Correlation Risk, Cross-Market Derivative Products, and Portfolio Performance[1]

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Abstract

We consider portfolios whose returns depend on at least three variables and show the effect of the correlation structure on the probabilities of the extreme outcomes of the portfolio return, using a multivariate binomial approximation. The portfolio risk is then managed by using derivatives. We illustrate this risk management both with simple options, whose payoff depends upon only one of the underlying variables, and with more complex instruments whose payoffs (and values) depend upon the correlation structure.

The question of benchmarking portfolio performance is complicated by the use of derivatives, especially complex derivatives, since these instruments fundamentally alter the distribution of returns. We use the multivariate binomial model to set performance benchmarks for multicurrency, international, portfolios. Our model is illustrated using a simple example where a German institution invests a proportion of its funds in German equities, and the remainder in UK equities. Portfolio performance is measured in Deutsche Marks and depends upon (1) the DAX index, (2) the FTSE index, and (3) the Deutsche Mark-Sterling exchange rate.

The output of the model is a simulation of possible outcomes from the portfolio hedging strategy. The difference in our methodology is that we are able to retain the simplicity of the binomial distribution, used extensively in the analysis of options, in a multivariate context. This is achieved by building three (or more) binomial trees for the individual variables and capturing the correlation structure with the use of varying *conditional* probabilities.

1 Introduction

In this paper, we illustrate how advances in the modelling of multivariate binomial processes can be used to facilitate the management of internationally diversified portfolios. The increasing use of derivatives for hedging the risks inherent in portfolios of equities has made the analysis and control of asset allocation decisions more complex. The methods outlined in the paper can be applied by professional portfolio managers to explain to clients (at quarterly review meetings, for example) the alternative risk management strategies that can be pursued and the possible returns from following those strategies.

The basic idea is as follows. Suppose that a German mutual fund invests a proportion of its initial resources in an indexed German portfolio of equities. It converts the remainder into Pounds Sterling and invests it in an indexed UK portfolio. The performance of the UK investment depends, we assume, on the FTSE index and the performance of the German investment depends upon the DAX index. For convenience, we assume that the indices are joint-lognormally distributed over some finite interval (up to the next performance reporting date, for example). Since the fund is German, the portfolio performance is to be reported in Deutsche Marks at the end of the time interval. We assume that the £:DM exchange rate is also distributed joint-lognormally with the two equity indices. This is an example of a classic asset allocation problem of the type faced by any investor who considers diversification into overseas markets.

We now add the following hedging possibilities: the UK investment can be hedged by buying put options on the FTSE index, and alternatively by buying complex, *Quanto* or multivariate-type put options on the FTSE denominated in DM. The problem facing the fund manager is how to explain and discuss with the client the possible effects of the alternative hedging strategies and to establish benchmarks for portfolio performance. Given either of the two option strategies, the diversified fund with options will underperform simple non-option strategies, if markets remain static or rise, and if the exchange rate does not move unfavourably. This under-performance is due to the fact that "insurance premia" have to be paid to the writers of the put options. Of course, if the FTSE goes down, either in Sterling or in Deutsche Mark terms, then the respective option strategies will come into play and the fund will tend to outperform the simple non-option hedging strategy.

The problem with the use of derivatives illustrated by the above example is one of benchmarking. How does the fund manager's client judge subsequent performance, given that a diversification and hedging strategy has been agreed upon? In some states of the world, the hedged portfolio would be *expected* to outperform the simple strategy, and in others, it would be *expected* to underperform. If the option strategy is agreed upon with the client, the professional manager cannot be applauded if the fund outperforms the simple weighted-benchmark portfolio in the event of the FTSE declining, and *vice-versa*, if the fund underperforms in rising markets. Hence, in general, the benchmark for portfolio performance depends on the particular scenarios (combinations of equity market performance and the exchange rate) that unfold. Thus, a state-by-state approach is required for benchmarking purposes. This paper describes such an approach.[1]

The key feature of the methodology described in this paper is parsimony. We build binomial distributions (i.e. binomial trees) for each of the three variables involved: for example, the DAX index, the FTSE index, and the £:DM exchange rate. We then capture the effect of the correlation between the variables by an appropriate choice of the *conditional* probabilities of the various variables' outcomes. The mathematical details of the methodology are developed fully in a related paper by the same authors.[2] In Section **??** of the present paper, we outline this methodology using examples where the number of binomial stages is small. In Section **??**, we apply the methodology to the international diversification example of asset allocation. In this case, where the number of variables is restricted to three, and the number of binomial stages (that is, the number of up and down movements in the

[1] Other methods of simulating the states using Monte-Carlo techniques are computationally very intensive and become virtually impossible beyond a few variables and possible outcomes, since the number of combinations explodes. Previous work by Bookstaber (1981) and Bookstaber and Clarke (1983) describe the distributions of portfolios of options using such a technique for the case where there is only one state variable.

[2] In Ho, Stapleton, and Subrahmanyam (1994), we describe how to fit a quite general multi-period, multivariate binomial distribution as an approximation to a multivariate lognormal diffusion process. The paper builds binomial distributions for the individual variables, relying on previous work by Cox and Rubinstein (1985) and Jarrow and Rudd (1983). The paper then modifies and extends to a multi-period world, the previous lattice model of Boyle (1988), and is closely related to the methodologies discussed in Amin (1991) and Nelson and Ramaswamy (1990).

binomial process) is also restricted to three, the outcome of the model is a set of $(4 \times 4 \times 4)$ benchmark returns. These returns correspond, approximately, to the inter-quartile range for the individual variables.[3]

The international asset allocation problem is an example of a generic problem where optimal hedging for an asset holder involves a Quanto option as opposed to simple options on individual prices. Other applications of our general approach would include treasury and commodity risk management for a corporation, the risk management of an equity or fixed income derivatives book (with several underlying assets whose returns are driven by a common factor structure) etc. In all these cases, the decision maker needs to look at three or more variables and analyse the "worst-case" scenarios, so that proper hedging action can be taken. For example, consider the case of a Japanese oil importer faced with the uncertainty of the oil price, denominated in dollars, and the dollar–yen exchange rate. Here again, a relevant hedging instrument could be a yen-denominated call option on the oil price, i.e. a Quanto call option as opposed to a dollar-denominated call option on the oil price. The state-by-state binomial approach used in Section **??** can be used to choose between alternative hedging strategies using simple and Quanto options.

