Supplementary Appendix for "Attention Allocation over the Business Cycle"

S.1 Proofs of Propositions

S.1.1 Mathematical Preliminaries

Expressing matrices in terms of fundamental variances To determine the effect of changes in aggregate shock variance on dispersion and profits, we need to express some of the matrices in terms of σ_a^{-1}. If we can decompose the matrices into components that depend on σ_a and those that do not, we can differentiate the expressions more easily.

First, we decompose the payoff precision matrices. To do this decomposition, we need to invert Σ. Doing it by hand yields

$$\Sigma^{-1} = \begin{bmatrix} \sigma_1^{-1} & 0 & -b_1\sigma_1^{-1} \\ 0 & \sigma_2^{-1} & -b_2\sigma_2^{-1} \\ -b_1\sigma_1^{-1} & -b_2\sigma_2^{-1} & \sigma_a^{-1} + b_1^2\sigma_1^{-1} + b_2^2\sigma_2^{-1} \end{bmatrix} \tag{S.1}$$

Similarly, the posterior precision matrix for investor j is

$$\hat{\Sigma}_j^{-1} = \begin{bmatrix} \hat{\sigma}_1^{-1} & 0 & -b_1\hat{\sigma}_1^{-1} \\ 0 & \hat{\sigma}_2^{-1} & -b_2\hat{\sigma}_2^{-1} \\ -b_1\hat{\sigma}_1^{-1} & -b_2\hat{\sigma}_2^{-1} & \hat{\sigma_a}^{-1} + b_1^2\hat{\sigma}_1^{-1} + b_2^2\hat{\sigma}_2^{-1} \end{bmatrix} \tag{S.2}$$

It is useful to separate out the terms that depend on σ_a from those that do not. Define

$$S \equiv \begin{bmatrix} \sigma_1^{-1} & 0 & -b_1\sigma_1^{-1} \\ 0 & \sigma_2^{-1} & -b_2\sigma_2^{-1} \\ -b_1\sigma_1^{-1} & -b_2\sigma_2^{-1} & b_1^2\sigma_1^{-1} + b_2^2\sigma_2^{-1} \end{bmatrix} \tag{S.3}$$

and let $\hat{S}$ be the posterior S, meaning that each σ_1 is replaced with the posterior variance $\hat{\sigma}_1$ and each σ_2 is replaced with the posterior variance $\hat{\sigma}_2$.

$$\Upsilon_a \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \Upsilon_1 \equiv \begin{bmatrix} 1 & 0 & -b_1 \\ 0 & 0 & 0 \\ -b_1 & 0 & b_1^2 \end{bmatrix} \tag{S.4}$$

so that $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma_a}^{-1} = \Upsilon_a$ and $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma_1}^{-1} = \Upsilon_1$.

Then,

$$\Sigma^{-1} = S + \sigma_a^{-1}\Upsilon_a \tag{S.5}$$

$$\hat{\Sigma}_j^{-1} = \hat{S} + \hat{\sigma_a}_j^{-1}\Upsilon_a \tag{S.6}$$

Define the average posterior precision matrix when a fraction χ of investment managers have capacity to be $(\bar{\Sigma})^{-1} \equiv \int_j \hat{\Sigma}_j^{-1} dj$. Similarly, let $S^a \equiv \int_j \hat{S}_j dj$ and let $\bar{K}_a$ be the average amount of capacity that an agent devotes to processing aggregate information. For example, if a fraction χ of investors are skilled, and all skilled investors devote all their capacity K to processing aggregate information, $\bar{K}_a = \chi K$. Then,

$$(\bar{\Sigma})^{-1} = S^a + (\sigma_a^{-1} + \chi K^a)\Upsilon_a \tag{S.7}$$

An expression that recurs frequently below is the difference between the precision of an informed manager's posterior beliefs and the precision of the average manager's posterior beliefs. This difference becomes

$$\Sigma^{-1} - (\bar{\Sigma})^{-1} = S - S^a + (1-\chi)K\Upsilon_a \tag{S.8}$$

Second, we decompose the variance matrices. In particular, we need to know average variance, which requires inverting $(\bar{\Sigma})^{-1}$. Replacing σ_a with $(\sigma_a^{-1}+\chi K)^{-1}$, and following the same inversion steps backwards, we get

$$\bar{\Sigma} = (\sigma_a^{-1} + \bar{K}_a)^{-1} bb' + \Phi, \tag{S.9}$$

where b is the 3×1 vector of loadings of each asset on aggregate risk, and if $\bar{K}_1$ and $\bar{K}_2$ represent the average amount of capacity devoted to processing information about assets 1 and 2,

$$\Phi \equiv \begin{bmatrix} (\sigma_1^{-1} + \bar{K}_1)^{-1} & 0 & 0 \\ 0 & (\sigma_2^{-1} + \bar{K}_2)^{-1} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Finally, let $\tilde{\sigma}_a \equiv (\sigma_a^{-1} + \bar{K}_a)^{-1}$.

Portfolio holdings

The optimal portfolio for investor j is

$$q_j = \frac{1}{\rho}\hat{\Sigma}_j^{-1}(\hat{\mu}_j - pr) \tag{S.10}$$

This comes from the first order condition and is a standard expression in any portfolio problem with CARA or mean-variance utility.

Next, compute the portfolio of the average investor. Let the average of all investors' signal precision be $(\bar{\Sigma})^{-1} \equiv \int \hat{\Sigma}_j^{-1} dj$. Use the fact that $\hat{\mu}_j = \hat{\Sigma}_j \Sigma^{-1}\mu + (I - \hat{\Sigma}_j\Sigma^{-1})\eta_j$ and the fact that the signal noise is mean-zero to get that $\int \hat{\Sigma}_j^{-1}\hat{\mu}_j dj = \Sigma^{-1}\mu + ((\bar{\Sigma})^{-1} - \Sigma^{-1})f$. This is true because the mean of all investors' signals are the true payoffs f and because the signal errors are uncorrelated with (but of course, not independent of) signal precision.

$$\bar{q} \equiv \int q_j dj = \frac{1}{\rho}\left(\Sigma^{-1}\mu + ((\bar{\Sigma})^{-1} - \Sigma^{-1})f - (\bar{\Sigma})^{-1}pr\right) \tag{S.11}$$

Using Bayes' rule for the posterior variance of normal variables, we can rewrite this as

$$\bar{q} \equiv \int q_j dj = \frac{1}{\rho}\left(\Sigma^{-1}\mu + (\Sigma_\eta^a)^{-1} f - (\bar{\Sigma})^{-1}pr\right) \tag{S.12}$$

S.1.2 Proof of Proposition 1

Proof. Following Admati (1985), we conjecture that the price vector p is linear in the payoff vector f and the supply vector x: $pr = A + Bf + Cx$. We now verify that conjecture by imposing market clearing

$$\int q_j dj = \bar{x} + x \tag{S.13}$$

Using (S.11) to substitute out the left hand side, and rearranging,

$$pr = -\rho\bar{\Sigma}(\bar{x} + x) + f + \bar{\Sigma}\Sigma^{-1}(\mu - f)$$

Thus, the coefficients A, B, and C are given by

$$A = -\rho\bar{\Sigma}\bar{x} + \bar{\Sigma}\Sigma^{-1}\mu \tag{S.14}$$

$$B = I - \bar{\Sigma}\Sigma^{-1} \tag{S.15}$$

$$C = -\rho\bar{\Sigma} \tag{S.16}$$

which verifies our conjecture. □

S.1.3 Proof of Proposition 2

If aggregate variance is not too high ($\sigma_a \leq 1$), then the marginal value of a given investor j reallocating an increment of capacity from stock-specific shock $i \,\epsilon\, \{1, 2\}$ to the aggregate shock is increasing in the aggregate shock variance: If $K_{aj} = \tilde{K}$ and $K_{ij} = K - \tilde{K}$, then $\partial^2 U / \partial\tilde{K}\partial\sigma_a > 0$.

Proof. From (9) in the main text, we know that expected utility is

$$U_{1j} = \frac{1}{2} trace(\hat{\Sigma}_j^{-1} V_1[\hat{\mu}_j - pr]) + \frac{1}{2} E_1[\hat{\mu}_j - pr]' \hat{\Sigma}_j^{-1} E_1[\hat{\mu}_j - pr]$$

The first step is to work out the variance $V_1[\hat{\mu}_j - pr]$.

$$\hat{\mu}_j - pr = \hat{\Sigma}_j(\Sigma^{-1}\mu + \Sigma_{\eta j}^{-1}\eta_j) - A - Bf - Cx$$

The signal η can be expressed as the true asset payoff f, plus orthogonal signal noise ϵ_j.

$$\hat{\mu}_j - pr = \hat{\Sigma}_j\Sigma^{-1}\mu - A + (\hat{\Sigma}_j\Sigma_{\eta j}^{-1} - B)f + \hat{\Sigma}_j\Sigma_{\eta j}^{-1}\epsilon_j - Cx$$

Since μ and A are known constants and f, ϵ_j, and x are independent, with variances Σ, $\Sigma_{\eta j}$, and σ_x respectively,

$$V_1[\hat{\mu}_j - pr] = (\hat{\Sigma}_j\Sigma_{\eta j}^{-1} - B)\Sigma(\hat{\Sigma}_j\Sigma_{\eta j}^{-1} - B)' + \hat{\Sigma}_j\Sigma_{\eta j}^{-1}\hat{\Sigma}_j + CC'\sigma_x$$

Substituting in for the price coefficients using (S.14), (S.15), and (S.16) yields

$$V_1[\hat{\mu}_j - pr] = (\bar{\Sigma} - \hat{\Sigma}_j)\Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)' + \hat{\Sigma}_j\Sigma_{\eta}^{-1}\hat{\Sigma}_j + \rho^2\bar{\Sigma}\bar{\Sigma}\sigma_x$$

Next, work out the second term by using the expression above for $\hat{\mu}_j - pr$ and taking the expectation: $E[\hat{\mu}_j - pr] = \hat{\Sigma}_j \Sigma^{-1}\mu - A + (\hat{\Sigma}_j \Sigma_{\eta j}^{-1} - B)\mu$. Substituting in the coefficients A and B, and simplifying reveals that $E[\hat{\mu}_j - pr] = \rho\bar{\Sigma}\bar{x}$. Thus,

$$E_1[\hat{\mu}_j - pr]'\hat{\Sigma}_j^{-1}E_1[\hat{\mu}_j - pr] = \rho^2 \bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}$$

Thus, expected utility is

$$U_{1j} = \frac{1}{2}trace\left(\hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)\Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)' + \Sigma_{\eta j}^{-1}\hat{\Sigma}_j + \rho^2\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{\Sigma}\sigma_x\right) + \frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}$$

In the next step, we want to take a cross-partial derivative of utility with respect to σ_a and $\tilde{K}$. To do this, we will substitute out the $\hat{\Sigma}_j^{-1}$ terms using (S.5), (S.6), and (S.7). Then, we will use the fact that, by the chain rule, $\partial U/\partial\tilde{K} = \partial U/\partial K_{aj} - \partial U/\partial K_{ij}$. Therefore, $\partial^2 U/\partial\tilde{K}\partial\sigma_a = \partial^2 U/\partial K_{aj}\partial\sigma_a - \partial^2 U/\partial K_{ij}\partial\sigma_a$ We consider each of these two cross-partial derivatives separately.

Part a: *The marginal value of a given investor j having additional capacity K_{aj} devoted to learning about the aggregate shock a is increasing in the aggregate shock variance: $\partial^2 U/\partial K_{aj}\partial\sigma_a > 0$.*

Sign last term: $\frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}$.

Note that K_{aj} appears only in $\hat{\Sigma}_j^{-1}$. Recall that $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma_a}^{-1} = \Upsilon_a$. Since $\hat{\sigma_a}^{-1} = \sigma_a{}^{-1} + K_{aj}$, the chain rule implies that $\partial\hat{\Sigma}_j^{-1}/\partial K_{aj} = \Upsilon_a$. Thus, the last term has derivative $(\rho^2/2)\bar{x}'\bar{\Sigma}\Upsilon_a\bar{\Sigma}\bar{x}$. The only term in this expression that varies in σ_a is $\bar{\Sigma}$. Since $\bar{\Sigma}$ has every entry increasing in σ_a (equation S.9), and $\bar{\Sigma}$ and Υ_a are positive semi-definite matrices, this term has a positive cross-partial derivative $\partial^2/\partial K_j\partial\sigma_a > 0$.

Thus, a sufficient condition for $\partial^2 U/\partial K_j\partial\sigma_a > 0$ is for the trace term to have a positive cross partial derivative. Since the trace of a sum is the sum of the traces, we can break this term up into 3 major parts.

Sign term 3: $Tr(\rho^2\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{\Sigma}\sigma_x)$.

This takes the same form as the term outside the trace. K_{aj} appears only in $\hat{\Sigma}_j^{-1}$. Its derivative is $\partial\hat{\Sigma}_j^{-1}/\partial K_{aj} = \Upsilon_a$. Thus, the term has derivative $\rho^2\Upsilon_a\bar{\Sigma}\bar{\Sigma}\sigma_x$. The only term in this expression that varies in σ_a is $\bar{\Sigma}$. Since $\bar{\Sigma}$ and Υ_a are positive semi-definite matrices, and ρ^2 and σ_x are positive constants, the trace must be positive. Since $\bar{\Sigma}$ has every entry increasing in σ_a (equation S.9), the $\partial Tr(\cdot)/\partial K_{aj}$ is increasing in σ_a. In other words, $\partial^2/\partial K_j\partial\sigma_a > 0$.

Sign term 2: $Tr(\Sigma_{\eta j}^{-1}\hat{\Sigma}_j)$

By Bayes' Law, the signal precision is the difference between the posterior and prior precisions, $\Sigma_{\eta j}^{-1} = \hat{\Sigma}_j^{-1} - \Sigma^{-1}$. Substituting this into the trace term yields $Tr(I - \Sigma^{-1}\hat{\Sigma}_j) = 3 - Tr(\Sigma^{-1}\hat{\Sigma}_j)$, where the 3 comes from the fact that the variance matrices are all (3×3). The $-Tr(\Sigma^{-1}\hat{\Sigma}_j)$ will cancel out with the first term.

Sign term 1: $Tr(\hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)\Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)')$

We can further break this term up into a sum of three parts, which I will call 1a, 1b and 1c.

