ON THE

VALUATION OF

CORPORATE BONDS

by

Edwin J. Elton,* Martin J. Gruber,*

Deepak Agrawal** and Christopher Mann**

* Nomura Professors, New York University

** Doctoral students, New York University

The valuation of corporate debt is an important issue in asset pricing. While there has been an enormous amount of theoretical modeling of corporate bond prices, there has been relatively little empirical testing of these models. Recently there has been extensive development of rating based models as a type of reduced form model. These models take as a premise that groups of bonds can be identified which are homogeneous with respect to risk. For each risk group the models require estimates of several characteristics such as the spot yield curve, the default probabilities and the recovery rate. These estimates are then used to compute the theoretical price for each bond in the group. The purpose of this article is to clarify some of the differences among these models, to examine how well they explain prices, and to examine how to group bonds to most effectively estimate prices.

This article is divided into four sections. In the first section we explore two versions of rating-based models emphasizing their differences and similarities. The first version discounts promised cash flows at the spot rates that are estimated for the group in question. The second version uses estimates of risk-neutral default probabilities to define a set of certainty equivalent cash flows which are discounted at estimated government spot rates to arrive at a model price. The particular variant of this second model we will use was developed by Jarrow, Lando and Turnbull (1997). In the second section of this paper we explore how well these models explain actual prices. In this section we accept Moody's ratings along with classification as an industrial or financial firm as sufficient metrics for grouping. In the next section, we examine what additional characteristics of bonds beyond Moody's classification are useful in deriving a homogeneous grouping. In the last section we examine whether employing these characteristics can increase the precision with which we can estimate bond prices.

I. Alternative Models:

There are two basic approaches to the pricing of risky debt: reduced form models, of which rating based models are a sub class, and models based on option pricing. Rating-based models are found in Elton, Gruber, Agrawal, and Mann (1999), Duffie and Singleton (1997), Jarrow, Lando and Turnbull (1997), Lando (1997), Das and Tufano (1996). Option-based models are found in Merton (1974) and Jones and Rosenfeld (1984). In this paper we will deal with a subset of reduced form models, those that are ratings based. Discussion of the efficacy of the second approach can be found in Jones and Rosenfeld (1984).

We now turn to a discussion of the two versions of rating-based models which have been advocated in the literature of Financial Economics and to a comparison of the bond valuations they produce. The simplest version of a rating-based model first finds a set of spot rates that best explain the prices of all corporate bonds in any rating class. It then finds the theoretical or model price for any bond in this rating class by discounting the promised cash flows at the spot rates estimated for the rating class. We refer to this approach as discounting promised payments or DPP model. The idea of finding a set of risky spots that explain corporate bonds of a homogeneous risk class has been used by Elton, Gruber, Agrawal and Mann (1999). While there are many ways to justify this procedure, the most elegant is that contained in Duffie and

Singleton (1997). They delineate the conditions under which these prices are consistent with no arbitrage in the corporate bond market. We refer to the DPP model as a rating based model under the reduced form category because, as shown in the appendix, DPP is equivalent to a model which uses risk neutral default probabilities (and a particular recovery assumption) to calculate certainty equivalent cash flows which are then discounted at riskless rates. To find the bonds model price the recovery assumption necessary for this equivalency is that at default the investor recovers a fraction of the market value of an equivalent corporate bond plus its coupon.

The second version of a rating-based model is the particular form of the risk-neutral approach used by Jarrow, Lando and Turnbull (1997), and elaborated by Das (1999) and Lando (1999). This version, referred to hereafter as JLT, like all rating based models involves estimating a set of risk-neutral default probabilities which are used to determine certainty equivalent cash flows which in turn can be discounted at estimated government spot rates to find the model price of corporate bonds[1]. Unlike DPP, the JLT requires an explicit estimate of risk neutral probabilities. To estimate risk neutral probabilities JLT start with an estimate of the transition matrix of bonds across risk classes (including default), an estimate of the recovery rate in the event of default, estimates of spot rates on government bonds and estimates of spot rates on zero coupon corporate bonds within each rating class. JLT select the risk-neutral probabilities so that for zero coupon bonds, the certainty equivalent cash flows discounted at the riskless spot

[1] As shown in Elton, Gruber, Agrawal and Mann (1999), state taxes affect corporate bond pricing. The estimated risk-neutral probability rates are estimated using spot rates. Since spot rates include the effect of state taxes. These tax effects will be impounded in risk-neutral probabilities.

rates have the same value as discounting the promised cash flows at the corporate spot rate. In making this calculation, any payoff from default, including the payoff from early default, is assumed to occur at maturity and the amount of the payoff is a percentage of par. This is mathematically identical to assuming that at the time of default a payment is received which is equal to a percentage of the market value of a zero coupon government bond of the same maturity as the defaulting bond.[2] Thus, one way to view the DPP and JLT models is that they are both risk neutral models but they make different recovery assumptions.

A. Comparison for zero coupon bonds

In this section we will show that for zero coupon bonds, the JLT and DPP procedures are identical. We will initially derive the value of a bond using the JLT procedure.

To see how these models compare, we defined the following symbols:

1. Q be the actual transition probability matrix.

---

[2] Many discussions of the JLT models describe this assumption as the recovery of an equivalent treasury. The equivalence occurs because all cash flows are discounted at the government bond spot rates.

2. $q_{id}(t)$ be the actual probability of going from rating class i to default sometime over *t* periods and is the appropriate element of Q^t .

3. $\Pi_i(t)$ be the probability risk adjustment for the t^{th} period for a bond initially in rating class *i*.

4. $A_i(t)$ be the risk adjusted (neutral) probability of going from rating class *i* to default at some time over *t* periods. It is equal to $\Pi_i(t)q_{id}(t)$.

5. V_{iT} be the price of a bond in rating class *i* at time zero that matures at time *T*.

6. r_{0t}^{g} be the government spot rate at time zero that is appropriate for discounting cash flows received at time *t*.

7. r_{0t}^{ci} be the corporate spot rate at time zero appropriate for discounting the cash flow at time *t* on a bond in risk category *i*.

8. b_i be the fraction of the face value for a bankrupt bond that is paid to the holder of a corporate bond in class *i* at the maturity.

Since zero coupon bonds have cash flows only at maturity and since, for JLT model, recovery is assumed to occur at maturity, we have only one certainty equivalent cash flow to determine. As shown in Das (1999) or Lando (1999), the probability risk adjustment for this cash flow in the JLT model is

$$\Pi_i(T) = \left[1 - \left| \frac{1 + r_{0T}^{g}}{1 + r_{0T}^{ci}} \right|^{T} \right| \frac{1}{(1 - b_i) q_{id}(T)}$$

Multiplying both sides of equation (1) by $q_{id}(T)$, we find that $A_i(T)$ is equal to

$$A_i(T) = \left| 1 - \frac{\left(1 + r_{0T}^{g}\right)^{T}}{\left(1 + r_{0T}^{ci}\right)^{T}} \right| \frac{1}{\left(1 - b_i\right)} \qquad (1)$$

From examining the right-hand side of the equation, $A_i(T)$ is independent of the value of $q_{id}(T)$. Thus unlike JLT's assertion, risk-adjusted probabilities are not a function of transition probabilities and , the results of their analysis are completely independent of the transition matrix used to price bonds.[3] Risk-adjusted probabilities are only a function of the spot rates on governments, the spot rates on corporates, and the recovery rate.[4]

The risk-neutral price of a zero coupon corporate bond maturing after *T* periods in rating class *i* where any payment for default is made at maturity is given by:

$$V_{iT}^{z} = \frac{100(1 - A_i(T)) + 100b_i\ A_i(T)}{\left(1 + r_{0T}^{g}\right)^{T}} \tag{2}$$

where the superscript Z has been added to V_{iT} to explicitly recognize that this equation holds only for zero coupon bonds. Substituting (1) into (2) yields

---

3 This also follows directly from noting that their results are equivalent to discounting promised cash flows at spot rates.

