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Problem set # 3

5.2 We have:

$$EW = -(1-r)\frac{\tau_y + (1-r)^2\tau_\omega + \tau_z}{(\tau_y + (1-r)\tau_\omega + \tau_z)^2}$$

$$\frac{\partial[EW]}{\partial\tau_y} = \frac{-(1-r)}{(\tau_y + (1-r)\tau_\omega + \tau_z)^3} \times \left[(\tau_y + (1-r)\tau_\omega + \tau_z) - (\tau_y + (1-r)^2\tau_\omega + \tau_z) \times 2\right]$$

$$= \frac{+(1-r)}{(\tau_y + (1-r)\tau_\omega + \tau_z)^3}\left[+\tau_y + \tau_z + [1-2r](1-r)\tau_\omega\right]$$

$$\Rightarrow \frac{\partial EW}{\partial\tau_y} > 0 \iff \frac{\tau_y + \tau_z}{\tau_\omega} > (2r-1)(1-r)$$

So more precision in the state is welfare improving if:
- coordination motives are weak
- common prior and public info sufficiently precise.

$\boxed{5-4}$

$$U_i = -(1-r)(a_i-s)^2 - r(a_i-\bar{a})^2 + (1-r)\int_0^1 (a_j-s)^2 dj$$

(a) Here perfect coordination: $a_i = \bar{a}\ \forall i$ maximizes social welfare. Because of the externality, this allocation is not Pareto optimal. So the amount of coordination is less the the socially efficient level.

$a_i = y + \alpha_\omega \frac{(1-r)}{1-\alpha_\omega r}(\omega_i - y) + \frac{\alpha_z}{1-\alpha_\omega r}(z-y) \Rightarrow a_i \neq a_j$ if $\omega_i \neq \omega_j$

(b) $$W = \int_0^1 U_i\, di$$

$$= -r\int_0^1 (a_i-\bar{a})^2 di$$

with $a_i = y + \frac{\alpha_\omega(1-r)}{1-\alpha_\omega r}(\omega_i-y) + \frac{\alpha_z}{1-\alpha_\omega r}(z-y)$

$$W = -r\int\left[\frac{1-\alpha_\omega-\alpha_z}{1-\alpha_\omega r}(y-\bar{a}) + \frac{\alpha_\omega(1-r)}{1-\alpha_\omega r}(\omega_i-\bar{a}) + \frac{\alpha_z}{1-\alpha_\omega r}(z-\bar{a})\right]^2 d_i$$

where: $\bar{a} = \int a_i\, d_i = \int\left(y + \frac{\alpha_\omega(1-r)}{1-\alpha_\omega r}(\omega_i-y) + \frac{\alpha_z}{1-\alpha_\omega r}(z-y)\right)d_i$

$$= y\left[\frac{1-\alpha_\omega-\alpha_z}{1-\alpha_\omega r}\right] + s\left[\frac{\alpha_\omega(1-r)}{1-\alpha_\omega r}\right] + z\left[\frac{\alpha_z}{1-\alpha_\omega r}\right]$$

$$\Rightarrow W = -r\int\left[\frac{\alpha_\omega(1-r)}{1-\alpha_\omega r}(\omega_i-s)\right]^2 d_i$$

$$\Rightarrow W = \frac{-r}{(1-\alpha_\omega r)^2}(\alpha_\omega(1-r))^2\tau_\omega^{-1} = -\frac{r(\tau_\omega(1-r)^2)}{(\tau_\omega(1-r)+\tau_y+\tau_z)^2}$$

- $\dfrac{\partial \omega}{\partial T_z} > 0$ since:

$$T_z \uparrow \Rightarrow \frac{T_Z(1-r)}{T_y+T_Z+T_\omega(1-r)} \downarrow \Rightarrow \omega \uparrow$$

(c)

- $$\frac{\partial \omega}{\partial T_\omega} = -2\left[-r\,T_\omega(1-r)\right]\left[T_y+T_Z+T_\omega(1-r)\right]^{-3}(1-r) + \left[T_y+T_Z+T_\omega(1-r)\right]^{-2}\left[-r(1-r)\right]$$

So: $\dfrac{\partial \omega}{\partial T_\omega} > 0$

$$\Leftrightarrow \frac{2\,T_\omega(1-r)}{T_y+T_z+T_\omega(1-r)} \geq 1$$

$$\Leftrightarrow (1-r) \geq \frac{T_y+T_Z}{T_\omega}$$

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