Life-Cycle Housing and Portfolio Choice with Bond Markets*

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Abstract

I study optimal life-cycle housing and portfolio choice under stochastic inflation and real interest rates. Investors can choose between an adjustable-rate mortgage (ARM) and a fixed-rate mortgage (FRM) to finance the purchase of a house. Together the mortgage choice and bond portfolio choice determine an investor’s strategy for the management of interest rate risk. Consistent with empirical evidence, I find that (i) many investors hold both long and short positions in fixed-income securities, and (ii) young investors more often take an ARM. Suboptimal mortgage choice leads to a large utility loss.

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1 Introduction

For most households the house is their largest asset.\footnote{Based on the 2001 Survey of Consumer Finances, Campbell (2006) shows that real estate is the largest asset class in the household portfolio from the 30th to the 95th percentile of the distribution of total assets.} Choosing whether to finance the house with an adjustable-rate mortgage (ARM) or a fixed-rate mortgage (FRM) is one of the most important financial decisions a household faces over the course of life.\footnote{Based on the 2004 Survey of Consumer Finances, 47.9\% of households holds a mortgage, and the median value for households holding a mortgage is $95,000. In comparison, only 20.7\% of households directly hold stocks, and the median value for stockholders is $15,000.} In this paper I study a dynamic life-cycle model with mortgage choice as an integral part of the overall household asset allocation problem. This allows for a close examination of a household’s interest rate risk management, as determined jointly by the mortgage choice and bond portfolio choice.

The model rationalizes two empirically-observed patterns in households’ allocation to fixed-income securities. First, many investors hold both a long and short position in fixed income securities. The long position typically comprises of bonds, held either directly, or indirectly in the pension account.\footnote{As of June 30th, 2000, TIAA-CREF, the largest private pension provider in the U.S., held 38.4\% of its assets in the TIAA traditional annuity investment account, which consists mainly of fixed-income investments (Ameriks and Zeldes (2004)).} The short position is the mortgage loan. I show that this can be understood in the light of a risk-averse investor who cares about real consumption and seeks to hedge changes in the real interest rate. To attain this hedge, the investor typically has to rely on nominally-denominated fixed-income securities, potentially leaving her exposed to inflation risk. I show that by holding short-term nominal bonds and taking a nominal FRM on the house, an investor can hedge against real interest changes without incurring exposure to inflation risk. That is, the inflation risk exposures of the long and short position cancel out. A second empirical pattern in the US is that young investors are more likely to take an ARM.\footnote{In the 18-24 age category, 32\% prefers an ARM, while in the 45-64 age category only 19\% prefer an ARM; see the opinion survey commissioned by the Consumer Federation of America (2004).} I show that for young investors the presence of large human capital creates a desire for leveraged risk taking in their financial wealth and therefore reaping the associated risk premium. With an ARM the investor attains this leverage without paying the bond risk premium associated with an FRM. In contrast, older households, who have accumulated substantial wealth, are more concerned about hedging against changes in the real interest rate, creating a demand for short-term bonds in combination with an FRM.

I study a dynamic life-cycle model where investors optimize over the housing and portfolio choice. To this end I combine the main model features of two recent strands in the portfolio choice literature. First, I follow Cocco (2005) and Yao and Zhang (2005a,b) by ex-
plicitly modeling the housing decision and incorporating a stochastic labor income stream.\textsuperscript{5} Households derive utility from both housing and other goods consumption. They acquire housing services by either renting or owning the house they live in. Investors can change their housing tenure and size only at a transaction, resulting in infrequent, endogenously-generated house moves. Both the house and human capital have a major impact on the investor’s willingness and ability to take risk in their financial portfolio. Cocco (2005) and Yao and Zhang (2005a,b) assume constant interest rates, do not consider bonds, and do not allow for a choice between different mortgage types. Consequently these papers do not address a household’s interest rate risk management.

Second, this paper follows Campbell and Viceira (2001) and Brennan and Xia (2002) by incorporating bonds in the financial portfolio. Nominal bonds are priced by a two-factor model for the term structure of interest rates with expected inflation and real interest rate as factors.\textsuperscript{6} Unlike Campbell and Viceira (2001) and Brennan and Xia (2002), I model labor income, housing, and mortgages, and am therefore able to study the life-cycle pattern in households’ interest risk management and the optimal mortgage choice. Renters choose how to allocate financial wealth to stocks, 3-year bonds, 10-year bonds, and cash. Negative positions are precluded. Homeowners also choose the mortgage type and size. A homeowner may take out a mortgage loan up to the market value of the house minus a down payment. I allow for an adjustable-rate mortgage (ARM), a fixed-rate mortgage (FRM), and a combination of the two (hybrid mortgage). A homeowner can adjust her mortgage type and size at zero cost. The ARM is modeled as a negative cash position, and the FRM as a negative position in the 10-year bond.\textsuperscript{7}