Term 1a is $Tr(\hat{\Sigma}_j^{-1}\hat{\Sigma}_j\Sigma^{-1}\hat{\Sigma}_j')$, which is equal to $Tr(\Sigma^{-1}\hat{\Sigma}_j')$. This term cancels out the $-Tr(\Sigma^{-1}\hat{\Sigma}_j)$ from term 2.

Term 1b is $-2Tr(\hat{\Sigma}_j^{-1}\hat{\Sigma}_j\Sigma^{-1}\bar{\Sigma}')$, which is equal to $-2Tr(\Sigma^{-1}\bar{\Sigma}')$. This term only depends on prior variance and average posterior variance, not on investor j's information choice. Since it has no K_j in it, its derivative with respect to K_{aj} is 0.

Term 1c is $Tr(\hat{\Sigma}_j^{-1}\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}')$. Investor j's information choice K_{aj} shows up only in $\hat{\Sigma}_j^{-1}$, where $\partial\hat{\Sigma}_j^{-1}/\partial K_{aj} = \Upsilon_a$. Thus $\partial Tr(\cdot)/\partial K_{aj} = Tr(\Upsilon_a\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}')$. Next, replace $\bar{\Sigma}$ with (S.9) and Σ^{-1} with (S.5) and then take the derivative with respect to σ_a^{-1}. That delivers

$$\frac{\partial^2 Tr(\cdot)}{\partial K_{aj}\partial\sigma_a^{-1}} = Tr\left[-2\Upsilon_a(\sigma_a^{-1}+\bar{K}_a)^{-2}bb'((\sigma_a^{-1}+\bar{K}_a)^{-1}bb'+\bar{\Phi})(S+\sigma_a^{-1}\Upsilon_a)\right.$$

$$\left.+\Upsilon_a((\sigma_a^{-1}+\bar{K}_a)^{-1}bb'+\bar{\Phi})((\sigma_a^{-1}+\bar{K}_a)^{-1}bb'+\bar{\Phi})'\Upsilon_a\right]$$

If we want the cross-partial derivative with σ_a to be positive, then we are looking for a negative sign here.

Since $\bar{\Phi}$ only has non-zero entries in the (1,1) and (2,2) spots and Υ_a has only a 1 in the (3,3) entry, $\Upsilon_a\bar{\Phi} = 0$. Thus,

$$\frac{\partial^2 Tr(\cdot)}{\partial K_{aj}\partial\sigma_a^{-1}} = (\sigma_a^{-1}+\bar{K}_a)^{-2}Tr\left[-2(\sigma_a^{-1}+\bar{K}_a)^{-1}\Upsilon_a bb'bb'(S+\sigma_a^{-1}\Upsilon_a)+\Upsilon_a bb'bb'\Upsilon_a\right]$$

Since Υ_a, bb' and S are all positive semi-definite matrices, their product must have a positive trace and $-2Tr((\sigma_a^{-1}+\bar{K}_a)^{-1}\Upsilon_a bb'bb'S) < 0$. That leaves $Tr(\Upsilon_a bb'bb'\Upsilon_a)$, which is positive since Υ_a and bb' are positive semi-definite, times a constant $1-2\sigma_a^{-1}/(\sigma_a^{-1}+\bar{K}_a)$. This is negative or zero as long as $\bar{K}_a \leq \sigma_a^{-1}$.

We can get an alternative sufficient condition for this term to be positive by using the fact that $\Sigma^{-1} = (S+\sigma_a^{-1}\Upsilon_a)$ and therefore $\Upsilon_a = (\Sigma^{-1}-S)\sigma_a$. Thus, we can rewrite

$$\frac{\partial^2 Tr(\cdot)}{\partial K_{aj}\partial\sigma_a^{-1}} = (\sigma_a^{-1}+\bar{K}_a)^{-2}Tr\left[\Upsilon_a bb'bb'((\Sigma^{-1}-S)\sigma_a - 2\frac{1}{\sigma_a^{-1}+\bar{K}_a}\Sigma^{-1}\right]$$

The term $Tr[-\Upsilon_a bb'bb'S\sigma_a]$ is negative, as before. The remaining term is $Tr[\Upsilon_a bb'bb'\Sigma^{-1}]$, which is positive, times $(\sigma_a - 2/(\sigma_a^{-1}+\bar{K}_a)$. This is negative or zero if $\bar{K}_a \geq \sigma_a$.

If $\sigma_a \leq 1$, then one of the sufficient conditions for the last term to have $\partial^2/\partial K_{aj}\partial\sigma_a > 0$ is always satisfied. In sum, when $\sigma_a \leq 1$, all the terms have non-negative cross-partial derivatives and therefore $\partial^2 U/\partial K_j\partial\sigma_a > 0$.

Part b: *The marginal value of a given investor j having additional capacity K_{ij} devoted to learning about stock-specific shock i is constant in the aggregate shock variance: $\partial^2 U/\partial K_{ij}\partial\sigma_a = 0$.*

Without loss of generality, we consider reallocating capacity from the asset 1 shock to the aggregate shock ($i = 1$). The same proof follows if it were asset 2 instead.

Sign last term: $\frac{\rho^2}{2}\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}$.

Note that K_{1j} appears only in $\hat{\Sigma}_j^{-1}$. Recall that $\partial\hat{\Sigma}_j^{-1}/\partial\hat{\sigma_1}^{-1} = \Upsilon_1$. Since $\hat{\sigma_1}^{-1} = {\sigma_1}^{-1} + K_{1j}$, using the chain rule, we get $\partial\hat{\Sigma}_j^{-1}/\partial K_{1j} = \Upsilon_1$. Therefore, $\partial/\partial K_{1j}(\bar{x}'\bar{\Sigma}\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}) = \bar{x}'\bar{\Sigma}\Upsilon_1\bar{\Sigma}\bar{x}$.

Because of the structure of the Υ_1 matrix, it turns out that using (S.4) and (S.9) to multiply out the three matrices $\bar{\Sigma}\Upsilon_1\bar{\Sigma}$ delivers

$$\bar{\Sigma}\Upsilon_1\bar{\Sigma} = \begin{bmatrix} (\sigma_1^{-1}+\bar{K}_1)^{-2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{S.17}$$

Since this has no σ_a term in it and both $\bar{x}$ and ρ are exogenous, the cross-partial derivative $\partial^2/\partial K_{1j}\partial\sigma_a$ of the last terms is zero.

Sign term 3: $Tr(\rho^2\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{\Sigma}\sigma_x)$.

This takes the same form as the term outside the trace. The term inside the trace has derivative $\partial/\partial K_{1j} = \rho^2\bar{\Sigma}\Upsilon_1\bar{\Sigma}\sigma_x$. Since ρ and σ_x are exogenous and $\bar{\Sigma}\Upsilon_1\bar{\Sigma}$ does not depend on σ_a, the derivative of the trace is invariant in σ_a. Its cross-partial derivative $\partial^2/\partial K_{1j}\partial\sigma_a = 0$.

Sign term 2: $Tr(\Sigma_{\eta j}^{-1}\hat{\Sigma}_j)$

As in part a, this term cancels out Term 1a.

Sign term 1: $Tr(\hat{\Sigma}_j^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)\Sigma^{-1}(\bar{\Sigma} - \hat{\Sigma}_j)')$

We can further break this term up into a sum of three parts, which I will call 1a, 1b and 1c.

Term 1a, $Tr(\hat{\Sigma}_j^{-1}\hat{\Sigma}_j\Sigma^{-1}\hat{\Sigma}_j')$, cancels out term 2.

Term 1b is $-2Tr(\hat{\Sigma}_j^{-1}\hat{\Sigma}_j\Sigma^{-1}\bar{\Sigma}')$, which is equal to $-2Tr(\Sigma^{-1}\bar{\Sigma}')$. This term only depends on prior variance and average posterior variance, not on investor j's information choice. Since it has no K_j in it, its derivative with respect to K_{1j} is 0.

Term 1c is $Tr(\hat{\Sigma}_j^{-1}\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}')$. Investor j's information choice K_{1j} shows up only in $\hat{\Sigma}_j^{-1}$, where $\partial\hat{\Sigma}_j^{-1}/\partial K_{aj} = \Upsilon_1$. Therefore, $\partial/\partial K_{1j}(\hat{\Sigma}_j^{-1}\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}') = \Upsilon_1\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}'$. As before, the sparse form of Υ_1 causes the matrix multiplication to turn out neatly. Using (S.1), (S.4) and (S.9) to multiply out the four matrices delivers

$$\Upsilon_1\bar{\Sigma}\Sigma^{-1}\bar{\Sigma}' = \begin{bmatrix} \sigma_1^{-1}(\sigma_1^{-1} + \bar{K}_1)^{-2} & 0 & 0 \\ 0 & 0 & 0 \\ -b_1\sigma_1^{-1}(\sigma_1^{-1} + \bar{K}_1)^{-2} & 0 & 0 \end{bmatrix}. \tag{S.18}$$

The trace of this matrix is $\sigma_1^{-1}(\sigma_1^{-1} + \bar{K}_1)^{-2}$. Since this has no σ_a term in it, the cross-partial derivative $\partial^2/\partial K_{1j}\partial\sigma_a$ is zero.

Since $\partial^2 U/\partial K_{aj}\partial\sigma_a > 0$ and $\partial^2 U/\partial K_{ij}\partial\sigma_a = 0$, the difference of the two terms is positive. Thus, the marginal value of a given investor j reallocating an increment of capacity from shock 1 to the aggregate shock is increasing in the aggregate shock variance: $\partial^2 U/\partial\tilde{K}\partial\sigma_a = \partial^2 U/\partial K_{aj}\partial\sigma_a - \partial^2 U/\partial K_{ij}\partial\sigma_a > 0$. □

S.1.4 Proof of Proposition 3

Part a: *If the average manager has sufficiently low capacity $\chi K < \sigma_a^{-1}$, then an increase in aggregate risk σ_a increases the dispersion of fund portfolios $E[(q_j - \bar{q})'(q_j - \bar{q})]$, where $\bar{q} \equiv \int q_j dj$.*

Proof. Using the optimal portfolio expressions, (S.10) and (S.12), and Bayes' rule ($\hat{\mu}_j = \hat{\Sigma}_j(\Sigma^{-1}\mu + \Sigma_\eta^{-1}\eta_j)$), the difference in portfolios $(q_j - \bar{q})$ is

$$(q_j - \bar{q}) = \frac{1}{\rho}\left[\Sigma_\eta^{-1}\eta_j - (\Sigma_\eta^a)^{-1}f + ((\bar{\Sigma})^{-1} - \hat{\Sigma}_j^{-1})pr\right]$$

where $(\Sigma_\eta^a)^{-1} \equiv \int \Sigma_{\eta j}^{-1} dj$ is the average manager's signal precision.

Next, we need to take into account that signals and payoffs are correlated. To do this, replace the signal η_j with the true payoff, plus signal noise: $\eta_j = f + e_j$,

$$(q_j - \bar{q}) = \frac{1}{\rho}\left[\Sigma_\eta^{-1}e_j + (\Sigma_\eta^{-1} - (\Sigma_\eta^a)^{-1})f + ((\bar{\Sigma})^{-1} - \hat{\Sigma}_j^{-1})pr\right]$$

Bayes' rule for variances of normal variables is $\hat{\Sigma}_j^{-1} = \Sigma^{-1} + \Sigma_\eta^{-1}$. Integrating the left and right sides of this expression over managers j yields $(\bar{\Sigma})^{-1} = \Sigma^{-1} + (\Sigma_\eta^a)^{-1}$. Subtracting one expression from the other yields $\hat{\Sigma}_j^{-1} - (\bar{\Sigma})^{-1} = \Sigma_\eta^{-1} - (\Sigma_\eta^a)^{-1}$. Define $\Delta \equiv \hat{\Sigma}_j^{-1} - (\bar{\Sigma})^{-1}$. Substituting this in and combining terms yields

$$(q_j - \bar{q}) = \frac{1}{\rho}\left[\Sigma_\eta^{-1} e_j + \Delta(f - pr)\right]. \tag{S.19}$$

Now replace pr with $A + Bf + Cx$, where A, B, and C are given by Appendix S.1.2,

$$(q_j - \bar{q}) = \frac{1}{\rho}\left[\Sigma_\eta^{-1} e_j + \Delta((I - B)f - A - Cx)\right] \tag{S.20}$$

Substituting in the coefficients in the pricing equation reveals that $(I - B)\mu - A = \rho\bar{\Sigma}\bar{x}$, that $I - B = \bar{\Sigma}\Sigma^{-1}$, and that $C = -\rho\bar{\Sigma}$.

To work out the expectation of this quantity squared, recognize that this is the square of a sum of one constant and three, independent, mean-zero, normal variables. Since e_j, $f - \mu$ and x are independent, all the cross terms drop out, leaving

$$E[(q_j - \bar{q})'(q_j - \bar{q})] = Tr(\Sigma_{\eta j}^{-1}) + Tr(\bar{\Sigma}\Sigma^{-1}\Delta\Delta\bar{\Sigma}) + \rho^2\bar{x}'\bar{\Sigma}\Delta\Delta\bar{\Sigma}\bar{x} + \rho^2\sigma_x Tr(\bar{\Sigma}\Delta\Delta\bar{\Sigma})$$

The first term depends only on information choice variables. So, holding choices fixed, the partial derivative with respect to σ_a is zero.

For the second term, it is easier to take the partial derivative with respect to σ_a^{-1} and show that it is negative. This is equivalent to showing that the derivative with respect to σ_a is positive. First, use (S.9) to show that $\partial\bar{\Sigma}/\partial\sigma_a^{-1}$ is $-(\sigma_a^{-1} + \bar{K}_a)^{-2}bb'$. Next, use (S.5) to show that $\partial\bar{\Sigma}/\partial\sigma_a^{-1} = \Upsilon_a$. Recall that Δ depends only on information choices, which we hold fixed.

