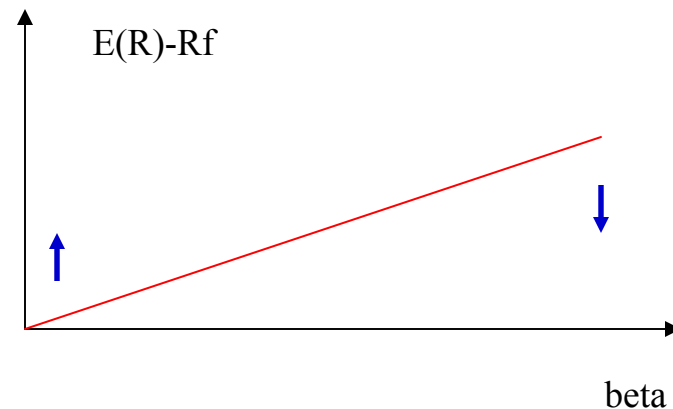
where  $\psi$  is the agents' average Lagrange multiplier, measuring the tightness of funding constraints, and  $\lambda$  is the risk premium:

$$\lambda_t = E_t \left( r_{t+1}^M \right) - r^f - \psi_t$$

### Implication:

Tighter portfolio constraints (i.e., larger  $\psi$ ) flattens the capital market line:

- increase the intercept  $\psi$  and
- decrease the slope  $\lambda$



## *High Beta = Low Alpha*

### **Proposition 1. (continued)**

**(ii)** *A security's alpha with respect to the market is.*

$$\alpha_t^s = \psi_t (1 - \beta_t^s)$$

*Alpha decreases in the security's market beta  $\beta$ .*

**(iii)** *For a diversified efficient portfolio, the Sharpe Ratio (SR) is highest for an efficient portfolio with beta less than 1 and decreases in for higher betas and increases for lower betas.*

## Betting Against Beta Factors

- Betting-Against-Beta (BAB) factors:
  - Long low-beta assets, levered to a beta of 1
  - Short high-beta assets, de-levered to a beta of 1

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L} (r_{t+1}^L - r^f) - \frac{1}{\beta_t^H} (r_{t+1}^H - r^f)$$

- A BAB factor is a market-neutral excess return on a zero-cost portfolio (like HML and SMB)
- Example: BAB factor for US stocks
  - Long \$1.5 worth of low-beta stocks
  - Short \$0.7 worth of high-beta stocks, on average
- BAB factor useful for studying:
  - the magnitude of the beta effect and its relation of other known factors
  - the time-series of the beta effect
  - the beta effect in different assets classes and in subsets of securities (e.g., stocks by size)
  - and pricing other portfolios

## Betting Against Beta Factors

### Proposition 2.

(i) *The expected excess return of the zero-cost BAB factor is positive:*

$$E_t(r_{t+1}^{BAB}) = \frac{\beta_t^H - \beta_t^L}{\beta_t^L \beta_t^H} \psi_t \geq 0$$

*and increasing in the “beta spread” and the funding tightness  $\psi$ .*

(ii) *A tighter portfolio constraint, that is, an increase in  $m_t^k$  for some of  $k$ , leads to a contemporaneous loss for the BAB factor*

$$\frac{\partial r_t^{BAB}}{\partial m_t^k} \leq 0$$

*and an increase in its future required return:*

$$\frac{\partial E_t(r_{t+1}^{BAB})}{\partial m_t^k} \geq 0$$

## Beta Compression

### Proposition 3.

*The percentage price sensitivity with respect to funding shocks  $\frac{\partial P_t^s}{P_t^s} / \partial \psi_t$  is the same for all securities  $s$ .*

*A higher independent variance of funding shocks compresses betas of all securities towards 1, and the beta of the BAB factor increases if this is unanticipated.*

## Equilibrium Positions

### Proposition 4.

*Unconstrained agents hold risk free securities and a portfolio of risky securities that has a beta less than 1; constrained agents hold portfolios of securities with higher betas.*

*If securities  $s$  and  $k$  are identical expect that  $s$  has a larger market exposure than  $k$ , then any constrained agent  $j$  with greater than average Lagrange multiplier,  $\psi^j > \psi$ , holds more shares of  $s$  than  $k$ , while the reverse is true for any agent with  $\psi^j < \psi$ .*

### Intuition:

- More constrained agents seek to achieve higher expected returns buy overweighting risky assets
- This pushes down the expected return of risky assets
- Less constrained exploit this buy underweighting or shortselling risky assets

## *Road Map*

- Theory and predictions
- Evidence: testing the main predictions of the model
  1. Beta-sorted portfolios: alphas and Sharpe ratios
    - US stocks
    - Global stocks
    - Treasuries
    - Credit markets
    - Futures: equity indices, bonds, currencies, commodities
  2. Positive abnormal returns on BAB factors
  3. Cross-sectional prediction of the model: beta compression
  4. Time series prediction of the model: BAB time varying returns and funding-liquidity proxies
- Conclusion

## *Data Sources*

### ➤ Equities (common stocks)

- CRSP 1927 – 2009.
- Xpressfeed Global 1984 – 2009
- 20 Countries (MSCI Developed Markets)

### ➤ Treasury bonds

- CRSP Fama Bond Portfolio Returns, monthly 1952 – 2009

### ➤ Credit

- Barclays Capital's Bond Hub database, 1973 – 2009
- US credit indices with maturity ranging from 1 to 10 years
- Corporate bond portfolios with credit risk ranging from AAA to Ca-D

