The Risk of a Currency Swap: A Multivariate-Binomial Methodology

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** Please send proofs to Marti G. Subrahmanyam.

Executive Summary

The concept of "value-at-risk" (VAR) - the maximum loss on a specified horizon date at a given level of confidence - is widely used at various levels of the financial system. Individual traders and trading desks use the concept to determine the range of their potential gains and losses over the next trading day. The managers of financial institutions apply the notion of VAR to measure and control the risks of traders. Regulatory authorities, concerned with the stability of the financial system and of the institutions under their jurisdiction, set capital adequacy standards based on measures of VAR. No matter what the specific criteria for disclosure and capital adequacy are, a financial intermediary needs to translate the regulatory requirements into a practical system for implementation. The system needs to take into account market data such as interest rates and exchange rates, as well as the detailed characteristics of the portfolio of assets and liabilities held by the intermediary, in order to value the portfolio on future dates and measure its response to changes in the market prices.

Although the broad approach to the measurement of the risk has to be consistently applied across financial instruments, the precise implementation depends on the instrument in question, and in particular, on the variables that affect its value on the future date. In this paper, the case of a widely-used contract, the currency swap, is examined in some detail, to illustrate our approach. The value of a currency swap on a future date depends on a number of variables: the interest rates in the two currencies and the foreign exchange rate. Hence, the probability distribution of the value of the swap on this date can be computed using the valuation model for the instrument and the joint probability distribution of these rates.

In order to estimate the joint distribution of these variables, it is useful to start from a model of the term structure of interest rates. The simplest possible model for a currency swap would involve only three variables: one determining the term structure in each of the two currencies, and one representing the exchange rate between the currencies. For some swaps, this might be a reasonable specification and we retain this as a special case of our model. However, as in the case of fixed income securities and derivatives even in a single currency, a two-factor model in at least one of the currencies will usually be required, in order to capture both shifts and tilts in the term structure. To be specific, we might assume that for each of the two currencies involved in the swap, two "factor" rates determine the whole term structure. Then, we need to take into account the correlation between these rates and the exchange rate between the two currencies. The future exchange rate is relevant since the valuation of the swap involves the conversion of the cash flows into the numeraire currency. Hence, we need to model the stochastic behavior of five variables.

We use a method developed in an earlier paper [Ho, Stapleton and Subrahmanyam (1995)] to obtain the joint distribution of the five variables. In particular, we compute a multivariate-binomial distribution that approximates a joint-lognormal distribution with given means, volatilities and correlations. Essentially, the method yields an approximation to the joint distributions of the variables using a joint binomial distribution, with the approximation getting better as the binomial density for each variable is increased. For example, with three binomial stages, each variable has a four state distribution and there are then 4^5 = 1024 scenarios of interest rates and exchange rates. The output is the joint distribution of the variables i.e., the set of interest rate and exchange rate scenarios and their probabilities. These are then used to compute values of the currency swaps. This methodology is applied in a single-period context,

producing a single five-dimensional vector of interest rates and currencies together with the joint-probability of each scenario. These inputs are then used to value the currency swap in each scenario and to produce a probability distribution for the value of the swap. The resulting probability distribution of the swap value can either be used to highlight particular, problematic scenarios, or to compute portfolio values at various confidence levels. Thus, the proposed methodology can be used to obtain conventional measures of VAR or to perform scenario-based stress tests.

The main advantage of the proposed methodology is that it provides a computationally fast and efficient scenario analysis, allowing a state-by-state valuation of the swap. Particular states can be identified where the swap is significantly exposed to market movements, so that the risk manager can take appropriate action. Since the binomial density can be chosen at a different level for each factor, it can be useful for sensitivity analysis of the valuation, after the most significant variables have been identified. If one of them is the exchange rate, for example, the simulation can be re-run with a greater density for the exchange rate. The other advantage is that relatively few parameters are required in order to implement the model.

In the case of currency swaps, the valuation involves discounting the future cash flow streams in the two currencies. Although this is fairly straight-forward conceptually, there is a practical problem in dealing with a large number of cash flows arising on different future dates. A common way of reducing the number of dates to be considered in measuring the market risk of a large swap portfolio is to use a "bucketing" methodology. Essentially, this is a way of reducing the number of maturity dates of the future cash flows and is based on interpolation of the term structure. Thus, a small number of discount factors for standard maturity dates is sufficient to value a complex portfolio with cash flows on several hundred dates.

Apart from computation of VAR, the methodology proposed in the paper can be used for practical problems of risk management such as stress testing and credit risk. Stress testing normally refers to the testing of model assumptions and parameters under extreme conditions. Illustrative examples used in the paper are departures from the assumption of joint-lognormality and high values of the volatilities and correlations. Credit risk can be analysed by examining the risk-time profile of the swap position i.e., the risk exposure on different dates in the future. This may be useful for analyzing the credit risk of the swap book.

Abstract

In general, the risk of a financial instrument on a future valuation date depends on several stochastic variables. In the case of a currency swap, its value on a future date, can be modelled as a function of five stochastic variables. These represent the factors that determine the term structure of interest rates in the two currencies, and the foreign exchange rate between the currencies. The joint-probability distribution of the the relevant variables on the horizon date of is approximated by a multivariate-binomial distribution. The proposed methodology provides a fast and flexible alternative to Monte-Carlo simulation of the swap value. The distributions of value produced by the method can be employed to assist with both market and credit risk management.

The Risk of a Currency Swap:
A Multivariate-Binomial Methodology

1. Introduction

There has been considerable interest in recent years in the measurement of the risk of financial contracts. The concept of "value-at-risk" (VAR) - the maximum loss on a specified horizon date at a given level of confidence- is widely used at various levels of the financial system.[1] Individual traders and trading desks use the concept to determine the range of their potential gains and losses over the next trading day. The managers of financial institutions apply the notion of VAR to measure and control the risks of traders. Regulatory authorities, concerned with the stability of the financial system and of the institutions under their jurisdiction, set capital adequacy standards based on measures of value-at-risk.

The regulatory concerns have manifested themselves in a global context in discussions under the auspices of the Bank of International Settlements (BIS). A typical comment in these deliberations is that "quantitative information about risk exposures and risk management performance can provide a framework for qualitative description and assessment."[2] It is clear that, no matter what the specific criteria for disclosure and capital adequacy are, a financial intermediary needs to translate these into a practical system for implementation. The system needs to take market data such as interest rates and exchange rates, as well as the detailed characteristics of the portfolio of assets and liabilities held by the intermediary into account, in order to value the portfolio on future dates and measure its response to changes in the market prices.