Then, using the product rule, the derivative is the sum of two terms:

$$\frac{\partial}{\partial\sigma_a^{-1}} Tr(\bar{\Sigma}\Sigma^{-1}\Delta\Delta\bar{\Sigma}) = \frac{-2}{(\sigma_a^{-1} + \bar{K}_a)^2} Tr(\bar{\Sigma}\Delta\Delta bb') + Tr(\bar{\Sigma}\Upsilon_a\Delta\Delta\bar{\Sigma})$$

Using (S.9), we can write $\bar{\Sigma}\Upsilon_a = (\sigma_a^{-1} + \bar{K}_a)^{-1}bb'\Upsilon_a + \Phi\Upsilon_a$. Recall that Φ has only non-zero $(1,1)$ and $(2,2)$ entries and the Υ_a has only a non-zero $(3,3)$ entry. Therefore, $\Phi\Upsilon_a = 0$. Letting $\bar{\sigma}_a \equiv (\sigma_a^{-1} + \bar{K}_a)^{-1}$, we can rewrite

$$\frac{\partial}{\partial\sigma_a^{-1}} Tr(\bar{\Sigma}\Sigma^{-1}\Delta\Delta\bar{\Sigma}) = \bar{\sigma}_a Tr(\bar{\Sigma}(\Upsilon_a - 2\bar{\sigma}_a\Sigma^{-1})\Delta\Delta bb')$$

This is negative iff $(\Upsilon_a - 2\bar{\sigma}_a\Sigma^{-1})$ is negative semi-definite. Using (S.5) to rewrite Σ^{-1} as $S + \sigma_a^{-1}\Upsilon_a$ and substituting in reveals that $((1 - 2\bar{\sigma}_a)\Upsilon_a - 2\bar{\sigma}_a S)$ must be negative semi-definite. Since S is a positive semi-definite matrix, a sufficient condition is $1 - 2\bar{\sigma}_a \leq 0$. Substituting back in the definition of $\bar{\sigma}_a$ and rearranging yields $\bar{K}_a \leq \sigma_a^{-1}$. Thus, if $\bar{K}_a \leq \sigma_a^{-1}$, the second term is decreasing in σ_a^{-1} and therefore increasing in σ_a.

<u>Term 3</u>: The product $\partial/\partial\sigma_a^{-1}(\bar{x}'\bar{\Sigma}\Delta\Delta\bar{\Sigma}\bar{x})$ is non-positive iff $\partial/\partial\sigma_a^{-1}(\bar{\Sigma}\Delta\Delta\bar{\Sigma})$ is negative semi-definite. As before Δ is a choice variable, which we hold fixed. $\partial\bar{\Sigma}/\partial\sigma_a^{-1} = -(\sigma_a^{-1} + \bar{K}_a)^{-2}bb'$. Therefore,

$$\frac{\partial}{\partial\sigma_a^{-1}}\bar{\Sigma}\Delta\Delta\bar{\Sigma} = \frac{-2}{(\sigma_a^{-1} + \bar{K}_a)^2}\bar{\Sigma}\Delta\Delta bb'$$

Since $\bar{\Sigma}$, Δ and bb' are positive semi-definite, this is negative semi-definite.

Term 4: As show in the previous step, $\partial/\partial\sigma_a^{-1}(\bar{\Sigma}\Delta\Delta\bar{\Sigma})$ is negative semi-definite. Therefore, the derivative of the trace, which is the trace of the derivative, is negative: $\partial/\partial\sigma_a^{-1}Tr(\bar{\Sigma}\Delta\Delta\bar{\Sigma}) < 0$.

Since all four terms in the expression for dispersion are decreasing in σ_a^{-1}, dispersion is increasing in σ_a. □

Part b: *If the average manager has sufficiently low capacity, $\chi K < \sigma_a^{-1}$, then an increase in aggregate risk, σ_a, increases the dispersion of funds' portfolio returns $E[((q_j - \bar{q})'(f - pr))^2]$.*

Proof. From the previous part (equation S.20), we know that we can use the optimal portfolio expressions, (S.10) and (S.12), and Bayes' rule to express the portfolio difference as a function of three underlying random variables, e_j, $f - \mu$ and x. The $f - pr$ can likewise be expressed as a function of $f - \mu$ and x:

$$(q_j - \bar{q})'(f - pr) = \frac{1}{\rho}\left[\Sigma_\eta^{-1}e_j + \Delta((I - B)f - A - Cx)\right]'((I - B)f - A - Cx) \tag{S.21}$$

Substituting in the price coefficients, this is

$$= \frac{1}{\rho}\left[\Sigma_\eta^{-1}e_j + \Delta\bar{\Sigma}(\rho\bar{x} + \Sigma^{-1}(f - \mu)\rho x)\right]'\bar{\Sigma}(\rho\bar{x} + \Sigma^{-1}(f - \mu)\rho x) \tag{S.22}$$

Since a linear combination of two normal variables is also a normal variable, we can write $\bar{\Sigma}(\Sigma^{-1}(f - \mu)\rho x) = Vz$ where $z \sim N(0, I)$ and $V \equiv \bar{\Sigma}\Sigma^{-1/2} + \rho\sigma_x^{1/2}\bar{\Sigma}$. Likewise, we can use a shorthand for the constant term $w \equiv \rho\bar{\Sigma}\bar{x}$. Then, dispersion in fund profits becomes

$$E[((q_j - \bar{q})'(f - pr))^2] = E\left[\left(\frac{1}{\rho}[\Sigma_\eta^{-1}e_j + \Delta Vz + \Delta w]'(w + Vz)\right)^2\right] \tag{S.23}$$

Only terms with even powers of the normal variables are non-zero. This leaves

$$= \frac{1}{\rho^2}\left[\Sigma_\eta^{-1} + (w'\Delta w)^2 + w'w(Tr(V\Delta\Delta V) + 4Tr(V\Delta V)w'\Delta w + Tr(VV)w'\Delta\Delta w + Tr(\Delta VVVV\Delta) + Tr(\Delta VV)^2)\right] \tag{S.24}$$

where the last two terms come from the expectation of a multivariate normal variable (z), raised to the fourth.

To sign the partial derivative of dispersion, with respect to σ_a, we proceed term-by-term. The first term Σ_η^{-1} depends only on choice variables, which we hold fixed. Similarly, Δ is also choice variables.

The constant w is $\rho\bar{\Sigma}\bar{x}$, where ρ and $\bar{x}$ are positive constants and every entry of $\bar{\Sigma}$ is increasing in σ_a. Therefore, all the $w'w$ and are increasing in σ_a. Furthermore, since wherever Δ appears, it shows up twice, whether it is positive or negative makes no difference. If it is negative, the two negative signs cancel. Thus, it remains to be shown that $\partial V/\partial\sigma_a$ is a positive semi-definite matrix. If so, its trace will be positive.

Recall that $V \equiv \bar{\Sigma}\Sigma^{-1/2} + \rho\sigma_x^{1/2}\bar{\Sigma}$. The second term is increasing in σ_a because every term of $\bar{\Sigma}$ is increasing in σ_a and $\rho\sigma_x^{1/2}$ is a positive constant. Specifically, $\partial\bar{\Sigma}/\partial\sigma_a = \sigma_a^{-2}/(\sigma_a^{-1} + \bar{K}_a)^2 bb'$, which is positive semi-definite.

The partial derivative of the first term is $\sigma_a^{-2}/(\sigma_a^{-1} + \bar{K}_a)^2 bb'\Sigma^{-1/2} - 1/2\sigma_a^{-2}\Upsilon_a\Sigma^{1/2}$. This is positive semi-definite if $1/(\sigma_a^{-1} + \bar{K}_a)^2 bb' - 1/2\Upsilon_a\Sigma$ is positive semi-definite. The trace of this is $(b_1^2 + b_2^2 + 1)/(\sigma_a^{-1} +$

$\bar{K}_a)^2 - 1/2\sigma_a$. This is positive if $\sigma_a(\sigma_a^{-1} + \bar{K}_a)^2 < 2b'b$.

□

S.1.5 Proof of Proposition 4

If some managers are uninformed $\chi < 1$, but all informed managers learn about aggregate risk, and the average manager has sufficiently low capacity $\chi K < \sigma_a^{-1}$, then an increase in aggregate risk σ_a increases the expected profit of an informed fund, $E[(q_j - \bar{q})'(f - pr)]$, where $\bar{q} \equiv \int q_j dj$.

Proof. Assume that all informed investors use their capacity K to learn about the aggregate risk. We show that when σ_a^{-1} falls (in recessions), that expected excess returns of the informed traders rise.

Begin by taking the expectation of (S.20) to get expected profits. Since the supply shocks and the signal noise are mean-zero and independent of all other shocks, we can take their expectations separately. Using the formula for the expectation of a chi-square variable,

$$E[(q_j - \bar{q})'(f - pr)] = \frac{-\sigma_x}{\rho} Tr[C'(1-\chi)K\Upsilon_a C] + \frac{(1-\chi)K}{\rho} E\left\{((I-B)f - A)'\Upsilon_a'((I-B)f - A)\right\} \quad \text{(S.25)}$$

Since $((I - B)f - A)$ is normally distributed, the remaining expectation is also the mean of a chi square

$$\begin{aligned} E[(q_j - x - \bar{x})'(f - pr)] &= \frac{\sigma_x(1-\chi)K}{\rho} Tr[C'\Upsilon_a C] + \frac{(1-\chi)K}{\rho}((I-B)\mu - A)'\Upsilon_a((I-B)\mu - A) \\ &+ \frac{(1-\chi)K}{\rho} Tr[(I-B)'\Upsilon_a \Sigma (I-B)] \end{aligned}$$

Finally, substitute in for A, B, and C from (S.14), (S.15), and (S.16).

$$E[(q_j - x - \bar{x})'(f - pr)] = (1-\chi)K\left\{\sigma_x \rho Tr[\bar{\Sigma}\Upsilon_a\bar{\Sigma}] + \rho\bar{x}'\bar{\Sigma}\Upsilon_a\bar{\Sigma}\bar{x} + \frac{1}{\rho}Tr[\bar{\Sigma}\Sigma^{-1}\Upsilon_a\bar{\Sigma}]\right\}$$

$\bar{\Sigma}$ is equal to $1/(\sigma_a^{-1} + \chi K)bb'$ in its 3rd column and 3rd row entries, which are the only entries that the Υ_a matrix does not zero out. Therefore, $\bar{\Sigma}\Upsilon_a\bar{\Sigma} = (1/(\sigma_a^{-1} + \chi K))^2 bb'\Upsilon_a bb'$, and

$$E[(q_j - x - \bar{x})'(f - pr)] = (1-\chi)K\left\{\frac{\sigma_x\rho}{(\sigma_a^{-1} + \chi K)^2} Tr[bb'\Upsilon_a bb'] + \frac{\rho}{(\sigma_a^{-1} + \chi K)^2}\bar{x}'bb'\Upsilon_a bb'\bar{x} + \frac{1}{\rho}Tr[\bar{\Sigma}\Sigma^{-1}\Upsilon_a\bar{\Sigma}]\right\}$$

Since $bb'\Upsilon_a bb'$ does not depend on σ_a, but is positive semi-definite, and $(1/(\sigma_a^{-1} + \chi K))^2$ is increasing in σ_a, the first two terms are increasing in σ_a.

The last term can be rewritten using the relationship that $\Sigma^{-1} = S + \sigma_a^{-1}\Upsilon_a$.

$$Tr[\bar{\Sigma}\Sigma^{-1}\Upsilon_a\bar{\Sigma}] = Tr[\bar{\Sigma}S\Upsilon_a\bar{\Sigma}] + \sigma_a^{-1}Tr[\bar{\Sigma}\Upsilon_a\Upsilon_a\bar{\Sigma}]$$

Note that $\bar{\Sigma}$ and S are both positive-definite matrices and therefore have positive eigenvalues. Even Υ_a has non-negative eigenvalues. Since the trace is the sum of the eigenvalues and sums and products of non-negative eigenvalues are non-negative, the first term is positive. Furthermore, $\bar{\Sigma}$ has every entry increasing in σ_a. Therefore, the first trace term is increasing in σ_a.

In the second trace term, the matrix $\Upsilon_a \Upsilon_a = \Upsilon_a$. Using the value derived for $Tr[\bar{\Sigma}\Upsilon_a\bar{\Sigma}]$ above, we can rewrite the remaining term as

$$\sigma_a^{-1} Tr[\bar{\Sigma}\Upsilon_a\Upsilon_a\bar{\Sigma}] = \frac{\sigma_a^{-1}}{(\sigma_a^{-1} + \chi K)^2} Tr[bb'\Upsilon_a bb']$$

This is increasing in σ_a if $\partial/\partial\sigma_a^{-1}(\sigma_a^{-1}/(\sigma_a^{-1} + \chi K)^2) < 0$, which is true if

$$\frac{\partial}{\partial\sigma_a^{-1}} \frac{\sigma_a^{-1}}{(\sigma_a^{-1} + \chi K)^2} = \frac{\chi K - \sigma_a^{-1}}{(\sigma_a^{-1} + \chi K)^3} < 0$$

Thus,

$$\chi K < \sigma_a^{-1}$$

is a sufficient, but not a necessary, condition for profits to be increasing in σ_a.

□

S.2 Model Simulation

In this Section, we use a numerical example to illustrate the model's predictions for the same measures of attention, portfolio dispersion, and performance as the ones we measure in the data. The goal of this exercise is to confirm that the model makes the same qualitative predictions for these observables as for the slightly different measures of attention allocation, portfolio dispersion, and fund performance for which we formally proved our propositions. Notably, we do not attempt to quantitatively account for all time-series and cross-sectional moments of actively managed fund portfolio holdings and returns. Such a task would be beyond the scope of this paper and indeed beyond the current state of the literature. Our model is too stylized along many dimensions to deliver on such a task. For example, it has only three assets and no heterogeneity in risk aversion, prior beliefs, or initial wealth among funds, and no heterogeneity in information capacity among skilled managers. Adding such features could improve the predictions, but only at the cost of obscuring the main mechanisms operating in the model.

S.2.1 Parameter Choices

The following explains how we choose the parameters of our model. The simplicity of the model prevents a full calibration. Instead, we pursue a numerical example that matches some salient properties of stock return data. Our benchmark parameter choices are listed in Table S.1. Section S.2.3 below shows that the qualitative results are robust to a wide range of parameter choices.

Our procedure is to simulate 3000 draws of the shocks $(x_1, x_2, x_c, s_1, s_2, a)$ in recessions and 3000 draws of the shocks in expansions. Since our model is static, each simulation is best interpreted as different draws of a random variable, and not as a period (months). The model's recessions differ from expansions in *two respects*.