### ➤ Futures markets

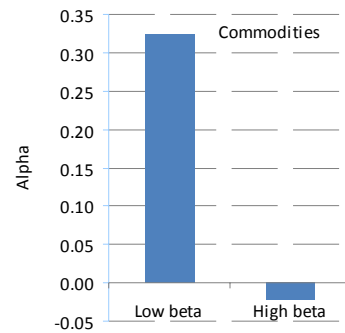
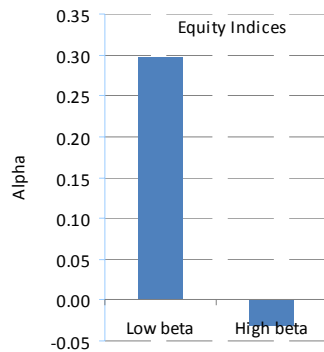
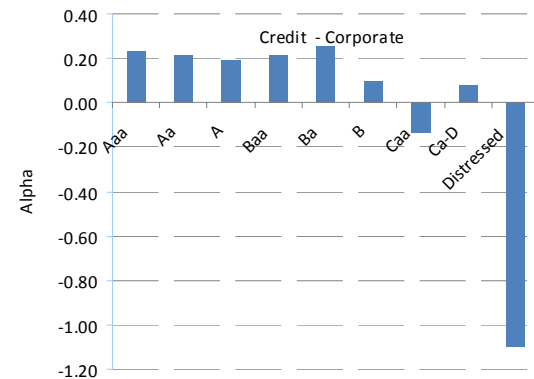
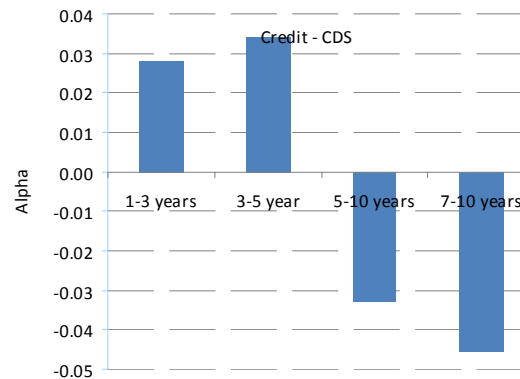
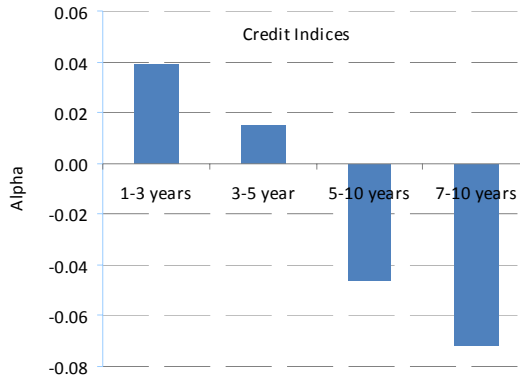
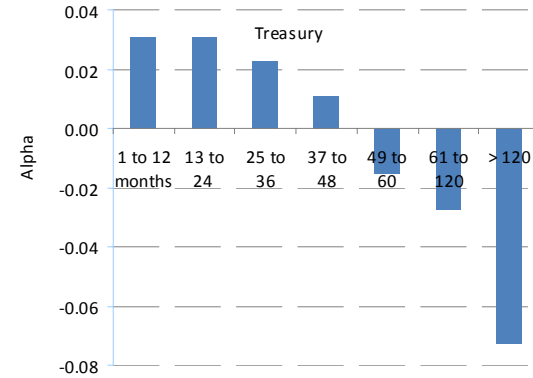
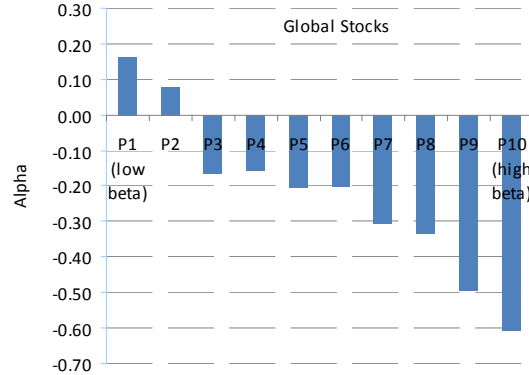
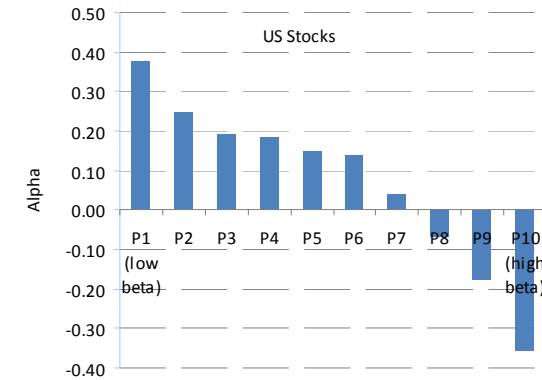
- Bloomberg, Datastream, Citigroup, various exchanges, 1965 – 2009
- Daily excess returns on rolled futures and forwards
- Equity indices: 13 developed markets
- Government Bonds : 9 developed markets, constant duration
- Foreign Exchange : 9 developed markets
- Commodities : 27 Commodities (Energy, Agricultural , Metal , Soft)

