Information Acquisition and Portfolio Under-Diversification

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Portfolio Facts

- Individual Investor Portfolios
  - Median retail investor holds 2.6 stocks
    (Barber and Odean, 2000).
  - Stocks held are positively correlated
    (Goetzman and Kumar, 2003).
  - Remaining 60% of portfolio is diversified
    (Polkovnichenko, 2003).

- Institutional Investor Portfolios
  - Diversified and alpha-funds (or hedge funds)
  - Median mutual fund: 65 stocks and high industry concentration
    (Kacperczyk, Sialm and Zheng, 2004).

⇒ bi-polar portfolios
Can Information Choice Rationalize Portfolios?

• Should investors specialize or learn about many assets?

• Does specialization cause under-diversification? Or, do all investors study the same assets?

• How do investors balance gains to specialization and diversification?

• Can we predict which assets will be learned about?
Outline

• A GE model of learning and investing
  ◦ Investors have limited capacity to learn asset-relevant information.
  ◦ Before forming asset portfolios, they optimally allocate this capacity.

• Results
  ◦ Investor specializes in learning about one risk.
  ◦ Ex-ante identical agents specialize in different risks. Reason: Risk factors that many investors learn about have lower risk premia, lower returns.
  ◦ Optimal portfolio: diversified fund, plus a set of correlated assets.
Modeling Learning Trade-offs

• How to measure information?
  ◦ Standard one-dimensional constraint: \( \frac{\hat{\sigma}^{-1}_i}{\sigma_i^{-1}} \leq e^{2K} \).
  ◦ Extension to N dimensions: \( \left| \frac{\hat{\Sigma}^{-1}}{\Sigma^{-1}} \right| \leq e^{2K} \).

• What is the choice variable?
  ◦ Signal precision
  ◦ Equivalent to variance of posterior beliefs.
  ◦ Unconditional variance is unchanged, but conditional variance (residual uncertainty) is reduced by learning.

• If assets are correlated, what is information about?
  ◦ An orthogonal principal components decomposition: \( \Sigma = \Gamma' \Lambda \Gamma \).
  ◦ Learn about risk factor payoffs \( f' \Gamma_i \Rightarrow \hat{\Sigma} = \Gamma' \hat{\Lambda} \Gamma \).
Setup: An Individual Investor’s Problem

- Information chosen
- Payoff \( f \) realized
- Signals and prices realized
- Asset shares \( (q) \) chosen

\[
f \sim N(\mu, \Sigma) \\
\hat{\mu} \sim N(\mu, \Sigma - \Sigma) \\
f \sim N(\hat{\mu}, \hat{\Sigma})
\]

Time:
- Time 1
- Time 2
- Time 3
Setup: The Investor’s Problem

- Objective: Maximize risk-adjusted profit
  \[
  U = E \left[ q' (f - pr) - \frac{\rho}{2} q' \hat{\Sigma} q \mid \mu, \Sigma \right]
  \]
  - Comes from \( U = E_1[\log(E_2[-e^{-\rho W}])] \).
    Preference for early resolution of uncertainty.

- Period 2: \( q = \frac{1}{\rho} \hat{\Sigma}^{-1} (\hat{\mu} - pr) \)
  - Asset supply \( \sim N(\bar{x}, \sigma^2_x) \).
  - Price \( p \) clears markets. \( (rp - A) \sim N(f, \Sigma_p) \)

- Period 1: \( \max_{\hat{\Lambda}} \frac{1}{2} E \left[ (\hat{\mu} - pr)' \Gamma \hat{\Lambda}^{-1} \Gamma' (\hat{\mu} - pr) \mid \mu - pr \right] \) s.t.
  - capacity constraint: \( \frac{|\hat{\Sigma}^{-1}|}{|\Sigma^{-1}|} \leq e^{2K} \),
  - no negative learning: signal var-cov. matrix positive semidefinite
Proposition 4 The optimal information portfolio with $N$ correlated assets uses all capacity to learn about one linear combination of asset payoffs $F'\Gamma_i$, associated with the highest learning index:

\[
(\Gamma_i'(\mu - pr))^2 \Lambda_i^{-1} + \Lambda_{pi}\Lambda_i^{-1}.
\]

- Learn about risks with high: (factor Sharpe ratio)\(^2 = \) expected factor return $\Gamma_i'(\mu - pr) \times \) expected factor portfolio share $\Gamma_i E[q]$.
- Learn about risks with high noise in prices: $\Lambda_{pi}$ is exploitable pricing error.
- Fully specialize in that risk factor: Increasing returns to information.
A Two-Asset Example

Objective is: \( \max_{\hat{\Lambda}} \sum_i \left( \Lambda_{pi} + (\Gamma_i' E[\hat{\mu} - pr])^2 \right) (\hat{\Lambda}_i)^{-1} \).

