# THE RATIONALITY OF ASSET ALLOCATION RECOMMENDATIONS

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In recent years, the literature of Financial Economics has contained numerous articles examining the reasonableness and accuracy of investment advice. Topics such as earnings estimates, security analysts recommendations, and recommendations for selecting mutual funds have been studied extensively. However, almost no attention has been paid to examining advice about the asset allocation decision (the allocation of funds across broad classes of assets). This is surprising because the asset allocation decision has been recognized as a major determinant of return and risk and because of this, advice on the optional allocation decision is provided by most brokerage firms and investment advisors

Given the importance of the asset allocation decision we anticipate that the rationality of asset allocation advice will be extensively examined. The purpose of the article is two fold. First, we will examine modern portfolio theory to see what we can learn about the general characteristics of advice that are necessary for consistency with theory. Second, we will examine the advice of some specific investment advisors to see if their advice is consistent with rational behavior.

We proceed in three steps. We first review some of the basic tenets of MPT, discuss alternative formulations of the problem, and examine which formulation is appropriate and consistent with advisor recommendations. Second, using efficient set mathematics we examine some criteria for judging rational asset allocation that have been suggested by others. Finally we examine the specific asset allocations proposed by a set of investment advisors to see if they are consistent with MPT under realistic estimates of inputs to the portfolio optimization problem.

## *I. Modern Portfolio Theory*

The simplest form of modern portfolio theory represents the investor's problem in mean return standard deviation space as a choice among efficient portfolios (minimum standard deviation for any expected return). Depending on the investor's tolerance for risk, a different efficient portfolio will be selected from among those in the efficient set. The shape, composition and characteristics of the efficient frontier depend on assumptions about the existence of a riskless asset and whether investors can short sell risky assets or not. Each of these assumptions leads back to a different model for determining the efficient frontier. We will briefly review these alternative models, their implications for the characteristics of investment advise, and the assumptions that make each one the correct problem to solve.

If investors can risklessly lend and borrow at the same rate, then the separation theorem holds and the investor's problem is to find the optimal mix of the riskless asset and the optimal risky portfolio. Investors with different risk tolerance simply hold different percentages of the riskless asset and the risky portfolio and the relative proportions invested in each of the risky assets remains constant. The separation theorem holds whether short sales of risky assets are allowed or disallowed.[1] If this is the appropriate model, asset allocation advice at different risk levels should be simply different linear combinations of the riskless asset and the tangency portfolio. This implies that the ratio of bonds to stocks (the two risky assets) should remain unchanged across all portfolios recommended by an investment advisor.

[1] See, for example, Elton and Gruber (1995).

If a riskless asset does not exist, the important assumption affecting the characteristics of the efficient frontier is whether short sales are allowed or not. If the standard definition of short sales is assumed, the two fund theorem holds and all portfolios on the efficient frontier are a linear combination of any two other efficient portfolios.[2] Assume we observe three efficient portfolios. If short sales are allowed, then the proportions invested in each risky asset for any of the three portfolios is a constant linear combination of the proportions held in the other two. All assets are held in positive or negative proportions except that for each asset there is a maximum of one efficient portfolio where it is held with zero weight. This implies that a necessary condition for an investment advisor to be making rational allocation decisions using the assumption of short sales allowed is that the allocation for any recommended portfolio must be a linear contribution in any two other recommended portfolios.

If short sales are not allowed, the nature of the efficient frontier changes. The two fund theorem no longer holds. Securities enter and leave the efficient frontier at different risk return tradeoffs. The points where they enter or leave are called corner portfolios. Securities may be held in zero weight for a range of risk tolerance and some assets are never held. Generally, the maximum return portfolio on the efficient frontier will consist of one asset and the minimum risk portfolio will consist of multiple assets. Thus, if short sales are not allowed and advisors are rational, any allocation recommendation should not be a linear combination of any two others unless all three lie at or between adjacent corner portfolios.