First, the variance of the aggregate payoff shock σ_a is higher. It is set to replicate the fact the market return volatility is about 25% higher in recessions than in expansions. In the numerical example, the volatility of the market return is 4.0% in expansions and 5.0% in recessions, straddling the observed market return volatility of 4.5%. Setting the variance of the asset supply vector $\sigma_x = .05^2\bar{x}$ allows us to match this level of market return volatility.

Second, recessions are also characterized by lower *realized* stock market returns (despite high expected returns). In order to generate lower realized market returns and higher expected returns in a static model, we have to assume that agents are surprised by unexpectedly low returns in recessions. We accomplish this in the numerical example by replicating the bottom $m = 2.5\%$ of market return realizations among the 3000 simulations of the model in recessions, in effect simulating the economy in recessions for $3000 * (1 + .025) > 3000$ draws. This choice for m is conservative because the 0.03% difference between market returns in expansions (0.87% per month) and recessions (0.84% per month) it generates is lower than the 0.20% per month difference in the data. In the robustness section below, we consider a case that generates a 0.20% return difference. The results are qualitatively and quantitatively similar.

To get the average market return right, we choose the mean of asset payoffs μ (equal for all assets) and the coefficient of absolute risk aversion, ρ, to achieve an average equilibrium market return of about 0.85% per month.

We think of assets 1 and 2 as two large industries and the composite asset as summarizing all other industries. Therefore, we normalize the mean asset supply of assets 1 and 2 to 1, and set the supply of the composite asset, $\bar{x}_c$, to 7. The variance of the firm-specific shocks is chosen to match the fact that individual industry returns are about 30% more volatile than the market return over our sample from 1980 to 2005. We use data from the 30 industry portfolios of Fama and French (1997). In the example, the average volatility of assets 1 and 2 is 6.5% in recessions and 5.8% in expansions, 29% and 45% higher than that of the market return. This choice matches the proportion of the average industry's return variance that is idiosyncratic. We choose the asset loadings on the aggregate payoff shock, b_1 and b_2, to be different from each other so as to generate some spread in asset betas. The chosen values generate average market betas of 0.9 and a dispersion in betas of 33%. This is reasonably close to the average beta of 0.95 and the dispersion of 23% for the 30-industry portfolios.

We set the average risk-free rate equal to 0.22% per month, the average of the 1-month yield minus inflation in our sample. We set initial wealth, W_0, to generate average holdings in the risk-free asset around 0%.

For simplicity, we set capacity K for skilled investment managers equal to 1. This implies that learning can increase the precision of one of the idiosyncratic shocks (or the aggregate shock) by 25% (by 18%). We will vary K in our robustness exercise below. Likewise, we have no strong prior on the fraction of skilled funds, χ. In our benchmark, we set it equal to 20%, and we will vary it for robustness. The model is simulated for 800 investors, of which 175 are skilled (20%). We assume that 20% of all investors are non-investment managers ("other investors"). The unskilled managers (60% of the populations) and other investors differ in name only. We note that the parameter conditions in Propositions 2 through 4 are satisfied by these parameter choices.

As in our empirical work on mutual funds in Section 2, we compute all statistics of interest as equally-weighted averages across all investment managers (i.e., without the 20% other investors). We also report results separately for skilled and unskilled investment managers.

S.2.2 Main Simulation Results

Every skilled manager ($K > 0$) solves for the choice of signal precisions $\tau_{aj}^{-1} \geq 0$ and $\tau_{1j}^{-1} \geq 0$ that maximize time-1 expected utility (9). We assume that these choice variables lie on a 25×25 grid in $\mathbb{R}_+^2$. The signal precision choice $\tau_{2j}^{-1} \geq 0$ is implied by the capacity constraint (6).

We simulate a sequence of $T = 3000$ draws (months) for the random variables in each of the recession and expansion states, as explained above. We form a $T \times 1$ time series for the three individual asset returns, for the market return, for each fund's return, and for each fund's (and the market's) portfolio weights in each asset. For each asset i, we then estimate a CAPM regression of the asset's excess return on the market excess return. This delivers the asset's CAPM beta, β_i; one value in expansions and one in recessions. We define the systematic component of returns as $\beta_i R_t^m$, for $t = 1, \cdots, T$ and $i = 1, 2, 3$. Stacking the different is and ts results in a $3T \times 1$ vector of systematic returns. Similarly, we define the idiosyncratic return as $R_t^i - \beta_i R_t^m$.

To compute RAI in equation (10) for fund j, we stack its portfolio weights in deviation from the market's weights for the three assets and the T draws into a $3T \times 1$ vector. We also create a $3T \times 1$ vector of aggregate shocks by stacking three identical repetitions of each aggregate shock realization a. We calculate RAI as the covariance between these two variables. Likewise, we form $Timing$ in equation (11) as the covariance between the time series of portfolio weights, in deviation from the market's weights, and the systematic component of returns. The procedure delivers one RAI and one $Timing$ measure per fund in recessions and one set of measures in expansions. We multiply RAI by 1000 and $Timing$ by 10,000 because the aggregate shocks are an order of magnitude larger than the systematic returns.

Table S.2 summarizes the predictions of the model for the main statistics of interest. The left panel shows the results for recessions, while the right panel shows the results for expansions. In each panel, we present three columns. Column *skilled* reports the equally weighted average of the statistic in question for the group of skilled investors (20% of investors have $K > 0$ in our benchmark parametrization). Column *unskilled* is the equally weighted average across the unskilled funds (60% of investors are unskilled investment managers). Column *all* is the equally weighted average across all funds (80% of investors). The 20% unskilled other investors are excluded from the table because we do not observe them in the data. However, the model's predictions for this group are identical to those for the unskilled funds. These two groups differ in name only.

Rows 1 and 2 of Table S.2 show that RAI and $Timing$ are higher for skilled investors in recessions (left panel) than in expansions (right panel). Because of market clearing, unskilled investors are the flip side of the skilled ones, their RAI and $Timing$ measures are negative. Since no investors learn about the aggregate shock in expansions, RAI and $Timing$ are essentially zero for both skilled and unskilled. The net effect of the skilled and unskilled is listed in Column *all*. This combination of all investment managers, skilled and unskilled, is what we have data on. Hence, the first testable implication of the model is that RAI and $Timing$ should be higher for all funds in recessions than in expansions.

In a similar fashion, we construct RSI measure, defined in equation (12), and the stock-picking measure $Picking$, defined in equation (13). That is, we stack the stock-specific shocks, s_i, the idiosyncratic returns, $R_t^i - \beta_i R_t^m$, and the fund's portfolio weights into $3T \times 1$ vectors and compute the respective covariances. Rows 3 and 4 summarize the predictions of the model for RSI (multiplied by 1000) and $Picking$ (multiplied by 10,000). Across all funds (skilled and unskilled), the model predicts lower RSI and $Picking$ in recessions.

Skilled funds have a high *RSI* and *Picking* ability in expansions, when they allocate their attention to stock-specific information. Unskilled investors exhibit a negative *Picking* in expansions for the same reason that they have a negative *Timing* in recessions: Price fluctuations induce them to buy when returns are low and sell when returns are high. The *RSI* and *Picking* measures are close to zero for all investors in recessions. Hence, the second testable implication of the model is that *RSI* and *Picking* should be lower for all funds in recessions than in expansions.

Next, we turn to the measures of portfolio and return dispersion. Row 5 of Table S.2 shows the results for the *Concentration* measure, defined in equation (15), in our numerical example. We calculate $Concentration^j$ for fund j by stacking all squared deviations of fund j's portfolio from the market portfolio into a $3T \times 1$ vector, and by summing over its entries, and dividing by T. We obtain one number for recessions and one for expansions. We find that *Concentration* is higher for all funds in recessions than in expansions. This increase is driven entirely by the informed; the uninformed are all holding the exact same portfolio because of common prior beliefs.

More concentrated portfolios are also less diversified. For each fund j, we estimate CAPM regression (16) by regressing the fund's excess return on the market's excess return. This delivers the fund's α^j, β^j, and σ_ε^j. We use the idiosyncratic risk σ_ε^j as our second measure of portfolio dispersion. If all funds held the market portfolio, their idiosyncratic risk would be zero, and there would be zero cross-sectional dispersion. In simulation, the skilled funds take on more idiosyncratic risk than the unskilled ones, and more in recessions than in expansions. As a result, idiosyncratic risk is higher in recessions than in expansions for all funds.

Rows 7 through 9 report the results for the dispersion across funds' abnormal returns, CAPM alphas, and CAPM betas. All three metrics show increasing dispersion in recessions, driven largely by the heterogeneity in the choices of the skilled investors.

Finally, we study performance measures. Rows 10 and 11 of Table S.2 show that skilled investment managers have large excess returns, as measured by abnormal fund returns or fund alphas ($R^j - R^m$ and α^j), at the expense of the uninformed. The average investment manager has a slightly higher alpha in recessions than in expansions. While quantitatively modest (4.6bp per month or 55bp per year), the positive difference in average alphas between recessions and expansions is a robust finding of the model.

S.2.3 Robustness of Simulation Results

This section discusses the robustness of the model to alternative parameter choices. We conduct several experiments in which we vary one key parameter at a time, while holding all other parameters fixed at their benchmark levels. Table S.3 summarizes these robustness checks. For brevity, we only report the results averaged over all investment managers and omit the results broken out for skilled and unskilled managers, separately. We find that none of the comparative statics are sensitive to variation in the key parameters of the model.

In our benchmark model, we assume that 20% ($\chi = .20$) of investors are skilled mutual funds (60% are unskilled mutual funds and 20% unskilled other investors). We first study two different values for the fraction of skilled investment managers: $\chi = 10\%$ and $\chi = 30\%$. When there are fewer skilled funds, they have a comparatively larger advantage over the unskilled. This results in investment choices that exploit their informational advantage more aggressively. *Timing* for the skilled increases from 156 to 210 in recessions while their *Picking* reading in expansions increases from 160 in the baseline to 181. At the same time, there are fewer skilled investors exploiting more unskilled investors than in the baseline, so that the unskilled investors have less negative average $Timing$ values in recessions and less negative $Picking$ values in expansions. As a result, the $Timing$ value in recessions and $Picking$ value in expansions, averaged across across *all* investment managers (80% of the investor population), fall relative to the benchmark (from 9.9 to 5.8). Similarly, RAI increases in recessions for all funds and RSI decreases, but the changes are smaller than in the benchmark case. Likewise, our measures of portfolio dispersion continue to be higher in recessions than in expansions, but all dispersion levels are somewhat lower than before. The reason is that there is no dispersion among the unskilled, and there are more of them than in the benchmark. Finally, the performance results remain intact as well. The skilled investors make higher abnormal returns and alphas than in the benchmark, which means the unskilled loose more in total. However, they loose less per unskilled investor. As a result, alphas averaged across all funds are lower than in the benchmark: 18.6bp per month in recessions (versus 35.3bp) and 15.6bp in expansions (versus 30.7bp).

The opposite effects occur when we increase the fraction of skilled investors to 30 percent. The increase in RAI and $Timing$ and the decrease in RSI and $Picking$ in recessions are larger than those in the benchmark model. The same is true for portfolio dispersion and performance. For example, the average alpha is now 50.6bp per month in recessions and 45.4bp in expansions; the difference is slightly higher than in the benchmark. In expansions, all skilled investors continue to learn about the stock-specific information. In recessions, about 70% of attention is allocated to the aggregate shock in recessions and 15% to each of the stock-specific shocks. This 70% is lower than the 87% of skilled managers who learn about the aggregate shock in recessions in our benchmark parametrization. This is a general equilibrium effect, which we label *strategic substitutability.* When many informed investors learn about the aggregate shock, and buy assets that load heavily on that shock, they push up the price of these assets, making it less desirable to learn about for other informed investors ceteris paribus. This leads some to learn about the stock-specific shocks instead. Hence, the higher average RSI of the informed in recessions compared to the benchmark. Why is the reverse not happening in expansions? Because the volatility of the aggregate shock is low enough in expansions that it turns out not to be optimal for any of the 30% informed investors to deviate from the full attention allocation to the idiosyncratic shocks.

The second variational experiment is to decrease and increase the amount of attention allocation capacity K that skilled investors have. In our benchmark, $K = 1$, which amounts to the ability to increase the precision on any one signal by 25% of the prior precision of the stock-specific information through learning. We now consider $K = .5$ and $K = 2$. When the 20% of skilled have twice as much capacity, their RAI and $Timing$ increase substantially in recessions ($Timing$ goes up from 157 in the benchmark to 220), and their RSI and $Picking$ increase in expansions ($Picking$ goes up from 160 in the benchmark to 311). In contrast to the previous exercise, the $Timing$ measure for the unskilled becomes more negative in recessions and their $Picking$ more negative in expansions than in the benchmark. The reason is that there are as many unskilled as in the benchmark, but they are now at a larger informational disadvantage. The net effect of the skilled and the unskilled is an increase in $Timing$ in recessions from 9.9 in the benchmark to 14.0. Likewise, $Picking$ in expansions increases from 10.0 to 19.5. Giving 30% of investors $K = 1$ has similar effects as giving 20% of investors $K = 2$. Portfolio dispersion increases substantially with higher K. The result is driven by the more concentrated portfolios of the skilled, which creates both more dispersion among the skilled and a bigger difference with the unskilled. The skilled investors make abnormal returns and alphas that are about twice as high as those in the benchmark, and the unskilled loose about twice as much. The net effect are average fund alphas that are substantially higher than in the benchmark: 67.2bp per month in recessions (versus 35.3bp) and 59.3bp in expansions (versus 30.7bp). The opposite happens when we lower K to 0.5.

We recall that recessions in the model are periods with not only a higher variance of the aggregate shock, but also with lower realized market returns. We implement the latter by first simulating the model in recessions for 3000 periods, then taking the bottom m% of return realizations, and adding them to the 3000 draws when calculating the moments of interest. In our third robustness check we verify how robust our results are to different values for m. We explore $m = 0$ and $m = 0.08$, while our benchmark is $m = 0.025$. When $m = .08$, realized market returns are 22 basis points per month lower in recessions than in expansions (0.54 versus 0.76% per month). This corresponds to the return difference in the data. The results for $Timing$, $Picking$, RAI, and RSI are slightly stronger, but the magnitudes are quite close to the benchmark. The same is true for all dispersion measures, except for the beta dispersion. The latter is quite a bit lower in recessions than in the benchmark (3.91 instead of 6.37), driven by a reduction in the beta dispersion of the skilled. Because of the lower returns in recessions, skilled managers have both lower betas and less differences in their betas compared to the unskilled in recessions. Finally, the performance results are similar to the benchmark. Alphas are slightly higher than in the benchmark: 38.5bp per month in recessions (versus 35.3bp) and 31.6bp in expansions (versus 30.7bp). The difference between recessions and expansions grows to 7bp per month.