Although the broad approach to the measurement of the risk has to be consistently applied across financial instruments, the precise implementation depends on the instrument in question, and in particular, on the variables that affect its value on the future date. In this paper, the case of a common contract, the currency swap, is examined in some detail. The value of a currency swap on a future date depends on a number of variables. For example, the value of a five-year fixed-for-fixed USD-DEM (U.S. Dollar-Deutsche Mark) swap in one year's time will depend upon the USD interest rates for up to four years maturity in one year's time, the DM interest rates for similar maturities and the foreign exchange rate of DEM for USD. The probability distribution of the value of the swap depends on the valuation model and the joint distribution of these rates. In order to estimate the joint probability distribution of these variables it is useful to start from a model of the term structure of interest rates. To be specific, we might assume that for each of the two currencies involved in the swap, two "factor" rates determine the whole term structure. Then, we need to estimate the relationship of these rates with the exchange rate between the two currencies. The future exchange rate is relevant since the valuation of the swap involves the conversion of the cash flows into the numeraire currency. is required in one of the two currencies of the swap. In this paper, we propose and illustrate a procedure for determining the value of such a swap, assuming that a simple two-factor model holds for the term structure of interest rates in each of the two currencies.

The simplest possible model for a currency swap would involve only three variables : one determining the term structure in each of the two currencies, and one representing the exchange rate between the currencies. For some swaps, this might be a reasonable specification and we retain this as a special case of our model. However, as in the case of fixed income securities and derivatives even in a single currency, a two-factor model in at least one of the currencies will usually be required, in order to capture both shifts and tilts in the term structure. This is because swaps often involve fixed cash flows over reasonably long periods of time such as five years. The approach here, therefore, allows for the possibility of a two-factor model in each of the currencies. Along with the currency factor, this means that we need to model the stochastic evolution of five variables.

Most of the term-structure models in the literature analyse the period-by-period movements of interest rates up to some terminal valuation date. This is true, for example, of the models of Ho and Lee (1986), Black, Derman and Toy (1990), Heath, Jarrow and Morton (1992), and Hull and White (1993). A five-dimensional version of any of those models would be prohibitively expensive from a computational point of view, and perhaps infeasible, given currently available computing power. Luckily, for the purpose of risk management such a model is not strictly required. In the context of risk management, it is sufficient to value a swap on a particular date in the future, termed the value date. Thus, it is usually only necessary to model the joint distribution of the five variables on that date, rather than to model the evolution of the variables at a series of intermediate dates, up to the value date. This makes the problem far more manageable.

A method for computing a multivariate-binomial distribution that approximates a joint-lognormal distribution of a specified number of variables, with given means, volatilities and correlations is proposed in Ho, Stapleton and Subrahmanyam (1995) (HSS). The method was applied in a portfolio management context in Ho, Stapleton and Subrahmanyam (1995a) and the application is extended here to the case of five variables. The distribution generated by this method converges to a joint-lognormal distribution, with the specified parameters, as the density of the binomial tree increases. In the present paper, we apply this methodology in a single-period context, producing a single five-dimensional vector of interest rates and currencies together with the joint-probability of each scenario. These inputs are then used to value the currency swap in each scenario and to produce a probability distribution for the value of the swap. The resulting probability distribution of the swap value can either be used to highlight particular, problematic scenarios or to compute portfolio values at various confidence levels. Thus, the proposed methodology can be used to obtain conventional measures of VAR or to perform scenario-based stress tests.

The outline of the paper is as follows. Section 2 presents a definition of the cash flows of a currency swap and provides a general valuation formula for the value of the swap in the numeraire currency given that a separate two-factor model of the term structure of interest rates holds for each currency. A practical approach to the valuation of a large currency swap book via the “bucketing” of the swap cash flows is described in section 3. Section 4 explains how to employ the multivariate-binomial approximation technique to generate scenarios of the relevant interest rates and exchange rates. Section 5 shows how to implement the two-factor model to value the swap cash flows in terms of the numeraire currency. Section 6 illustrates the output of the analysis using a numerical example. Section 7 concludes.

2. **The Cash Flows of Currency Swaps**

A currency swap involves cash flows in two currencies, which will be indexed by the subscripts j and k. In the Figure 1 below, we illustrate the flows for a swap where cash flows are paid in currency j and received in currency k.

Figure 1 here

Figure 1 summarises the cash flows of a currency swap. The relevant dates are shown at the top of the diagram. We assume that a swap contract is in place at time 0 and is to be valued at time t. The payment dates for the swap cash flows are $t_1, t_2, ..., t_i, ..., t_n$ Hence, the currency swap involves the exchange of a stream of cash flows $X_{k,t_1}, X_{k,t_2}, \ldots, X_{k,t_i}, ..., X_{k,t_n}$ in currency k for the stream $X_{j,t_1}, X_{j,t_2}, ..., X_{j,t_i}, \ldots, X_{j,t_n}$ in currency j. The cash flows shown on the time line are all non-stochastic. If the swap involves floating payments we assume that these are valued, at par, on the first reset date.

We assume that valuation is required in the numeraire currency j. The value of the swap at time t in currency j is

$$\begin{aligned} V_{j,t} = &\left[X_{k,t_1} B_{k,t,t_1} + X_{k,t_2} B_{k,t,t_2} + \ldots + X_{k,t_n} B_{k,t,t_n} \right] P_{j,k,t} \\ &- \left[X_{j,t_1} B_{j,t,t_1} + X_{j,t_2} B_{j,t,t_2} + \ldots + X_{j,t_n} B_{j,t,t_n} \right] \end{aligned} \qquad \text{equation (1)}$$

where

$V_{j,t}$ is the value of the swap in currency j at time t
B_{h,t,t_i} is the discount factor,at time t, for maturity t_i in currency h, h = j,k
$P_{j,k,t}$ is the spot exchange rate, the price in terms of currency j, of currency k, at time t

We now assume, that the discount factor B_{h,t,t_i} , for maturity t_i, is a known function of two 'factor' rates of interest, i.e.

$$B_{h,t,t_i} = f_{i,h}(r_{h,t,n_1}, r_{h,t,n_2}) \qquad \text{equation (2)}$$

where r_{h,t,n_1} and r_{h,t,n_2} are the 1st and 2nd factor interest rates in currency h, for maturity t_i. Hence, for the swap involving currencies j and k , $V_{j,t}$, the value in the numeraire currency j, is a function of the five variables:

r_{j,t,n_1}	1st 'factor' rate for currency j
r_{j,t,n_2}	2nd 'factor' rate for currency j
r_{k,t,n_1}	1st 'factor' rate for currency k
r_{k,t,n_2}	2nd 'factor' rate for currency k
$P_{j,k,t}$	Exchange rate : price of k in units of j.

The function in equation (2) relates the discount factor or zero-bond price in currency h to the two factor rates, which could be, for example, the 3-month libor rate and the 10 year bond rate. However, both the factor rates and the functional relationship itself could be different for the two currencies involved in the swap.