Fixed-q objective: \( \max_{\hat{\Lambda}} C - \sum_i (q'\Gamma)^2 \hat{\Lambda}_i \)
Strategic Substitutability: An Equilibrium Effect

- Equilibrium return on factor $i$: $\Gamma_i' E[f - rp] = \rho(\Gamma_i \bar{x})\hat{\Lambda}_{ai}$.
  
  - Average uncertainty: $\hat{\Lambda}_{ai}$. As more agents learn about risk factor $i$, $\hat{\Lambda}_{ai}$ decreases.
  
  - Expected excess returns are low (prices are high) for assets that load heavily on risk factors many investors learn about.

- More learning = more informative prices.
  Explicable pricing errors $\Lambda_{pi}$ falls when $\hat{\Lambda}_{ai}$ decreases.

- Both effects reduce the learning index when average uncertainty falls.

  $\Rightarrow$ Learn what others don’t know.
Which Risk Factors Are Learned in Equilibrium?

With sufficient capacity, identical investors study different risks. High learning index risks are studied more.
The optimal portfolio has **bi-polar structure**:

- **Diversified fund** $q^{\text{div}}$: optimal portfolio without learning ($K=0$).
- **Learning fund** $q^{\text{learn}}$: contains assets $\propto$ loading on risk factor learned about.

**Proposition 2** As capacity $K$ increases, $E[|q^{\text{learn}}|]$ increases and diversification falls.

**Corollary 3** An investor who optimally chooses a less diversified portfolio earns a higher expected return than an investor who chooses a more diversified portfolio.
Data Example - Uncorrelated Assets

Optimal Risky Asset Allocation

- Diversified Portf.
- Learning Portf.
- Total Risky Portf.

Portfolio Share
**Data Example - Correlated Assets**

**Optimal Risky Asset Allocation**

- **Portfolio Share**
- **Optimal Risky Asset Allocation**
- **Diversified Portf.**
- **Learning Portf.**
- **Total Risky Portf.**

### Assets
- **ATT**
- **Chevron**
- **JP Morgan**
- **Cisco**
Extension: Un-Learnable Risk

• Of the total variance $\Sigma$, only $(1 - \alpha)\Sigma$ can be learned.

$$\log(|\Sigma - \alpha\Sigma|) - \log(|\hat{\Sigma} - \alpha\Sigma|) \leq 2K$$

• Eliminating learnable risk requires $K = \infty$

• Benefits to specialization are bounded.

• Proposition 5 As capacity rises, investors learn about multiple factors.
Heterogenous Information Endowments and Home Bias

- If investors are endowed with small initial information advantage, will learning undo it?

- Increasing returns makes it optimal to specialize learning in assets/risk factors with initial advantage.

- Applications (Van Nieuwerburgh and Veldkamp, 2004b):
  - Home bias - add a 2-country environment. Determines who learns what.
  - Local bias. Excess returns imply capacity $K$ sufficient to explain home bias.
  - Asymmetry in market size $\Rightarrow$ Asymmetry in home bias.
  - Asymmetry in capacity $K \Rightarrow$ Patterns of international investing.
 Increasing Returns to Information and Home Bias

- Hold more assets you’re initially better informed about.
- Learn about assets you expect to hold.
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Testing Asymmetric Information Theories

• Problem with asymmetric information theories: We can’t test them, because we can’t observe information.

• Solution: A model that ties information to observable variables. Learning indices do that. Paper describes an algorithm to estimate them.

• 2 strategies to test this theory
  1. High learning index → more analyst / newsletter /press coverage.
  2. High learning index → positive CAPM pricing error.
     Size premium is consistent (Fama and French, 1992)
Conclusions

- Investors specialize: Learning and investing reinforce each other.

- Substitutability: Under-diversification arises because investors learn about different risks.

- Bipolar portfolios: Balance specialization and diversification.

- Learning indices: Make unobservable information predictable. Decouple variance from risk.

- Extending the theory: Markets for information could explain mutual funds/portfolio delegation facts.
Work in Progress: A Theory of Mutual Funds

- What if a portfolio manager can process information for many investors?

- Portfolio managers will have the same incentives to specialize learning.

- How to prevent information leakage?

- How to price information services?

- Will under-diversification still be optimal for investors?
  
  Yes, because of *non-linear fee structure!*
Risk Factor Choice

- For symmetric learning, all agents must have identical prior beliefs. A Mondria (2006) agent who knows more, learns more about that risk.
- Symmetric learning is only sustainable for parameters where learning has little effect on price.