The case of $m = 0$ corresponds to a world in which assets have realized payoffs that are symmetrically distributed around the same mean in expansions and in recessions. However, because recessions are times in which returns are more volatile, *expected* (and unconditional average) returns must be higher to compensate the investors for bearing higher risk. In particular, the average market return is 1.30% in recessions and 0.95% in expansions. The results on the fund moments are opposite from the case with higher m, but still quantitatively similar to our benchmark case. For example, the difference in average alphas between recessions and expansions is 4.1bp per month compared to 4.6bp in the benchmark.

Table S.1: **Numerical Example**

The first column lists the parameter in question, the second column is its symbol, the third column lists its numerical value, and the last column briefly summarizes how we chose that value.

Parameter	*Symbol*	*Value*	*How Chosen?*
CARA	ρ	0.525	*Asset return mean*
mean of payoffs 1,2,c	μ_1, μ_2, μ_c	10, 10, 10	*Asset return mean*
variance aggr. payoff comp. a	σ_a	0.1225 (E), 0.2625 (R)	*Market return vol in expansions vs. recessions*
variance idio. payoff comp. s_i	σ_i	0.25	*Asset return vol vs. market return vol*
a-sensitivity of payoffs	b_1, b_2	0.25,0.50	*Asset beta level + dispersion*
mean asset supply 1,2	$\bar{x}_1 = \bar{x}_2$	1,1	*Normalization*
mean asset supply 1,2	$\bar{x}_c$	7	*Asset return volatility*
variance asset supply	σ_x	$(.05 * \bar{x})^2$	*Asset return idio vol*
risk-free rate	r	0.0022	*Average T-bill return*
initial wealth	W_0	90	*Average cash position*
difficulty learning aggr. info	ψ	1	*Simplicity*
information capacity	K	1	
skilled fraction	χ	0.20	

Table S.2: **Benchmark Simulation Results from the Model**

This table provides the main statistics for a simulation of the model under the benchmark parameter values summarized in Table S.1. Panel A reports moments related to attention allocation, Panel B reports the moments related to portfolio dispersion, and Panel C reports moments related to performance. The first column lists the moments in question, as defined in the main text, the next three columns report the predictions for the model simulated in a recession, the last three columns report the results for the model simulated in an expansion. All moments are generated from a simulation of 3,000 draws and 800 investors. For both recessions and expansions, we list the equally-weighted average across *all* investment managers (the 20% skilled and the 60% investment managers), and separately for the *skilled* and the *unskilled* investment managers.

	Recessions			**Expansions**		
	All managers	Skilled	Unskilled	All managers	Skilled	Unskilled
Panel A: Attention Allocation						
1. RAI	10.55	162.54	-40.11	0.07	0.94	-0.22
2. Timing	9.91	155.78	-38.72	0.06	3.31	-1.02
3. RSI	2.15	33.46	-8.28	15.61	249.72	-62.42
4. Picking	1.66	25.84	-6.40	10.02	160.11	-40.01
Panel B: Dispersion						
5. Concentration	3.75	13.15	0.00	3.12	11.48	0.00
6. Idiosyncratic volatility	5.09	15.21	1.72	4.33	13.50	1.28
7. Dispersion in abnormal return	3.54	10.05	1.36	3.37	9.82	1.22
8. Dispersion in CAPM alpha	2.52	5.05	1.68	2.28	4.55	1.52
9. Dispersion in CAPM beta	6.37	13.20	4.09	1.46	4.30	0.51
Panel C: Performance						
10. Abnormal return	0.346	5.471	-1.363	0.302	4.867	-1.220
11. CAPM Alpha	0.353	5.401	-1.330	0.307	4.861	-1.211

S.2.4 Endogenous Capacity Model

Finally, we consider an extended model in which skilled managers can freely choose not only how to allocate their information processing capacity, but also how much capacity to acquire. We let the cost of acquiring K units of capacity be $\mathcal{C}(K)$. Each skilled fund solves for the choice of signal precisions $\tau_{aj}^{-1} \geq 0$ and $\tau_{1j}^{-1} \geq 0$, and capacity K that maximize time-1 expected utility, as in (9) but adjusted for a penalty term $-\mathcal{C}(K)$. In our numerical work, we assume that these choice variables lie on a 25×25 grid in $\mathbb{R}_+^3$. The choice of signal precision $\tau_{2j}^{-1} \geq 0$ is implied by the capacity constraint (6).

In our numerical exercise, we consider two different functional forms for $\mathcal{C}(K)$. The first one is $\mathcal{C}_1(K) = c_1 \exp(K)$ and the second one is $\mathcal{C}_2(K) = c_2 K^{\psi}$. For ease of comparison with our exogenous K results, we choose the scalars c_1 and c_2 such that the optimal capacity choice is $K = 1$ on average across expansions and recessions. This is the same capacity choice we assume in our benchmark parametrization. Clearly, increasing (lowering) the scalars c_1 and c_2 will lead to lower (higher) optimal capacity choice. These scalars can be interpreted as (shadow) prices of capacity. All other parameters are the same as in our benchmark model.

More interesting than the level of K that is chosen is how that choice differs between recessions and expansions. We find that for both cost functions, investors acquire more capacity in recessions than in expansions. Nothing in the cost function makes it cheaper to acquire capacity in either expansions or recessions. This result is solely driven by the fact that the higher (aggregate) uncertainty in recessions makes it optimal to acquire more capacity and to allocate it to the aggregate shock. This extensive-margin effect acts as an amplification to our intensive-margin effect. How elastic capacity choice is to changes in prior aggregate uncertainty, and hence how large the amplification effect is, *does* depend on the functional form of the cost function. For cost function 1, we find that capacity choice is 1.02 in recessions and 0.97 in expansions. For cost function 2, the elasticity is much higher, with a capacity choice of 1.15 in recessions and 0.92 in expansions. The reason for the higher elasticity is that the marginal cost function 2 is less steep in capacity. As a result, a given change in the marginal benefit of acquiring information leads to larger equilibrium changes in capacity. Since we have no strong prior over the functional form, we conduct our numerical simulation for both cost functions.

Table S.4 summarizes the main moments of interest for the endogenous K model, alongside the benchmark, exogenous K results. For brevity, we only report the results averaged over all investment managers and omit the results broken out for skilled and unskilled managers, separately. Overall, we find that the results are very similar to those in our exogenous K model, not only qualitatively, but also quantitatively. The moments for cost function 2 (two most right columns) tend to be higher in recessions than do the benchmark numbers, and lower in expansions. Hence, there is amplification of the difference between recessions and expansions. For example, average fund alphas are somewhat higher than in the benchmark in recessions (40.7bp per month versus 35.3bp) and somewhat lower in expansions (27.7bp versus 30.7bp). The resulting difference between recessions and expansions grows substantially from 4.6bp to 13bp per month. For cost function 1 (two middle columns), the moments are slightly higher in recessions since the skilled investment managers choose to acquire slightly more capacity than what they are endowed with in the benchmark ($K = 1.02$ versus 1). The moments are slightly lower in expansions, since they have slightly lower capacity ($K = 0.97$ versus 1). Overall, the difference in our key variables between recessions and expansions is usually very similar to that in our benchmark model.

Table S.3: **Robustness of Predictions of the Model**

Panel A reports moments related to attention allocation, Panel B reports the moments related to dispersion, and Panel C reports moments related to performance. The first column lists the moments in question, as defined in the main text. The other pairs of columns report results for the benchmark parameters and for six robustness exercises. In each pair of columns, the first column reports the predictions for the model simulated in a recession and the second column for the model simulated in an expansion. All moments are generated from a simulation of 3000 draws and 750 investors. For both recessions and expansions, we list the equally weighted average across all investment managers (the 20% skilled and the 60% investment managers). The parameters are the same as in the benchmark model, except for the parameter listed in the first row.

	baseline		$\chi = .10$		$\chi = .30$		$K = 0.5$		$K = 2$		$m = 0$		$m = .08$	
	R	E	R	E	R	E	R	E	R	E	R	E	R	E
Panel A: Attention Allocation														
1. RAI	10.55	0.07	6.11	0.04	12.96	-0.08	6.23	0.01	14.53	-0.08	9.30	-0.04	11.01	-0.02
2. Timing	9.91	0.06	5.77	-0.09	11.56	0.12	5.93	-0.05	13.96	0.13	9.34	0.09	10.93	0.06
3. RSI	2.15	15.61	0.01	7.76	7.23	23.21	-0.03	7.89	12.66	30.14	2.08	16.16	2.14	15.76
4. Picking	1.66	10.02	0.13	5.04	5.10	14.83	0.18	5.04	8.19	19.45	1.68	9.31	1.65	10.28
Panel B: Dispersion														
5. Concentration	3.75	3.12	1.99	1.59	5.27	4.62	1.78	1.50	7.71	6.74	3.74	3.12	3.77	3.14
6. Idiosyncratic volatility	5.09	4.33	2.90	2.27	6.66	6.07	3.34	2.80	7.38	6.71	4.83	4.09	5.29	4.33
7. Dispersion in abnormal return	3.54	3.37	1.93	1.82	4.94	4.71	2.21	2.13	5.97	5.51	3.42	3.21	3.74	3.44
8. Dispersion in CAPM alpha	2.52	2.28	1.52	1.35	3.06	2.84	1.32	1.15	4.85	4.46	2.22	2.15	2.76	2.35
9. Dispersion in CAPM beta	6.37	1.46	4.70	1.03	5.76	2.45	3.45	0.98	10.84	2.67	21.03	2.08	3.91	1.38
Panel C: Performance														
10. Abnormal return	0.346	0.302	0.178	0.150	0.500	0.450	0.184	0.151	0.664	0.589	0.330	0.283	0.379	0.312
11. CAPM Alpha	0.353	0.307	0.186	0.156	0.506	0.454	0.192	0.156	0.672	0.593	0.329	0.288	0.385	0.316

Table S.4: **Endogenous Capacity Model**

This table provides the results from an extension of the model where skilled funds endogenously choose how much capacity to acquire. It reports on the main predictions of the model. Panel A reports moments related to attention allocation, Panel B reports the moments related to dispersion, and Panel C reports moments related to performance. The first column lists the moments in question, as defined in the main text. The other pairs of columns report results for the benchmark parameters and for two versions of the endogenous K model with different cost functions. The cost function in the first one is $\mathcal{C}_1(K) = c_1 \exp(K)$, while the cost function in the second one is $\mathcal{C}_2(K) = c_2 K^{\psi}$. We set $c_1 = 1.057$, $c_2 = 2.4$, and $\psi = 1.2$. All other parameters are the same as in the benchmark model. In each pair of columns, the first column reports the predictions for the model simulated in a recession (R) and the second column for the model simulated in an expansion (E). All moments are generated from a simulation of 2000 draws and 100 investors. For both recessions and expansions, we list the equally weighted average across all investment managers (the 20% skilled and the 60% investment managers).

	Baseline		$\mathcal{C}_1(K) = c_1 \exp(K)$		$\mathcal{C}_2(K) = c_2 K^{\psi}$	
	R	E	R	E	R	E
Panel A: Attention Allocation						
1. RAI	10.55	0.07	10.53	-0.12	11.58	-0.01
2. Timing	9.91	0.06	9.83	0.10	10.65	0.13
3. RSI	2.15	15.61	2.35	15.43	3.69	14.37
4. Picking	1.66	10.02	1.76	9.91	2.65	8.87
Panel B: Dispersion						
5. Concentration	3.75	3.12	3.82	3.13	4.37	2.85
6. Idiosyncratic volatility	5.09	4.33	5.20	4.53	5.34	4.15
7. Dispersion in abnormal return	3.54	3.37	3.59	3.34	3.97	3.16
8. Dispersion in CAPM alpha	2.52	2.28	2.57	2.29	2.92	2.06
9. Dispersion in CAPM beta	6.37	1.46	5.18	3.01	5.28	1.94
Panel C: Performance						
10. Abnormal return	0.346	0.302	0.348	0.302	0.399	0.271
11. CAPM Alpha	0.353	0.307	0.355	0.307	0.407	0.277

S.3 Robustness Checks for Empirical Results

S.3.1 Attention Allocation Results

Table S.5 reports several robustness checks. First, we compute an alternative RAI measure in which the aggregate shock is proxied by surprises in non-farm employment growth, another salient macroeconomic variable, instead of industrial production growth. Second, we compute an alternative RSI measure in which earnings surprises are defined as the residual from a regression of earnings per share in a given year on earnings per share in that same quarter one year earlier (instead of one quarter earlier), as in Bernard and Thomas (1989). Third, to check the market-timing results, we also study the R^2 from a CAPM regression at the fund level (as in equation 16). It measures how the funds' excess returns (as opposed to their portfolio weights) covary with the aggregate state, as measured by the market's excess return. The average R^2 across all funds rises from 77% in expansions to 80% in recessions. All three findings are consistent with the hypothesis that recessions are times when funds learn about the aggregate shock, making their portfolio choices and therefore their fund returns more sensitive to changes in market returns.

To get further insight into why the RAI and the market-timing measure increase in recessions, we conduct several other exercises. First, we ask whether managers actively change their cash holdings in recessions. Cash is measured either as Reported Cash, from CRSP, or Implied Cash, backed out from fund size and its equity holdings. Table S.6 shows a slightly higher cash position in recessions than in expansions. In expansions, funds hold about 5% of their portfolio in cash. In recessions, the fraction of their holdings in cash rises by about 0.3% for the Reported Cash measure and by about 3% for the Implied Cash measure. Both increases are statistically significant, and each represents about a 0.1 standard-deviation change. The last two columns report the month-over-month change in the Implied Cash position. In recessions, cash holdings increase by 0.5%. The effect is modest, but measured precisely. In sum, one way in which funds lower their portfolio beta in recessions is to increase their cash positions.

The second question we ask is whether fund managers also invest in lower-beta stocks in recessions. For each individual stock, we compute the beta (from twelve-month rolling-window regressions). Based, on the individual stock holdings of each mutual fund, we construct the funds' (value-weighted) *equity betas*. Table S.7 shows that this beta is 1.11 in expansions and 0.99 in recessions; the 0.118 difference has a t-statistic of 4.5. This means that funds not only keep more cash in recessions, but also hold different types of stocks, namely lower-beta stocks.