3. **Valuation of the Swap : A Practical Approach**

In the case of currency swaps, the valuation involves discounting the future cash flow streams in the two currencies. Although this is fairly straight-forward conceptually, there is a practical problem in dealing with a large number of cash flows arising on different future dates. A common way of reducing the number of dates to be considered in measuring the market risk of a large swap portfolio is to use a "bucketing" methodology. Essentially, this is a form of interpolation of the term structure. For any swap in the portfolio, a standard grid of discount factors is used, rather than the general set B_{t,t_i}. For illustrative purposes, we will assume here that a grid of sixteen bucket rates is used for each of the currencies involved in the swap.

The first step in the procedure is to bucket the cash flows. This has to be done on a *forward*, rather than on a spot, basis since the valuation is required at a specified future date. Forward bucketing means that if the standard buckets are, for example [1 day, 1 week, 1 month, 3 months, 6 months, ... , 30 years], the cash flows are bucketed into pots of cash occurring at the dates :

$$0 \text{———} t \text{———} t+\tau_1 \text{———} t+\tau_2 \text{———} \ldots \text{———} t+\tau_i \ldots \text{———} t+\tau_{16}$$

Note that the current date is 0 and the bucket dates are defined given the value date t, at $t+\tau_1, t+\tau_2, \ldots, t+\tau_i, \ldots, t+\tau_{16}$.[3]
The cash flows resulting from the bucketing process are shown in Figure 2. Here, it is not necessary to assume that the buckets have the same maturity length for each currency. Hence $t_{k,i}$ is not necessarily equal to $t_{j,i}$.

Figure 2 here

Having bucketed the cash flows, we now require the discount factors for the standard grids in the two currencies and the exchange rates. We next describe the generation of scenarios of interest rates in the following section.

4. **The Joint-Probability Distribution of Factor Interest Rates and the Exchange Rate**

In order to value the swap, we need to generate a joint-probability distribution of the five variables that determine its value. The most popular method employed in practice is to assume that the variables are lognormal with given means, variances and covariances and to construct a multivariate lognormal distribution.[4] Here we use a binomial approximation approach where the distribution limits to the lognormal as the density of the binomial tree increases. This method has the advantage of allowing a scenario analysis, of extreme states, to be performed.[5] This is advantageous if we wish to use the output for an analysis of the credit risk of a book of currency swaps.

Given that we wish to analyse individual states by building a binomial tree, a number of approaches have been suggested in the literature. Most of these are surveyed in the recent paper by Amin (1995). One approach, suggested by He (1990) and Boyle (1989), models N variables using a binomial distribution with just N+1 nodes over each sub-interval. This method is efficient in the context of valuing options whose payoff depends on more than one variable. However, in the context of risk analysis, we require to distinguish all possible states. We therefore favour the methodology of Ho, Stapleton and Subrahmanyam (1995). They build an approximation to the joint distributions of the variables using a joint binomial distribution which produces $(J+1)^N$ nodes, where J is the binomial density for each variable.[6] There are two ways of building such a distribution. First, it is possible to orthogonalize the variables and build the joint distribution of a set of orthogonal vectors. This method is illustrated in Amin (1991). However, the HSS methodology builds vectors of the individual variables with just J+1 nodes for each variable. It captures the covariance structure by adjusting the conditional probabilities. This feature of the HSS method makes it somewhat simpler to employ in the context of risk management, where the distributions of the individual variables, as well as the multivariate distribution may be relevant. To summarise, there are a number of advantages of the HSS approach in this context. The main ones are :

- It is an efficient method of generating scenarios of the relevant interest rates and exchange rates.
- If greater accuracy is required, at the expense of more computation time, then the number of binomial stages can be increased. This increases the fineness or density of the binomial tree. It can also be done on a variable-by-variable basis.
- It allows specific scenarios to be investigated. The swap value can be computed for particular, unusual combinations of interest rates and exchange rates. Thus, it gives more qualitative information to the risk manager than a simple mean-variance analysis, for example.

The HSS methodology assumes that the underlying variables, in this application, the factor interest rates and the exchange rate are joint-lognormally distributed. As a first approximation, this is perhaps not a bad assumption, since it is the assumption underlying most of the option pricing models used in practice. Later, when we perform a stress test, the effect of relaxing this assumption is investigated. The required inputs for the joint-lognormal distribution are the means, volatilities and correlations of the variables. These are specified in Table 1 below

Table 1 here

The input data can be estimated in many different ways, depending on the information available. However, a reasonable procedure may be as follows :

1. Estimate the means of the factor interest rates from the forward rates at time 0 (the current date) for delivery at the value date t.[7] Also, estimate the mean of the exchange rate from the forward rate, at time 0, for deliveries of the currency at time t.

2. Estimate the volatilities using either historical data on the factor interest rates or by backing out implied volatilities from option prices. Based on the results in the empirical literature, the implied volatilities may be upward-biased estimates of the true volatility.
3. Estimate the correlation matrix from historical data.[8]

The HSS methodology requires the choice of J, the binomial density, J, determines how many factor interest rates, exchange rates and scenarios are generated. For example, with J=3, each variable has a four state distribution. There are then $4^5 = 1024$ scenarios of factor interest rates and exchange rates. The output is the joint distribution of the variables. Note that J can be chosen at a different level for each factor, if required. This can be useful for sensitivity analysis of the valuation, after the most significant variables have been identified. If one of them is the exchange rate, for example, the simulation can be re-run with a greater density for the exchange rate. The output at this stage of the simulation is the set of factor interest rates and exchange rate scenarios and their probabilities. These numbers can now be used to compute value of the currency swap in each scenario.

The methodology fits the input mean vector and the variance-covariance matrix by changing the conditional probabilities of the binomial tree.[9] Details of the implementation of the approach are provided in the Appendix.

5. **Swap Valuation in Each Scenario**

In order to value the swap cash flows, we first need to specify the functional relationship between the discount factors and the generated factor interest rates for each currency, shown generally in equation (2). We then need to compute the swap value in each scenario using equation (1). The choice of the particular interest rate model to be used depends upon how closely we wish to model interest rate movements. Given the complexity of the problem situation, we choose here a rather simple linear model for the spot rates of interest. Suppose that n_1 is the maturity in years of the first factor interest rate and n_2 is the maturity of the second factor rate, then, for each currency, we assume that the yield rate for the kth bucket maturity is[10]

$$r_{t,t_i} = \left[\frac{n_2 - t_i}{n_2 - n_1}\right] r_{t,n_1} + \left[\frac{t_i - n_1}{n_2 - n_1}\right] r_{t,n_2} \qquad \text{equation (3)}$$

We then compute the required discount factors from the relationship

$$B_{t,t+t_i} = \frac{1}{(1+r_{t,t_i})^{t_i}} \qquad \text{equation (4)}$$

The two-factor model of the interest rates in equation (3) is a simple “shift-tilt” model. The term structure shifts, if both the factor interest rates move in the same direction, and tilts, if one of the rates increases while the other falls. Also, given equation (3), the rate for any intermediate maturity is just a weighted average of the two factor rates, where the weights depend on the distance of the maturity from each of the factor rate maturities.