4 Thus if bond pricing is the purpose of the analysis, the various estimation techniques developed for estimating transition matrixes are vacuous in that they lead to identical pricing. See Lando (1997)for a review of these techniques.

$$V_{iT}^{z} = \frac{100}{\left(1 + r_{0T}^{ci}\right)^{T}} \tag{3}$$

Thus, as stated earlier, employing the JLT methodology yields exactly the same model price for any zero coupon bond (where payment for default only occurs at maturity) as discounting the promised cash flow at the corporate spot rates that were used as input to the analysis. If the only bonds we were interested in were zero coupon bonds where payment for default occurred at maturity, it would not matter in terms of pricing bonds whether we discounted promised payments at the corporate spot rate or used the JLT procedure. Why, then, bother with both models? The reason is that they produce very different answers if we examine coupon-paying bonds, or in fact any bond where the pattern of cash flows in any period is different from that of a zero coupon bond that pays off as a percentage of par in default at the horizon.

B. Comparison for Coupon Bonds

If we examine a two-period bond with a coupon of c dollars, the value of the bond using the corporate spot rate to discount promised payments is

$$V_{i2} = \frac{c}{\left(1 + r_{01}^{ci}\right)} + \frac{c + 100}{\left(1 + r_{02}^{ci}\right)^{2}} \tag{4}$$

Using risk-adjusted probabilities and continuing the assumption that the recovery of cash flows on defaulted bonds occurs at the maturity of the bond.[5]

$$V_{i2} = \frac{c(1 - A_i(1))}{\left(1 + r_{01}^g\right)} + \frac{(c + 100)\left[1 - \left(A_i(2)\right)\right] + b_i 100\, A_i(2)}{\left(1 + r_{02}^g\right)^2} \qquad (5)$$

It is easy to see that these two equations (4) and (5) are not equal to each other for the definition of risk adjustment given by equation (1), and in fact that there is no risk-adjustment expression that will equate them for a group of coupon paying bonds with different coupons using JLT's assumption about recovery.

However, we can be more precise concerning the direction of the differences. We will now show that the JLT procedure will produce model prices which are lower for coupon paying debt than those produced by discounting promised cash flows at corporate spot rates. The JLT risk adjustment factor was arrived at by finding the factor that produced the same value for zero coupon debt as discounting promised cash flows at the corporate spot rate.[6]

---

5 JLT assume that at bankruptcy the investor recovers a fraction of the face value of the bond at the horizon or equivalently an amount equal to the fraction of an equal maturity government bond at the time of bankruptcy. In the appendix we show that if an investor recovers an amount equal to a fraction of the market value of an equal maturity corporate bond in the same risk class plus the same fraction of the coupon, then the risk-neutral valuation gives the same valuation as discounting promised cash flows at corporate spot rates.

6 This is the procedure employed by JLT. An alternative might be to solve for the factor that produced the same value for a bond with an average coupon. However, since the

The risk-neutral valuation of zeros is

$$V_{iT}^{z} = \frac{\left(1 - A_i(T)\right)100 + b_i\, A_i(T)\ 100}{\left(1 + \mathrm{r}_{0T}^{\mathrm{g}}\right)^{T}}$$

The valuation from discounting promised cash flows is

$$V_{iT}^{z} = \frac{100}{\left(1 + r_{0T}^{ci}\right)^{T}}$$

Equating the two and solving for $A_i(T)$ [7]

$$A_i(T) = \left[1 - \frac{\left(1 + r_{0T}^{g}\right)^{T}}{\left(1 + r_{0T}^{ci}\right)^{T}} \right] \frac{1}{\left(1 - b_i\right)} \qquad (6)$$

correct factor in the JLT procedure is a function of coupon, this would misprice bonds in a manner analogous to that shown in the following analysis.

[7] All our empirical work uses continuous compounding. However, it is easier to follow the discussion, and the comparisons are more obvious, using discrete compounding.

Note this is identical to the definition of $A_i(T)$ from Das (1999) and Lando (1999) presented earlier.

If there was a coupon in the last period, the present value of the last period's cash flow would be

$$v_{iT} = \frac{\left(1 - A_i(T)\right)\left(c + 100\right) + b_i\, A_i(T)\ 100}{\left(1 + r_{OT}^{g}\right)^T}$$

Where the lower case *v* indicates it is the present value of a single cash flow rather than the complete bond value.

Discounting the last period's promised cash flow at the corporate spot rate yields

$$v_{iT} = \frac{c + 100}{\left(1 + r_{0T}^{c}\right)^T}$$

Equating these two equations and solving for $A_i(T)$ yields

$$A_i(T) = \left| 1 - \frac{\left(1 + r_{0T}^{g}\right)^T}{\left(1 + r_{0T}^{ci}\right)^T} \right| \frac{1}{1 - b_i\left(\frac{100}{100 + c}\right)} \tag{7}$$

If we examine the cash flows for any period prior to the period in which a bond matures, the present value of the t^{th} period cash flow using risk-neutral probabilities is

$$v_{it} = \frac{\left(1 - A_i(t)\right)c}{\left(1 + r_{0t}^{g}\right)^t}$$

and for promised cash flows the present value of the t^{th} period cash flow is

$$v_{it} = \frac{c}{\left(1 + r_{0t}^{ci}\right)^t}$$

Equating and solving for $A_i(t)$ yields

$$A_i(t) = \left| 1 - \frac{\left(1 + r_{0t}^{g}\right)^t}{\left(1 + r_{0t}^{ci}\right)^t} \right| \tag{8}$$

By inspection, equation (6) results in a higher value for any $A_i(t)$ than equation (7) or (8). Thus using zeros to define $A_i(t)$ under the JLT procedure leads to estimates of $A_i(t)$ that are larger than those obtained by determining $A_i(t)$ using coupon paying bonds. From equation (5) using higher $A_i(t)'s$ results in lower prices. Thus using the JLT procedure will always result in lower estimated prices than discounting promised cash flows at corporate spot rates. Later we will estimate and examine the size of this difference for coupon paying corporate bonds.

Since the JLT methodology leads to different values for coupon-paying corporate debt than discounting promised cash flows at corporate spot rates, the question remains as to which provides more accurate valuation. Discounting promised payments at corporate spot rates is an approximation except under restrictive conditions. The defense of using spots is an arbitrage argument, and the arbitrage argument in terms of promised payments is an approximation which is only exactly correct under certain assumptions.[8] On the other hand, the structure of the JLT model insures that coupon paying bond prices can't be reproduced exactly even over a fit period. The choice between these models then becomes an empirical matter, one to which we now turn.

II. TESTING THE MODELS

---

[8] See Duffie and Singleton (1997) for a detailed discussion of assumptions under which it is exactly correct to discount promised payments at spot rates. See Appendix A for a discussion of the recovery assumption necessary for discounting promised cash flows at the spot rate to be the same as risk-neutral valuation.

A. DATA

Our bond data is extracted from the Lehman Brothers Fixed Income database distributed by Warga (1998). This database contains monthly price, accrued interest, and return data on all investment grade corporate and government bonds. In addition, the database contains descriptive data on bonds including coupon, ratings, and callability.

A subset of the data in the Warga database is used in this study. First, any bond that is matrix-priced rather than trader-priced in a particular month is eliminated from the sample for that month. Employing matrix prices might mean that all our analysis uncovers is the formula used to matrix price bonds rather than the economic influences at work in the market. Eliminating matrix priced bonds leaves us with a set of prices based on dealer quotes. This is the same type of data contained in the standard academic source of government bond data: the CRSP government bond file.[9]

[9] The only difference in the way CRSP data is constructed and our data is constructed is that over the period of our study CRSP used an average of bid/ask quotes from five primary dealers called randomly by the New York Fed rather than a single dealer. However, comparison of a period when CRSP data came from a single dealer and also from the five dealers surveyed by the Fed showed no difference in accuracy (Sarig and Warga, (1989)). See also the discussion of pricing errors in Elton, Gruber, Agrawal and Mann (1999).Thus our data should be comparable in accuracy to the CRSP data.