2 The Multivariate Binomial Distribution: A Simplified Example

The Ho, Stapleton, and Subrahmanyam (1994) (HSS) technique takes as inputs, the mean and volatility of a number of variables together with the correlation matrix. It then builds binomial approximating distributions with N up and down movements for each variable. For the case where $N = 1$, an example of the resulting joint distribution is shown in Figure 1.

[Insert Figure 1 Here]

In the international asset allocation problem, there are three variables that are of interest: the level of the DAX index at a point of time in the

[3] The model is implemented using the "Visual Basic" programming tool in the Microsoft Windows for Workgroups 3.11 operating system platform. This produces a cascade of four (4×4) matrices to illustrate the output of the system. In principle, it is possible to increase the number of binomial stages. However, this would be at the expense of both computational speed and intuitive appeal.

future, the level of the FTSE index and the £:DM exchange rate. In the case where the binomial parameter $N = 1$, there are just two possible outcomes for each of the variables and eight combinations of the variables. In general, $N + 1$ outcomes are generated for each variable and $(N+1)^3$ combinations.

In Figure 1, q is the probability of an up–movement in the DAX index and $q_1, q_2, \ldots, q_6$ are *conditional* probabilities. We fix $q = 0.5$ and then let q_1 to q_6 be chosen so as to capture the correlations between the variables. The methodology is based on the linear regression property of the normal distribution. For example, to determine q_1, let

$$\begin{aligned} x &= \ln(\text{DAX}), \\ \text{and } y &= \ln(\text{FTSE}), \end{aligned}$$

then the regression

$$y = a + bx + \varepsilon \tag{1}$$

is linear. In HSS, we show that if q_1 is chosen so that[4]

$$q_1 = \frac{a + bx^u - N \ln d_y}{N(\ln u_y - \ln d_y)}, \tag{2}$$

where u_y and d_y are the proportionate up and down movements respectively in the DAX index and the FTSE index, and a and b are the constant and slope term of the regression, then the specified correlation between the variables is satisfied in the limit as $N \to \infty$.[5]

For small values of N, the method produces binomial distributions that only approximate the variance and covariance characteristics of the variables. However, unless the correlations are high (that is, above say 0.7) the accuracy is acceptable even for small N (for example, for $N = 3$, as in the examples shown in Sections 3 and 4).[6]

In Figure 2, we show the structure of model output in the case where $N = 1$. First, since there are only $N + 1$ outcomes for each of the variables, we can represent the output in the form of $N + 1$ matrices. Each of these is

[4]The formula for the general conditional probability, q_i, is given in the Appendix A.

[5]Note that the up and down factors, u_y and d_y, are determined by equations (7) and (8) in HSS (1994), given here in Appendix B.

[6]The variances and covariances of the approximating distribution converge rapidly to those of the true distribution. This is demonstrated in the simulations for the cases where $N = 10$ and $N = 20$ for various correlations in HSS (1994), Table 2.

an $(N+1) \times (N+1)$ matrix showing the outcomes and their probabilities. Note that the joint probabilities are the simple products of the conditional probabilities.

[Insert Figure 2 Here]

The principal feature of our approach is the parsimonious nature of the outcome space, which leads to a substantial improvement in computational efficiency compared with other alternatives such as Amin (1991) or Amin and Bodurtha (1994). This is because, in our case, the number of nodes increases only *linearly* with the number of variables since the binomial trees "recombine". For example, in the other methods, with three state variables, $(N+1)^3$ nodes are generated for the third variable, whereas in our method each variable has just $(N+1)$ nodes. This can lead to a dramatic improvement in computational speed when valuing complex options, for example.

3 An Illustration of the Method: International Asset Allocation

In order to illustrate the use of the joint binomial distributions in the context of portfolio management, we consider the following asset allocation problem:

A German equity portfolio has a proportion α (taken as given) of its funds invested in a DAX-indexed portfolio. The remainder is invested in a FTSE-indexed portfolio of UK shares. The portfolio manager is due to report on the performance of the investments in 3 months time (at time t). We will adopt the following notation:

Rate of return on the DAX index, $r(\mathrm{DM}, t)$.

Rate of return on the FTSE index, $r(\pounds, t)$.

The £:DM exchange rate (DM for £1) at time 0 and t, by EX(£:DM,0) and EX(£:DM,t), respectively.

The rate of return on the overall, internationally diversified, portfolio in this example is:

$$R(t) = \alpha[1 + r(\mathrm{DM}, t)] + (1-\alpha)[\mathrm{EX}(\pounds{:}\mathrm{DM}, 0)]\frac{1 + r(\pounds, t)}{\mathrm{EX}(\pounds{:}\mathrm{DM}, t)} - 1 \qquad (3)$$

In equation (**??**), the rate of return on the portfolio $R(t)$ is measured in DM. The return is computed on the assumption that $(1-\alpha)$ of the portfolio capital is converted from DM into £–Sterling at time 0 and invested in the FTSE-indexed portfolio. This investment yields a stochastic rate of return, $r(£, t)$. The period t, £–Sterling value, of this proportion of the fund is then reconverted into DM at the future stochastic exchange rate, EX(£:DM, t). Note that this will normally represent only an accounting, i.e. a paper, transaction at the end of the reporting period, rather than a series of actual transactions.

The simulation of portfolio returns requires data inputs of the mean, volatility, and the correlation of the three variables that affect the portfolio return: in this case, the DAX, FTSE, and the £:DM exchange rate. We will assume that the means of the two indices are given exogenously; in this context, they can be regarded as subjective estimates that have been agreed upon between the portfolio manager and the client. The mean of the exchange rate is assumed to equal the current forward rate in the market. On the other hand, the input values of the volatilities and correlations of the variables are estimated using past observations of the variables modified by judgement about changed market conditions. It should be noted, in passing, that an alternative to empirical estimation from historic data would be to take implied volatilities from options on the stock indices and the exchange rate. However, for the purpose of the current simulations we used historical estimates based on daily data for the period September 1st, 1993 to September 30th, 1994. The various data input estimates are summarised in Table 1 on an annualised basis.