[2] See Black (1972). The discussion of short sales allowed follows the standard definition of short sales, namely that the proceeds are fully available for investment in other assets. A more realistic characterization of short sales that restricts their use (the Lintner definition; see Lintner (1965)) results in an efficient frontier with characteristics more like the one described for no short sales. Under Lintner's definition with no riskless lending and borrowing the two fund theorem no longer holds.

In order to choose among alternative models, we must first decide on whether the analysis is performed in terms of real or nominal returns and then decide on whether short sales are allowed or not. If the analysis is done in real returns, then all bonds with a fixed coupon (including T-bills) are risky. The only potential risk free asset is an inflation indexed bond. If the analysis is done in nominal terms then a riskless asset can exist.

This leaves us with a choice of whether the standard definition of short sales should be allowed or not. In examining general rationality criteria for the asset allocation decision we will examine the case where short sales are allowed and where they are not allowed. However, we believe that many investment advisors including the ones we have chosen to examine in the third part of this paper (Merrill Lynch, Jane Bryant Quinn and the *New York Times)* do not consider short sales.[3] We believe this is appropriate for two reasons. First, the investment advisors are concerned with the allocation across a stock portfolio, a bond portfolio and a money market portfolio. What are these portfolios? For Fidelity it is certainly the mutual funds of these types they offer. For Merrill Lynch the portfolios can be mutual funds or bond or stock accounts managed by Merrill Lynch. Finally, Quinn explicitly discusses mutual funds as the investment vehicle. For open-end funds and managed accounts, short sales were not possible at the time of the advice.[4] Even if we assume mutual funds or managed accounts could be sold short, there is a second problem. The definition of short sales assumes that the investors can short sell at no cost,

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[3] The Merrill Lynch recommendations are in Underwood and Brown (1993). The Fidelity recommendations are in Mark (1993). As explained latter, we selected these advisors to compare our results with those of Canner Mankiw and Weil (1997)

[4] The only possibility of short sales of asset classes is for large closed end funds. Insofar as the assets are held in the form of pension funds, e.g., 401K plans, there are additional legal and self-imposed constraints on short sales.

and that the full proceeds are available for investment in other assets. For a stock or bond managed account, the standard definition of short sales requires that investors short sell a managed account, again at no cost and have the entire proceeds available for investment in another account. The standard definition of short sales assumes the investor, by employing short sales, can create portfolios of extreme risk and return through this costless leveraging of starting capital. But investors cannot do this. Individual investors pay a high fee for short sales, cannot short sell a managed portfolio, and the use of funds that arise from short sales of individual securities is restricted by brokerage firms.[5] Thus an assumption of no short sales is the only realistic assumption for the asset allocation decision in general and for specific advisors we study in the latter section.

In summary, it seems reasonable to assume that the most appropriate model to use for investors allocating money across a money market fund, a bond fund and a stock fund is one assuming no short sales of risky assets. Whether a riskless asset exists depends on whether the analysis is done in real or nominal returns.

## II *A Proposed Rationality Test*

In this and the following section of this article we shall repeatedly refer to a recent article on asset allocation by Canner, Mankiw and Weil (1997) (CMW). CMW have produced the first study to explicitly try to develop tests of investor rationality and to examine the rationality of the

[5] Lintner recognized this and reformulated portfolio theory under more realistic assumptions about the use of proceeds from short sales. While Lintner's definition of short sales leads to an alternative proof of the CAPM. Two fund separation does not hold under his definition of short sales.

advice of a specific set of well-known advisors. The complexity of the issues involved and the difficulty and ambiguity of applying the tenants of portfolio theory outlined in the previous section can best be illustrated by employing the CMW study as a concrete example. CMW, in their article, advocate the following as their test of rationality for asset allocation advice: the ratio of bonds to stocks should rise as an investor is willing to take more risk. In this section, we will examine this rationality criteria both in the case where short sales are allowed and the case where short sales are not allowed (the framework we feel is the one used by the advisors). We show that (a) when unrestricted short sales are allowed an increase in the ratio of bonds to stocks as risk increases does not follow from efficient set mathematics but only holds under particular estimates of expected returns, variances and covariances; and (b) in the case of short sales not allowed it is impossible for a continuously rising bond to stock ratio to be rational over the entire range of increasing risk. Thus, CMW's test for rationality of asset allocation proposals is at best inclusive.