Next, we eliminate all bonds with special features that would result in their being priced differently. This means we eliminate all bonds with options (e.g., callable or sinking fund), all corporate floating rate debt, bonds with an odd frequency of coupon payments, government flower bonds and index-linked bonds. Next, we eliminate all bonds not included in the Lehman Brothers bond indexes because researchers in charge of the database at Shearson-Lehman indicated that the care in preparing the data was much less for bonds not included in their indexes. Finally, we eliminate bonds where the data is problematic.[10] For classifying bonds we use Moody's ratings. In the few cases where Moody's ratings do not exist, we classify using the equivalent S&P rating.

B. Testing the Approaches

In this section we discuss the comparison of model errors produced by discounting promised cash flows at corporate spot rates with those produced by discounting risk-adjusted cash flows at the riskless government rates.

Calculating model prices using the discounting of promised cash flows is relatively straightforward. First, spots rates must be calculated. In order to find spot rates, we used the

[10] Slightly less than 3% of the sample was eliminated because of problematic data. The eliminated bonds had either a price that was clearly out of line with surrounding prices (pricing error) or involved a company or bond undergoing a major change.

Nelson Siegal (1987) procedure for estimating spots from a set of coupon paying bonds. For each rating category, including governments, spots can be estimated as follows:[11]

$$D_t = e^{-r_{0t} t}$$

$$r_{0t} = a_o + (a_1 + a_2)\left|\frac{1 - e^{-a_3 t}}{a_3 t}\right| - a_2 e^{-a_3 t}$$

Where

D_t is the present value as of time zero for a payment that is received t periods in the future

r_{0t} is the spot rate at time zero for a payment to be received at time t

[11] See Nelson and Siegal (1987). For comparisons with other procedures, see Green and Odegaard (1997) and Dahlquist and Svensson (1996). We also investigated the McCulloch cubic spline procedures and found substantially similar results throughout our analysis. The Nelson and Siegal model was fit using standard Gauss-newton non-linear least squared methods. The Nelson and Siegal (1987) and McCulloch (1971) procedures have the advantage of using all bonds outstanding within any rating class in the estimation procedure, therefore lessening the effect of sparse data over some maturities and lessening the effect of pricing errors on one or more bonds. The cost of these procedures is that they place constraints on the shape of the yield curve. We used Moodys categories where they existed to classify bonds. Otherwise we used the equivalent S&P categories.

a_0, a_1, a_2 and a_3 are parameters of the model

Discounting the promised cash flows at these estimated rates produces the model prices for this technique.

The estimation for the JLT procedure is more complicated. The first step is to estimate risk-neutral probabilities based on equation (1). So that the models can be directly compared, we use the same estimated spots that were used to discount promised cash flows as input to equation (1). We used historical recovery rates for Aa, A, and Baa rated corporate bonds.[12]

In Table I we report the risk-adjusted probabilities we arrive at using this procedure. While risk-adjusted probabilities are derived each month, in the interest of brevity we report them once a year (January) for each year in our sample period and only for industrial Baa bonds. It is interesting that the risk-neutral probabilities we arrive at are quite well-behaved relative to the risk-neutral probabilities reported by other authors (e.g., Jarrow, Lando, Turnbull (1997). In particular, our risk-neutral probabilities are all positive, and increase with maturity. We attribute the greater plausibility of our results to the large sample we use as well as the procedure we employ to extract spot rates.

---

[12] For a discussion of historical rates see Elton, Gruber, Agrawal and Mann (1999). We us continuous compounding in estimating risk neutral possibilities.

As shown earlier, if one uses the JLT model, the risk-adjusted probabilities from zero coupon bonds should understate the price of any coupon-paying bond. In addition, we would expect that the absolute errors (a measure of dispersion) should be higher for the errors themselves should be function of the coupon and coupons vary within any rating class.

Table II shows that the empirical results are consistent with the implications of the theory. Note, as shown in Table II Panel A, that when bonds are priced by discounting promised payments at corporate spot rates, the average error for each class of bonds is very close to zero and overall the average error is less than one cent per $100 bond. When we look at the average pricing errors from the JLT procedure, we see that they are negative and quite large for any class of bonds. Errors are measured as JLT model price minus invoice price. The negative error shows that the JLT procedure applied to coupon-paying bonds understates their market value. In addition, as shown in Table II Panel B the average absolute error is much higher for the JLT procedure. The average absolute error is affected by both the mean error and the dispersion across bonds. Table 2C corrects for the mean error by computing the average absolute error around the mean. Since the average error for DPP is close to zero, this correction has little effect on DPP and the average absolute errors in 2B and 2C are similar. Since there is a large mean error for JLT, calculating average absolute errors around the mean does make a difference for JLT. Even after this correction, however, absolute JLT errors are much higher than absolute DPP errors. Thus, the JLT procedure not only has a mean bias, but also results in greater dispersion of errors around the mean across bonds. These results are exactly what our analytical examination of the models lead us to expect.

It is worth examining one more point before we end this section. We would not expect the average error to be a function of maturity when we discount promised payments. With the JLT procedure we would expect the error to increase as maturity increases. This pattern occurs because each coupon is systematically undervalued and the more coupons a bond pays, the larger the mispricing. This is exactly what happens, as shown in Table III. For example, for the JLT procedure applied to BBB industrial bonds, the error increases from thirteen cents per $100 to over $3 per $100 as maturity increases.

In the next section of this paper we examine the ability of additional bond and/or company characteristics to improve the pricing of corporate bonds. We will conduct this examination employing the model which discounts promised cash flows at a rate which is appropriate for the risk of the promised payments rather than the JLT model since it produced lower errors.

III. Getting a Homogeneous Sample

When estimating spot rates, one has to make a decision as to how to construct a group of bonds that is homogeneous with respect to risk. In the prior section we accepted the major classifications of rating agencies. In this section we explore the use of additional data to form more meaningful groups.

In general, when dividing bonds into subsets, one faces a difficult tradeoff. The more subsets one has, the less bonds are present in any subset. Bond prices are subject to idiosyncratic noise as well as systematic influences. The more bonds in a subset, the more the idiosyncratic noise is averaged out. This suggests larger groupings. However, if the subset is not homogeneous, one may be averaging out important differences in underlying risk and mis-estimating spot rates because they are estimated for a group of bonds where subsets of the group have different yield curves.

What are the characteristics of bonds that vary within a rating class that could lead to price differences? We will examine the following possibilities:

(A) Default risk

(B) Liquidity

(C) Tax liability

(D) Recovery rates

(E) Age

A. Differential Default Risks:

All bonds within a rating class may not be viewed as equally risky. There are several characteristics of bonds which might be useful in dividing bonds within a rating class into new groups. We will examine several of these in this section. We start by examining the subcategories of risk within a rating class which Moodys and Standard & Poors have both introduced. We then examine whether either past changes in rating category or a difference in rating by Standard & Poors and Moodys convey information.

We start by examining the finer breakdown of ratings produced by the rating agencies themselves. Standard & Poors and Moodys have introduced plus and minus categories within each letter rating class. One obvious possibility is that bonds that are rated as a plus or a minus are viewed as having different risk than bonds that receive a flat letter rating. If this is true, then estimating one set of spot rates for all bonds in a class should result in consistent pricing errors for bonds rated "plus" (too low a model price and hence negative errors) or bonds rated "minus" (too high a model price and hence positive errors).