Given the discount factors for the bucket maturities, we can now value the bucketed cash flows using the valuation equation :

$$\begin{aligned} V_{j,t} = & \left[X_{k,t_{1,k}} B_{k,t,t_{1,k}} + X_{k,t_{2,k}} B_{k,t,t_{2,k}} + \ldots + X_{k,t_{n,k}} B_{k,t,t_{n,k}}\right] P_{j,k,t} \\ & - \left[X_{j,t_{1,j}} B_{j,t,t_{1,j}} + X_{j,t_{2,j}} B_{j,t,t_{2,j}} + \ldots + X_{j,t_{n,j}} B_{j,t,t_{n,j}}\right] \end{aligned} \qquad \text{equation (5)}$$

Here, the general valuation equation (1) has been adjusted so that all the cash flows occur only on the bucket days. This revised valuation equation can now be used to compute the swap value in each scenario of the interest rate and exchange rate.

6. **Scenario Analysis of the Value of a Currency Swap : An Illustration**

Equation (5) above can be evaluated in each scenario, given its probability of occurrence, to yield a probability distribution of the value of the swap. We can obtain an estimate of the VAR by ranking the states by the value of the swap and reading off the value at any given confidence level. Of more relevance, however, is the state by state analysis of possible values of the swap position. This is given by the joint-probability distribution produced by the binomial model.

As an illustration, we take a JPY/ DEM currency swap with the cash flows shown in Table 2.

Table 2 here

The swap is a three year "receive floating-DEM, pay fixed-JPY" swap with a principal of 50,000,000DM. The current date 11/5/95. The exchange rate is 65Yen per DM on the current date, 11/5/95. The term structures for the two currencies on 11/5/95 are as follows.

Table 3 here

Note that the yields in Table 3 are yields on zero-coupon bonds for the given maturities. Here, we have assumed, for simplicity, that the bucket maturities are the same for each currency. The yield curves for both currencies are generally upward sloping with the Yen yields somewhat lower than those for the DM.

The problem is to calculate value of the swap cash flows estimates on a value date 90 days hence. The first step is to bucket the swap cash flows into standard buckets. It yields the cash flows shown in Table 4.

Table 4 here

The first swap payment date is 11/09/96, which is four months from the current date, or one month from the valuation date. Hence, the first cash flow is allocated to the one-month bucket. The later cash flows for the fixed Yen payments are bucketed in the 6 month, 1 year, 2 year and 3 year buckets.

We now proceed to generate scenarios for the interest rate term structures and exchange rate on the valuation date. The input data for the simulation is shown in Table 5.

Table 5 here

The HSS model assumes that the variables are joint-lognormally distributed with given means, volatilities and correlations. The means for the short and long rates and the exchange rates are taken to be the respective forward rates. In turn, these are computed from the zero-coupon bond yields and the given spot exchange rate. Reasonable estimates for the volatilities and correlations have been assumed.

We run the HSS model with, for illustrative purposes, J=1 for each variable. This produces $2^5 = 32$ scenarios. These are shown in Table 6. The probability of each scenario is shown in the last column of Table 6.

Table 6 here

Note that the methodology captures the correlation structure by building a binomial distribution for the first variable (yen short rate factor) with two nodes, for the second variable (yen long rate factor) with four nodes, and so on up to thirty-two nodes for the fifth variable (the exchange rate). Also, the joint probabilities are chosen so as to approximate the means, variances and correlations of the variables. The methodology is tested and shown to converge, as N increases, to the given parameter value. For each scenario, we now model the term structure for the two currencies using equation (3). The resulting rates are shown in Table 7.

Table 7 here

In Table 7, we show only the rates from six months to five years for each currency. (Note that all sixteen bucket rates are actually computed for each currency, although only some of them are relevant for this example.) In the cross-sectional model for the term structure using equation (3), the computed rates are simple weighted- averages of the two factor rates, where the weights reflect the distance of the bucket maturity from the two factor rates.

We now use the term structure scenarios to value the two sets of bucketed cash flows, using equations (4) and (5). The results are shown in the following Table 8.

Table 8 here

Columns six and eight of Table 8 show the cumulative probability distribution of the swap value in Yen. This is computed by valuing the Yen and DM cash flows, in each scenario, and then converting the DM value into Yen at the exchange rate prevailing in the scenario. The low values of the swap (from 133-155 million Yen) correspond to low Yen:DM exchange rates. The higher values are for the higher exchange rate. This shows that the major risk, in this case, is the exchange rate volatility. The table can also be used to compute the swap value at a variety of confidence levels. For example, there is at least a 99% chance of the swap being of greater value than 134,678,753 Yen.

Table 9 here

In Table 9, we show the effect of increasing the number of binomial stages. The confidence level values are quite sensitive to the binomial-density chosen. In Figure 3, we show the cumulative density in the case where J = 4. In the case of this currency swap, it is clear from Table 8 that the swap value is most sensitive to the exchange rate variable. An increase in the binomial density, J, has the effect of including a wider range of possible exchange rates. This effect can be seen in Figure 3, where the range has been considerably extended compared with the range for J = 1 shown in Table 9. This also has a radical effect on the percentile VAR estimate.

Figure 3 here

Stress Testing of the Swap Value-at-Risk model

Stress testing normally refers to the testing of model assumptions and parameters. How sensitive are the results to the assumption that the interest rate factors and the exchange rate are joint-lognormally distributed, for example? There is considerable empirical evidence that exchange rates, for example, are fat tailed compared to the lognormal distribution. Also, a related fact is that the volatilities of both interest rates and exchange rates are stochastic and correlated to the rates themselves. In table 9 we show the effect on the VAR estimates of re-running the simulation, for the case of J=2, assuming all volatilities are increased by 50%. Suppose that we expect volatility to be high at the same time that an observation in the 5% tail of the distribution occurs. The appropriate VAR at the 95% confidence level is that shown for the high volatility distribution.

Conclusions

This paper has illustrated how the HSS methodology, for approximating joint distributions of variables with binomial distributions, can be applied in the case of a currency swap. The swap has first to be reduced to a set of non-stochastic cash flows, in the two currencies. The inputs and outputs of the system are summarised in the diagram in Figure 4 below. The main inputs are the current term structures of the two currencies and the exchange rate, their respective volatilities, and the non-stochastic swap cash flows.

