Homework #4

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1. $$\Lambda_{t,t} = \begin{cases} 1 & \text{if } t \text{ is even} \\ 0 & \text{if } t \text{ is odd.} \end{cases}$$

$$\Lambda_{t,\tau} = 1 \qquad \forall \tau \leq t-1$$

Then,

$$P_t^* = \begin{cases} \sum_{\tau=0}^{\infty} \varepsilon_{t-\tau} & \text{if } t \text{ even} \\ \sum_{\tau=1}^{\infty} \varepsilon_{t-\tau} + (1-r)\varepsilon_t & \text{if } t \text{ odd} \end{cases}$$

and

$$\mathbb{E}\left[P_t^* \mid I_{t-1}\right] = \sum_{\tau=1}^{\infty} \varepsilon_{t-\tau}$$

$$\mathbb{E}\left[P_t^* \mid I_t\right] = \begin{cases} \sum_{t=0}^{\infty} \varepsilon_{t-\tau} & \text{if } t \text{ even} \\ \sum_{t=1}^{\infty} \varepsilon_{t-\tau} + (1-r)\varepsilon_t & \text{if } t \text{ odd.} \end{cases}$$

Then, loss from updating in odd periods is:

$$\sum_{t=0}^{\infty} \left[\beta^{2t} \, \mathbb{E}(\varepsilon_t^2) + \beta^{2t+1} c\right] = \sum_{t=0}^{\infty} \beta^{2t} \sigma^2 + \sum_{t=0}^{\infty} \beta^{2t+1} c$$

$$= \frac{\sigma^2}{1-\beta^2} + \frac{\beta c}{1-\beta^2}$$

Loss from updating in even periods:

$$\sum_{t=0}^{\infty}\left[\beta^{2t+1}\left((1-r)^2\,\mathbb{E}(\varepsilon_t^2)\right) + \beta^{2t}c\right] =$$

$$(1-r)^2\sum_{t=0}^{\infty}\beta^{2t+1}\sigma^2 + \sum_{t=0}^{\infty}\beta^{2t}c = \frac{(1-r)^2\beta\sigma^2}{1-\beta} + \frac{c}{1-\beta^2}$$

Then, it's better to update in even periods if:

$$(1-r)^2\beta\sigma^2 + c < \sigma^2 + \beta c$$

If c is small enough (or we forget about it)

we know that $(1-r)^2\beta < 1$ $\Rightarrow$ updating in even periods is better than doing so in odd periods.

and there is complementarity in updating dates

2) Additive precision constraint

$$\sigma_\varepsilon^{-2} + \sigma_\varphi^{-2} \leq k$$

We know that the optimal price is

$$p_{it} = E\left[\bar{p}_t - \frac{\Pi_{py}}{\Pi_{pp}} y_t - \frac{\Pi_{pz}}{\Pi_{pp}} z_{it} \,\middle|\, s_i^t \right].$$

$$= E\left[\Delta_t - \frac{\Pi_{pz}}{\Pi_{pp}} z_{it} \,\middle|\, s_i^t \right)$$

where $\Delta_t = \bar{p}_t - \frac{\Pi_{py}}{\Pi_{pp}} y_t$.

and $$EL = \frac{|\Pi_{pp}|}{2} E\left[(p_{it} - p_{it}^*)^2 \right].$$

Signals: $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$; $\varphi_{it} \sim N(0, \sigma_\varphi^2)$

EL can be rewritten as:

$$EL = \frac{|\Pi_{pp}|}{2} \left(\hat{\sigma}_\Delta^2 + \left(\frac{\Pi_{pz}}{\Pi_{pp}} \right)^2 \hat{\sigma}_z^2 \right)$$

Since we know the constraint will always bind (EL is increasing in $\hat{\sigma}_\Delta^2$ and $\hat{\sigma}_z^2$) we can substitute out $\hat{\sigma}_z^2$

$$\hat{\sigma}_\Delta^{-2} = \sigma_\Delta^{-2} + \sigma_\varepsilon^{-2} \qquad \sigma_\varepsilon^{-2} = \hat{\sigma}_\Delta^{-2} - \sigma_\Delta^{-2}$$

$$\hat{\sigma}_z^{-2} = \sigma_z^{-2} + \sigma_\psi^{-2} \qquad \sigma_\psi^{-2} = \hat{\sigma}_z^{-2} - \sigma_z^{-2}.$$

Then the precision constraint becomes.

$$\hat{\sigma}_\Delta^{-2} + \hat{\sigma}_z^{-2} - (\sigma_\Delta^{-2} + \sigma_z^{-2}) \le k.$$

and since it binds,

$$\hat{\sigma}_z^{-2} = k + (\sigma_\Delta^{-2} + \sigma_z^{-2}) - \hat{\sigma}_\Delta^{-2}$$

The EL function becomes:

$$EL = \frac{|\pi_{pp}|}{2}\left(\hat{\sigma}_\Delta^{2} + \left(\frac{\pi_{pz}}{\pi_{pp}}\right)^2\left(\sigma_\Delta^{-2} + \sigma_z^{-2} + k - \hat{\sigma}_\Delta^{-2}\right)^{-1}\right)$$

FOC with respect to $\hat{\sigma}_\Delta^2$:

$$0 = \frac{|\pi_{pp}|}{2}\left[1 + \left(\frac{\pi_{pz}}{\pi_{pp}}\right)^2 \frac{\hat{\sigma}_\Delta^{-4}}{(\sigma_\Delta^{-2} + \sigma_z^{-2} + k - \hat{\sigma}_\Delta^{-2})^2}\right]$$

$$\sigma_\Delta^{-2} + \sigma_z^{-2} + k - \hat{\sigma}_\Delta^{-2} = -\left|\frac{\pi_{pz}}{\pi_{pp}}\right| \hat{\sigma}_\Delta^{-2}$$

$$\hat{\sigma}_\Delta^{-2}\left(1-\left|\frac{\Pi_{pz}}{\Pi_{pp}}\right|\right)=\sigma_\Delta^{-2}+\sigma_z^{-2}+k$$

$$\hat{\sigma}_\Delta^{-2}=\left(\sigma_\Delta^{-2}+\sigma_z^{-2}+k\right)\left(1-\left|\frac{\Pi_{pz}}{\Pi_{pp}}\right|\right)^{-1}$$

$$\hat{\sigma}_z^{-2}=\left(\sigma_\Delta^{-2}+\sigma_z^{-2}+k\right)\left(1-\left(1-\left|\frac{\Pi_{pz}}{\Pi_{pp}}\right|\right)^{-1}\right)$$

A Excellent job.