## *Estimating Betas and Constructing BAB portfolios*

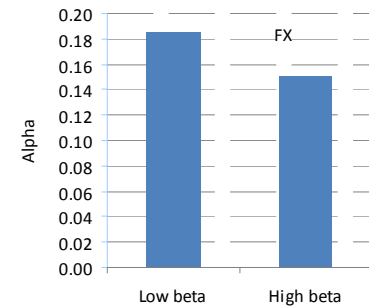
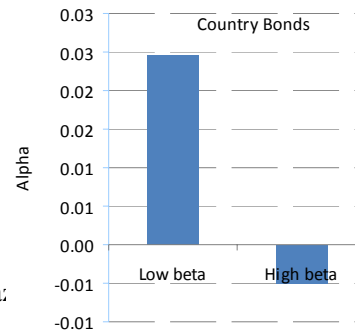
- Betas are computed from 1-year rolling regression of daily excess returns on market excess return
  - Markets excess return computed as value weighted index
  - Include 1 week lags on the RHS to account for small/illiquid securities and sum the slopes
  - Use a simplified Vasicek (1973) estimator: shrink betas towards one:  $0.5*1 + 0.5*\hat{\beta}$
  
- We form monthly portfolios by sorting stocks in deciles.
  - Base currency USD. Returns, risk free rate, and alphas are in USD, no currency hedging
  
- To form zero-beta zero-costs BAB factors
  - Assign stocks to two portfolios: low beta and high beta
  - Rescale portfolios to have a beta of 1 at portfolio formation.
  - Long the (levered) low-beta portfolio and shorts the (de-levered) high-beta portfolio

# Alphas by Beta-Sorted Portfolios

## All Asset Classes, 1964 – 2009

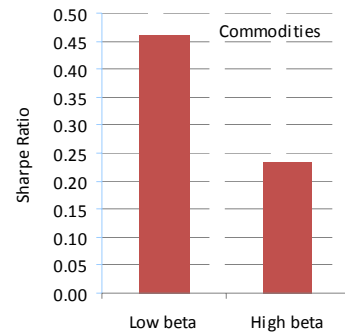
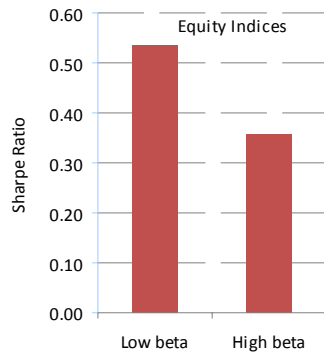
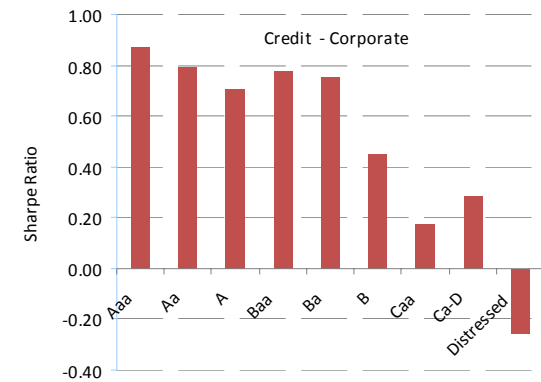
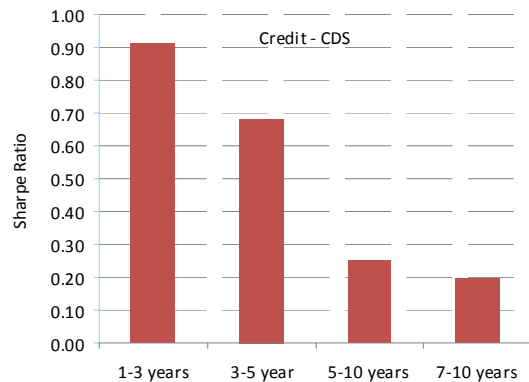
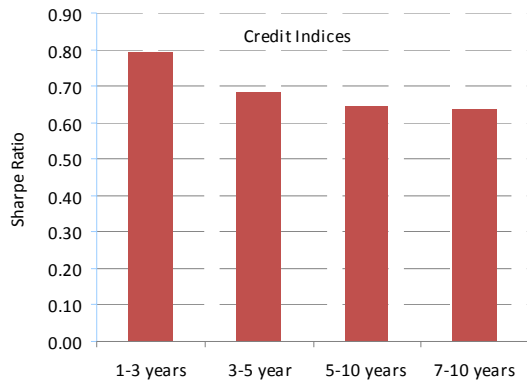
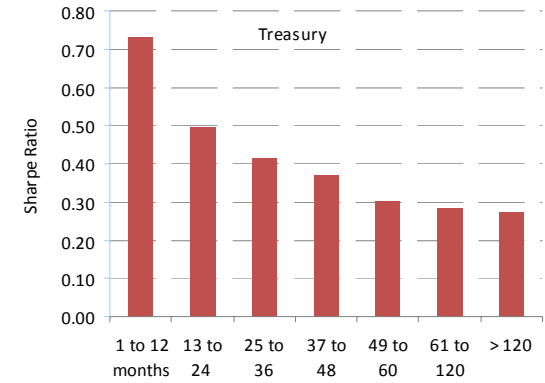
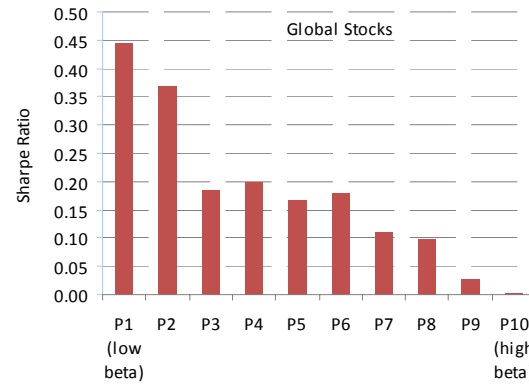
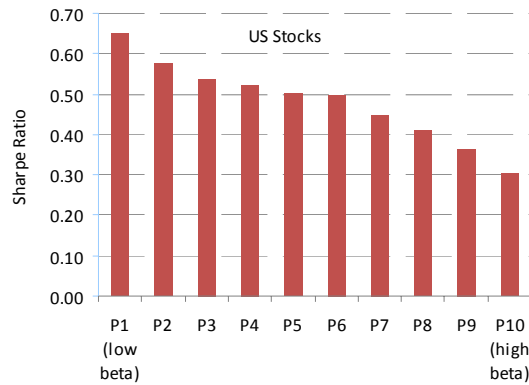