## A. Short Sales Allowed

If short sales are allowed, the two fund separation theorem holds and the proportion invested in any asset is linear when plotted against expected return (or risk). This means that the ratio of bonds to stocks must be monotonic when plotted against the proportion invested in stock. However, as proved in the appendix, it can be monotonically increasing or decreasing. For the particular inputs CMW use, it is monotonically increasing as shown in Figure 1-A.[6]

[6] CMW's analysis results in bonds being sold short at low risk levels and T-bills sold short at high risk levels. In fact at the high risk levels, T-bills must be sold short in the amounts in excess of the initial capital of the investor. The assumption here is that investors can borrow unlimited sums of money at the same rate and with the same risk as the US Government.

While there are many sets of historic data and subjective estimates of the future for which the ratio is monotonically increasing, there are also many sets of historic data and plausible estimates of the future for which it is monotonically decreasing. For example if CMW had used as estimates of the future the history of real monthly returns over the final five years of their sample (more recent data), they would have found that the ratio of bonds to stock was monotonically decreasing. We present this input data in Table 1-A and the relationship between the ratio of debt to equity and risk (expressed as CMW does as the fraction of the portfolio in stock) in Figure 1-B. In section III, we develop sets of input data that are consistent with the recommendation of Fidelity, Jane Bryan Quinn, and Merrill. These are shown in Tables 1-B to 1-D. In Figures 1-C, D and E we have graphed the relationship between the debt to equity ratio and the fraction of the portfolio in common stock for data consistent with the recommendations of these three advisors when short sales are allowed. Notice that for Fidelity and Jane Bryant Quinn, the ratio of bonds to stocks is monotonically decreasing, while for Merrill it is monotonically increasing.

As we have shown with short sales allowed the relationship between the ratio of bonds to stocks and risk (expressed as the fraction of stock in the portfolio) can be monotonically increasing or decreasing depending on the choice of input data. This means that the shape of this relationship cannot be used as a test for rationality of investment asset allocation advice when short sales are allowed.

## B. Short Sales Not Allowed

When short sales are not allowed, the efficient set mathematics are more complicated. Efficient portfolios are only mixtures of adjacent efficient portfolios between corner portfolios. Thus only between corner portfolios need there be a monotonic relationship. Each corner portfolio is an inflection point and changes the slopes of the lines plotting asset proportions versus the expected return of the efficient portfolios.

However, there are some general statements we can make about the bond-to-stock ratio and the proportion in stocks. As long as stocks have the highest expected return, the ratio of bonds to stocks must decrease for high regions of expected return. This follows since if short sales are not allowed and stocks have the highest expected return, the highest return efficient portfolio is 100% in common stocks. If the curve is monotonic it must decrease over its entire length. The most common shapes for the relationship are humped (increasing ratio of bonds to stocks followed by a decreasing ratio as risk increases) and a continuously decreasing curve.