Finally, we investigate whether funds change their portfolio allocations towards *defensive sectors* sectors over the business cycle. Table S.8 shows that, in recessions, funds increase their portfolio weights (relative to those in the market portfolio) in low-beta sectors such as Healthcare, Non-Durables (which includes Food and Tobacco), Wholesale, and Utilities. They reduce their portfolio weight (relative to those in the market portfolio) in high-beta sectors such as Telecom, Business Equipment and Services, Manufacturing, Energy, and Durables. Hence, funds do engage in sector rotation over the course of the business cycle.

S.3.2 Dispersion Results

Table S.9 considers additional measures of portfolio and return dispersion. For example, managers shift their investment styles more in recessions, consistent with them pursuing a more active portfolio management strategy. The results are again highly significant, both economically and statistically. Investment managers also display a somewhat greater industry concentration in recessions.

Next, we show that the dispersion of abnormal returns (fund returns minus the market return) nearly doubles in recessions. In unreported results, we obtain similar results for the dispersion in CAPM alpha and beta when funds' alphas and betas are calculated not by using twelve-month rolling-window regressions, but by estimating their dependence on several state variables (the dividend-price ratio of the aggregate stock market, the term spread, the short-term interest rate, and the default spread) in one full-sample regression (Avramov and Wermers 2006).

Finally, we study the dispersion of the information ratio, defined as the ratio of the CAPM alpha to the CAPM residual volatility. Its dispersion increases from 0.31 in expansion to 0.39 in recessions. The increase is measured precisely (t-statistic of 4.7). These results bolster our evidence for increased dispersion in recessions.

S.3.3 Performance Results

The cross-sectional regression model allows us to include a host of fund-specific control variables, thereby making use of rich panel data. But because it measures performance using past twelve-month rolling-window regressions, a given month observation for the dependent variable can be classified as a recession when some or even all of the remaining eleven months of the window are expansions. To avoid this problem and verify our results, we also employ a time-series approach which is free of this issue and potentially provides a cleaner estimate of the economic magnitudes of interest. In each month, we form the equally weighted portfolio of funds and calculate its (net) return in excess of the risk-free rate. This procedure results in a series of portfolio returns which we then use to estimate a time-series regression model of the portfolio returns on *Recession* and common risk factors. We adjust standard errors for any heteroscedasticity and autocorrelation using the procedure in Newey and West (1987). Table S.10 presents the results.

In Column 1, we control for the excess market return. The intercept of the regression, equal to -6bp per month, measures the CAPM alpha in expansions. The coefficient on *Recession* measures the incremental CAPM alpha in recessions, and is equal to 27bp per month. This result is equivalent to an economically large and statistically positive alpha in recessions of around 2.5% per year. In Columns 2 and 3, we include additional two and three factors. The resulting three- and four-factor alphas are both estimated to be 16bp per month higher in recessions than in expansions; they are also fairly negative in expansions. The time-series approach allows us to include an additional illiquidity factor, defined as in Pástor and Stambaugh (2003). The results, in Column 4, show that fund returns do not load significantly on this factor; consequently, our previous results remain largely unchanged.

To allow for the possibility that factor loadings may vary over the business cycle, we re-estimate the regression specifications in Columns 1 through 4, allowing for interactions of the factors with *Recession*. The results, in Columns 5 through 8, show that the coefficients on the interaction terms are typically statistically insignificant, with the exception of the interaction term with the value factor. Our results are largely unaffected by the time-varying factor exposures of fund returns. If anything, the magnitudes of the incremental returns in recessions become slightly stronger. As an example, in Column 8, the risk-adjusted excess return is 1.7% per year in recessions, 2.7% higher than the -1% return in expansions. This difference is statistically and economically significant. The magnitudes of the estimates in Table S.10 are reassuring as they are consistent with our panel-regression estimates in Table 4.

We also study the robustness to additional ways of measuring performance. Table S.11 uses *gross* fund returns and alphas instead of returns and alphas net of fees. They are constructed by adding back the management fees. Gross returns are about zero in expansions and substantially higher in recessions. In unreported results, we also examine the information ratio, defined as the ratio of the CAPM alpha to the CAPM residual volatility. The results are, if anything, stronger. The information ratio is -0.07 in expansions and increases by 0.14 (t-statistic of 7.8) to 0.072 in recessions. The 0.14 increase in recessions can be interpreted as a Sharpe ratio gain. Finally, we study a specification in which we lead the alpha on the left-hand side of equation (19) by one month, instead of using its contemporaneous value. The increase in alpha in recessions falls slightly, compared to the benchmark specification, but the effect remains statistically and economically significant.

Table S.12 reports the results of the performance analysis based on the pooled-regression specification. It shows that the average fund outperformance continues to be higher in recessions when we replace the NBER recession indicator with the Chicago Fed National Activity Index (CFNAI). The CFNAI is a weighted average of 85 existing monthly indicators of national economic activity. It is constructed to have an average value of zero and a standard deviation of one. Since economic activity trends, a positive index reading corresponds to growth above trend and a negative index reading corresponds to growth below trend. The CFNAI has a correlation of -70% with the NBER recession indicator. The CFNAI has the advantage that it is a continuous variable, which measures the *strength* of economic activity, i.e., how deep the recession or how strong the expansion is. Table S.12 shows that, on average (when CFNAI is zero), the average fund alpha is negative. But, in recessions, when economic activity is low (CFNAI is negative), an average fund's performance increases significantly. The CFNAI is constructed to have a standard deviation of one in the full sample (1967-2008). In our 1980-2005 sample, it has a standard deviation of 0.644. Hence, a one standard-deviation decrease in CFNAI increases the CAPM alpha by 12.8bp per month and the four-factor alpha by 5.7bp points per month. These effects are measured precisely and quantitatively similar to our benchmark results.

Next, we find performance results that are similar, and somewhat stronger, when we replace the NBER recession indicator with an indicator which is one when real consumption growth is negative. We also find similar results when we use a dummy that captures the 25% lowest stock market returns as a recession indicator. The latter two tables are omitted for brevity.

As a second alternative recession indicator we use a measure of fundamental volatility. More specifically, we calculate the twelve-month rolling-window standard deviation of aggregate earnings growth. Aggregate earnings growth is the year-to-year (e.g., March to March) log change in aggregate earnings. Aggregate earnings are based on the earnings of all firms in the S&P 500 index; the aggregate earnings data are from Robert Shiller for the period from 1926 until 2008. The variable $Volatility$ equals one if the standard deviation of aggregate earnings growth is in the highest 10% of months in the 1926-2008 sample. Twelve percent of months in our 1980-2005 sample are such high-volatility months. Table S.13 shows that alphas are negative in low-volatility months and substantially higher in high-volatility months. The incremental CAPM alpha in recessions is 54bp per month and the incremental four-factor alpha is 14.8bp per month. Both are statistically significant. All results point in the same direction: Outperformance clusters in periods of recessions.

S.3.4 Identifying Skilled Fund Managers

Table S.14 reports the estimation results from the linear probability regression model of the SP indicator variable on fund age, TNA, expenses, and turnover. The R^2 of the baseline regression is 14%.

The existence of a group of skilled mutual funds who switch in terms of their learning and investment strategies between recessions and expansions is not very sensitive to the exact specification. First, we investigate a different cutoff level for the inclusion in the SP portfolio. Including more than 25% of funds in the SP portfolio considerably weakens the funds' average market-timing ability in recessions as well as their unconditional alpha, because skill dissipates with the size of the group. Conversely, including fewer than 25% of funds strengthens both results.

Second, we examine funds that are among the top 25% in terms of their RSI metric in expansions. These funds have higher RAI in recessions and higher unconditional alphas (Tables S.15 and S.16).

Third, we verify our results using alternative definitions of market timing (CT) and stock picking (CS), originally proposed in Daniel, Grinblatt, Titman, and Wermers (1997). The funds that are among the 25% best in terms of their CS metric in expansions have significantly higher CT in recessions. They also have higher unconditional alphas. We also find the same results for fund and manager characteristics as those in Table 7, but using the SP portfolio based on the CT and CS variables instead. Fund age, size, expenses, and turnover explain 8% of the selection into the SP portfolio. Adding stock concentration, beta deviation, and RAI improves that R^2 to 18%. For brevity, we omit these tables. They are available upon request.

Fourth, we perform the reverse sort. We verify that funds in the top 25% of market-timing ability in recessions, have statistically higher stock-picking ability, $Picking$, in expansions (Table S.17) and higher unconditional alphas (Table S.18). Finally, Table S.19 shows that a one-standard-deviation increase in a skill index, based on RAI and RSI, instead of $Timing$ and $Picking$, increases one-month ahead alphas by 0.3-0.5% per year, a statistically significant effect.

S.3.5 Alternative Explanations

We consider labor market explanations for the variations in fund strategies and returns we observe. The data show that outside labor market options of investment managers deteriorate in recessions. Not only do their assets under management and therefore their wages shrink, they are also more likely to get fired or demoted. Table S.20 shows less turnover in the labor market for investment managers in recessions (Columns 1 and 2), and a smaller incidence of promotion to a larger mutual fund in a different fund family (Columns 3 and 4), a higher incidence of demotion to a smaller mutual fund in a different fund family (Columns 5 and 6), and a lower incidence of departure to a hedge fund (Columns 7 and 8). Finally, Table S.21 shows that an average manager's experience is not higher in recessions.

Table S.5: **Robustness: Alternative RAI and RSI Measures**

The dependent variables are funds' reliance on aggregate information $RAI2$, funds' reliance on stock-specific information $RSI2$, and the CAPM $R-squared$. A fund j's $RAI2_t^j$ is defined as the (twelve-month rolling window time series) covariance between the funds' portfolio holdings in deviation from the market ($w_{it}^j - w_{it}^m$) in month t and changes in non-farm employment growth between t and $t+1$. A fund j's $RSI2_t^j$ is defined as the (across stock) covariance between the funds' holdings in deviation from the market ($w_{it}^j - w_{it}^m$) in month t and changes in earnings growth between $t-11$ and $t+1$. $R-squared$ is obtained from the twelve-month rolling-window regression model of a fund's excess returns on excess market returns. $RAI2$, and $RSI2$ are multiplied by 10,000 and $R-squared$ is multiplied by 100 for ease of readability. *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Turnover* is the fund turnover ratio. *Flow* is the percentage growth in a fund's new money. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)
	RAI2		RSI2		R-squared	
Recession	0.004	0.004	-0.886	-0.897	3.040	2.891
	(0.001)	(0.001)	(0.201)	(0.191)	(1.451)	(1.315)
Log(Age)		-0.001		0.452		2.126
		(0.000)		(0.076)		(0.190)
Log(TNA)		0.000		-0.229		0.258
		(0.000)		(0.034)		(0.074)
Expenses		-0.158		111.982		-582.087
		(0.058)		(12.954)		(26.684)
Turnover		0.000		-0.329		-1.242
		(0.000)		(0.074)		(0.110)
Flow		-0.001		2.570		-6.614
		(0.003)		(0.723)		(2.885)
Load		0.021		-12.614		68.883
		(0.007)		(2.317)		(5.434)
Constant	-0.001	-0.001	3.962	3.962	77.361	77.331
	(0.000)	(0.000)	(0.089)	(0.089)	(0.854)	(0.846)
Observations	224,257	224,257	166,328	166,328	227,159	227,159

Table S.6: **Robustness: Cash Holdings**

The dependent variables are three measures of funds' cash holdings. *ReportedCash* is the cash position reported by mutual funds to CRSP in their quarterly statements, relative to the size of the fund (expressed as a percent). *ImpliedCash* is based on the portfolio holdings of the fund. In particular, it is the difference between the total size of the fund (monthly) as reported in the data and the implied size of the equity portio based on the observed holdings and their prices. It is also expressed as a percent of total holdings. *%ChangeCash* is defined as the percentage change in the implied cash measure. *Recession* equals one for every month the economy is in the recession according to the NBER, and zero otherwise. We use the following control variables. *Log(Age)* is the natural logarithm of fund age. *Log(TNA)* is the logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the fund new money growth. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)
	Implied Cash		Reported Cash		% Change Cash	
Recession	2.491	3.278	0.230	0.362	0.643	0.545
	(0.537)	(0.535)	(0.089)	(0.087)	(0.051)	(0.050)
Log(Age)		-0.453		0.309		-0.075
		(0.517)		(0.081)		(0.037)
Log(TNA)		1.676		-0.047		-0.092
		(0.277)		(0.040)		(0.018)
Expenses		163.772		-46.153		24.280
		(119.092)		(18.459)		(6.859)
Turnover		-0.059		-0.168		0.119
		(0.413)		(0.064)		(0.031)
Flow		13.794		3.893		0.189
		(2.840)		(0.315)		(0.301)
Load		-5.033		15.169		-1.144
		(14.366)		(2.837)		(1.196)
Constant	5.316	5.252	4.672	4.656	-1.505	-1.495
	(0.481)	(0.479)	(0.065)	(0.062)	(0.033)	(0.031)
Observations	230,185	230,185	209,516	209,516	225,374	225,374

Table S.7: **Robustness: Equity Betas**

The dependent variable is the fund's *Equity Beta*. For each individual stock, we compute the market beta from a twelve-month rolling-window regression. We then construct the funds' equity beta as the value-weighted average of the individual stock betas, where the weights are the fund's dollar holdings in that stock divided by the dollar holdings in all stocks. *Recession* equals one for every month the economy is in the recession according to the NBER, and zero otherwise. We use the following control variables. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the fund new money growth. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)
	Equity Beta	
Recession	-0.118	-0.106
	(0.026)	(0.027)
Log(Age)		0.013
		(0.002)
Log(TNA)		0.008
		(0.001)
Expenses		3.113
		(0.385)
Turnover		0.035
		(0.002)
Flow		0.056
		(0.037)
Load		0.561
		(0.062)
Constant	1.112	1.111
	(0.006)	(0.008)
Observations	226,094	226,094