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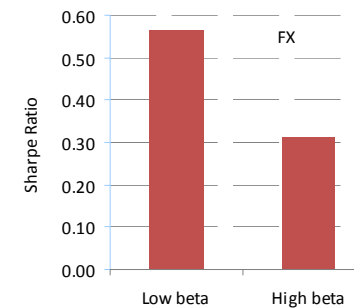
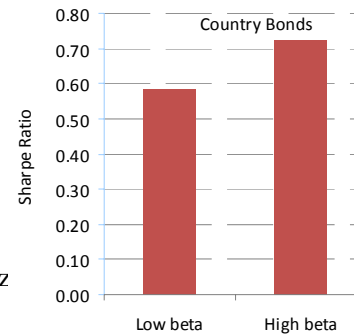


# Annualized SR by Beta-Sorted Portfolios

## All Asset Classes, 1964 – 2009



a Fraz



## *BAB - US Treasury Bonds, 1952 – 2009*

This table shows average monthly excess returns of Fama bond portfolios by maturity. Returns are in percent and 5% statistical significant is indicated in bold. BAB is a portfolio short (de-levered) long maturity and long (levered) low maturity

	P1 (low beta)	P2	P3	P4	P5	P6	P7* (high beta)	BAB Factor
Maturity (months)	1 to 12	13 to 24	25 to 36	37 to 48	49 to 60	61 to 120	> 120	
Excess return	<b>0.05</b> (5.57)	<b>0.09</b> (3.77)	<b>0.11</b> (3.17)	<b>0.12</b> (2.82)	<b>0.12</b> (2.30)	<b>0.14</b> (2.17)	0.21 (1.90)	<b>0.16</b> (6.37)
Alpha	<b>0.03</b> (5.87)	<b>0.03</b> (3.42)	<b>0.02</b> (2.21)	0.01 (1.10)	-0.02 (-1.59)	<b>-0.03</b> (-2.66)	<b>-0.07</b> (-2.04)	<b>0.16</b> (6.27)
Beta (ex ante)	0.14	0.46	0.75	0.99	1.22	1.44	2.17	0.00
Beta (realized)	0.17	0.49	0.77	0.99	1.17	1.43	2.06	0.02
Volatility	0.83	2.11	3.23	4.04	4.76	5.80	9.12	2.32
Sharpe ratio	0.73	0.50	0.42	0.37	0.30	0.29	0.27	0.85