[Insert Table 1 Here]

In order to highlight the effects of correlation on the probability distribution of portfolio returns, we first generate the simulation output assuming that the three variables are uncorrelated. The annualised mean and the annualised volatility of the DAX are 10% and 17% respectively. The corresponding estimates for the FTSE are 8% and 15%. The 90-day forward £:DM exchange rate is taken as 2.45DM and its volatility is estimated as 7%. The spot exchange rate on the day the simulation is run is 2.46DM. The future date for which the distribution of portfolio returns is required is 90 days hence. Hence, the data in Table 1 have to be converted into a 90-day basis.

To summarise, the mean and volatility input data are as follows:
The means of the three variables in 90 days are :

$$\begin{aligned} E[r(\mathrm{DM},t)] &= 0.025, \\ E[r(\pounds,t)] &= 0.02, \\ E[\mathrm{EX}(\pounds{:}\mathrm{DM},t)] &= 2.45. \end{aligned}$$

The volatilities (over 90 days) of the three variables are:

$$\begin{aligned} \sigma(\mathrm{DM},t) &= 0.085, \\ \sigma(\pounds,t) &= 0.075, \\ \sigma(\mathrm{EX}(\pounds{:}\mathrm{DM},t)) &= 0.035. \end{aligned}$$

In this international diversification example, there are three relevant variables. We now also restrict the number of state outcomes by assuming $N = 3$. Although, in principle, the simulations could be run for any size of N, we choose the binomial parameter $N = 3$, because it leads to a manageable set of four (4×4) matrices. The four outcomes for each individual variable roughly correspond to the inter-quartile range values commonly employed in portfolio management discussions. The model first generates binomial distributions for the three individual variables. For the case where $N = 3$, the binomial distributions for each of the variables in three months time are shown in Table 2.

[Insert Table 2 Here]

In Table 3, we illustrate the joint distribution of the three variables and the distribution of the portfolio return, computed for each state using equation (3) and the values of the variables from Table 1. In Table 3, the node number indicates the number of down movements in the binomial process for each individual variable. Hence, the state (2,1,0), for example, is the state where the first variable, the DAX, has two down movements, the second variable, the FTSE, has one down movements, and the third variable, the £:DM exchange rate, has zero down movements. The unhedged portfolio return in this state is 1.02%, with a joint probability of 0.0176.

[Insert Table 3 Here]

The portfolio rate of return has a maximum value of 19.25% at node (0,0,0), when the DAX is up 18.2%, the FTSE is up 15.7%, the exchange rate is 2.6005DM. At the other extreme, the portfolio has a minimum value of −12.87% at node (3,3,3), when the DAX is down 11.7%, the FTSE index is down 10.6%, and the exchange rate is 2.3055DM. Thus, the portfolio return varies from a maximum of 19.25% to a minimum of −12.87%.

We now introduce the effect of correlation. The correlation matrix used is the one estimated from historical data and reported in Table 1. Table 4 shows the effect of the positive correlation on the joint probabilities. Table 4 reveals the following points. For example, the probability of node (0,0,0) where the DAX, FTSE, and the Exchange Rate are at their highest values is 0.01, five times its value in the uncorrelated case. Secondly, the 'disaster' states, where both the DAX and the FTSE are down have higher probabilities. This increases the need for and the benefits of hedging. A summary of the effects of correlation on the unhedged portfolio is given in Table 5.

[Insert Table 4 Here]

[Insert Table 5 Here]

Hedging Strategies in International Asset Allocation

The matrices of outcomes and joint probabilities produced by the HSS methodology can be used to show the possible effects of an asset allocation strategy to the client of the portfolio manager. However, the real advantage of this state-by-state, binomial approach, lies in its ability to show the possible effects of agreed hedging strategies. As an example, we now consider two possible ways of insuring against a fall in the FTSE index.

For simplicity, we consider, in this simulation, only ways of hedging against falls in the foreign index. We ignore possible hedging of the exchange rate, or of the domestic index. However, similar methods apply to the analysis of these risks. We look at the effect of two alternative hedge strategies.[7] These are:

[7] The methodology illustrated here can be used to value and hedge complex, multivariate derivative instruments. However in that case, the distributions of the prices of the assets have to be centred around (i.e. the means have to be set equal to) their respective forward prices. In other words, we need to use the risk-neutral rather than the actual distributions.

- Hedge 1: a put option on the FTSE index (which will be referred to as option 1), and
- Hedge 2: a put option on the value of the DM value of the FTSE index (which will be referred to as option 2).

The first hedge is a straightforward insurance contract against falls in the FTSE. The second is a Quanto option which could be purchased on the over-the-counter market.[8] Both the payoff and the option premium are denominated in the domestic currency, but the payoff depends on the foreign index. The payoff function for option 1 is:

$$\beta\{\max[K(\pounds) - \mathrm{FTSE}, 0] - P(\pounds)\}, \tag{4}$$

where $K(\pounds)$ is the strike price in Sterling, $P(\pounds)$ is the put option premium in Sterling (quoted on a forward basis), and β is the proportion of the FTSE portfolio insured. For option 2, the payoff function is:

$$\beta\left\{\max\left[K(\mathrm{DM}) - \mathrm{FTSE}\left(\frac{\mathrm{EX}(\pounds{:}\mathrm{DM}, t)}{\mathrm{EX}(\pounds{:}\mathrm{DM}, 0)}\right), 0\right] - P(\mathrm{DM})\right\}, \tag{5}$$

where $K(\mathrm{DM})$ is the strike price in Deutsche Marks, $P(\mathrm{DM})$ is the put option premium in Deutsche Marks (quoted on a forward basis), and β is the proportion of the FTSE portfolio insured.

The result of using Hedge 1 with $\beta = 1$, $K(\mathrm{DM}) = 1$, and $P(\mathrm{DM}) = 3\%$ is shown in Table 6. Comparing the second percentage return with the first in each cell of the matrices, we see the effect of this strategy. The problem is highlighted in the states where the exchange rate rises. This produces a loss in the DM value of the FTSE index, which is not protected by the put option. Note that the cost of the put option is approximately 1% of the portfolio return (on a forward basis) and that it pays off approximately 3% when the FTSE goes down.