The parameter values for the asset price dynamics are calibrated to US data and partially based on estimates by Van Hemert, De Jong, and Driessen (2005). In accordance with Brennan and Xia (2002) and Campbell and Viceira (2001), the mean reversion in the real interest rate is found to be faster than the mean reversion in the expected inflation rate. I show that this implies that a portfolio consisting of a positive position in a short-term bond and a negative position in a long-term bond can be constructed with the property

\textsuperscript{5} Hu (2005) investigates housing and portfolio choice in a five-period model. Cauley, Pavlov, and Schwartz (2005) assume a fixed housing position, and study a model where homeowners can sell a fractional interest in their house. Brueckner (1997) and Flavin and Yamashita (2002) focus on the housing and financial portfolio choice in a static, one-period, mean-variance setting.

\textsuperscript{6} Sangvinatsos and Wachter (2005) and Koijen, Nijman, and Werker (2006) also allow for time variation in risk premia.

\textsuperscript{7} I abstract from the prepayment option that is associated with FRMs in some countries, most notably the US. The prepayment behavior of US households is often sluggish compared to what option theory would imply, see e.g. Schwartz and Torous (1989).
that it has a negative exposure to real interest rate shocks and a zero exposure to expected inflation rate shocks. An investor who desires to hedge real interest rate shocks can create this hedge portfolio by holding short-term bonds and taking an FRM, even when negative positions in bonds are precluded.

The main results of the paper can be summarized as follows. The motivation to hold risky assets varies over an investor’s lifetime, giving rise to a clear life-cycle pattern in her optimal house, stock, bond, and mortgage choice. An investor starts adult life with little financial wealth and large human capital, making her severely borrowing constrained. The investor starts out renting the house she lives in. Over time more labor income is earned and the investor starts to save for the down payment on an owner-occupied house. In this period she becomes less borrowing constrained, but is still very short-sale constrained. Taking into account her large human capital, the investor chooses an almost 100% stock allocation in order to exploit the equity premium, which is set at 4%.

Per-period housing costs for a given house size are smaller when owning than when renting. This makes the investor so eager to buy her first house that the move from a rental to an owner-occupied house often involves moving to a smaller house, for which she is just able to pay the required down payment. The young homeowner optimally chooses an ARM to save on the bond risk premium paid on an FRM. She chooses the maximum allowed loan size to leverage up the riskiness of the financial portfolio and exploit the associated risk premium.

As a homeowner builds up more financial wealth, she typically decides to buy a bigger house. With the larger physical (financial plus housing) capital and smaller human capital, the desire to take risk and exploit risk premia decreases, while the desire to hedge against falling real interest rates becomes more important. Initially a homeowner chooses a long-term bond for this hedge. Long-term bonds also have substantial exposure to expected inflation risk and investors capture the associated risk premium. When approaching retirement age the allocation shifts towards short-term bonds which have smaller exposure to expected inflation shocks. A more risk-tolerant homeowner still holds a considerable amount of long-term bonds and stocks at retirement. A more risk-averse homeowner is mostly concerned with hedging real interest rate risk and completely shifts to short-term bonds. Moreover, she changes her mortgage from a pure ARM to a hybrid mortgage, a mix between an FRM and an ARM, modeled as a short position in both cash and a long-term bond. She chooses the size of the FRM part of the hybrid mortgage such that the inflation risk exposure cancels out against the inflation risk exposure on the short-term bond.
Towards the end of her lifetime, the investor sells her house and starts renting again. This enables her to consume all her wealth, including the down payment on the previously owned house. In anticipation of this sell, the investor adjusts her financial portfolio to hedge against house price falls. As an extension I consider an investor with a strong bequest motive. This investor does not sell her house at the end of life.