\* Return missing from 196208 to 197112

## *BAB - Equities, 1926 - 2009*

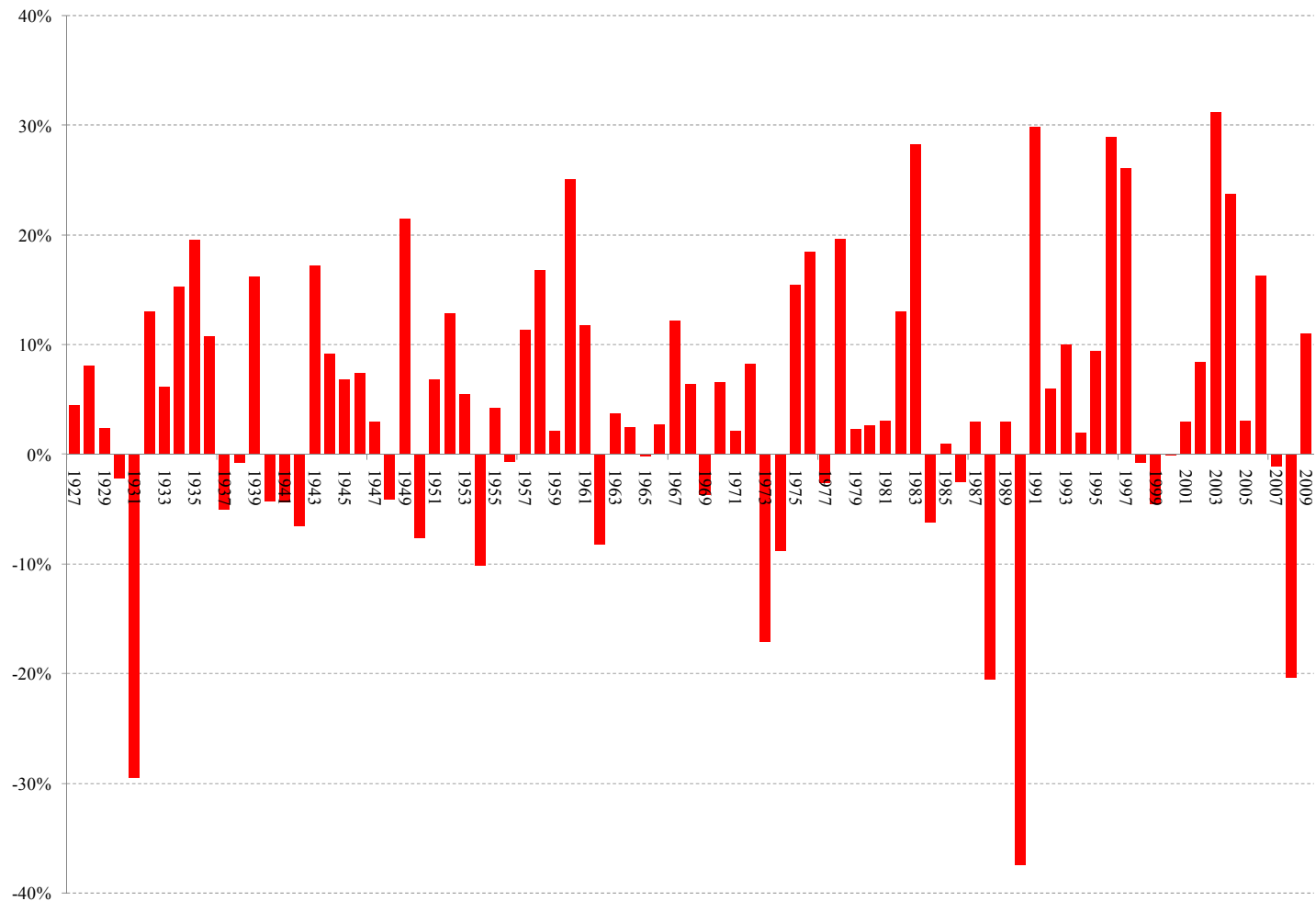
This table shows calendar-time portfolio returns. BAB is a portfolio short (de-levered) high beta stocks and long (levered) low beta stocks. Returns and alphas are in monthly percent, t-statistics are shown below the coefficient estimates, and 5% statistical significance is indicated in bold.

	US equities 1926 - 2009				Global Equities 1984 - 2009			
	P1 (Low beta)	...	P10 (high beta)	BAB Factor	P1 (Low beta)	...	P10 (high beta)	BAB Factor
Excess return	<b>0.99</b> (5.90)	...	<b>1.02</b> (2.77)	<b>0.71</b> (6.76)	<b>0.55</b> (2.13)	...	0.01 (0.01)	<b>0.72</b> (3.79)
CAPM alpha	<b>0.54</b> (5.22)	...	-0.05 (-0.29)	<b>0.69</b> (6.55)	0.33 (1.46)	...	-0.55 (-1.30)	<b>0.71</b> (3.72)
3-factor alpha	<b>0.38</b> (5.24)	...	<b>-0.36</b> (-3.10)	<b>0.66</b> (6.28)	0.16 (0.78)	...	-0.61 (-1.47)	<b>0.60</b> (3.18)
4-factor alpha	<b>0.42</b> (5.66)	...	-0.07 (-0.59)	<b>0.55</b> (5.12)	0.10 (0.46)	...	-0.37 (-0.88)	<b>0.45</b> (2.47)
5-factor alpha*	<b>0.23</b> (2.37)	...	0.01 (0.07)	<b>0.46</b> (2.93)	-0.03 (-0.13)	...	-0.77 (-1.80)	<b>0.42</b> (2.22)
Beta (ex ante)	0.57	...	1.64	0.00	0.50	...	1.44	0.00
Beta (realized)	0.75	...	1.82	0.03	0.48	...	1.18	0.02
Volatility	18.2	...	40.0	11.5	14.9	...	30.3	10.9
Sharpe Ratio	0.65	...	0.31	0.75	0.44	...	0.00	0.79

\* Pastor and Stambaugh (2003) liquidity factor only available between 1968 and 2008.

## US Equity BAB : 4-Factor Alphas 1926 - 2009

This figure shows calendar-time annual abnormal returns. This figure plots the annualized intercept in a regression of monthly excess return. The explanatory variables are the monthly returns from Fama and French (1993) mimicking portfolios and Carhart (1997) momentum factor. A separate factor regression is run for each calendar year. Alphas are annualized.