Table S.8: **Robustness: Sector Rotation**

The dependent variable is the portfolio weight of fund j in sector l in deviation from the market portfolio's weight in sector l: $w_{lt}^j - w_{lt}^m$. Each column represent a different sector. The sectors are the ten Fama-French industry sectors: (1) Consumer non-durables, (2) Consumer durables, (3) Healthcare, (4) Manufacturing, (5) Energy, (6) Utilities, (7) Telecom, (8) Business Equipment and Services, (9) Wholesale and Retail, (10) Finance. *Recession* equals one for every month the economy is in the recession according to the NBER, and zero otherwise. We use the following control variables. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the logarithm of a fund total net assets. *Expenses* is the fund expense ratio. $Flow$ is the fund new money growth. $Turnover$ is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	NDRBL	DRBL	HLTH	MFCT	ENER	UTIL	TEL	BUSEQ	WHLS	FIN
Recession	0.817	0.123	0.541	-0.278	-0.311	0.246	-0.493	-1.565	0.384	0.741
	(0.085)	(0.111)	(0.215)	(0.121)	(0.397)	(0.269)	(0.111)	(0.563)	(0.081)	(0.154)
Log(Age)	0.301	-0.022	0.728	-0.086	-0.557	-0.702	0.037	0.278	0.085	0.036
	(0.028)	(0.016)	(0.032)	(0.031)	(0.044)	(0.035)	(0.022)	(0.064)	(0.027)	(0.054)
Log(TNA)	-0.246	-0.076	-0.387	-0.114	0.270	0.247	0.257	0.001	-0.060	-0.003
	(0.014)	(0.013)	(0.013)	(0.013)	(0.025)	(0.016)	(0.010)	(0.025)	(0.015)	(0.016)
Expenses	42.310	2.132	-54.607	-72.195	67.829	28.997	22.623	-63.774	11.964	29.247
	(3.641)	(2.849)	(4.630)	(5.043)	(7.958)	(4.180)	(3.902)	(10.138)	(4.611)	(8.483)
Turnover	-0.181	-0.134	0.330	-0.586	0.328	0.040	0.307	1.276	0.021	-1.325
	(0.030)	(0.025)	(0.040)	(0.039)	(0.025)	(0.017)	(0.031)	(0.075)	(0.044)	(0.041)
Flow	-0.256	-0.423	-0.320	-1.241	-1.554	-1.298	1.308	3.289	0.057	0.450
	(0.314)	(0.296)	(0.443)	(0.431)	(0.602)	(0.503)	(0.423)	(1.219)	(0.377)	(0.580)
Load	-4.833	0.764	6.123	8.589	-12.164	-9.617	3.758	23.523	-1.886	-15.841
	(0.656)	(0.563)	(0.658)	(0.894)	(1.219)	(0.705)	(0.473)	(1.729)	(0.871)	(0.978)
Constant	-0.471	-0.777	0.173	2.499	-1.171	-1.769	-1.489	3.600	2.404	-2.660
	(0.046)	(0.065)	(0.050)	(0.073)	(0.080)	(0.084)	(0.053)	(0.201)	(0.059)	(0.101)
Observations	207,382	207,382	207,382	207,382	207,382	207,382	207,382	207,382	207,382	207,382

Table S.9: **Robustness: Dispersion in Funds' Portfolio Strategies and Returns**

The dependent variables are *Style Shifting*, *Sector Deviation*, and $|X_t^j - \bar{X}_t|$, where X_t^j is the *Abnormal Return* or $3-Factor\ Alpha$ and $\bar{X}$ denotes the (equally weighted) cross-sectional average. *Style Shifting* for fund j at time t is the absolute value of the change between time t and time $t-1$ in the the fund's investment style index, defined in footnote 15. *Sector Deviation* for fund j at time t is calculated as the mean square root of the sum of squared differences between the share of fund j's assets in each of 10 industry sectors and the mean share in each sector in quarter t among all funds in fund j's objective class (aggressive growth, growth, or value). To identify the investment objectives, we use the Thomson Financial's style categories 2, 3, and 4. Industry sectors are defined using a modified 10-industry classification of Fama and French, as in Kacperczyk, Sialm, and Zheng (2005). The three-factor alphas are obtained from twelve-month rolling window regressions of fund-level excess returns on excess market returns, SMB, and HML. The abnormal return is the fund return minus the market return, also from a twelve-month rolling-window regression. *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the percentage growth in a fund's new money. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. *Flow* and *Turnover* are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Style Shifting		Sector Deviation		Abnormal Return		3-Factor Alpha	
Recession	5.246	5.082	0.003	0.003	0.530	0.561	0.201	0.212
	(0.826)	(0.798)	(0.002)	(0.001)	(0.108)	(0.102)	(0.042)	(0.039)
Log(Age)		0.014		0.000		-0.064		-0.013
		(0.080)		(0.002)		(0.007)		(0.003)
Log(TNA)		-0.214		-0.006		0.029		-0.001
		(0.047)		(0.001)		(0.004)		(0.001)
Expenses		101.35		2.084		13.816		9.178
		(13.908)		(0.301)		(1.152)		(0.465)
Turnover		1.111		-0.001		0.074		0.064
		(0.109)		(0.002)		(0.006)		(0.004)
Flow		-1.749		0.008		0.479		0.227
		(1.101)		(0.007)		(0.088)		(0.034)
Load		-4.517		-0.175		-1.738		-0.622
		(2.284)		(0.052)		(0.182)		(0.076)
Constant	14.725	14.807	0.185	0.186	0.661	0.659	0.497	0.497
	(0.283)	(0.284)	(0.001)	(0.001)	(0.029)	(0.027)	(0.010)	(0.010)
Observations	191,109	191,109	72,708	72,708	226,745	226,745	226,745	226,745

Table S.10: **Robustness: Time-Series Approach to Fund Performance**

Each month, we form an equally weighted portfolio of all funds. *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. We regress the portfolio (net) return in excess of the risk-free rate on *Recession* and a set of four risk factors: the market excess return (MKTPREM), the return on the small-minus-big portfolio (SMB), the return on the high-minus-low book-to-market portfolio (HML), the return on the up-minus-down momentum portfolio (UMD), and the return on the illiquid-minus-liquid stock portfolio (LIQ) of Pástor and Stambaugh (2003). We also consider specifications in which the risk factors are interacted with Recession. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are corrected for autocorrelation and heteroscedasticity.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Recession	0.274	0.151	0.156	0.157	0.265	0.215	0.198	0.222
	(0.105)	(0.079)	(0.076)	(0.074)	(0.104)	(0.094)	(0.097)	(0.081)
MKTPREM	1.000	0.976	0.978	0.981	0.998	0.979	0.980	0.984
	(0.010)	(0.010)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)
SMB		0.169	0.168	0.168		0.170	0.169	0.169
		(0.024)	(0.025)	(0.026)		(0.026)	(0.027)	(0.027)
HML		-0.007	-0.003	-0.001		0.001	0.005	0.007
		(0.030)	(0.030)	(0.030)		(0.032)	(0.033)	(0.033)
UMD			0.014	0.014			0.015	0.015
			(0.019)	(0.020)			(0.021)	(0.022)
LIQ				-0.008				-0.007
				(0.007)				(0.008)
Recession*MKT					0.011	-0.029	-0.018	-0.025
					(0.024)	(0.023)	(0.025)	(0.024)
Recession*SMB						-0.027	-0.024	-0.013
						(0.047)	(0.049)	(0.041)
Recession*HML						-0.088	-0.098	-0.124
						(0.049)	(0.050)	(0.044)
Recession*UMD							0.017	0.027
							(0.033)	(0.035)
Recession*LIQ								-0.046
								(0.026)
Constant	-0.064	-0.059	-0.074	-0.075	-0.062	-0.065	-0.079	-0.081
	(0.057)	(0.038)	(0.047)	(0.047)	(0.058)	(0.038)	(0.048)	(0.048)
Observations	309	309	309	309	309	309	309	309

Table S.11: **Robustness: Using Gross Returns**

Each month, we calculate value-weighted or equally weighted gross portfolio returns (returns + expenses) of all mutual funds. *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. We regress the returns on *Recession* and a set of four risk factors: the market excess return (MKTPREM), the return on the small-minus-big portfolio (SMB), the return on the high-minus-low book-to-market portfolio (HML), the return on the up-minus-down momentum portfolio (UMD), and the return on the illiquid-minus-liquid stock portfolio (LIQ) of Pástor and Stambaugh (2003). We also consider specifications in which the risk factors are interacted with Recession. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are corrected for autocorrelation and heteroskedasticity.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Recession	0.268	0.146	0.150	0.151	0.259	0.210	0.195	0.218
	(0.104)	(0.078)	(0.076)	(0.073)	(0.103)	(0.092)	(0.096)	(0.081)
MKTPREM	1.000	0.976	0.978	0.981	0.998	0.979	0.980	0.984
	(0.010)	(0.010)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)
SMB		0.169	0.168	0.168		0.170	0.169	0.169
		(0.024)	(0.025)	(0.026)		(0.026)	(0.027)	(0.027)
HML		-0.007	-0.003	-0.002		0.001	0.005	0.007
		(0.030)	(0.030)	(0.030)		(0.032)	(0.033)	(0.033)
UMD			0.014	0.014			0.015	0.015
			(0.019)	(0.020)			(0.021)	(0.022)
LIQ				-0.008				-0.007
				(0.007)				(0.008)
Recession*MKTPREM					0.011	-0.029	-0.019	-0.026
					(0.024)	(0.023)	(0.025)	(0.024)
Recession*SMB						-0.028	-0.025	-0.014
						(0.047)	(0.048)	(0.040)
Recession*HML						-0.090	-0.099	-0.125
						(0.049)	(0.049)	(0.044)
Recession*UMD							0.016	0.026
							(0.033)	(0.035)
Recession*LIQ								-0.045
								(0.026)
Constant	0.037	0.042	0.027	0.026	0.039	0.036	0.022	0.020
	(0.058)	(0.038)	(0.047)	(0.047)	(0.058)	(0.038)	(0.048)	(0.048)
Observations	309	309	309	309	309	309	309	309

Table S.12: **Robustness: Performance Using the CFNAI as Recession Indicator**

The dependent variables are funds' $CAPM\ Alpha$, $3-Factor\ Alpha$, and $4-Factor\ Alpha$. All alphas are obtained from twelve-month rolling-window regressions of fund-level excess returns on excess market returns for the CAPM alpha, additionally on the SMB and the HML factors for the three-factor alpha, and additionally on the UMD factor for the four-factor alpha. The abnormal return is the fund return minus the market return. $CFNAI$, or Chicago Fed National Activity Index, is a weighted average of 85 existing monthly indicators of national economic activity. It is constructed to have an average value of zero and a standard deviation of one. Since economic activity tends toward trend growth rate over time, a positive index reading corresponds to growth above trend and a negative index reading corresponds to growth below trend. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. $Expenses$ is the fund expense ratio. $Flow$ is the percentage growth in a fund's new money. $Turnover$ is the fund turnover ratio. $Load$ is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimension, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. $Flow$ and $Turnover$ are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)
	CAPM Alpha		3-Factor Alpha		4-Factor Alpha	
CFNAI	-0.163	-0.198	-0.046	-0.061	-0.071	-0.088
	(0.034)	(0.033)	(0.024)	(0.021)	(0.028)	(0.025)
Log(Age)		-0.035		-0.028		-0.038
		(0.007)		(0.006)		(0.006)
Log(TNA)		0.032		0.009		0.012
		(0.004)		(0.003)		(0.003)
Expenses		-3.407		-7.996		-7.967
		(0.922)		(0.786)		(0.747)
Turnover		-0.045		-0.075		-0.066
		(0.010)		(0.010)		(0.008)
Flow		2.439		1.695		1.541
		(0.169)		(0.096)		(0.095)
Load		-0.611		-0.064		-0.278
		(0.169)		(0.128)		(0.136)
Constant	-0.031	-0.031	-0.055	-0.055	-0.042	-0.042
	(0.022)	(0.021)	(0.018)	(0.016)	(0.021)	(0.019)
Observations	226,745	226,745	226,745	226,745	226,745	226,745

Table S.13: **Robustness: Performance Using Volatility as Recession Indicator**

The dependent variables are funds' *CAPM Alpha*, $3-$*Factor Alpha*, and $4-$*Factor Alpha*. All are obtained from twelve-month rolling-window regressions of fund-level excess returns on excess market returns for the CAPM alpha, additionally on the SMB and the HML factors for the three-factor alpha, and additionally on the UMD factor for the four-factor alpha. The abnormal return is the fund return minus the market return. Volatility is a dummy variable indicating periods of high volatility in fundamentals. We calculate the twelve-month rolling-window standard deviation of aggregate earnings growth. Aggregate earnings growth is the year-to-year log change in the earnings of S&P 500 index constituents; the aggregate earnings data are from Robert Shiller for the period from 1926 until 2008. Volatility equals one if the standard deviation of aggregate earnings growth is in the highest 10% of months in the 1926-2008 sample. Twelve percent of months in our 1985-2005 sample are such high volatility months. *Log(Age)* is the natural logarithm of fund age. *Log(TNA)* is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the percentage growth in a fund's new money. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimension, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. *Flow* and *Turnover* are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)
	CAPM Alpha		3-Factor Alpha		4-Factor Alpha	
Volatility	0.449	0.540	0.022	0.060	0.106	0.148
	(0.060)	(0.043)	(0.058)	(0.048)	(0.072)	(0.061)
Log(Age)		-0.023		-0.026		-0.035
		(0.008)		(0.006)		(0.006)
Log(TNA)		0.023		0.008		0.009
		(0.004)		(0.003)		(0.003)
Expenses		-5.373		-8.090		-8.396
		(0.955)		(0.843)		(0.803)
Turnover		-0.053		-0.074		-0.067
		(0.011)		(0.010)		(0.008)
Flow		2.424		1.690		1.534
		(0.167)		(0.099)		(0.097)
Load		-0.227		-0.035		-0.185
		(0.163)		(0.139)		(0.145)
Constant	-0.086	-0.097	-0.058	-0.062	-0.055	-0.060
	(0.023)	(0.022)	(0.020)	(0.018)	(0.023)	(0.021)
Observations	226,745	226,745	226,745	226,745	226,745	226,745