As another extension I consider an investor who is restricted to finance her house with an FRM, the predominant mortgage type in the US. This means that the investor pays the bond risk premium associated with the FRM. I find that this restriction has large associated utility losses.8

Few other papers study optimal mortgage choice. Campbell and Cocco (2003) study the choice between an FRM and an ARM as a trade off between what they refer to as wealth and income risk. My mortgage analysis differs from Campbell and Cocco (2003) in several important ways. I study mortgage choice as integral part of the overall household asset allocation problem. Unlike Campbell and Cocco (2003), I model the stock, bond, and housing allocation. Moreover, Campbell and Cocco (2003) incorporate persistent shocks to the expected inflation only, while I allow for persistent shocks in the real interest rate as well. As I show, hedging real interest rate risk has important implications for the optimal mortgage type. Moreover, I study the optimal mortgage size, taking into account the funds needed for the optimal positions in stocks and bonds. Van Hemert, De Jong, and Driessen (2005) study a homeowner’s optimal portfolio choice assuming (i) utility of terminal wealth, (ii) no labor income, and (iii) fixed housing investment. This paper provides a richer set up by examining a life-cycle setting with stochastic labor income, which allows me to uncover the life-cycle pattern in the optimal portfolio choice, including the mortgage choice. In contrast to both Campbell and Cocco (2003) and Van Hemert, De Jong, and Driessen (2005), I model the housing tenure and house size choice and therefore endogenize the housing wealth available to use as collateral for mortgage loan.

The structure of this paper is as follows. Section 2 presents the model. Section 3 discusses the estimation of the model parameters. Section 4 contains the main results, and section 5 provides additional analyses. Section 6 concludes.

8This is consistent with a remark from Alan Greenspan, former chairman of the Federal Reserve, that an FRM might be an expensive method of financing a home; see the speech to the Credit Union National Association, Greenspan (2004).
2 The Model

I study optimal financial planning for an investor from time 0 to time $T = 60$ years, corresponding to age 20 to 80. The investor dynamically chooses: (i) housing tenure, (ii) house size, (iii) financial portfolio, and (iv) consumption. I interpret the house size as a one-dimensional representation of the overall quality of the house. When the investor decides to move, i.e. change house size or housing tenure, transaction costs are incurred.

2.1 Preferences

The investor derives utility from housing services and other goods consumption, $c$. The real price of consumption goods is chosen to be the numeraire and the real price of a unit of housing is denoted $q$. I denote the house size at time $t$ by $H_t$. In accordance with Cocco (2005), and Yao and Zhang (2005a), preferences over housing and other goods consumption are represented by the Cobb-Douglas function. Lifetime utility, $U_t$, satisfies

$$U_t = \int_t^T \beta^{s-t} u(c_s, H_s) \, ds,$$

$$u(c, H) = \frac{(c^{1-\psi} H^\psi)^{1-\gamma}}{1-\gamma},$$

where time $T$ is the time of death, which is assumed to be known in advance, $u$ is the Cobb-Douglas utility function, $\beta$ is the subjective discount rate, $\gamma$ is the coefficient of relative risk aversion, and $\psi$ is the relative preference for housing consumption. In the baseline case, I abstract from a bequest motive. Section 5 revisits the bequest motive.

The \textit{intra}temporal elasticity of substitution between housing and other goods consumption equals 1, which is a well-known property of the Cobb-Douglas function. However, the \textit{inter}temporal elasticity of substitution is governed by $1/\gamma$. In this paper, I focus on investors with a stronger desire to smooth consumption than the log investor ($1/\gamma < 1$), or, in terms of willingness to take risk, investors more risk averse than the log investor ($\gamma > 1$).

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9 In contrast, Lustig and Van Nieuwerburgh (2005), and Piazessi, Schneider, and Tuzel (2006) use the more general constant elasticity of substitution (CES) utility function in their studies on the role of housing in asset pricing. Piazessi, Schneider, and Tuzel (2006) estimate a value for the intratemporal elasticity parameter only slightly above 1; the value that corresponds to the special case of Cobb-Douglas preferences. To facilitate comparison with Cocco (2005), and Yao and Zhang (2005a), I use Cobb-Douglas preferences, even though either utility specification would be computationally feasible.

10 The empirical evidence for a strong, intentional bequest motive is mixed. See for example Hurd (1989) and Bernheim (1991) for negative and positive evidence respectively.
We have $1/\gamma < 1 \iff u_{eH} < 0$, where $u_{eH}$ is the cross derivative to housing and other goods consumption. Suppose that frictions in the housing market cause the investor to live in a small house in period 1 and to move to a larger house in period 2. With an intertemporal elasticity of substitution smaller than 1, the investor optimally spends more on other goods consumption while in the small house in period 1, at the cost of spending on other goods in period 2.