[Insert Table 6 Here]

[8] More complex derivatives can also be considered using this methodology where the option payoff net of the price compounded to the expiration date is represented by the function $f[r(\mathrm{DM}, t), r(\pounds, t), \mathrm{EX}(\pounds{:}\mathrm{DM}, t)]$. These may be relevant if the client is extremely concerned about particular states.

The use of the simple put on the FTSE provides some down-side protection for the portfolio. For example, in the worst case scenario node (3,3,3) — the DAX goes down, the FTSE goes down, and the £:DM exchange rate declines — the unhedged portfolio return is −11.94%, whereas the hedged portfolio return is −10.16%. Note also that the hedged return in the favourable states is less than the unhedged return reflecting the cost of the put option. Thus, the maximum return in node (0,0,0), goes down from 18.12% to 17.33%.

Now, consider the effect of using a put option on the DM value of the FTSE index: Hedge 2. In this contract, the premium is paid in Deutsche Marks and the payoff depends upon the exchange rate adjusted DM value of the FTSE. We assume the following data for this Quanto option. First, the put option is again at the money, i.e. $K = 1$. Second, the proportion, β, is equal to one; that is, the DM portfolio is fully hedged. Finally, we assume that the cost of the option is again 3%.[9] The hedged returns using this Quanto are shown in Table 7. In this case, the effective cost of the option is 0.75% of the portfolio return in the state with the best return, state (0,0,0). At the same time, the effect of the hedge is to produce a floor for the portfolio return at −8.64%.

[Insert Table 7 Here]

The Quanto put provides better down-side protection than the simple put, since it takes into account the effect of the correlation between the FTSE and the £:DM exchange rate. In the worst case scenario, the hedged return is −8.64% for the Quanto put against −10.16% for the simple put. On the up-side in the best state, node (0,0,0), the return is 17.37% which is slightly better for the Quanto put compared to 17.33% for the simple put.

4 Correlation and the Design of Multivariate Hedging Strategies

The effect of introducing positive correlation between the indices and the exchange rate has been to increase the probability of the "disaster" states,

[9]In fact, this cost should be lower, given the correlation effect. However, for purposes of comparison we ran the simulation with the same 3% price.

where all three variables have unfavourable outcomes. In fact, comparing the joint probabilities in Table 3, for the uncorrelated case, with those in Table 4, for the correlated case, we find that the probabilities of nodes (3,3,3), (3,3,2), (3,3,1), (3,3,0) have all increased. These are the states where both the DAX and the FTSE are at their lowest levels. However, the probability of states (3,2,3) and (3,2,2), where the FTSE is at node 2 (slightly down) and the exchange rate is low, have also increased significantly. In all these cases, the unhedged portfolio loss exceeds 10%. In fact, as shown previously in Table 5, the probability of a loss in excess of 10% has increased from 0.04 to 0.09 in the correlated case.

We have already seen that the Quanto put concentrates its payoff in those states where either the exchange rate or the FTSE is down. Comparing the results in Tables 6 and 7, for the state where the DAX index is at its lowest level (node 3), we find that the Quanto put has a significant payoff in all the states mentioned above, putting a floor of −8.64% under the portfolio return. It achieves this by earning a relatively large return in those states (where the exchange rate is down) when it is really needed. In contrast, the FTSE put option pays off when FTSE is down at node 3 and ignores the adverse exchange rate movement.

The existence of positive correlation increases the demand for (and the price) of Quanto options, because it increases the probability of exactly those states where low returns can occur, in DM terms, and where this type of derivative instrument pays off. In comparison, separate options on the FTSE and on the £:DM exchange rate are relatively inefficient and therefore more expensive hedge vehicles. However, the state-by-state analysis of portfolios, which can be performed with the binomial approximation methodology illustrated here, also allows the decision maker to design hedging instruments and strategies which are most appropriate to the situation of the fund under management. In our international diversification example above, it is apparent that even more efficient multivariate payoff options would have payoffs that were contingent also on the level of the DAX index. For example, in Table 7, node 0 for the DAX, we find that the Quanto option pays off 3% even when the unhedged portfolio return is +8.5%. A more efficient hedging vehicle would be one that paid off only when the DAX was also at a low level. Such an option should have a cost which is correspondingly less because of its concentrated payoff.

A portfolio manager is concerned with reducing the probability of achieving low returns, even at the cost of giving up some of the portfolio's upside potential in the high return states. However, it is important that the hedge pays off in precisely those states where it is most needed and not in others. Only in this way can the hedging policy be cost effective. The state-by-state approach allows the manager to negotiate hedging vehicles in the over-the-counter market which suit the particular needs of the fund.

5 Conclusions

The correlation between different asset groups is one of the most important determinants of portfolio behaviour. If markets are uncorrelated, for example, diversification may be a sufficient form of risk reduction behaviour. In international asset allocation problems, the correlation between market indices and the relevant exchange rates are the important parameters. The existence of positive correlation between market indices, however, increases the incentive for funds to hedge market movements with options and forwards/futures. Diversification is less beneficial given positive correlation, and hedging with derivatives is essential if adverse combinations of index outcomes are to be avoided. Hedging instruments to reduce the impact of these adverse outcomes range from simple put options on individual market indices to complex, multivariate options. The more complex instrument can be used to counter the effects of the correlation between the market indices and the exchange rates.

In this paper, we have illustrated a system for generating a parsimonious, multivariate binomial distribution of three variables. The system by which we generate the $(4 \times 4 \times 4)$ matrices of the portfolio returns and the hedged portfolio returns could be used in meetings between portfolio managers and their clients. The projected outcomes in each state can be used as benchmarks for discussion of subsequent portfolio performance. The important point is that the performance benchmarks are *non-linear* in the underlying variables if option positions are included. In portfolio analysis, the traditional mean-variance analysis has to be replaced by a state-by-state approach, where the non-linear effects can be taken into account. The state-by-state approach also allows the decision-maker to design multivariate options which meet the objective of protection against specified adverse outcomes.