Tables IVA and IVB explore this possibility. For each rating class the table is split into two sections. The top section shows the number of bond months in each rating class for varying maturity and across all maturities.[13] The bottom section shows the average of the model price minus the invoice price (market price plus accrued interest) for each rating category. For all rating categories, plus-rated bonds have, on average, too low a model price, and minus-rated

[13]For all bonds rated by Moodys we use Moodys' classification. For the few bonds not rated by Moody's, we use S&P's classification.

bonds too high a model price. The difference between the pricing error of plus rated, flat, and negative rated bonds is statistically significant at the 5% level. Furthermore, the differences are of economic significance (e.g., for minus versus flat Baa industrial bonds the difference is almost 1% of the invoice price). The same pattern is present for most of the maturities. In addition, the size of the average pricing error increases as rating decreases. Thus, it is most important for Baa bonds. This would suggest that one should estimate a separate spot curve for these subclasses of ratings. However, for much of the sample, the paucity of bonds in many of the subclasses makes it difficult to estimate meaningful spot rates for a subclass. Instead, we propose to directly estimate the price impact of the finer gradation of rankings on errors (which is a function of maturity). The ability to correct the model price for these differences will be examined in the next section.

There is a second reason why investors might consider bonds within the same rating class to have different risk. Investors might believe that a particular bond is likely to be downgraded or upgraded. One predictor of this might be past rating changes. Past rating changes might predict future rating changes, either because rating agencies tended to make changes in steps or because a company whose risk has increased or decreased in the past is more likely to experience similar changes in the future. In Table V we explore whether past rating changes contain information about future rating changes. As shown in the table, bonds that have been upgraded in the past are more than twice as likely to be upgraded in the future than they are to be downgraded, and bonds that have been downgraded in the past are about twice as likely to be downgraded than upgraded in the future.

Although there is evidence that past rating changes predict future rating changes, it is unclear if the tendency is strong enough to show up in price data. We examined differences between model price and invoice price for all bonds which had a past change in ratings. Pricing errors were examined in the month of the change, the next three months after the change, and the period 4 to 15 months after the change. These results are shown in Table VI. Despite the fact that past rating changes contain information about future rating changes, we find no evidence that bonds with past rating changes have prices that are systematically different from model prices. Our sample of bonds with rating changes was quite small, for there were few bonds which had rating changes. Thus the failure to find a relationship between past rating changes and errors could arise either because investors do not take the predictability of past rating changes into account when they price bonds, or simply because the number of rating changes is so small that the effect is swamped by random pricing errors. In any case, examining past rating changes provides no evidence that the Markoff assumption used in calculating the transition probability matrix found in many studies is violated.

In Table VII we explore whether bonds that are given a higher (lower) rating by S&P than by Moody's are considered less (more) risky by investors. In considering differences we use pluses and minuses. Thus, if Moodys rates a bond as Baa and S&P rates the bond BBB+, we count this as a difference in ratings. Once again the upper half of the table shows the number of bonds in each category, and the lower half the difference between model price and invoice price. In presenting the data we do not sub-classify by maturity since we found no pattern in pricing errors across maturity.

Investors clearly take the difference in rating into account. If the S&P rating is lower than Moodys, then investors act as if the bond is higher risk than is implied by the Moodys rating and they will set a lower market price, and this results in a model price above invoice price and a positive error. Likewise, if S&P rates the bond higher than Moodys the bond is considered by investors as lower risk compared to bonds where they agree and the pricing error is negative. The errors when the rating agencies disagree is statistically different from the errors when they agree.

B. Different Liquidity

The second reason why bonds within a rating class might be valued differently is because they have different liquidity. Data is not available on bid/ask spread, the most direct measure of liquidity, nor is there data on trading volume which is a natural proxy for liquidity. Thus we had to use two indirect measures of liquidity: volume outstanding and percentage of months a bond was matrix priced. Our logic behind the latter measure was that dealers priced the more active issues more often. Thus bonds that were always dealer-priced were likely to be more liquid than bonds that were dealer-priced only part of the time. Neither of these measures showed any significant patterns, and so we have not presented a table of results. Thus while there may be liquidity differences between bonds, and these may be priced, we are unable to find reasonable proxies to demonstrate this influence.

C. Different Tax Treatment

The third possible reason why bonds within a risk class might be viewed by investors differently is because they have different tax treatment of coupons and capital gains. Throughout most of the period used in our study the tax rate on capital gains and interest was the same. However, since capital gains are paid at the time of sale, lower coupon bonds may be more valuable because some taxes are postponed until the time of sale and because the holder of the bond has control over when these taxes are paid (tax timing option). In order to examine the effect of taxes, we group bonds by coupon and examined the model errors. Table VIII shows the results for Baa rated industrial bonds. The results for other ratings are similar. The entries in Panel B represent model prices minus invoice price across six coupon categories and different maturities. Panel A shows the number of bond months in each category.

If taxes matter, we would expect to see a particular pattern in this table. Recall that for any risk class, spot rates are fitted across all bonds. This means that for the average bond the tax effect on pricing errors should be zero (because it is averaged out), and if taxes don't matter it should not vary with maturity. If taxes matter, high coupon bonds should be disadvantaged relative to the average bond, and these bonds would have to offer the investor a higher return. But since we are discounting all bonds in a risk class at the same rate, this implies that if taxes matter we are discounting high coupon bonds at too low a rate, and thus are computing a model price which is too high. This translates into a positive value for the pricing error, and this is what we see in Table VIII. In addition, as shown in Table VIII, the longer the maturity of the bond, the more significant the pricing error becomes. For bonds with coupons below the average coupon in a risk class we should get the opposite sign (a negative sign) on the pricing error and the size of the error should become more negative with the maturity of the bond. This is the pattern shown in Table VIII.

D. Different Recovery Rates

The fourth reason investors might rate bonds differently within a risk class is because of different expectations about recovery. Firms go bankrupt, not individual bonds. Bonds of the same firm with different ratings imply that the rating agency believes they will have different recovery rates. Thus investors should believe that an A bond of an Aa firm has different expected recovery rate than an Aa bond of the same firm.

Moodys and S&P ratings for any bond are a combination of default risk for the company issuing the bond and the recovery rate on the bond if the firm goes bankrupt. If their implicit weighting is the same as investors, then sorting a bond rating class by different company ratings should not result in pricing errors being related to the company rating. Examining Table IX shows that bonds where the company rating is lower than the bond rating have model prices above invoice prices. When the model price is above the invoice price, investors are requiring a higher rate of return in pricing the bond. Bonds whose ratings are above companies ratings (e.g., Aa and A respectively), have more default risk than bonds which have company and bond rating both equal to that of the bond (e.g. both AA). Since, from Table IX, investors price these bonds lower, investors are placing more weight on bankruptcy probability and less on estimated recovery rates than Moodys does. The same logic holds for bonds ranked below the company rating.

This raises another question. Could pricing be improved by discounting bonds at spot rates estimated from groups formed by using company rating rather than formed by bond rating? When we use company ratings to form groups and estimate spots the pricing errors are much larger. Bonds should be priced from discount rates estimated from groups using bond rating. However, taking into account the difference between bond rating and company rating adds information.

E. Bond Age:

We explored one other reason why bonds in a particular rating class might be viewed differently by investors – age of the bond. While the finance literature presents no economic reason why this might be true, it is a common way to present data in the corporate bond area, and it is an important consideration if one wants to model rating drift as a Markov process.[14] The issue is whether a bond with 15 years to maturity rated A, and ten years old, is different from a bond with the same characteristics but two years old. When we examined this issue, the only age category that mattered was for bonds under one year old. Table X shows the difference between first year bonds and older bonds. Once again the top half is the number of bond months in each cell, and the bottom half is the average difference between model price and invoice price. As shown in Table X, newly issued bonds sell at a premium compared to model prices. These results are consistent with newly issued bonds being more liquid. Thus there is no definitive evidence

[14] For example, Moodys typically presents data on the default rates as a function of the age of the bonds.

that the Markov assumption is being violated, and no definitive evidence that age of the bond is an important characteristic for classification. However there is strong evidence that bonds of one year of age or less should be discounted differently than bonds of longer maturity. We may, in fact, have found our liquidity measure.