Figure 4 here

The procedure is then as follows :

1. Compute forward rates using the term-structure inputs, and exchange rates.
2. Generate scenarios for the variables and probabilities using the HSS method. Use a two-factor model to generate the term structure of zero-bond yields for each scenario.
3. Given the future valuation date, bucket the swap cash flows into maturity buckets. Do this for each currency.
4. Value the bucketed cash flows in each currency using the relevant term structure.
5. Convert the values into a common currency value and form the cumulative density of the swap value

The approach suggested for the risk evaluation of the currency swap, or book of currency swaps has the following advantages :

a) It provides a computationally fast and efficient scenario analysis, allowing a state-by-state valuation of the swap. Particular states can be identified where the swap is particularly exposed to market movements.
b) Since the time to the value date can be varied, the method allows the analyst to build a risk-time profile of the swap position. This may be useful for analysing the credit risk of the swap book. The use of the methodology in the context of credit risk analysis is pursued in Stapleton and Subrahmanyam (1997)
c) Relatively few parameters need to be estimated in order to implement the model. Also the estimation and use of statistically significant and stable parameters is possible.

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Appendix

Methodology used for the multivariate-binomial approximation

We present below the procedure for estimating the multivariate-binomial distribution, based on the methodology proposed in HSS 1995).

a) The first step is to estimate the conditional volatility of the variables, given the means, volatilities, and covariances. Consider the system of regressions, for the five variables, *v,x,w,y* and *z*.

$$v = a_{0,v} + e_v$$
$$w = a_{0,w} + a_{1,w}v + e_w$$
$$x = a_{0,x} + a_{1,x}v + a_{2,x}w + e_x$$
$$y = a_{0,y} + a_{1,y}v + a_{2,y}w + a_{3,y}x + e_y$$
$$z = a_{0,z} + a_{1,z}v + a_{2,z}w + a_{3,z}x + a_{4,z}y + e_z$$

where $k = \ln(K / K_0)$ for $k = v, w, x, y, z$ and where K_0 is the current value of variable K, and where $a_{i,k}$ are the multiple regression coefficients, and e_k.

Let $\hat{s}_k$ be the given volatility of variable k.[We assume for ease of notation that the time period over which the volatility is measured is one year.] Then, let s_k be the volatility of e_k, i.e. the conditional volatility of variable k.

b) The next step is to compute the nodal values of the variables. Let J_k be the number of binomial steps used in the case of variable k. The J_k +1 element vectors of the variables are computed using up- and down-binomial movements, where, for each variable, the down-movement d_k and the up-movement u_k are given by

$$d_k = 2m_k^{(1/J_k)} / [1 + \exp(2s_k \sqrt{1/J_k})],$$
$$u_k = 2m_k^{(1/J_k)} - d_k.$$

where $m_k = E(K_t / K_0)$ is the mean drift of variable k .The value of variable k at node r at time t is then $K_{t,r} = E(K_t) m_k^{(J_k - r)} d_k^{\,r}, r = 0,1,\ldots, J_k$

c) The third step is the computation of the HSS conditional probabilities, which capture the covariances between the variables. Let

$$q(w|v = v_r) =_k \frac{(a_{0,w} + a_{1,w}v_r - J_w \ln d_w)}{J_w(\ln u_w - \ln d_w)},$$

$$q(x|v = v_r, w = w_s) = \frac{(a_{0,x} + a_{1,x}v_r + a_{2,x}w_s - J_x \ln d_x)}{J_x(\ln u_x - \ln d_x)},$$

and, in general,

$$q(k|v = v_r, w = w_s, \ldots) = \frac{(a_{0,k} + a_{1,k}v_r + a_{2,k}w_s + \ldots - J_k \ln d_k)}{J_k(\ln u_k - \ln d_k)}.$$

Figure 1 : Currency Swap Cash Flows

0	t	t_1	t_2	...	t_n
today	value date	first payment	second payment		n th payment
		$+\,X_{k,t_1}$	$+\,X_{k,t_2}$		$+\,X_{k,t_n}$
		$\downarrow$	$\downarrow$		$\downarrow$
0	t	t_1	t_2	...	t_n
		$\uparrow$	$\uparrow$		$\uparrow$
		$+\,X_{j,t_1}$	$+\,X_{j,t_2}$		$+\,X_{j,t_n}$

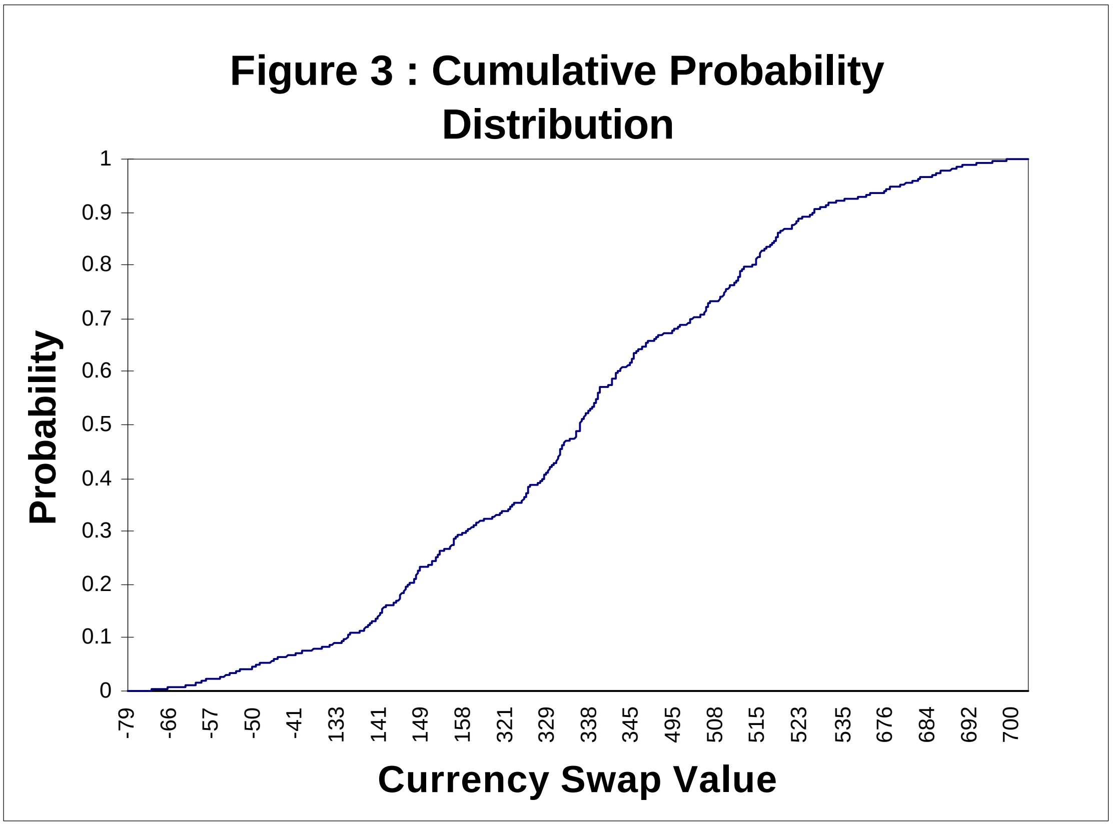
Notes to Figure 1:

1. For a floating / fixed currency swap, the floating cash flows can be valued, at par, on the 1st payment date.
2. A currency swap normally involves an exchange of principal amounts at time t_n

Figure 2 : Currency Swap : Bucketed Cash Flows

$$
\begin{array}{ccccccccc}
 & & & & +X_{k,t_{k,1}} & & +X_{k,t_{k,2}} & & \\
 & & & & \downarrow & & \downarrow & & \\
0 & \text{———} & t & \text{———} & t+\tau_{k,1} & \text{———} & t+\tau_{k,2} & \text{———} & \ldots
\end{array}
$$

$$
\begin{array}{ccccccccc}
0 & \text{———} & t & \text{———} & t+\tau_{j,1} & \text{———} & t+\tau_{j,2} & \text{———} & \ldots \\
 & & & & \uparrow & & \uparrow & & \\
 & & & & -X_{j,t_{j,1}} & & -X_{j,t_{j,2}} & &
\end{array}
$$

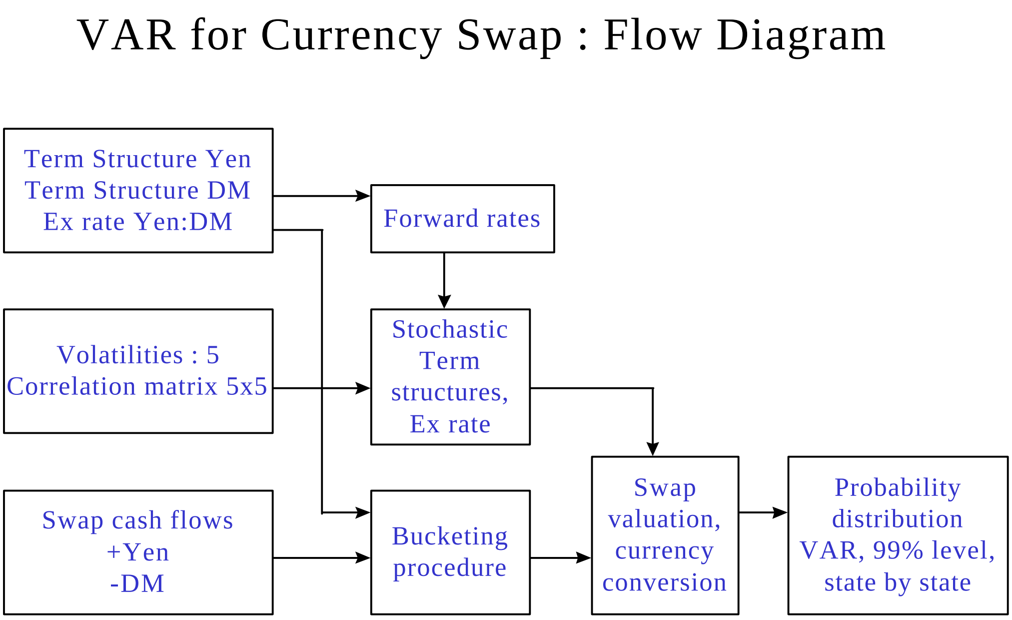


Notes to Figure 3

1. In this case the number of binomial stages is J=4 for each of the five variables. Hence there are 3125 possible states in this case.

2. The 98% tile in this case is -60 million Yen, as shown in Table 9

Figure 4



Notes to Figure 4

1. The flow chart shows the inputs, computations and the output of the currency swap risk model.

Table 1 : Inputs for HSS Simulation

Variable	Mean	Volatility	Correlation Matrix				
			1	2	3	4	5
1: r_{j,t,n_1}	μ_1	σ_1		$\rho_{1,2}$	$\rho_{1,3}$	$\rho_{1,4}$	$\rho_{1,5}$
2: r_{j,t,n_2}	μ_2	σ_2			$\rho_{2,3}$	$\rho_{2,4}$	$\rho_{2,5}$
3: r_{k,t,n_1}	μ_3	σ_3				$\rho_{3,4}$	$\rho_{3,5}$
4: r_{k,t,n_2}	μ_4	σ_4					$\rho_{4,5}$
5: $P_{j,k,t}$	μ_5	σ_5					

Table 2 : Swap Cash Flows

payment dates (Yen cash flows)	**Yen**	**payment dates (DM cash flows)**	**DM**
11/09/95	98,583	11/09/95	51,453
11/03/96	98,583		
11/09/96	99,667		
11/03/97	98,042		
11/09/97	99,667		
11/03/98	3,348,042		

Notes to Table 2

1. Columns 1 and 3 show the exact dates on which payments will be made. These are at six-monthly intervals.
2. Column 2 shows the amounts in 000's, adjusted for the exact day-count convention, that will be paid in Yen.
3. Column 4 shows the amounts in 000's, adjusted for the exact day-count convention, that will be received in Deutsche Marks. These floating payments have been valued on the first reset date of the swap.

Table 3 : Zero-Coupon Bond Yields

	Term Structures on the analysis date	
maturity	**Yen yield curve**	**DM yield curve**
O/N	1.35%	2.15%
1W	1.40%	2.15%
1M	1.40%	2.17%
2M	1.41%	2.28%
3M	1.41%	2.30%
6M	1.46%	2.30%
1Y	1.50%	2.50%
2Y	1.88%	3.04%
3Y	2.34%	3.59%
4Y	2.68%	4.08%
5Y	2.99%	4.33%
7Y	3.47%	4.53%
10Y	3.68%	4.73%
15Y	3.65%	4.77%
20Y	3.64%	4.81%
30Y	3.62%	4.85%

Notes to Table 3

1. Columns 1 shows the maturity of the rate in weeks (W), months (M) and years (Y). The first rate maturity is the overnight (O/N) rate.
2. Columns 2 and 3 show the zero-coupon bond interest rates for Yen and DM on 11, May, 1995.

Table 4 : Bucketed Cash Flows

fixed cash flows-Yen bucket maturity	gridded cash flows	floating cash flows- DM bucket maturity	gridded cash flows
O/N	0	O/N	0
1W	0	1W	0
1M	98,583	1M	51,453
2M	0	2M	0
3M	0	3M	0
6M	82,310	6M	0
1Y	147,797	1Y	0
2Y	1,533,861	2Y	0
3Y	1,980,494	3Y	0
4Y	0	4Y	0
5Y	0	5Y	0
7Y	0	6Y	0
10Y	0	7Y	0
15Y	0	8Y	0
20Y	0	9Y	0
30Y	0	10Y	0

Notes to Table 4

1. Columns 1 and 3 show the maturity of the buckets in weeks (W), months (M) and years (Y). The first rate maturity is the overnight (O/N) rate.
2. Columns 2 and 4 show the Yen and DM cash flows respectively in 000's.