As an alternative to our methodology, it is possible to generate portfolio return scenarios using Monte Carlo simulation. However, the Monte Carlo method is relatively cumbersome and time consuming and even infeasible, if the number of state-variables becomes large. The system described here, based on the linear property of the lognormal distribution, provides a flexible method for instantaneous analysis of portfolio strategies. A significant advantage of our method is the small number of states used to reflect the underlying lognormally distributed variables. It should be possible to employ the methodology to generate scenarios and test strategies, in real time, at manager/client meetings. An alternative use of the methods described in this paper is in the valuation and hedging of complex multivariate options. We can turn around the focus of the analysis to value the payoffs from the option. In this case, however, the distributions have to be "centered" around the respective forward prices of the variables, so that we obtain "risk-neutral" densities. For the valuation and hedging of such complex options, the improved efficiency due to our method can represent a significant saving in computational time.

Appendix A: The Binomial Probability in the case of a Multivariate Lognormal Stochastic Process

The General Problem

The method used in the paper can be applied to construct a binomial tree as a discrete-time approximation for any multivariate lognormal distribution. We first consider the general problem of approximating variables with a given co-variance structure. We then apply this general method to the problem in the text. To see the specific details of the method consider the case of a trivariate lognormal distribution of three variables X, Y, and Z, with the following variance–covariance matrix (between the logarithms of the variables):[10]

$$\Omega = \begin{pmatrix} \sigma_x^2 & \sigma_{x,y} & \sigma_{x,z} \\ \sigma_{y,x} & \sigma_y^2 & \sigma_{y,z} \\ \sigma_{z,x} & \sigma_{z,y} & \sigma_z^2 \end{pmatrix}.$$

For notational convenience we use lower case letters to denote natural logarithms (that is, $x = \ln X$, $y = \ln Y$, $z = \ln Z$). Since x, y, and z are normally distributed, the multiple regression

$$z = \alpha_z + \beta_z x + \gamma_z y + \varepsilon,$$

where

$$\beta_z = \frac{\sigma_{z,x}\sigma_y^2 - \sigma_{y,z}\sigma_{x,y}}{\sigma_x^2\sigma_y^2 - \sigma_{x,y}^2},$$

$$\gamma_z = \frac{\sigma_{y,z}\sigma_x^2 - \sigma_{x,y}\sigma_{z,x}}{\sigma_x^2\sigma_y^2 - \sigma_{x,y}^2},$$

and

$$\alpha_z = E(z) - \beta_z E(x) - \gamma_z E(y)$$

is linear, and the conditional expectation of z is

$$E(z|x,y) = \alpha_z + \beta_z x + \gamma_z y. \tag{6}$$

[10] The method is readily generalised to the n–variable case. However, the notation is complex.

First, we construct separate binomial trees for the variable x, y, z, using the method described in Ho, Stapleton and Subrahmanyam (1994). We then choose the conditional probability of an up movement in z so that equation (**??**) is satisfied at each node. Given that z is a binomial process this implies that

$$\alpha_z + \beta_z x_r + \gamma_z y_s = n_z\{q(z)\ln u_z + [1 - q(z)]\ln d_z\}, \tag{7}$$

where $q(z) = q(z|x = x_r, y = y_s)$ is the probability of an up movement in z given that x is at node r and y is at node s of their respective binomial distributions. In equation (**??**), n_z is the number of stages in the binomial process of z and u_z and d_z are the up and down movements of z. Solving equation (**??**) we find

$$q(z) = \frac{\alpha_z + \beta_z x_r + \gamma_z y_s - n_z \ln d_z}{n_z(\ln u_z - \ln d_z)}. \tag{8}$$

Appendix B: The Proportionate Up and Down Factors of the Multivariate Binomial Process

The up and down movements u_i and d_i of the multivariate binomial process used in the paper are as follows:

$$d_i = \frac{2(E(X_i)/X_0)^{\frac{1}{N_i}}}{1+\exp(2\sigma_{i-1,i}\sqrt{(t_i-t_{i-1})/n_i})}, \tag{9}$$

$$u_i = 2(E(X_i)/X_0)^{\frac{1}{N_i}} - d_i, \tag{10}$$

where

$$i = 1, 2, \ldots, m \tag{11}$$

and

$$N_i = \sum_{l=1}^{i} n_l, \quad l = 1, \ldots, i. \tag{12}$$

In the above equations, X_i is the price of the ith underlying asset, n_i is the number of binomial stages between any two date points t_{i-1} and t_i, $\sigma_{i-1,i}$ is the conditional volatility of the logarithmic asset return over the period $i-1$ to i, and $\sigma_{0,i}$ is the unconditional volatility.

The up and down movements, u_i and d_i are analogous to those in Cox, Ross and Rubinstein (1979), in that they are chosen to match the true unconditional mean and conditional volatility.

References

Amin, K.I. (1991), "On the Computation of Continuous Time Option Prices Using Discrete Approximations," *Journal of Financial and Quantitative Analysis*, **26**, 4, December, 477–495.

Amin, K.I. and J.N. Bodurtha, Jnr (1994), "Pricing American Options with Stochastic Interest Rates: International Money Market Applications," *Review of Financial Studies*, forthcoming.

Bookstaber, R.M. (1981), *Option Pricing and Strategies in Investing*, Addison–Wesley, Reading, Massachusetts.

Bookstaber, R.M. and R.G. Clarke (1983), *Option Strategies for Institutional Investment Management: A Guide to Portfolio Performance*, Addison–Wesley, Reading, Massachusetts.

Boyle, P. (1988), "A Lattice Framework for Options Pricing when there are Two State Variables," *Journal of Financial and Quantitative Analysis*, **23**, 1, March, 1–12.

Cox, J.C. and M. Rubinstein (1985), *Options Markets*, Prentice–Hall, Englewood Cliffs, New Jersey.

Cox, J.C., S.A. Ross and M. Rubinstein (1979), "Option Pricing: A Simplified Approach," *Journal of Financial Economics*, **7**, September, 229–263.