CMW, in their Table 2, present the set of data which they use to construct their efficient frontier. Employing this data to solve for the efficient frontier with no short sales and no riskless lending and borrowing leads to the portfolio composition graphed in Figure 2. We can see from this figure that an investor who starting with a low return portfolio and seeking a higher return, first increases the ratio of bonds to stocks. But to obtain still higher returns, the investor

substitutes stock for bonds, decreasing the ratio of bonds to stocks until the ratio falls to zero.[7] Thus, even using the data of CMW, the downward sloping bond/stock mix of the advisors is not evidence of inconsistencies with MPT. In particular, over the high risk segment which CMW discusses, it is downward sloping, and in fact it must be downward sloping for any input data where stocks offer the highest expected return. Using the input data consistent with the recommendations of the advisors shows that with no short sales the Merrill estimates result in a relationship like Figure 2 while for Jane Bryant Quinn and Fidelity the ratio of bonds and stocks declines monotonically throughout. These relationships are shown in Table 2. Thus, for many plausible sets of inputs, there isn't any range of risk where the ratio of bonds to stocks increases.

## III *Consistency with Specific Recommendations*

In this section, we will examine the specific asset allocation advice of the advisors examined by CMW. So that our own analysis can be compared to CMW we will make their assumption that the analysis should be done with real returns. CMW in their article argued that the advisors advice was inconsistent with modern portfolio theory and hence irrational. In this section, we will show these conclusions are wrong. To examine the appropriateness of the specific recommendations of each of the advisors analyzed by CMW we need estimates of mean returns, variances of returns and correlation coefficients. An examination of the reports of the advisors makes it clear they are concerned with the risk and return that will arise in the future

[7] This is not surprising, since the highest obtainable point in the efficient frontier is 100% investment in the asset (or assets in the case of a tie) with the highest expected return.

from asset allocation across a diversified portfolio of assets and more specifically a money market fund, a stock fund and a bond fund.[8] CMW use as a proxy for the future return on these portfolios the return from 1926 to 1992 for the S&P Index, the long term bond index, and the 30-day Treasury bill index as constructed by Ibbotson. As we will show shortly, the use of Ibbotson data and MPT exactly reproduces the recommendations of one of the advisors quoted by CMW. However, the Ibbotson data starting in 1926 is not the only or necessarily even the best data to use for forecasts of the future behavior of capital markets. Forecasts can differ from the history presented in Ibbotson for several reasons. First, the portfolios being analyzed by the investment advisors may be poorly represented by the Ibbotson indexes. Second, forecasters should employ more recent market data to modify simple extrapolation of the past.

We start by discussing why investment advisors might rationally choose to use alternative indexes to represent fund returns. While the S&P Index (the Ibbotson stock index) is the most commonly used measure of the return on large stock portfolios, several of the advisors recommend portfolios including smaller stocks and foreign stocks. The risk and return characteristics of these portfolios will differ from the S&P Index. Ibbotson long term bond index is computed using the monthly return on a single bond with a maturity of approximately 20 years. Bond funds hold portfolios of bonds with a much shorter maturity than 20 years and this affects the distribution of returns. Expectations about average returns can be lowered or raised by holding shorter maturity bonds depending on the analyst's beliefs about interest rate movements relative to those impounded in the yield curve. The expectations about the size of the standard deviation are clearly smaller for a shorter maturity portfolio. In addition, the fact that a mutual

[8] For example, Mark Lary (1993).

funds holds a portfolio of bonds rather than a single bond also reduces risk. The Shearson Lehman Government Corporate Index, a value weighted index of all bonds with a maturity of more than one year, is a much better proxy for bond mutual funds and a frequent target which bond index funds try to replicate. If one wishes to use the Ibbotson data, a mixture of 80% of their intermediate government and 20% of their long term corporate bond index, has a beta of one with the average bond mutual fund (see Elton, Gruber, Das and Hlavka (1993)). In any case, actual bond mutual funds are less risky than the Ibbotson long term bond index. The opposite is true for money market funds. The 30 day T-bill rate likely understates the risk on money market mutual funds. Actual money market funds generally invest longer and in more risky instruments than the 30 day T-bill in an attempt to attract investors by outperforming the return on short-term government debt.