Table 5 : Input data for Scenario Analysis

HSS model: Input Data					
	Short (yen)	**Long (yen)**	**Short (DM)**	**Long (DM)**	**Exrate (Y:DM)**
Means	1.51%	3.73%	2.30%	4.79%	64.86
Volatilities	0.14	0.10	0.16	0.12	0.12
Correlation matrix					
Short (yen)	1	0.4	0.3	0	-0.2
Long (yen)	0.4	1	0.1	0.2	-0.1
Short (DM)	0.3	0.1	1	0	0.2
Long (DM)	0	0.2	0	1	0.1
Exrate (Y:DM)	-0.2	-0.1	0.2	0.1	1

Notes to Table 5.

1. The short and long rates are the 3 month and 10 year zero coupon bond rates in each case.
2. The means are computed as the forward rates for 90 days forward. The mean exchange rate is the forward exchange rate, assuming a current exchange rate of 65 Yen per DM.

Table 6 : Scenarios of Interest Rates and the Exchange Rate:
Output of the HSS simulation

Short(yen)	Long(yen)	Short(DM)	Long(DM)	Exrate(Y:DM)	joint prob
1.62	3.90	2.48	5.22	68.51	0.07
1.62	3.90	2.48	5.22	61.21	0.06
1.62	3.90	2.48	4.64	68.51	0.04
1.62	3.90	2.48	4.64	61.21	0.06
1.62	3.90	2.12	5.22	68.51	0.02
1.62	3.90	2.12	5.22	61.21	0.05
1.62	3.90	2.12	4.64	68.51	0.01
1.62	3.90	2.12	4.64	61.21	0.05
1.62	3.56	2.48	5.22	68.51	0.02
1.62	3.56	2.48	5.22	61.21	0.01
1.62	3.56	2.48	4.64	68.51	0.03
1.62	3.56	2.48	4.64	61.21	0.03
1.62	3.56	2.12	5.22	68.51	0.00
1.62	3.56	2.12	5.22	61.21	0.01
1.62	3.56	2.12	4.64	68.51	0.01
1.62	3.56	2.12	4.64	61.21	0.03
1.40	3.90	2.48	5.22	68.51	0.02
1.40	3.90	2.48	5.22	61.21	0.01
1.40	3.90	2.48	4.64	68.51	0.01
1.40	3.90	2.48	4.64	61.21	0.00
1.40	3.90	2.12	5.22	68.51	0.03
1.40	3.90	2.12	5.22	61.21	0.03
1.40	3.90	2.12	4.64	68.51	0.01
1.40	3.90	2.12	4.64	61.21	0.02
1.40	3.56	2.48	5.22	68.51	0.05
1.40	3.56	2.48	5.22	61.21	0.01
1.40	3.56	2.48	4.64	68.51	0.05
1.40	3.56	2.48	4.64	61.21	0.02
1.40	3.56	2.12	5.22	68.51	0.06
1.40	3.56	2.12	5.22	61.21	0.04
1.40	3.56	2.12	4.64	68.51	0.06
1.40	3.56	2.12	4.64	61.21	0.07

Notes to Table 6

1. Columns 1 and 3 show the short Yen and DM rates, in each case the rate maturity is three months.
2. Columns 2 and 4 show the long Yen and DM rates, in each case the rate maturity is ten years.
3. Column 5 shows the Yen:DM exchange rate.
4. Column 6 shows the joint probability of the rates occurring.

Table 7 : Yen and DM Term Structures

		Yen	rates					DM	rates		
0.5 yr	**1yr**	**2 yr**	**3 yr**	**4 yr**	**5 yr**	**0.5 yr**	**1yr**	**2 yr**	**3 yr**	**4 yr**	**5 yr**
1.67	1.79	2.03	2.26	2.50	2.73	2.55	2.69	2.97	3.25	3.53	3.81
1.67	1.79	2.03	2.26	2.50	2.73	2.55	2.69	2.97	3.25	3.53	3.81
1.67	1.79	2.03	2.26	2.50	2.73	2.53	2.64	2.86	3.09	3.31	3.53
1.67	1.79	2.03	2.26	2.50	2.73	2.53	2.64	2.86	3.09	3.31	3.53
1.67	1.79	2.03	2.26	2.50	2.73	2.20	2.36	2.68	3.00	3.31	3.63
1.67	1.79	2.03	2.26	2.50	2.73	2.20	2.36	2.68	3.00	3.31	3.63
1.67	1.79	2.03	2.26	2.50	2.73	2.19	2.32	2.58	2.83	3.09	3.35
1.67	1.79	2.03	2.26	2.50	2.73	2.19	2.32	2.58	2.83	3.09	3.35
1.67	1.77	1.96	2.16	2.36	2.56	2.55	2.69	2.97	3.25	3.53	3.81
1.67	1.77	1.96	2.16	2.36	2.56	2.55	2.69	2.97	3.25	3.53	3.81
1.67	1.77	1.96	2.16	2.36	2.56	2.53	2.64	2.86	3.09	3.31	3.53
1.67	1.77	1.96	2.16	2.36	2.56	2.53	2.64	2.86	3.09	3.31	3.53
1.67	1.77	1.96	2.16	2.36	2.56	2.20	2.36	2.68	3.00	3.31	3.63
1.67	1.77	1.96	2.16	2.36	2.56	2.20	2.36	2.68	3.00	3.31	3.63
1.67	1.77	1.96	2.16	2.36	2.56	2.19	2.32	2.58	2.83	3.09	3.35
1.67	1.77	1.96	2.16	2.36	2.56	2.19	2.32	2.58	2.83	3.09	3.35
1.47	1.60	1.85	2.11	2.36	2.62	2.55	2.69	2.97	3.25	3.53	3.81
1.47	1.60	1.85	2.11	2.36	2.62	2.55	2.69	2.97	3.25	3.53	3.81
1.47	1.60	1.85	2.11	2.36	2.62	2.53	2.64	2.86	3.09	3.31	3.53
1.47	1.60	1.85	2.11	2.36	2.62	2.53	2.64	2.86	3.09	3.31	3.53
1.47	1.60	1.85	2.11	2.36	2.62	2.20	2.36	2.68	3.00	3.31	3.63
1.47	1.60	1.85	2.11	2.36	2.62	2.20	2.36	2.68	3.00	3.31	3.63
1.47	1.60	1.85	2.11	2.36	2.62	2.19	2.32	2.58	2.83	3.09	3.35
1.47	1.60	1.85	2.11	2.36	2.62	2.19	2.32	2.58	2.83	3.09	3.35
1.46	1.57	1.79	2.01	2.23	2.45	2.55	2.69	2.97	3.25	3.53	3.81
1.46	1.57	1.79	2.01	2.23	2.45	2.55	2.69	2.97	3.25	3.53	3.81
1.46	1.57	1.79	2.01	2.23	2.45	2.53	2.64	2.86	3.09	3.31	3.53
1.46	1.57	1.79	2.01	2.23	2.45	2.53	2.64	2.86	3.09	3.31	3.53
1.46	1.57	1.79	2.01	2.23	2.45	2.20	2.36	2.68	3.00	3.31	3.63
1.46	1.57	1.79	2.01	2.23	2.45	2.20	2.36	2.68	3.00	3.31	3.63
1.46	1.57	1.79	2.01	2.23	2.45	2.19	2.32	2.58	2.83	3.09	3.35
1.46	1.57	1.79	2.01	2.23	2.45	2.19	2.32	2.58	2.83	3.09	3.35