Ho, T.S., R.C. Stapleton and M.G. Subrahmanyam (1994), "Multivariate Binomial Approximations for Asset Prices with Non-Stationary Variance and Covariance Characteristics," Proceedings of the European Finance Association Conference, Lisbon, Portugal, August 1992, (revised).

Jarrow, R. and A. Rudd (1983), *Option Pricing*, Irwin, Homewood, Illinois.

Nelson, D.B. and K. Ramaswamy (1990), "Simple Binomial Processes as Diffusion Approximations in Financial Models," *Review of Financial Studies*, **3**, 393–430.

Figure 1
Binomial Distribution Outcomes

Figure 1 illustrates the Ho, Stapleton and Subrahmanyam (1994) technique of building binomial approximating distributions for the case of $N = 1$ up and down movements of three variables: the level of the DAX index at a future date, the level of the FTSE index, and the £:DM exchange rate. In this case, there are just two outcomes for each variable, yielding eight combinations of the three variables. Note that, q is the probability of an up-movement in DAX, q_1 is the conditional probability of an up-movement in the FTSE given that the DAX is up, q_2 is the conditional probability of an up-movement in the FTSE given that the DAX goes down, q_3 is the conditional probability of an up-movement in the exchange rate given that the DAX is up and the FTSE is up, q_4 is the conditional probability of an up-movement in the exchange rate given that the DAX is up and the FTSE is down, q_5 is the conditional probability of an up-movement in the exchange rate given that the DAX is down and the FTSE is up, q_6 is the conditional probability of an up-movement in the exchange rate given that the DAX is down and the FTSE is down; and u stands for the up-state, d for the down-state for all the variables.

Figure 2
Joint Probabilities in the System

Matrix A: DAX^u		
	EX^u	EX^d
FTSE^u	$q \times q_1 \times q_3$	$q \times q_1 \times (1 - q_3)$
FTSE^d	$q \times (1 - q_1) \times q_4$	$q \times (1 - q_1) \times (1 - q_4)$

Matrix B: DAX^d		
	EX^u	EX^d
FTSE^u	$(1 - q) \times q_2 \times q_5$	$(1 - q) \times q_2 \times (1 - q_5)$
FTSE^d	$(1 - q) \times (1 - q_2) \times q_6$	$(1 - q) \times (1 - q_2) \times (1 - q_6)$

Matrix A shows the joint probabilities of the DAX index being up, DAX^u, and the remaining FTSE and £:DM exchange rate variables up or down. Matrix B shows the joint probabilities of the DAX index being down, DAX^d, and the remaining variables up or down.

Table 1
Data Input for Portfolio Simulation[a]

Variable	Annualised Mean	Annualised Volatility[d]	Correlation Matrix[d] DAX	FTSE	EX(£:DM)
DAX Index	10%[b]	17%	1	0.37	0.22
FTSE100	8%[b]	15%	0.37	1	0.08
EX(£:DM)	2.46DM[c]	7%	0.22	0.08	1

Notes:

a. The time period for the simulation is $t = 90$ days. The number of binomial periods is $N = 3$.

b. The mean rates of return are subjective estimates, assumed to be given exogenously.

c. The expected exchange rate is the forward rate, for 90 day delivery, on the day of the simulation.

d. Volatilities and correlations were estimated using data from the on-line *DATASTREAM* financial database, for the period September 1st, 1993 to September 30th, 1994. The time-series of logarithms of the daily price relative is computed, and then the standard deviation and correlations estimated from the data.

Table 2
Summary Univariate Probability Distribution:
Uncorrelated Case

Node	(1) DAX	(2) Pr(DAX)	(3) FTSE	(4) Pr(FTSE)	(5) EX(£:DM)	(6) Pr(EX(£:DM))
0	1.1822	0.1250	1.1572	0.1250	2.6005	0.1250
1	1.0724	0.3750	1.0619	0.3750	2.4982	0.3750
2	0.9728	0.3750	0.9744	0.3750	2.3999	0.3750
3	0.8824	0.1250	0.8941	0.1250	2.3055	0.1250

The distributions of the outcomes for the three variables, the DAX index, the FTSE index and the £:DM exchange rate are based on the data inputs in Table 1. Column 1 shows the price relative of the DAX index in each of four states. Column 3 shows the price relative of the FTSE index in each of four states. Column 5 shows the £:DM exchange rate in each of four states. Columns 2, 4, and 6 show the probabilities of the states. Node r for each variable shows its outcome given r down movements of the binomial process.

Table 3
Distribution of Portfolio Returns: Uncorrelated Case

Node=0, DAX=1.1822				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6005	19.25%	16.72%	14.41%	12.29%
	0.0020	0.0059	0.0059	0.0020
2.4982	18.04%	15.62%	13.40%	11.36%
	0.0059	0.0176	0.0176	0.0059
2.3999	16.89%	14.56%	12.43%	10.47%
	0.0059	0.0176	0.0176	0.0059
2.3055	15.78%	13.54%	11.49%	9.61%
	0.0020	0.0059	0.0059	0.0020

Node=1, DAX=1.0724				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6005	11.01%	8.49%	6.18%	4.06%
	0.0059	0.0176	0.0176	0.0059
2.4982	9.81%	7.39%	5.16%	3.13%
	0.0176	0.0527	0.0527	0.0176
2.3999	8.65%	6.33%	4.19%	2.23%
	0.0176	0.0527	0.0527	0.0176
2.3055	7.54%	5.31%	3.26%	1.37%
	0.0059	0.0176	0.0176	0.0059

The joint distribution of the outcomes of the three variables, the DAX index, the FTSE index and the £:DM exchange rate are shown node-by-node. Each panel of the table represents a different outcome of the DAX index. The outcomes of the FTSE index are given at the top of each panel, while those for the £:DM exchange rate are indicated on the left hand side column. In each matrix segment, row 1 is the value of the unhedged portfolio return computed using equation (3), row 2 shows the joint probability of the portfolio return occurring.