IV Adjusting for Differences

We have now shown that a number of factors cause bonds within the same Moody's classification to have systematic price differences. The simplest way to adjust for this would be to take these factors into account in defining new classifications and to estimate the spot rates for each new category. However, this would result in such fine classifications that there would be too few bonds within a group to accurately estimate spot rates. Instead, we will estimate the price differences due to these factors and adjust for the average effect.

Prior analysis has shown the following influences are important:

1. A plus or minus rating within each risk letter classification. Furthermore, the importance is a function of maturity.

2. Differences in S&P and Moody's rating.

3. The coupon on a bond.

4. Differences in bond and company ratings.

5. An age of less than one year.

To estimate the adjustment function we regressed model errors on a series of variables to capture simultaneously the impact of the influences discussed above. The variables are discreet except for coupon which is continuous. The regression we estimated is

$$E_j = \alpha_j + \sum_{i=1}^{8} B_{ij} V_i + e_j \qquad (9)$$

Where

$E_j =$ the error measured as model price minus invoice price for bond j

$V_1 =$ the maturity of a bond if it is rated plus otherwise zero

V_2 = the maturity of a bond if it is rated minus, otherwise zero

V_3 = dummy variable which is 1 if S&P rates a bond higher than Moody's, otherwise zero

V_4 = dummy variable which is 1 if Moody's rates a bond higher than S&P, otherwise zero

V_5 = the coupon on the bond minus the average coupon across all bonds[15]

V_6 = dummy variable which is 1 if the company has a higher rating than the bond, otherwise zero

V_7 = a dummy variable which is 1 if the bond has a higher rating than the company, otherwise zero

V_8 = a dummy variable which is 1 if the bond is less than 1 year of age, otherwise zero

The regression is estimated for bonds within each rating class for industrials and financials separately. Results are shown in Table XI. Almost all regression coefficients are statistically significant at the 1% level in every sample and have the sign that we would expect to see. The adjusted R^2 vary between .05 and .32 and average .18.

If we examine the regression coefficients one at a time we see very strong results. Each of the variables measuring finer rating categories (plus or minus) have the right sign in five of the

[15] This variable was demeaned as not to transfer the average tax effect to the intercept.

six groups with each coefficient significant at the 1% level. In the one group where the sign is inconsistent with what we would expect the coefficient is both small and not statistically significantly different from zero at the 5% level. When interpreting the signs, recall that plus rated bonds are expected to have a negative error since the model price overestimates their risk.

Turning to bonds which have a S&P rating different from their Moody's rating, we find that the S&P rating contains added information about prices. For differences in ratings in either direction, the coefficient has the appropriate sign and is significantly different from zero at the 1% level in every case.

We have hypothesized that high coupon bonds were less desirable due to taxes. The coupon variable has the correct sign and a coefficient which is significantly different from zero (at the 1% level) in every case. While we reasoned that the impact of company and bond ratings were ambiguous because it depends on the weight the investor places on recovery rate versus probability of bankruptcy, the results tell a very consistent story. Of the 11 groups examined, 10 had consistent signs and of these 10, 8 had coefficients which were statistically significantly different from zero at the 1% level. The one coefficient with the inconsistent sign was not significantly different from zero at the 5% level. These results indicate that investors place more emphasis on bankruptcy risk than the relative weight it is given in bond ratings. Finally, new bonds sell at a premium. All the estimates have the right sign and are statistically different from zero at the 1% level.

We have just shown that adding a set of bond and company characteristics to Moodys ratings helps explain the differences between model price and invoice price and leads to better model prices in sample. The issue is whether the relationship is sufficiently stable to lead to improvement out of sample. That is, can we improve model prices by estimating the relationship using only data which is available at the time model prices are determined.

To answer this we fit equation (9) to the first five years of our bond data. We then used the coefficient of this regression along with the actual value of each of the variables for each month in the sixth year to adjust model prices. The we fit equation (9) to years two and six of our sample and made forecasts for the seventh year. This produced a set of model prices and pricing errors for each month from January to December. The pricing errors from these adjusted model prices are then compared with the errors from unadjusted model prices. One additional adjustment was made. Because the mean pricing error was different over the fit period than over the forecast period, model prices were adjusted to have the same mean error before and after the adjustment. The procedure uses only data that was available at the time of the forecast. The mean adjustment made almost no difference. The absolute error produced by the two models is shown in Panel A, Table 12. Adjusting for additional risk characteristics clearly results in lower pricing errors on average but the results differ across rating categories.

For AA and A rated financials the adjustment does not seem to affect the size of the absolute pricing error. However, for the lowest rated financials and for all three industrial categories, adjustment improved the model prices. The largest improvement occurred for BBB industrials where adjusted model prices led to about a 20% reduction in model errors. Panel B presents the root mean squared error for the same data. In all cases the root mean squared error is less for the adjusted model prices. Once again, the major improvement was in the lower rated groups. Examining Panel A and B together suggests that the adjustment we made to model prices is most successful in improving estimates for those risk categories of bonds where the unadjusted model performs poorly.

Conclusion

In this paper we explore the ability of two rating-based models to price corporate bonds. These models involve discounting promised cash flows at the estimated spot rates and risk-neutral valuation using the definition of risk-neutral probabilities proposed by JLT. We show that the JLT risk-neutral probabilities are independent of estimates of the transition matrix and default probability. We then show that the JLT risk-neutral probabilities result in lower model prices than discounting promised cash flows at the spot rates. Discounting promised cash flows at the corporate spot rates is unbiased, and has lower errors and a smaller dispersion than the JLT model.

All rating-based techniques involve working with a homogeneous population of bonds. In the last section we explore what characteristics of bonds are priced differently by the market. We find that several characteristics of bonds and bond rating beyond the simple rating categories of Moody's and Standard and Poor convey information about the pricing of corporate bonds. In particular the following five influences are important:

1. The finer rating categories introduced by both rating agencies when combined with maturity measures.
2. Differences between S&P and Moody's ratings.
3. Differences in the rating of a bond and the rating of the company which issued that bond.
4. The coupon on the bond.
5. Whether a bond is new and has traded for more than one year.

We adjust for these characteristics and show the improvement in pricing error. Bond pricing models which are based on ratings whether the models involve discounting cash flows or determining risk neutral probabilities need to be adjusted for these influences.

APPENDIX

Bankruptcy Assumptions and Risk Neutral Valuation

In this section we make the following recovery assumption: At the time of bankruptcy the investor receives a constant fraction of the market value of a similarly rated non-bankrupt bond of the same maturity, and the same fraction of the coupon payment. We will prove that with this definition of recovery, a risk-neutral probability (of default) exists that produces the same value for any bond whether one uses corporate spot rates to discount promised payments or government spot rates to discount cash flows which are derived from certainty equivalent risk-neutral probabilities.

We will use the notation employed earlier with the following additions:

1. $A'(t) =$ the probability of going bankrupt during the time period t given that the bond has not gone bankrupt before period t.

2. Superscripts *P* and *R* indicate the bond is valued by discounting promised cash flows (P) or risk-neutral valuation (R).

3. $r_{t,t+1}$ Is the forward rate as of the time the bond is being valued from *t* to *t+1*.

$$A'(t) = \frac{A(t+1) - A(t)}{(1 - A(t))}$$

Note the subscript i has been dropped as all equations are written for a bond in risk class i.

The value of an T period bond at time t given that it exists at t, based on discounting promised payments is

$$V_{t,T}^{P} = \frac{c}{\left(1 + r_{t,t+1}^{c}\right)} + \frac{V_{t+1,T}^{p}}{\left(1 + r_{t,t+1}^{c}\right)} \qquad \text{(A-1)}$$

Where $V_{t,T}^{P}$ is the value of a bond at time t that matures at period *T*.