## *BAB – US Corporate Bonds*

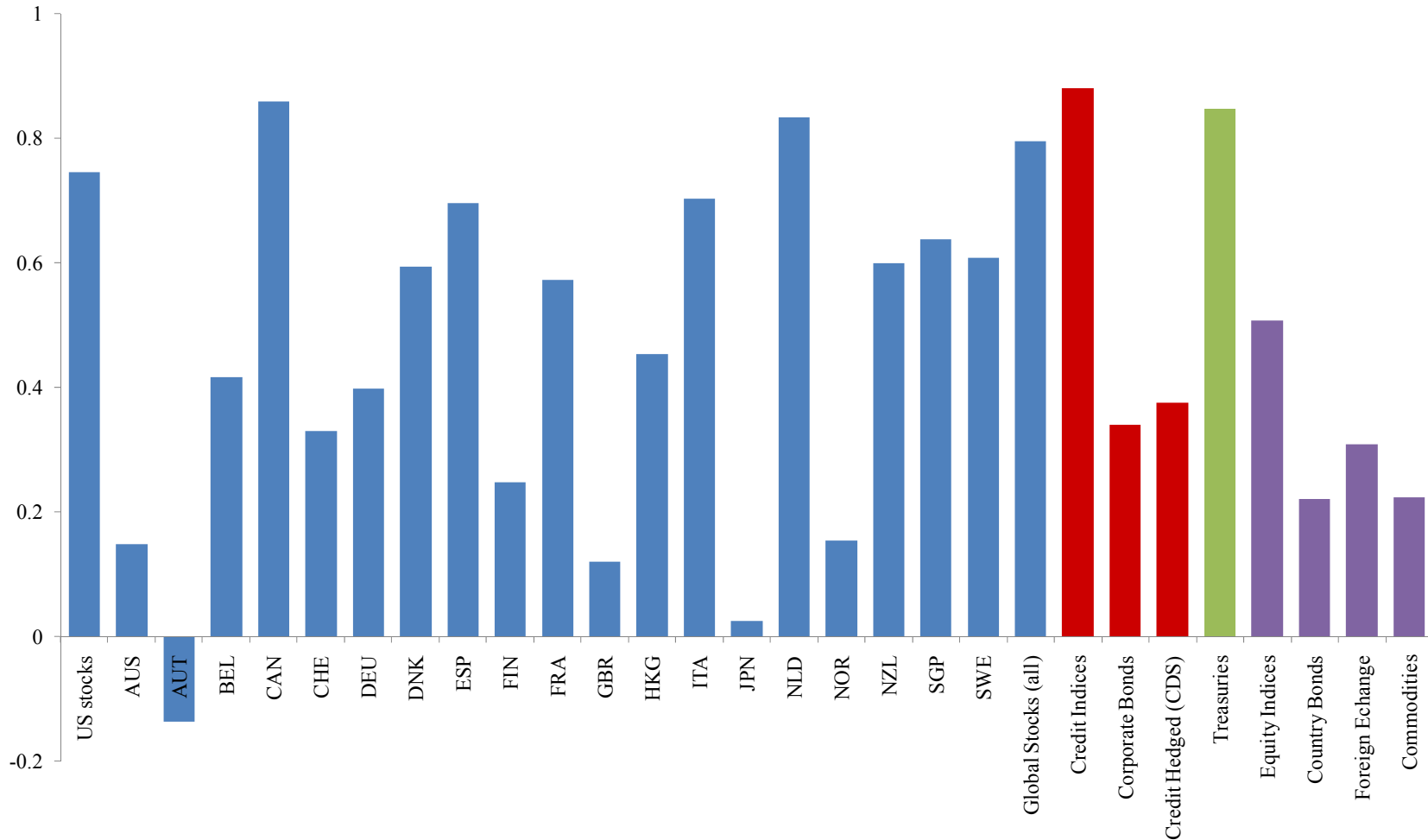
This table shows average monthly excess returns of US credit indices by maturity and US corporate bond. Returns are in percent and 5% statistical significant is indicated in bold. BAB is a portfolio short (de-levered) high beta bonds and long (levered) low beta bonds

US Credit indices 1976 - 2009		1-3 years	3-5 year	5-10 years	7-10 years	BAB Factor
Unhedged returns	Alpha	<b>0.04</b> (2.77)	<b>0.01</b> (0.96)	<b>-0.05</b> (-4.01)	<b>-0.07</b> (-4.45)	<b>0.13</b> (4.91)
	Beta (ex ante)	0.60	0.85	1.39	1.52	0.00
	Beta (realized)	0.62	0.85	1.37	1.48	-0.01
Hedged returns (CDS)	Alpha	<b>0.04</b> (3.62)	<b>0.04</b> (3.23)	<b>-0.03</b> (-2.38)	<b>-0.04</b> (-2.16)	<b>0.08</b> (3.33)
	Beta (ex ante)	0.70	0.78	1.14	1.38	0.00
	Beta (realized)	0.58	0.72	1.34	1.37	-0.34

US Corporate Bonds 1952 - 2009	Aaa	Aa	A	Baa	Ba	B	Caa	Ca-D	CSFB Distressed	BAB Factor
Alpha	<b>0.23</b> (4.09)	<b>0.21</b> (3.62)	<b>0.19</b> (3.13)	<b>0.21</b> (3.69)	<b>0.26</b> (4.20)	0.10 (1.40)	-0.13 (-0.95)	0.08 (0.26)	<b>-1.10</b> (-5.34)	<b>0.56</b> (4.02)
Beta (ex ante)	0.67	0.70	0.72	0.77	0.89	1.01	1.25	1.74	1.66	0.00
Beta (realized)	0.13	0.24	0.33	0.40	0.69	0.95	1.39	2.77	2.49	-0.94

## BAB Factor SRs - All Asset Classes 1964 – 2009

This table shows annualized Sharpe ratios of BAB factors across asset classes. BAB is a portfolio short (de-levered) high beta assets and long (levered) low beta assets



## *BAB - All Asset Classes 1964 – 2009*

This table shows calendar-time BAB portfolio returns. Returns are in monthly percent and 5% statistical significant is indicated in bold. BAB is a portfolio short (de-levered) high beta assets and long (levered) low beta assets