Whether using the Ibbotson data or other indexes of performance, historical data should only be a starting place. The analyst should modify historical values to reflect beliefs about how current conditions differ from the past. If historical data is being used as a starting place, it is necessary for an analyst to select an observation interval. In finance, it is common to use monthly intervals to measure returns used in estimating expected returns, variances, and covariances. There is substantial evidence that monthly returns are fairly independent, and using them increases the number of data points and the precision of the estimates.

In inferring the data on which an analyst may base recommendations, we have a serious problem. We can never determine exactly what a particular analyst used for estimating parameters because there are an infinite number of estimates that can produce a given set of

recommendations.[9] Thus, we can never be sure we are using the same inputs the advisor used. What we can, and will do, is to show that there exists a set of reasonable inputs for each advisor discussed by CMW such that his or her advice is consistent with modern portfolio theory. In our analysis, we will use real returns and assume short sales are not allowed. We use real returns because the investment advisors discussion suggests this is what they were using and because this is what CMW used.

The easiest results to validate are those presented by Merrill Lynch. Given the date of publication, Merrill Lynch most likely observed data through 1992.[10] To obtain the Merrill Lynch recommendations we assume that the historic monthly joint distribution of real returns (from 1926 to 1992) is predictive of future returns. As a first step in analyzing the Merrill Lynch recommendations we followed the procedure of CMW and assumed that the appropriate proxies were the Ibbotson, S&P Index, long term bond index and 30 day Treasury bills. The input data computed from Ibbotson is shown in Panel B of Table 1.

Based on this data, we can solve for the efficient frontier. Since the Merrill Lynch guide discusses in great detail the importance of always holding some cash in a portfolio, and since the recommendation showed no less than five percent in cash for the most aggressive portfolio, we solved for a set of efficient portfolios with the constraint that cash could not fall below 5% of the

[9] The input parameters that can be selected are mean returns (3), standard deviation of returns (3),correlations (3), and risk tolerances (3), or a total of 12 parameters. The output is the proportions for each recommendation. Since the proportions invested in each type of security add to one, this leaves six variables to solve for (two proportions for each of the three recommendations of the advisors). While the proportions must be positive, there are an infinite number of solutions.

[10] We also computed data ending in 1991 and 1993. The numbers are essentially the same.

portfolio.[11] The ratio of bonds to stocks as a function of the proportion in stocks is presented in Panel A of Table 2.

The three Merrill Lynch recommendations discussed by CMW (marked by x's) all lie on the efficient frontier.[12] Thus the Merrill Lynch recommendations are totally consistent with employing inputs calculated from the full history of Ibbotson and employing modern portfolio theory. The principal difference between our analysis and CMW is that we assume no short sales and they assume unrestricted short sales are allowed.

Figure 2 plots the bond/stock ratio. Note that consistent with CMW data analyzed earlier, the bond/stock ratio first rises and then falls. Depending on which section we take recommendations from, we can find the bond/stock ratio as rising or falling. For the risk return levels reported by Merrill Lynch (marked by x's), the bond/stock mix should, and does, fall. Thus, the CMW conjecture that the bond-stock mix should rise throughout does not hold with the data used to produce the Merrill Lynch recommendations.

In examining the recommendations of Jane Bryant Quinn and Fidelity, it was necessary to make more sophisticated assumptions about the data they were using. After all, if they had simply used historic Ibbotson data from 1926 on they would have gotten exactly the same results as Merrill Lynch and the same results as each other. Fortunately, the author's discussion which

[11] Merrill Lynch states that there are at least 3 reasons why an investor should always hold a portion of the portfolio in cash: in case of emergencies, to take advantage of opportunities, and to pay taxes.

[12] We, like the four analysts discussed by CMW, rounded investment weights to the nearest 5%.

accompanies their recommendation gives us an idea of what data the authors may have been using.

First consider the recommendations of Jane Bryant Quinn. In Panel C of Table 1, we present one set of inputs which are reasonable given the timing of Jane Bryant Quinn's book and which lead to her recommendations. Her book has a 1991 publication date. However, the data quoted in the book ends in 1989. Given production time and her data, it is likely that her analysis was done in early 1990.