Table S.14: **Robustness: Characteristics of the Picking-Skill Funds**

Expansion is every month the economy is not in recession. We define the stock picking ability of a fund as $Picking_t^j = \sum_{i=1}^{N}(w_{it}^j - w_{it}^m)(R_{t+1}^i - \beta_i R_{t+1}^m)$. *Switching Portfolio* is an indicator variable equal to one for all funds whose *Picking* in Expansion is in the highest 25th percentile of the distribution, and zero otherwise. *Age* is the fund's age. *TNA* is the fund 's total net assets. *Expenses* is the fund's expense ratio. *Turnover* is the fund's turnover ratio. *Industry Concentration* is the industry concentration of the fund's portfolio. *Stock Concentration* is the stock concentration of the fund's portfolio. *Beta Deviation* is the absolute difference between the fund beta and the average beta in its style category. *Number of Stocks* is the number of stocks in the fund portfolio. *RAI* is the fund manager's reliance on aggregate information (RAI), defined as the R-squared from the regression of fund portfolio returns on contemporaneous changes in industrial production. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)
	Picking Skill	Picking Skill
Log(Age)	-0.073	-0.069
	(0.010)	(0.010)
Log(TNA)	0.010	0.017
	(0.006)	(0.005)
Expenses	21.677	18.715
	(2.151)	(2.095)
Turnover	0.126	0.117
	(0.011)	(0.011)
Industry Concentration		1.063
		(0.170)
Stock Concentration		1.338
		(0.497)
Beta Deviation		0.146
		(0.054)
Number of Stocks		-0.002
		(0.002)
RAI		0.844
		(0.075)
Constant	0.255	0.255
	(0.010)	(0.010)
Observations	180,997	180,997
R-squared	0.14	0.19

Table S.15: **Robustness: Same Managers with High RSI in Expansions Have High RAI in Recessions**

We divide all mutual fund-month observations into Recession and Expansion subsamples. *Recession* is an indicator variable equal to one every month the economy is in recession according to the NBER, and zero otherwise; *Expansion* is one every month the economy is not in recession. The dependent variables are fund managers' reliance on aggregate information (RAI) and fund managers' reliance on stock-specific information (RSI). RAI is defined as the R-squared from the regression of fund portfolio returns on contemporaneous changes in industrial production. RSI is defined as the R-squared from the regression of changes in a mutual fund's stock holdings on contemporaneous changes in equity analysts' stock recommendations. *Skill Picking* 2 is an indicator variable equal to one for all funds whose RSI measure in Expansion is in the highest 25th percentile of the distribution, and zero otherwise. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the percentage growth in a fund's new money. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. *Flow* and *Turnover* are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)
	RAI		RSI	
	Expansion	Recession	Expansion	Recession
Skill Picking 2	0.138	0.811	25.899	22.030
	(0.124)	(0.411)	(0.602)	(1.164)
Log(Age)	-0.178	-1.372	0.195	2.020
	(0.091)	(0.257)	(0.374)	(0.734)
Log(TNA)	0.122	0.530	-0.728	-1.459
	(0.038)	(0.110)	(0.185)	(0.374)
Expenses	111.586	140.244	217.636	505.439
	(16.334)	(41.952)	(75.370)	(164.300)
Turnover	0.054	2.304	-0.268	0.068
	(0.085)	(0.246)	(0.312)	(0.663)
Flow	1.189	-13.400	4.402	18.798
	(0.553)	(1.327)	(2.047)	(9.011)
Load	-11.771	4.776	-68.071	-91.943
	(2.366)	(6.244)	(13.560)	(30.653)
Constant	8.318	13.943	31.903	33.704
	(0.083)	(0.225)	(0.263)	(0.623)
Observations	206,204	18,264	111,198	8,463

Table S.16: **Robustness: Unconditional Performance of the Skill Picking Funds using RAI/RSI**

We divide all mutual fund-month observations into Recession and Expansion subsamples. *Expansion* is one every month the economy is not in recession according to the NBER, and zero otherwise. *Skill Picking* 2 is an indicator variable equal to one for all funds whose RSI measure in Expansion is in the highest 25th percentile of the distribution, and zero otherwise. The dependent variable is the CAPM alpha, three-factor alpha, or four-factor alpha of the mutual fund, obtained from a twelve-month rolling-window regression. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the percentage growth in a fund's new money. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. *Flow* and *Turnover* are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)
	CAPM Alpha	3-Factor Alpha	4-Factor Alpha
Skill Picking 2	0.144	0.009	0.037
	(0.039)	(0.013)	(0.015)
Log(Age)	-0.024	-0.030	-0.036
	(0.007)	(0.006)	(0.006)
Log(TNA)	0.027	0.013	0.013
	(0.005)	(0.004)	(0.003)
Expenses	-4.480	-6.791	-7.530
	(1.187)	(0.877)	(0.797)
Turnover	-0.005	-0.041	-0.036
	(0.016)	(0.013)	(0.010)
Flow	2.571	1.753	1.600
	(0.173)	(0.102)	(0.101)
Load	-0.412	-0.125	-0.251
	(0.150)	(0.130)	(0.129)
Constant	-0.091	-0.058	-0.057
	(0.013)	(0.015)	(0.017)
Observations	227,183	227,183	227,183

Table S.17: **Robustness: Same Funds with Market-Timing Ability in Recessions Have Stock-Picking Ability in Expansions**

We divide all fund-month observations into Recession and Expansion subsamples. *Recession* is one every month the economy is in recession according to the NBER; *Expansion* is one every month the economy is not in recession. The dependent variables are $Timing_t^j$ and $Picking_t^j$. They are defined as follows: $Timing_t^j = \sum_{i=1}^N (w_{it}^j - w_{it}^m)(\beta_i R_{t+1}^m)$ and $Picking_t^j = \sum_{i=1}^N (w_{it}^j - w_{it}^m)(R_{t+1}^i - \beta_i R_{t+1}^m)$. *Skill Picking* 3 is an indicator variable equal to one for all funds whose *Timing* measure in recessions is in the highest 25th percentile of the distribution, and zero otherwise. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the percentage growth in a fund's new money. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. *Flow* and *Turnover* are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)
	Market Timing		Stock Picking	
	Expansion	Recession	Expansion	Recession
Skill Picking 3	0.000	0.017	0.056	-0.096
	(0.004)	(0.009)	(0.004)	(0.017)
Log(Age)	0.009	-0.025	-0.001	0.029
	(0.002)	(0.006)	(0.002)	(0.007)
Log(TNA)	-0.001	0.005	0.000	-0.023
	(0.001)	(0.003)	(0.001)	(0.003)
Expenses	0.868	1.374	-1.291	-4.434
	(0.321)	(1.032)	(0.376)	(1.378)
Turnover	0.009	-0.011	0.017	-0.006
	(0.003)	(0.007)	(0.004)	(0.012)
Flow	0.056	-0.876	0.138	-0.043
	(0.024)	(0.112)	(0.037)	(0.093)
Load	0.094	-0.076	0.131	0.615
	(0.049)	(0.151)	(0.055)	(0.195)
Constant	0.016	0.059	-0.021	-0.148
	(0.001)	(0.004)	(0.001)	(0.005)
Observations	204,330	18,354	204,330	18,354

Table S.18: **Robustness: Unconditional Performance of the Reverse-Sort Funds**

We divide all fund-month observations into Recession and Expansion subsamples. *Expansion* equals one every month the economy is not in recession according to the NBER. $Picking_t^j = \sum_{i=1}^{N}(w_{it}^j - w_{it}^m)(R_{t+1}^i - \beta_i R_{t+1}^m)$. *Skill Picking* 3 is an indicator variable equal to one for all funds whose *Timing* measure in recessions is in the highest 25th percentile of the distribution, and zero otherwise. The dependent variable is the CAPM alpha, three-factor alpha, or four-factor alpha of the mutual fund, obtained from a twelve-month rolling-window regression of excess gross fund returns on a set of various risk factors. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the percentage growth in a fund's new money. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. *Flow* and *Turnover* are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)
	CAPM Alpha	3-Factor Alpha	4-Factor Alpha
Skill Picking 3	0.076	0.056	0.064
	(0.040)	(0.021)	(0.018)
Log(Age)	-0.039	-0.028	-0.038
	(0.008)	(0.006)	(0.006)
Log(TNA)	0.032	0.013	0.014
	(0.005)	(0.004)	(0.004)
Expenses	4.956	0.627	0.241
	(1.066)	(0.793)	(0.739)
Turnover	-0.009	-0.047	-0.041
	(0.014)	(0.012)	(0.009)
Flow	2.579	1.754	1.602
	(0.173)	(0.102)	(0.101)
Load	-0.744	-0.090	-0.289
	(0.214)	(0.136)	(0.145)
Constant	0.057	0.038	0.049
	(0.017)	(0.015)	(0.018)
Observations	227,183	227,183	227,183

Table S.19: **Robustness: Skill Index using RAI/RSI Predicts Performance**

The dependent variable is the fund's cumulative CAPM, three-factor, or four-factor alpha, calculated from a twelve-month rolling regression of observations in month $t+2$ in the three left columns and in month $t+13$ in the three most right columns. For each fund, we form the following skill index in month t. $Skill\,Index\,2_t^j = w(z_t)RAI_t^j + (1-w(z_t))RSI_t^j, z_t \in \{Expansion, Recession\}$, w(Recession)=0.8 > w(Expansion)= 0.2, where RAI is the fund manager's reliance on aggregate information and RSI is the fund manger's reliance on stock-specific information. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. $Expenses$ is the fund expense ratio. $Flow$ is the percentage growth in a fund's new money. $Turnover$ is the fund turnover ratio. $Load$ is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. $Flow$ and $Turnover$ are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)
	One Month Ahead			One Year Ahead		
	CAPM Alpha	3-Factor Alpha	4-Factor Alpha	CAPM Alpha	3-Factor Alpha	4-Factor Alpha
Skill Index 2	0.021	0.035	0.020	0.005	0.011	0.011
	(0.009)	(0.009)	(0.007)	(0.008)	(0.006)	(0.007)
Log(Age)	-0.037	-0.025	-0.036	-0.024	-0.012	-0.027
	(0.009)	(0.006)	(0.007)	(0.009)	(0.006)	(0.007)
Log(TNA)	0.028	0.011	0.012	-0.015	-0.017	-0.010
	(0.005)	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)
Expenses	-2.489	-7.040	-7.199	-4.985	-8.916	-8.979
	(1.600)	(0.988)	(0.945)	(1.586)	(0.908)	(0.863)
Turnover	0.000	-0.043	-0.035	0.011	-0.035	-0.029
	(0.017)	(0.014)	(0.010)	(0.018)	(0.014)	(0.010)
Flow	2.483	1.691	1.546	0.329	0.252	0.270
	(0.173)	(0.106)	(0.104)	(0.115)	(0.083)	(0.067)
Load	-0.818	-0.074	-0.280	-0.698	0.251	0.001
	(0.238)	(0.144)	(0.157)	(0.223)	(0.129)	(0.148)
Constant	-0.034	-0.058	-0.044	-0.042	-0.070	-0.055
	(0.024)	(0.019)	(0.022)	(0.025)	(0.019)	(0.022)
Observations	218,104	218,104	218,104	183,845	183,845	183,845

Table S.20: **Alternatives: Managerial Turnover**

The dependent variables are: The manager's departure (*Departure*) for any reason, the manager's promotion to a larger mutual fund in a different fund family (*Promotion*), the manager's departure to a smaller mutual fund in a different fund family (*Demotion*), and the manager's departure to a hedge fund (*Hedge Fund*). *Departure* includes those managers in the variables *Promotion*, *Demotion*, *Hedge Fund*, as well as the managers that disappear from the investment management industry. *Recession* is an indicator variable that equals one for every month the economy is in the recession according to the NBER, and zero otherwise. $Log(Age)$ is the natural logarithm of a fund's age. $Log(TNA)$ is the natural logarithm of a fund's total net assets. *Expenses* is the fund expense ratio. *Flow* is the percentage growth in a fund's new money. *Turnover* is the fund turnover ratio. *Load* is the total fund load. $Log(Manage)$ is the natural logarithm of the fund manager's age in years. $Log(Experience)$ is the natural logarithm of a manager experience in years. *Ivy* is an indicator variable equal to one if the manager graduated from an Ivy League academic institution, and zero otherwise. All control variables are demeaned. *Flow* and *Turnover* are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Departure		Promotion		Demotion		Hedge Fund	
Recession	-1.118	-1.470	-0.247	-0.263	0.333	0.326	-0.037	-0.041
	(0.616)	(0.602)	(0.095)	(0.099)	(0.130)	(0.131)	(0.013)	(0.014)
Log(Age)		-1.079		0.097		0.021		0.001
		(0.125)		(0.043)		(0.044)		(0.009)
Log(TNA)		0.600		-0.065		0.018		-0.001
		(0.094)		(0.021)		(0.022)		(0.005)
Expenses		239.641		-9.060		12.793		0.771
		(41.513)		(7.985)		(8.238)		(1.730)
Turnover		2.823		0.061		0.052		0.021
		(0.197)		(0.037)		(0.035)		(0.009)
Flow		-5.175		0.170		-0.605		-0.073
		(1.871)		(0.537)		(0.403)		(0.180)
Load		56.096		4.587		-1.945		0.322
		(5.244)		(1.601)		(1.203)		(0.308)
Log(Manage)		-5.210		-0.497		-0.699		-0.139
		(1.854)		(0.403)		(0.289)		(0.086)
Log(Experience)		4.157		-0.477		0.473		0.020
		(0.787)		(0.236)		(0.165)		(0.034)
Ivy		1.103		0.137		0.085		0.036
		(0.433)		(0.071)		(0.072)		(0.020)
Constant	7.598	7.624	0.548	0.549	0.419	0.420	0.037	0.038
	(0.332)	(0.320)	(0.056)	(0.058)	(0.056)	(0.056)	(0.013)	(0.013)
Observations	86,823	86,823	86,823	86,823	86,823	86,823	86,823	86,823

Table S.21: **Alternatives: Managers' Age, Experience, and Education**

The dependent variables are: The natural logarithm of the fund manager's age in years ($Log(Manage)$); the natural logarithm of a manager experience in years ($Log(Experience)$); an indicator variable (Ivy) that is equal to one if the manager graduated from an Ivy League University, and zero otherwise. *Recession* is an indicator variable equal to one for every month the economy is in the recession according to the NBER, and zero otherwise. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at both fund and time dimensions.

	(1)	(2)	(3)
	Log(Manage)	Log(Experience)	Ivy
Recession	-0.011	-0.023	0.003
	(0.009)	(0.017)	(0.004)
Constant	3.974	3.287	0.253
	(0.003)	(0.006)	(0.001)
Observations	91,879	91,879	91,879