Panel A: Equity indices, country Bonds, Foreign Exchange and Commodities		Excess Return	T-stat Excess Return	Alpha	T(alpha)	\$Short	\$Long	Volatility	SR
Equity Indices	EI	<b>0.78</b>	2.90	<b>0.69</b>	2.56	0.93	1.47	18.46	0.51
Country Bonds	CB	0.08	0.99	0.06	0.73	0.95	1.69	4.47	0.22
Foreign Exchange	FX	0.2	1.45	0.14	1.08	0.61	1.61	7.72	0.31
Commodities	COM	0.42	1.44	0.38	1.26	0.78	1.56	22.65	0.22
All Futures*	EI + CB + FX + COM	<b>0.47</b>	3.99	<b>0.52</b>	4.50			9.02	0.62
Country Selection*	EI + CB + FX	<b>0.64</b>	3.78	<b>0.71</b>	4.42			11.61	0.66
Panel B: All Assets									
All Bonds and Credit*		<b>0.73</b>	6.00	<b>0.72</b>	5.88			11.06	0.79
All Equities*		<b>0.77</b>	8.10	<b>0.78</b>	8.16			10.31	0.89
All Assets*		<b>0.71</b>	8.60	<b>0.73</b>	8.84			8.95	0.95

\* Equal risk, 10% ex ante volatility

## *Beta Compression and BAB Conditional Market Beta*

Cross-sectional dispersion of betas in US and global stocks. *P1* to *P3* report coefficients on a regression of the dispersion measure on TED spread dummies (low, neutral and high) based on full sample breakpoints

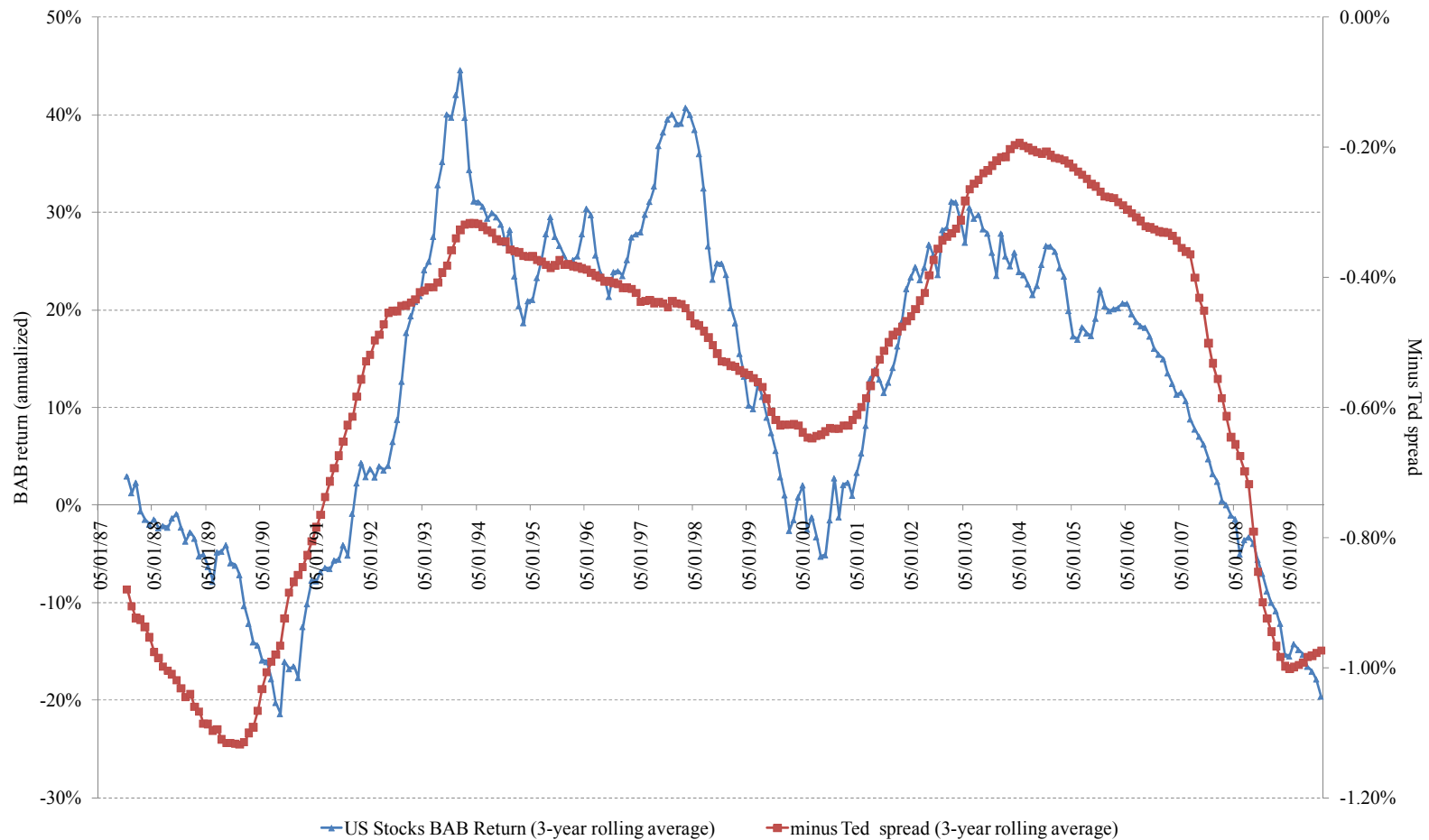
	Panel A Cross-Sectional Beta Dispersion - US			Panel B Cross sectional Beta Dispersion - Global		
	Standard deviation	Mean Absolute Deviation	Inter-quintile Range	Standard deviation	Mean Absolute Deviation	Inter-quintile Range
All	0.42	0.33	0.67	0.27	0.21	0.44
P1 (low TED)	0.47	0.36	0.74	0.29	0.23	0.46
P2	0.43	0.34	0.69	0.27	0.21	0.43
P3 (high TED)	0.35	0.28	0.58	0.25	0.20	0.42
P3 minus P1	<b>-0.11</b>	<b>-0.08</b>	<b>-0.16</b>	<b>-0.04</b>	<b>-0.03</b>	<b>-0.04</b>
t-statistics	<b>-10.72</b>	<b>-10.48</b>	<b>-10.04</b>	<b>-7.31</b>	<b>-6.59</b>	<b>-5.07</b>