Portfolio management requires the analyst's best estimate of future values. Analysts start with history and modify it. Jane Bryant Quinn examines Ibbotson data and in her book puts more emphasis on recent data. Analysts need to develop estimates of means, standard deviation, and correlation coefficients. However, the one type of input where they are most likely to accept historic data without modifying it is in estimating future correlation coefficients. Since Quinn relies on Ibbotson data and given that Ibbotson recommends using twenty years of data to compute correlation coefficients, correlation coefficients were calculated from twenty years of Ibbotson data ending in December 1989. These are the numbers shown for correlation coefficients in Panel C of Table 1. The remaining numbers are one set of values of expected return and standard deviation of return which are consistent with Quinn's results. The expected stock return, the standard deviation of stock returns and the standard deviation of bond returns are at or close to their historical values. The mean return on bonds, the T-bill return and the T-

bill standard deviation are further from historical values. Are the deviations from historical values for these inputs reasonable in early 1990?

In early 1990, long-term government yields were about 9% and corporates yielded about 1% more.[13] Inflation for the year 1990 was 6.1%. If yields are used as a proxy for expected return, the estimate of 3% a year or .24% per month employed for real returns on bond funds in Table 1 seems quite reasonable. The expected real return for treasury bills is significantly higher than the historic mean calculated from 1926 on. However, real returns on Treasury bills for the more recent years prior to Quinn's forecast were much higher. Whether one used 1, 5, 10 or 20-year holding periods, the historical mean return was close to or above the estimate used to produce Quinn's recommendations. For example, T-bills offered a real return of about twenty-five basis points per month over the ten years starting in 1980, and offered a real return of 14 basis points in the year in which Quinn prepared her forecast. The estimate of 12 basis points per month is reasonable given the long-term history of treasury bill returns and their more recent performance.

The final number that is different from its historical value is the standard deviation of Treasury bills. The number we used is much higher than historic values. There are two reasons why this is reasonable. First, Quinn is offering advice to investors on a mixture of a stock fund, bond fund and money market fund. Money market funds do not invest exclusively in 30 day Treasury bills. Their actual portfolio is generally of longer duration and includes riskier assets.

[13] We fully recognize yields need not be expected returns. However, assuming that the expected price change on bonds is zero and bonds are at par (the assumptions needed for expected return to be yield) are quite common.

Second, compared to history, real returns on Treasury bills were more volatile in 1990. Thus, assuming the standard deviation of real returns on money market funds was much higher than the historical standard deviation of 30 day Treasury bills was very rational in 1990. The data produces the results shown in Panel B of Table 2. The ratio of bonds to stocks falls throughout including the three recommendations discussed by CMW and marked by x's. We have shown that Quinn's results are consistent with modern portfolio theory and a sensible set of estimates. Again, the key difference between our analysis and CMW is that we assume short sales are not allowed.

The third set of recommendations presented by CMW is those of Fidelity. Fidelity is making recommendations concerning proportions in three types of Fidelity funds (stock funds, bond funds and money market funds). As discussed earlier, while Ibbotson indexes are useful for representing the performance of stock funds, some care must be used to represent bond or money market funds. The Ibbotson index chosen by CMW for bond funds is the Ibbotson Long Term Bond Index. Since Fidelity's recommendations are clearly concerning allocation across mutual funds, we used indexes that better represent returns on bond funds. As discussed earlier, an 80/20 mix of the Ibbotson Intermediate Term Government Bond Index and the Long Term Corporate Index is a good proxy for a typical bond mutual fund.