Notes to Table 7

1. Columns 1-6 show the Yen rates, only rates for maturities of a half year to five years are shown.
2. Columns 7-12 show the DM rates, again only rates for maturities of a half year to five years are shown.

Table 8 : Present Value Distribution of the Swap

PV Yen	PV DM	DM in Yen	swap value	joint prob	swap value	joint prob	cum prob
3,650,752	51,352	3,518,119	132,738	0.07	131,726	0.02	0.02
3,650,752	51,352	3,143,274	507,582	0.06	131,754	0.01	0.02
3,650,752	51,352	3,518,091	132,766	0.04	132,738	0.07	0.09
3,650,752	51,352	3,143,249	507,607	0.06	132,766	0.04	0.14
3,650,752	51,367	3,519,131	131,726	0.02	138,813	0.00	0.14
3,650,752	51,367	3,144,178	506,678	0.05	138,841	0.01	0.15
3,650,752	51,366	3,519,103	131,754	0.01	139,825	0.02	0.17
3,650,752	51,366	3,144,153	506,703	0.05	139,853	0.03	0.20
3,657,780	51,352	3,518,119	139,825	0.02	145,432	0.03	0.23
3,657,780	51,352	3,143,274	514,669	0.01	145,460	0.01	0.24
3,657,780	51,352	3,518,091	139,853	0.03	146,444	0.02	0.27
3,657,780	51,352	3,143,249	514,695	0.03	146,472	0.01	0.28
3,657,780	51,367	3,519,131	138,813	0.00	152,560	0.06	0.34
3,657,780	51,367	3,144,178	513,765	0.01	152,589	0.06	0.40
3,657,780	51,366	3,519,103	138,841	0.01	153,572	0.05	0.45
3,657,780	51,366	3,144,153	513,791	0.03	153,600	0.05	0.50
3,664,308	51,352	3,518,119	146,444	0.02	506,678	0.05	0.55
3,664,308	51,352	3,143,274	521,289	0.01	506,703	0.05	0.60
3,664,308	51,352	3,518,091	146,472	0.01	507,582	0.06	0.66
3,664,308	51,352	3,143,249	521,314	0.00	507,607	0.06	0.72
3,664,308	51,367	3,519,131	145,432	0.03	513,765	0.01	0.73
3,664,308	51,367	3,144,178	520,384	0.03	513,791	0.03	0.76
3,664,308	51,366	3,519,103	145,460	0.01	514,669	0.01	0.77
3,664,308	51,366	3,144,153	520,410	0.02	514,695	0.03	0.80
3,671,376	51,352	3,518,119	153,572	0.05	520,384	0.03	0.83
3,671,376	51,352	3,143,274	528,417	0.01	520,410	0.02	0.85
3,671,376	51,352	3,518,091	153,600	0.05	521,289	0.01	0.86
3,671,376	51,352	3,143,249	528,442	0.02	521,314	0.00	0.86
3,671,376	51,367	3,519,131	152,560	0.06	527,513	0.04	0.90
3,671,376	51,367	3,144,178	527,513	0.04	527,538	0.07	0.98
3,671,376	51,366	3,519,103	152,589	0.06	528,417	0.01	0.98
3,671,376	51,366	3,144,153	527,538	0.07	528,442	0.02	1.00

Notes to Table 8.

1. Column 1 shows the Present value of the Yen fixed cash flows in each scenario.
2. Column 2 shows the present value of the DM floating cash flows in DM.
3. Column 3 shows the Yen value of the DM flows.
4. Column 4 shows the value of the swap in each scenario.
5. Column 5 shows the joint probability of the events, i.e. the probability of the scenario.
6. Column 6 shows the ranking of the swap value by size, and column 7 shows the associated probability.
7. Column 8 shows the cumulative probability of the swap values.

Table 9 Sensitivity Analysis of Value-at -Risk:
Binomial Density and Volatility

J-size	Nodes	98%	95%	90%	75%
1	32	132	132	133	146
2	243	45	55	60	74
3	1024	-12	-6	2	220
4	3125	-60	-49	135	152
2	243	-101*	63*	77*	242*

Notes

1. All values in Yen million.
2. Values marked* are for the case where all volatilities are + 50%

Footnotes

[1] For example, the Working Group of the Euro-currency Standing Committee of the central banks of the Group of Ten countries provides the following definition: “Value-at-Risk is a concept derived from statistical estimates of the losses or gains a portfolio could experience, due to changes in underlying prices, over a given holding period, for given confidence intervals.” (BIS Discussion Paper, September 1995)

[2] See BIS Discussion Paper, September 1995.

[3] The bucketing procedure is designed to maintain the present value of the cash flows and the sensitivity of theirvalue to changes in interest rates.

[4] The effect of relaxing the assumption of lognormality is analysed in the stress testing that is described in section 6 below.

[5] An alternative, popular methodology is Monte Carlo simulation. However this is computationally expensive.

[6] More accurately, the number of nodes is $(J_1 + 1)(J_2+1)\ldots(J_N+1)$ where J_i is the binomial density chosen for variable i. An alternative approach is Monte Carlo simulation, which tends however to be computationally expensive. Also a full covariance matrix approach is possible. However this does not typically yield information on a state-by-state basis.

[7] If these are not directly available they can be computed from the spot rates for the two currencies.

[8] As it is for example in the JP Morgan Riskmetrics system.

[9] It is shown in HSS (1995) that the means, variances and covariances converges in the limit to the true values as the binomial density increases.

[10] This model is a simplification of the no-arbitrage model of the term structure developed in HSS (1996) where the curvature of the term structure results partially from mean-reversion in the short rate factor. In the model used in (3), simplicity is preferred to the more sophisticated mean reversing model in HSS (1996), since the issue here is not valuation, as such, but risk management.