Table 3 (Con't)
Distribution of Portfolio Returns: Uncorrelated Case

Node=2, DAX=0.9728				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6005	3.54%	1.02%	−1.29%	−3.41%
	0.0059	0.0176	0.0176	0.0059
2.4982	2.34%	−0.08%	−2.31%	−4.34%
	0.0176	0.0527	0.0527	0.0176
2.3999	1.18%	−1.14%	−3.28%	−5.24%
	0.0176	0.0527	0.0527	0.0176
2.3055	0.07%	−2.16%	−4.21%	−6.09%
	0.0059	0.0176	0.0176	0.0059

Node=3, DAX=0.8824				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6005	−3.24%	−5.76%	−8.07%	−10.19%
	0.0020	0.0059	0.0059	0.0020
2.4982	−4.44%	−6.86%	−9.08%	−11.12%
	0.0059	0.0176	0.0176	0.0059
2.3999	−5.59%	−7.92%	−10.05%	−12.01%
	0.0059	0.0176	0.0176	0.0059
2.3055	−6.70%	−8.94%	−10.99%	−12.87%
	0.0020	0.0059	0.0059	0.0020

The joint distribution of the outcomes of the three variables, the DAX index, the FTSE index and the £:DM exchange rate are shown node-by-node. Each panel of the table represents a different outcome of the DAX index. The outcomes of the FTSE index are given at the top of each panel, while those for the £:DM exchange rate are indicated on the left hand side column. In each matrix segment, row 1 is the value of the unhedged portfolio return computed using equation (3), row 2 shows the joint probability of the portfolio return occurring.

Table 4
Joint Probabilities: The Effect of Correlation

Node=0, DAX=1.1673				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6000	0.0100	0.0151	0.0067	0.0008
2.4980	0.0194	0.0308	0.0140	0.0016
2.4000	0.0122	0.0201	0.0091	0.0010
2.3059	0.0025	0.0041	0.0018	0.0002
Node=1, DAX=1.0683				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6000	0.0076	0.0225	0.0173	0.0036
2.4980	0.0218	0.0622	0.0478	0.0097
2.4000	0.0190	0.0540	0.0412	0.0079
2.3059	0.0052	0.0147	0.0110	0.0019
Node=2, DAX=0.9777				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6000	0.0019	0.0112	0.0149	0.0052
2.4980	0.0081	0.0419	0.0544	0.0190
2.4000	0.0099	0.0483	0.0625	0.0217
2.3059	0.0036	0.0175	0.0225	0.0075
Node=3, DAX=0.8948				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6000	0.0002	0.0019	0.0043	0.0025
2.4980	0.0010	0.0094	0.0206	0.0124
2.4000	0.0017	0.0144	0.0315	0.0198
2.3059	0.0009	0.0069	0.0154	0.0102

The joint distribution of the outcomes of the three variables, the DAX index, the FTSE index and the £:DM exchange rate are shown node-by-node. Each panel of the table represents a different outcome of the DAX index. The outcomes of the FTSE index are given at the top of each panel, while those for the £:DM exchange rate are indicated on the left hand side column. The joint probabilities assume that the correlation of the DAX and FTSE is 0.37, the FTSE and the £:DM exchange rate is 0.08, and that the DAX and the £:DM exchange rate are correlated 0.22. All probabilities are generated using the method of Ho, Stapleton and Subrahmanyam (1994).

Table 5
Summary Statistics: Correlation versus Uncorrelated

	Uncorrelated	Correlated[c]
1st quartile[a]	−2.3%	−2.9%
2nd quartile	+2.3%	+1.9%
3rd quartile	+7.8%	+7.1%
Probability of −10%[b]	0.04	0.09

Notes:

a. The first quartile means that there is a 25% chance of the portfolio return being less than −2.3%, for example.

b. The last line shows the probability of a return of less than −10%.

c. The correlated case uses the correlation estimates reported in Table 1.

Table 6
Distribution of Unhedged and Hedged Portfolio Returns Using a Put Option on the FTSE Index: Correlated Case

Node=0, DAX=1.1673				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6000	18.12%	15.60%	13.29%	11.17%
	17.33%	14.81%	13.18%	13.18%
	0.0100	0.0151	0.0067	0.0008
2.4980	16.92%	14.50%	12.28%	10.24%
	16.16%	13.74%	12.17%	12.17%
	0.0194	0.0308	0.0140	0.0016
2.4000	15.77%	13.45%	11.31%	9.35%
	15.04%	12.71%	11.21%	11.21%
	0.0122	0.0201	0.0091	0.0010
2.3059	14.67%	12.43%	10.38%	8.50%
	13.96%	11.73%	10.28%	10.28%
	0.0025	0.0041	0.0018	0.0002

Node=1, DAX=1.0683				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6000	10.70%	8.18%	5.87%	3.75%
	9.91%	7.39%	5.75%	5.75%
	0.0076	0.0225	0.0173	0.0036
2.4980	9.50%	7.08%	4.86%	2.82%
	8.74%	6.32%	4.75%	4.75%
	0.0218	0.0622	0.0478	0.0097
2.4000	8.35%	6.02%	3.89%	1.93%
	7.62%	5.29%	3.78%	3.78%
	0.0190	0.0540	0.0412	0.0079
2.3059	7.24%	5.01%	2.95%	1.07%
	6.54%	4.30%	2.85%	2.85%
	0.0052	0.0147	0.0110	0.0019

The joint distribution of the outcomes of the three variables, the DAX index, the FTSE index and the £:DM exchange rate are shown node-by-node. Each panel of the table represents a different outcome of the DAX index. The outcomes of the FTSE index are given at the top of each panel, while those for the £:DM exchange rate are indicated on the left hand side column. In each matrix segment, row 1 is the value of the unhedged portfolio return computed using equation (3). Row 2 shows the hedged portfolio return using a put option on the FTSE index. Row 3 shows the joint probability of the portfolio return occurring. Node r for each variable shows its outcome given r down movements of the binomial process.