While the value of the bond based on risk-neutral probabilities at time t and the recovery assumption discussed above is:

$$V_{t,T}^{R} = \frac{\left(c + V_{t+1,T}^{R}\right)\left(1 - A'(t)\right) + A'(t) b \left(V_{t+1,T}^{R} + c\right)}{\left(1 + r_{t,t+1}^{g}\right)} \tag{A-2}$$

Equation (A-2) can be rewritten as

$$V_{t,T}^{R} = \frac{\left(c + V_{t+1,T}^{R}\right)\left[1 - A'(t)(1 - b)\right]}{\left(1 + r_{t,t+1}^{g}\right)} \tag{A-3}$$

The first question to answer is: is there a value of $A'(t)$ such that $V_{t,T}^{P} = V_{t,T}^{R}$ when $V_{t+1,T}^{p} = V_{t+1,T}^{R}$ Setting these values equal and solving for $A'(t)$ yields

$$A'(t) = \frac{\left| 1 - \frac{1 + r_{t,t+1}^{g}}{1 + r_{t,t+1}^{c}} \right|}{1 - b} \tag{A-4}$$

We show below that for the definition of bankruptcy we are examining and the definition of risk-neutral probabilities given by equation (A-4), the value of any coupon or zero coupon bond will be exactly the same whether we use corporate rates to discount promised cash payments or government spot rates to discount risk-neutral cash flows.

By using ending conditions and solving iteratively backwards in time, we will show that if we define $A'(t)$ as in equation (A-4), then equation (A-3) reduces to equation (A-1) in every period. Consider a *T* period bond. If the bond is solvent at the horizon, its promised payment is the same as its risk-adjusted payment. Therefore, $V_{T,T}^{R} = V_{T,T}^{P}$.

Then for period *T-1* we can write equation (A-3) as

$$V_{T-1,T}^{R} = \frac{\left(c + V_{T,T}^{P}\right)\left[1 - A'(T-1)(1-b)\right]}{\left(1 + r_{T-1,T}^{g}\right)}$$

Substituting in equation (A-4) yields

$$V^R_{T-1,T} = \frac{\left(c + V^P_{T,T}\right)}{\left(1 + r^c_{T-1,T}\right)}$$

Since the right-hand side of this equation is identical to the right-hand side of equation (A-1), we have $V^P_{T-1,T} = V^R_{T-1,T}$.

We could now write out the expression for $V^R_{T-2,T}$ and $V^P_{T-2,T}$ and with identical substitution prove they were equal. This will hold for any period for any bond. Furthermore, setting c equal to zero does not change the identities so the definition of risk-neutral probabilities given by equation (A-4) holds for zero coupon bonds as well as coupon-paying bonds.

Thus, with the recovery rate defined as a fraction of the value of the firm plus the coupon, the value of the bond is identical whether one discounts promised payments at the corporate spot rate or discounts risk-neutral payments at the government rate. The results are independent of maturity or coupon on the bonds. Thus, discounting promised cash flows at corporate rates is exactly equivalent to a risk neutral valuation model where the recovery assumption is a fraction of the market value of an equivalent non-defaulted bond plus the same percentage of its coupon.

Bibliography

F. Black and M. Scholes (1973). "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81: 637-654.

S. Das (1999). "Pricing Credit Derivatives" *in* J. Francis, J. Frost and J.G. Whittaker (editors) Handbook of Credit Derivatives, 101-138.

S. Das and P. Tufano (1996). "Pricing Credit Sensitive Debt when Interest Rates and Credit Spreads are Stochastic." *Journal of Financial Engineering* 5:161-198.

D. Duffie and D. Lando (1998). "Term Structures of Credit Spreads with Incomplete Accounting Information." Working Paper, Stanford University and University of Copenhagen.

D. Duffie and K. Singleton (1997). "Modeling Term Structures of Defaultable Bonds." Forthcoming in *Review of Financial Studies.*

R. Jarrow, D. Lando and S. Turnbull (1997). "A Markov Model for the Term Structure of Credit Spreads." *Review of Financial Studies* 10:481-523.

E. Jones, S. Mason and E. Rosenfeld (1984). "Contingent Claims Analysis of Corporate Capital Structures: An Empirical Investigation." *The Journal of Finance* 39:611-625.

D. Lando (1997). "Modeling Bonds and Derivatives with Default Risk" *in* M. Dempster and S. Pliska (editors) *Mathematics of Derivative Securities,* pp. 369-393, Cambridge University Press.

D. Lando (1999). "Some Elements of Rating-Based Credit Risk Modeling." Unpublished paper, University of Copenhagen.

R. Merton (1974). "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." *The Journal of Finance* 29:449-470.

O. Sarig and A. Warga (1989). "Bond Price Data and Bond Market Liquidity." *Journal of Financial and Quantitative Analysis* 24: 367-378.

Table I
Risk Neutral Probabilities of Default

This table shows $A_i(t)$, the cumulative risk neutral probabilities of default after t years, conditional on initial rating being i. These numbers are computed from the spot rates of treasury (r_{0t}^{g}) and corporate bonds (r_{0t}^{c}) using the expression,

$$A_i(t) = (1/1-b_i)[1-exp(-t(r_{0t}^{c}-r_{0t}^{g}))]$$

where b_i is the recovery rate for rating i. The numbers shown are for Baa rated bonds of Industrial category with maturity t = 1 to t = 11 years.

	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9	t=10	t=11
Jan-87	0.0230	0.0519	0.0802	0.1070	0.1328	0.1578	0.1823	0.2064	0.2302	0.2536	0.2766
Jan-88	0.0433	0.0775	0.1031	0.1236	0.1416	0.1582	0.1742	0.1898	0.2052	0.2205	0.2356
Jan-89	0.0178	0.0340	0.0544	0.0779	0.1028	0.1284	0.1540	0.1794	0.2045	0.2293	0.2538
Jan-90	0.0233	0.0450	0.0711	0.1002	0.1309	0.1619	0.1928	0.2234	0.2535	0.2832	0.3123
Jan-91	0.0196	0.0711	0.1185	0.1587	0.1941	0.2267	0.2578	0.2878	0.3171	0.3458	0.3740
Jan-92	0.0277	0.0719	0.1060	0.1308	0.1500	0.1665	0.1815	0.1959	0.2099	0.2237	0.2374
Jan-93	0.0191	0.0662	0.1035	0.1300	0.1501	0.1667	0.1818	0.1960	0.2099	0.2235	0.2370
Jan-94	0.0143	0.0381	0.0607	0.0812	0.1003	0.1185	0.1362	0.1536	0.1708	0.1878	0.2046
Jan-95	0.0133	0.0300	0.0485	0.0678	0.0874	0.1069	0.1264	0.1456	0.1647	0.1836	0.2023
Jan-96	0.0113	0.0248	0.0416	0.0603	0.0798	0.0997	0.1196	0.1393	0.1589	0.1784	0.1976

Table II

DPP Errors and JLT Errors

This table compares the average pricing errors when promised payments are discounted at the corporate rates called DPP prices and JLT fitted prices for coupon bonds. Discounted rates on promised payments were fitted each month separately for each rating category of bonds and DPP errors are the DPP fitted prices minus the invoice prices of coupon bonds. JLT model parameters were estimated using the zero coupon prices obtained from the DPP estimation. These parameter estimates were then used to compute the JLT fitted prices of the coupon bonds. JLT errors are the JLT fitted prices minus the invoice prices of coupon bonds. Errors are expressed in dollars on bonds with face value of 100 dollars.

Panel (A): Average pricing errors with signs preserved

Estimation Method	Financial Sector			Industrial Sector		
	Aa	A	Baa	Aa	A	Baa
DPP	-0.0094	-0.0104	-0.0149	-0.0162	-0.0082	0.0094
JLT	-0.6256	-1.0954	-1.3308	-0.7414	-1.2216	-1.3485

Panel (B): Average absolute pricing errors

Estimation Method	Financial Sector			Industrial Sector		
	Aa	A	Baa	Aa	A	Baa
DPP	0.2983	0.5042	0.8584	0.4537	0.5905	1.1348
JLT	0.6913	1.2177	1.5766	0.8985	1.3366	1.7489

Panel (C): Average absolute pricing error around mean

Estimation Method	Financial Sector			Industrial Sector		
	Aa	A	Baa	Aa	A	Baa
DPP	0.2977	0.5028	0.8566	0.4505	0.5897	1.1367
JLT	0.5793	0.9734	1.2706	0.6976	1.0208	1.3941

Table III

DPP and JLT Errors sorted by Maturity

Panel (B) and (C) of this table show the average errors from discounting the promised payments and from using the JLT procedure respectively, sorted by maturity. The errors are model prices minus the invoice prices. Panel (A) shows the number of coupon bonds over which the averages were taken. All errors are in dollars on bonds with face value of one hundred dollars.