The date given for the Fidelity recommendations is winter 1993. Therefore, to get an estimate of the correlation matrix we used monthly data from the beginning of available data through December 1992, where the bond index is an 80/20 mix. This is the data shown in Panel D of Table 1. Once again we modified mean returns and standard deviation of returns. We

modified the historical data to reflect the nature of the market instruments being recommended by Fidelity. The largest changes involved the forecasts for T-bills. The mean return on T-bills is again well above the mean return using the full data set and close to the one used for Quinn. This is appropriate given the recent history of real returns on T-bills at the time of the forecast. The estimate of T-bill standard deviation consistent with Fidelity recommendations is higher than the historical average. This is also appropriate given the duration and security risk characteristics of money market funds relative to T-bills. The mean return on the bond proxy is increased slightly, 1 bp over historical levels (using an 80/20 mix). The mean return on stocks is increased by 20 bp. This increase reflects the higher return of non-S&P stocks and the tendency of mutual funds to hold non-S&P shares in 1992. Finally, the standard deviation for stocks is the historical standard deviation while for bonds it is only slightly higher than historic, given an 80/20 mix. With the data that produces the Fidelity recommendations the bond stock ratio falls throughout (see Panel C of Table 2 marked by x's), and CMW's rationality test fails.

Once again, with a sensible set of assumptions the recommendations of an advisor can be obtained using standard mean variance analysis.[14]

[14] We imposed one other constant. Like Merrill Lynch, we believe Fidelity is recommending a minimum amount in money market funds for liquidity needs. In solving the problem we imposed a minimum of 5% in the money market fund. We do not discuss the *New York Times* numbers for two reasons. First, the framework is unclear. Second, their recommendations seem to be a consensus of people they interviewed. Averaging across different recommendations does not produce a set of recommendations consistent with average expectations. Thus we do not expect to be able to find a consistent set of inputs using reasonable input data.

## CONCLUSION

The asset allocation decision is clearly a key decision in the investment process and one that has been largely neglected in the literature of Financial Economics. A recent article by CMW is the first to develop tests of rationality and to apply the tests to actual sets of advice. CMW single out the fact that the financial community advocates a decrease in the ratio of bonds to stocks as investors take on more risk as evidence of irrationality. We have shown that a decrease in the ratio of bonds to stock is not a result of the efficient set mathematics, but only holds with particular data sets. If short sales are allowed, the ratio of bonds to stocks must either monotonically increase or monotonically decrease, depending on the particular estimates of input data employed. We have shown that if short sales are not allowed than the bond stock mix can either increase or decrease as risk increases over low risk levels but the bond stock mix must decrease as risk increases over high risk levels. Thus whether short sales are allowed or not the sign of the relationship between the bond stock ratio and risk hypothesized by CMW can not be used as a rationality test.15

We have also discussed why we believe that the short sales not allowed case is the relevant case for most investment planning decisions and is almost certainly the case that is relevant for the investment advisors analyzed by CMW. We then show that the recommendation on asset allocation presented by the investment advisors analyzed by CMW are consistent with Modern

15 We have confined our discussion to standard mean variance return analysis - the base case in portfolio management. Can assuming a multi-period world with a set of utility functions or state space such that optimization can not be performed in terms of a derived single period utility function validate CMW's test of rationality? The answer is no as shown in Brennan and Xia (1999).

Portfolio Theory under reasonable assumptions about how capital markets might behave in the future.

It is extremely difficult to form guidelines as to when a set of investment advice is irrational. The problem is that given the mathematics of portfolio theory, there are an infinite set of inputs which are consistent with a set of asset allocation decisions when we only have the asset allocations for a few risk levels. Some of these sets of inputs would seem reasonable others would not. How can we improve tests of rationality. We have two suggestions. Have the advisor supply the input data on which suggested allocations are made. Then the rationality of the input data itself can be judged as well as whether the advisor used portfolio theory correctly. If this can not be accomplished, the advisor should be asked to supply more suggested portfolios (points on the efficient frontier). The more portfolios supplied to a person seeking to investigate advice, the smaller the set of input data which is consistent with that advice.

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