Conditional market betas of BAB portfolios based on the TED spread. Full set on regressors included, only market loadings reported

	Panel C: Conditional Market Loading - US				Panel D: Conditional Market Loading - Global			
	P1 (Low TED)	P2	P3 (High TED)	P3 - P1	P1 (Low TED)	P2	P3 (High TED)	P3 - P1
CAPM	-0.21 (-1.77)	0.10 (1.04)	<b>0.30</b> (3.99)	<b>0.51</b> (3.64)	<b>-0.33</b> (-3.96)	-0.01 (-0.17)	<b>0.19</b> (3.33)	<b>0.51</b> (5.15)
Control for 3 Factors	-0.07 (-0.66)	0.38 (4.14)	<b>0.33</b> (4.84)	<b>0.41</b> (3.24)	<b>-0.29</b> (-3.57)	0.09 (1.09)	<b>0.19</b> (3.46)	<b>0.49</b> (5.00)
Control for 4 Factors	0.06 (0.50)	<b>0.42</b> (4.55)	<b>0.36</b> (5.34)	<b>0.31</b> (2.46)	<b>-0.19</b> (-2.16)	0.11 (1.37)	<b>0.23</b> (4.09)	<b>0.41</b> (4.24)

## US equity BAB and TED Spread

This figure shows annualized 3-year return of the US stocks BAB factor (left scale) and 3-year (negative) average rolling TED spread (right scale). BAB is a portfolio short (de-levered) high beta stocks and long (levered) low beta stocks



## Regression Results: BAB Returns and Funding Liquidity

This table shows results from time series (pooled) regressions. The left-hand side is the month t return on the BAB factors. The explanatory variables include the TED spread (level and changes) and a series of controls. Asset fixed effects are include where indicated, t-statistics are shown below the coefficient estimates and 5% statistical significance is indicated in bold. Standard errors are clustered by date

	US - Stocks				Global Stocks - pooled				All Assets pooled (Equities, Bonds and Futures)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
TED Spread	<b>-0.036</b> -(6.17)	<b>-0.023</b> -(3.47)			<b>-0.022</b> -(5.02)	<b>-0.017</b> -(3.74)			<b>-0.014</b> -(5.30)	<b>-0.012</b> -(4.09)		
Change in TED Spread			<b>-0.033</b> -(5.23)	<b>-0.019</b> -(2.68)			<b>-0.021</b> -(4.84)	<b>-0.017</b> -(3.75)			<b>-0.014</b> -(5.04)	<b>-0.011</b> -(3.92)
Lagged TED Spread			<b>-0.046</b> -(4.48)	<b>-0.036</b> -(3.40)			<b>-0.030</b> -(3.92)	<b>-0.020</b> -(2.21)			<b>-0.018</b> -(3.98)	<b>-0.015</b> -(3.14)
Short Volatility Returns		0.295 (0.29)		<b>0.300</b> (3.48)		-0.044 -(0.04)		-0.044 -(0.64)		-0.068 -(0.07)		-0.069 -(1.45)
Beta Spread		0.018 (0.02)		<b>0.020</b> (2.82)		0.025 (0.02)		<b>0.024</b> (2.51)		0.010 (0.01)		<b>0.009</b> (3.54)
Market return		-0.027 -(0.03)		-0.022 -(0.36)		0.009 (0.01)		0.009 (0.22)		0.001 (0.00)		0.001 (0.04)
Lagged BAB return		0.186 (0.19)		<b>0.173</b> (2.86)		0.060 (0.06)		0.060 (1.14)		0.073 (0.07)		0.072 (1.50)
Asset Fixed Effects	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Num of observations	295	295	295	295	4,393	4,393	4,393	4,393	7,271	7,271	7,271	7,271
Adjusted R2	11.2%	20.9%	11.3%	21.3%	1.5%	2.4%	1.5%	2.3%	1.0%	1.9%	1.0%	1.9%

## Conclusion

- High beta = low alpha and SR
- Market neutral *Beta-Against-Beta factor*:
  - Long levered low-beta securities, short high-beta securities
  - Surprisingly high and consistent performance in each of the major global markets and asset classes
    - U.S. stocks
    - Global stocks
    - Treasuries
    - Corporate bonds
    - Futures
- Betas compression and time-varying expected returns on BAB portfolios
  - Market betas compress towards 1 when credit constraints are likely to be binding
  - BAB factors loads on market and has drawdowns when credit is contracting
- Evidence points toward a theory with
  - Certain investors cannot (or are unwilling to) use leverage
  - Other investors subject to margin requirements and funding liquidity risk
- Additional predictions for portfolio selection