Panel (A): Number of bonds over which the averages were taken

Maturity Range	Financial Sector			Industrial Sector		
	Aa	A	Baa	Aa	A	Baa
[1,2) years	1862	5036	1357	638	2069	1110
[2,4) years	2948	9309	2357	1214	4065	2841
[4,6) years	1993	7141	2169	1091	4602	2995
[6,8) years	1201	6002	2270	980	4153	2930
[8,10) years	897	6072	2700	1135	4502	3759
[10,11) years	62	667	224	167	731	346

Panel (B): Average DPP errors

Maturity Range	Financial Sector			Industrial Sector		
	Aa	A	Baa	Aa	A	Baa
[1,2) years	-0.0414	-0.0501	-0.1000	-0.0744	-0.0428	-0.0190
[2,4) years	0.0393	0.0341	0.0685	0.0159	0.0134	0.0192
[4,6) years	-0.0707	-0.0870	-0.1022	-0.0661	-0.0541	-0.0558
[6,8) years	-0.0560	-0.0137	-0.0516	-0.0135	-0.0009	0.0660
[8,10) years	0.1047	0.0584	0.0611	0.0451	0.0364	0.0298
[10,11) years	-0.1367	-0.1094	-0.0777	-0.1334	-0.0568	-0.1153

Panel(C): Average JLT errors

Maturity Range	Financial Sector			Industrial Sector		
	Aa	A	Baa	Aa	A	Baa
[1,2) years	-0.1188	-0.1520	-0.2245	-0.1403	-0.1236	-0.1268
[2,4) years	-0.2506	-0.3377	-0.3035	-0.1865	-0.3180	-0.3760
[4,6) years	-0.7849	-0.9660	-1.0123	-0.5806	-0.8852	-1.0144
[6,8) years	-1.2003	-1.5959	-1.7349	-0.9487	-1.4886	-1.5303
[8,10) years	-1.6716	-2.4409	-2.5563	-1.4735	-2.3305	-2.3878
[10,11) years	-2.2844	-3.4263	-3.0599	-1.9294	-3.1267	-3.3157

Table IV (a)

Model Errors due to Maturity and Gradations within Ratings
Industrial Sector

Moody's rates bonds using broad categories as well as finer gradations (+, 0, and -.) Plus securities are designated as less risky than minus securities. This table separates bonds into groups according to these finer gradations (along the left-hand side.) It further separates the bonds according to maturity (in years from left to right.) The first column represents bonds with maturity between 1.0 and 2.0 years, inclusive. Model price is calculated by discounting promised cash flows at estimated corporate spot rates. Average error is defined as model price minus invoice price.

			AA				
			Number of Bonds				
	1.0 - 2.0	2.01-4.0	4.01-6.0	6.01-8.0	8.01-10.0	10.01-10.99	Overall
+	34	130	129	108	172	18	591
0	360	634	509	365	398	62	2328
-	228	452	448	502	559	75	2264
			Average Error				
+	0.112	-0.152	0.360	0.255	0.517	-0.113	0.245
0	0.045	-0.015	0.004	0.065	0.009	-0.216	0.010
-	0.084	0.030	0.061	-0.095	-0.227	0.378	-0.038
			A				
			Number of Bonds				
	1.0 - 2.0	2.01-4.0	4.01-6.0	6.01-8.0	8.01-10.0	10.01-10.99	Overall
+	707	1364	1425	1176	1173	178	6023
0	752	1549	1692	1423	1641	200	7257
-	511	1092	1423	1481	1613	275	6395
			Average Error				
+	0.171	0.288	0.504	0.524	0.622	0.531	0.443
0	-0.005	-0.111	-0.078	-0.145	-0.133	0.160	-0.096
-	-0.095	-0.237	-0.225	-0.279	-0.391	-0.355	-0.277
			BBB				
			Number of Bonds				
	1.0 - 2.0	2.01-4.0	4.01-6.0	6.01-8.0	8.01-10.0	10.01-10.99	Overall
+	361	866	889	864	1257	66	4303
0	324	938	1068	965	1255	149	4699
-	393	1037	1039	1094	1236	93	4892
			Average Error				
+	0.374	0.684	0.932	0.839	1.009	1.415	0.846
0	0.242	0.039	0.116	0.266	0.278	0.500	0.196
-	-0.391	-0.567	-0.662	-1.013	-1.287	-1.509	-0.873

Table IV (b)

Model Errors due to Maturity and Gradations within Ratings
Financial Sector

	AA						
	Number of Bonds						
	1.0 - 2.0	2.01-4.0	4.01-6.0	6.01-8.0	8.01-10.0	10.01-10.99	Overall
+	218	207	36	47	44	0	552
0	306	616	642	420	294	12	2290
-	1284	2081	1283	705	551	44	5948
	Average Error						
+	-0.044	-0.055	-0.131	-0.283	-0.369	-	-0.100
0	-0.049	0.014	-0.066	-0.055	0.046	-0.707	-0.029
-	-0.025	0.056	-0.062	-0.024	0.166	0.064	0.014
	A						
	Number of Bonds						
	1.0 - 2.0	2.01-4.0	4.01-6.0	6.01-8.0	8.01-10.0	10.01-10.99	Overall
+	1838	3131	2146	1486	1475	110	10186
0	2100	4014	2604	2134	2378	222	13452
-	903	2112	2352	2352	2168	262	10149
	Average Error						
+	-0.112	-0.179	-0.491	-0.575	-0.646	-0.288	-0.359
0	-0.065	-0.025	-0.143	-0.127	-0.038	-0.163	-0.075
-	0.163	0.460	0.368	0.417	0.608	0.173	0.426
	BBB						
	Number of Bonds						
	1.0 - 2.0	2.01-4.0	4.01-6.0	6.01-8.0	8.01-10.0	10.01-10.99	Overall
+	843	1562	1092	1157	1499	123	6276
0	333	568	831	758	836	64	3390
-	131	228	254	350	365	4	1332
	Average Error						
+	-0.168	0.020	-0.255	-0.227	-0.224	-0.128	-0.160
0	0.062	0.118	-0.231	-0.135	0.110	0.142	-0.031
-	0.225	0.349	0.982	0.799	1.036	0.766	0.765

Table V

Predictability of Rating Changes by Past Rating Changes

This table examines whether the direction of rating change (i.e. upgrade or downgrade) in year *t-1* can predict the direction of rating change in year *t*. Each year, each issuer was put into one of the nine cells depending on the direction of rating change in year *t-1*and year *t*. This procedure was repeated for all the active issuers in a given year to arrive at a 3 by 3 table showing the number of issuers in each cell. The table shown below is the average of these tables over the 10-year period 1987 to 1996. It shows the average number of issuers per annum undergoing the particular type of rating transitions.

	year t upgrade	Year t no change	year t downgrade
year t-1 upgrade	24.7	123.4	9.4
year t-1 no change	135.2	1192.9	197.0
year t-1 downgrade	25.9	157.2	56.7

Table VI

Model Errors due to Recent Company Rating Changes

For each risk class (e.g., Financial Sector, AA bonds), specific bonds were chosen for which the issuing company experienced a rating change. The pricing errors, model price minus invoice price (using equation 4), were then placed into six bins. The bins are separated along two diffent dimensions: direction of rating change (Up or Down) and number of months after rating change (month of rating change, the following three months, and the subsequent fifteen months.) Panel A gives the number of observations in each bin. Because this table covers the entire sample of ten years, some bonds are observed many times. This results in the number of bonds in a given risk class being roughly proportional to the number of months in the bin. Panel B gives the average error for each bin.