Table 6 (Con't)
Distribution of Unhedged and Hedged Portfolio Returns Using a Put Option on the FTSE Index: Correlated Case

Node=2, DAX=0.9777				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6000	3.90%	1.38%	−0.93%	−3.05%
	3.11%	0.59%	−1.04%	−1.04%
	0.0019	0.0112	0.0149	0.0052
2.4980	2.70%	0.28%	−1.94%	−3.98%
	1.94%	−0.48%	−2.05%	−2.05%
	0.0081	0.0419	0.0544	0.0190
2.4000	1.55%	−0.77%	−2.91%	−4.87%
	0.82%	−1.51%	−3.01%	−3.01%
	0.0099	0.0483	0.0625	0.0217
2.3059	0.45%	−1.79%	−3.84%	−5.72%
	−0.26%	−2.49%	−3.94%	−3.94%
	0.0036	0.0175	0.0225	0.0075

Node=3, DAX=0.8948				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6000	−2.31%	−4.83%	−7.15%	−9.27%
	−3.11%	−5.63%	−7.26%	−7.26%
	0.0020	0.0019	0.0043	0.0025
2.4980	−3.51%	−5.94%	−8.16%	−10.20%
	−4.28%	−6.70%	−8.27%	−8.27%
	0.0010	0.0094	0.0206	0.0124
2.4000	−4.67%	−6.99%	−9.13%	−11.09%
	−5.40%	−7.72%	−9.23%	−9.23%
	0.0017	0.0144	0.0315	0.0198
2.3059	−5.77%	−8.01%	−10.06%	−11.94%
	−6.48%	−8.71%	−10.16%	−10.16%
	0.0009	0.0069	0.0154	0.0102

The joint distribution of the outcomes of the three variables, the DAX index, the FTSE index and the £:DM exchange rate are shown node-by-node. Each panel of the table represents a different outcome of the DAX index. The outcomes of the FTSE index are given at the top of each panel, while those for the £:DM exchange rate are indicated on the left hand side column. In each matrix segment, row 1 is the value of the unhedged portfolio return computed using equation (3). Row 2 shows the hedged portfolio return using a put option on the FTSE index. Row 3 shows the joint probability of the portfolio return occurring. Node r for each variable shows its outcome given r down movements of the binomial process.

Table 7
Hedging the FTSE with a DM Denominated Put (Quanto) Option: Correlated Case

Node=0, DAX=1.1673				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6000	18.12%	15.60%	13.29%	11.17%
	17.37%	14.85%	12.54%	11.80%
	0.0100	0.0151	0.0067	0.0008
2.4980	16.92%	14.50%	12.28%	10.24%
	16.17%	13.75%	11.80%	11.80%
	0.0194	0.0308	0.0140	0.0016
2.4000	15.77%	13.45%	11.31%	9.35%
	15.02%	12.70%	11.80%	11.80%
	0.0122	0.0201	0.0091	0.0010
2.3059	14.67%	12.43%	10.38%	8.50%
	13.92%	11.80%	11.80%	11.80%
	0.0025	0.0041	0.0018	0.0002

Node=1, DAX=1.0683				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6000	10.70%	8.18%	5.87%	3.75%
	9.95%	7.43%	5.12%	4.37%
	0.0076	0.0225	0.0173	0.0036
2.4980	9.50%	7.08%	4.86%	2.82%
	8.75%	6.33%	4.37%	4.37%
	0.0218	0.0622	0.0478	0.0097
2.4000	8.35%	6.02%	3.89%	1.93%
	7.60%	5.27%	4.37%	4.37%
	0.0190	0.0540	0.0412	0.0079
2.3059	7.24%	5.01%	2.95%	1.07%
	6.49%	4.37%	4.37%	4.37%
	0.0052	0.0147	0.0110	0.0019

The joint distribution of the outcomes of the three variables, the DAX index, the FTSE index and the £:DM exchange rate are shown node-by-node. Each panel of the table represents a different outcome of the DAX index. The outcomes of the FTSE index are given at the top of each panel, while those for the £:DM exchange rate are indicated on the left hand side column. In each matrix segment, row 1 is the value of the unhedged portfolio return. Row 2 shows the effect of hedging with the DM denominated put option (Quanto) on the FTSE index. Row 3 shows the joint probability of the portfolio return occurring.

Table 7 (Con't)
Hedging the FTSE with a DM Denominated Put (Quanto) Option: Correlated Case

Node=2, DAX=0.9777				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6000	3.90%	1.38%	−0.93%	−3.05%
	3.15%	0.63%	−1.68%	−2.42%
	0.0019	0.0112	0.0149	0.0052
2.4980	2.70%	0.28%	−1.94%	−3.98%
	1.95%	−0.47%	−2.42%	−2.42%
	0.0081	0.0419	0.0544	0.0190
2.4000	1.55%	−0.77%	−2.91%	−4.87%
	0.80%	−1.52%	−2.42%	−2.42%
	0.0099	0.0483	0.0625	0.0217
2.3059	0.45%	−1.79%	−3.84%	−5.72%
	−0.30%	−2.42%	−2.42%	−2.42%
	0.0036	0.0175	0.0225	0.0075

Node=3, DAX=0.8948				
FTSE	1.1572	1.0619	0.9744	0.8941
EX(£:DM)				
2.6000	−2.31%	−4.83%	−7.15%	−9.27%
	−3.06%	−5.58%	−7.90%	−8.64%
	0.0002	0.0019	0.0043	0.0025
2.4980	−3.51%	−5.94%	−8.16%	−10.20%
	−4.26%	−6.69%	−8.64%	−8.64%
	0.0010	0.0094	0.0206	0.0124
2.4000	−4.67%	−6.99%	−9.13%	−11.09%
	−5.42%	−7.74%	−8.64%	−8.64%
	0.0017	0.0144	0.0315	0.0198
2.3059	−5.77%	−8.01%	−10.06%	−11.94%
	−6.52%	−8.64%	−8.64%	−8.64%
	0.0009	0.0069	0.0154	0.0102

The joint distribution of the outcomes of the three variables, the DAX index, the FTSE index and the £:DM exchange rate are shown node-by-node. Each panel of the table represents a different outcome of the DAX index. The outcomes of the FTSE index are given at the top of each panel, while those for the £:DM exchange rate are indicated on the left hand side column. In each matrix segment, row 1 is the value of the unhedged portfolio return. Row 2 shows the effect of hedging with the DM denominated put option (Quanto) on the FTSE index. Row 3 shows the joint probability of the portfolio return occurring.