		Financial Sector			Industrial Sector		
Direction	Months after Change	Aa	A	Baa	Aa	A	Baa
Panel A: Number of Pricing Error Observations in Bin							
Up	4 to 15	24	1904	476	258	645	422
Up	1 to 3	6	506	138	58	139	131
Up	0	2	161	53	12	42	50
Down	0	1	104	19	12	38	62
Down	1 to 3	3	307	63	36	112	213
Down	4 to 15	12	1296	267	162	475	737
Panel B: Average Error in Bin							
Up	4 to 15	0.937	-0.081	-0.469	-0.241	-0.120	0.065
Up	1 to 3	0.838	-0.078	-0.486	-0.137	-0.043	-0.344
Up	0	1.306	-0.019	-0.502	-0.446	-0.015	-0.166
Down	0	0.465	0.096	-0.569	-0.185	0.567	0.431
Down	1 to 3	0.413	-0.057	-0.090	-0.231	0.725	0.263
Down	4 to 15	0.201	0.113	0.065	-0.157	0.847	0.174

Table VII

Model Errors due to Differences between Moody's and Standard and Poors

This table examines whether bonds whose S&P rating is different from Moody's rating are viewed by the market as having different risks. Model errors are model price minus invoice price.

	Financial Sector			Industrial Sector		
	Aa	A	Baa	Aa	A	Baa
Panel A: Number of Pricing Error Observations						
S&P Lower	2075	4557	1720	841	4281	3111
S&P Same	5198	18537	3481	2906	9459	6639
S&P Higher	1456	10465	5702	1432	5875	4062
Panel B: Average Error						
S&P Lower	0.015	0.253	0.117	0.080	0.010	0.212
S&P Same	-0.020	-0.085	0.009	0.063	0.052	0.000
S&P Higher	-0.086	-0.000	-0.066	-0.232	-0.138	-0.237

Table VIII

Model errors for Industrial Baa Bonds sorted by coupon and maturity

Panel (B) of this table shows the errors from discounting the promised payments for Baa rated bonds of industrial category. The errors are model prices minus the invoice prices. The columns are different maturity ranges and the rows are different coupon ranges. Panel (A) shows the number of bonds over which the averaging was done in each cell.

Panel (A): Number of bonds

	[1,2) years	[2,4) years	[4,6) years	[6,8) years	[8,10) years	[10,11) years
[0,5)%	57	58	0	0	0	0
[5,6.5)%	112	279	156	84	190	1
[6.5,8)%	144	501	584	774	1562	115
[8,9.5)%	470	1200	1185	1149	1273	125
[9.5,11)%	258	624	954	853	722	103
[11,15)%	69	179	116	70	12	2

Panel (B): Average errors

	[1,2) years	[2,4) years	[4,6) years	[6,8) years	[8,10) years	[10,11) years
[0,5)%	-0.4363	-0.6707	.	.	.	.
[5,6.5)%	-0.0381	-0.5762	-1.1603	-0.9723	-1.3549	-1.4769
[6.5,8)%	-0.0575	0.2403	-0.1202	-0.1021	-0.3126	-0.2746
[8,9.5)%	0.0497	0.0646	-0.0820	-0.0968	0.0789	-0.6200
[9.5,11)%	-0.0937	-0.0415	0.0991	0.4165	1.0066	0.6395
[11,15)%	0.2479	0.4590	0.7475	1.5713	2.5329	2.4079
Weighted Average	-0.0190	0.0192	-0.0558	0.0660	0.0298	-0.1153

Table IX

Model Errors due to Differences in Bond and Company Rating

Each risk class is separated into three groups, one in which the bond is rated higher than the issuing company, one in which the bond is rated lower than the issuing company, and one in which the bond and the issuing company are equally rated. Panel A gives the number of bond price observations for each group of bonds. Panel B gives the average error, defined as model price minus invoice price (using equation (4)).

	Financial Sector			Industrial Sector		
Bond Rating is …	Aa	A	Baa	Aa	A	Baa
Panel A: Number of Pricing Error Observations						
Higher	3385	1737	145	1211	4355	1108
Same	5086	19261	1839	3420	14201	9537
Lower	2	11396	8344	0	888	2604
Panel B: Average Error						
Higher	0.006	0.588	0.887	0.306	0.147	0.854
Same	-0.040	-0.025	0.427	-0.168	-0.027	0.093
Lower	-0.097	-0.105	-0.135	-	-0.615	-0.866

Table X

DPP Errors and JLT Errors

This table compares the average pricing errors for the DPP prices and JLT fitted prices of the coupon bonds. Discounted rates on promised payments were fitted each month separately on each rating category of bonds and DPP errors are the actual minus the DPP fitted prices of coupon bonds. JLT model parameters were estimated using the zero coupon prices obtained from the DPP estimation. These parameter estimates were then used to compute the JLT fitted prices of the coupon bonds. JLT errors are the actual minus the JLT fitted prices of coupon bonds.

Panel (A) : Average pricing errors with signs preserved

Estimation Method	Financial Sector			Industrial Sector		
	AA	A	BBB	AA	A	BBB
DPP	0.0094	0.0104	0.0149	0.0162	0.0082	-0.0094
JLT	0.6256	1.0954	1.3308	0.7414	1.2216	1.3485

Panel (B) : Average absolute pricing errors

Estimation Method	Financial Sector			Industrial Sector		
	AA	A	BBB	AA	A	BBB
DPP	0.2983	0.5042	0.8584	0.4537	0.5905	1.1348
JLT	0.6913	1.2177	1.5766	0.8985	1.3366	1.7489

Table XI

Model Errors versus Board and Firm Characteristics

This table presents regression coefficients. The variables are defined as follows: Age < 1.0 is one if the bond age is less than 1.0 years. Company > Bond is one if the company rating is better than the bond rating. Bond > Company is one if the bond rating is better than the company rating. Plus is one if the bond has a plus rating (e.g. Aa+.) Minus is one if the bond has a negative rating. S&P > Moody's is one if Standard and Poor rated the bond as less risky than Moody's did. Moody's > S&P is one if Moody's rated the bond as less risky than Standard and Poor did. Coupon is the bond's coupon rate.

	Financial Sector			Industrial Sector		
Variable	Aa	A	Baa	Aa	A	Baa
Intercept	-0.022*	-0.018*	0.423*	-0.093*	0.082*	-0.195*
Plus * maturity	-0.008	-0.055*	-0.005*	-0.010*	-0.069*	-0.071*
Minus * maturity	0.014*	0.061*	0.123*	-0.003	0.030*	0.159*
S&P > Moody's	-0.274*	-0.283*	-0.124*	-0.109*	-0.257*	-0.086*
Moody's > S&P	0.035**	0.147*	0.456*	0.333*	0.167*	0.982*
Coupon	0.051	0.059*	0.071*	0.110*	0.101*	0.155*
Company > Bond	0.059	-0.010	-0.570*	-	-0.222*	-0.407*
Bond > Company	0.018	0.487*	0.183	0.379*	0.075*	0.686*
Age < 1.0	-0.135*	-0.119*	-0.083*	-0.224*	-0.155*	-0.210*
Adjusted R2	0.053	0.219	0.109	0.182	0.184	0.325

* indicates the coefficient is different from zero at the 1% level of significance ** 5% level of significance

Table XII

Out of Sample Results

This table reports the results of out of sample regressions.

		Financial Sector			Industrial Sector	
Panel A: Average Absolute Errors						
Original	0.305	0.476	0.834	0.410	0.569	1.199
Adjusted	0.307	0.477	0.809	0.404	0.520	0.998
Panel B: Root Mean Square Errors						
Original	0.450	0.719	1.110	0.631	0.793	1.549
Adjusted	0.440	0.660	1.061	0.582	0.734	1.289