2.2 Asset Price Dynamics

I consider an economy with six sources of uncertainty represented by innovations in six Brownian motions. I assume the investor takes the price processes as a given. Furthermore, I assume that the risk premia on the sources of uncertainty are constant. Financial asset and house prices are determined by the dynamics of the first five sources of uncertainty. For these dynamics, I extend Brennan and Xia (2002) with an additional source of uncertainty to capture house price risk. The five variables that determine asset prices are: nominal stock return $S$, instantaneous real interest rate $r$, instantaneous expected inflation rate $\pi$, nominal house price $Q$, and the price level $\Pi$. The equations driving these variables are given by

\[
\frac{dS}{S} = \left[R_f + \sigma_S \lambda_S\right] dt + \sigma_S dz_S, \quad (3)
\]
\[
dr = \kappa_r (\bar{r} - r) dt + \sigma_r dz_r, \quad (4)
\]
\[
d\pi = \kappa_\pi (\bar{\pi} - \pi) dt + \sigma_\pi dz_\pi, \quad (5)
\]
\[
\frac{dQ}{Q} = \left[R_f + \sigma_Q \lambda_Q - r^{imp}\right] dt + \sigma_Q dz_Q, \quad (6)
\]
\[
\frac{d\Pi}{\Pi} = \pi dt + \sigma_\Pi dz_\Pi, \quad (7)
\]

where $R_f$ is the return on cash (the nominal risk free asset), $\lambda_S$ and $\lambda_Q$ are the nominal prices of risk, the $dz$’s are changes in standard Brownian motions $z$, the $\sigma$’s capture the volatility of the processes, the $\kappa$’s are the mean-reversion parameters, and $r^{imp}$ is the imputed rent. The $\sigma_S \lambda_S$ term in equation (3) represents the nominal equity risk premium. The imputed rent term in (6) represents the benefits from the housing services, as measured by the market.\footnote{The imputed rent is the value the market attaches to the net benefits provided by the house. Equation (6) is simply the first-order condition for the house price being equal to the present value of all future imputed rents. Put differently, the infinitesimal expected total return, as set by the market, equals $E \left[ dQ/Q + r^{imp} dt \right] = [R_f + \sigma_Q \lambda_Q] dt$, where the right-hand side captures the familiar compensation for the time value of money, $R_f$, and risk, $\sigma_Q \lambda_Q$.}
changes, driven by $dz_v$, and orthogonal price level changes, driven by $dz_u$. Specifically we define the vector of Brownian motions $z = (z_S, z_r, z_\pi, z_v, z_u)$, such that $dz_v$ is orthogonal to $dz_S, dz_r, \text{and } dz_\pi$, and $dz_u$ is orthogonal to $dz_S, dz_r, dz_\pi$ and $dz_v$. We get

$$dQ/Q = [R_f + \theta'\lambda - r^{imp}] dt + \theta'dz,$$  

$$d\Pi/\Pi = \pi dt + \xi'dz,$$  

with $\theta = (\theta_S, \theta_r, \theta_\pi, \theta_v, 0)'$ and $\xi = (\xi_S, \xi_r, \xi_\pi, \xi_v, \xi_u)'$ the vectors of loadings on the sources of risk for the nominal house price and price level respectively, and with $\lambda = (\lambda_S, \lambda_r, \lambda_\pi, \lambda_v, \lambda_u)'$ the vector of nominal prices of risk associated with the sources of risk. We have $\sigma_Q^2 = \theta'\rho\theta$ and $\sigma_\Pi^2 = \xi'\rho\xi$, where $\rho$ is the covariance matrix of $dz$

$$\rho = \begin{pmatrix} \rho_{S,r,\pi} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (10)$$

Brennan and Xia (2002) show that the nominal price of a discount bond with a $1 nominal payoff and maturity $\tau$, denoted as $P(\tau)$, satisfies

$$dP(\tau)/P(\tau) = [R_f - B_r(\tau)\sigma_r\lambda_r - B_\pi(\tau)\sigma_\pi\lambda_\pi] dt - B_r(\tau)\sigma_r dz_r - B_\pi(\tau)\sigma_\pi dz_\pi. \quad (11)$$

$$B_r(\tau) = \kappa_r^{-1}(1 - e^{-\kappa_r\tau}), \quad (12)$$

$$B_\pi(\tau) = \kappa_\pi^{-1}(1 - e^{-\kappa_\pi\tau}), \quad (13)$$

where $B_r$ and $B_\pi$ are functions of the time to maturity $\tau$. The return processes for bonds with different maturities differ only in their loadings on $dz_r$ and $dz_\pi$. When there are no constraints on position size, any desired combination of loadings on $dz_r$ and $dz_\pi$ can be accomplished by taking positions in any two bonds with different maturities.

The real asset price dynamics can be easily obtained from the nominal asset dynamics, the dynamics for the price level, equation (9), and applying Ito’s lemma. I use uppercase letters for nominal variables and the corresponding small case letters for their real counterpart. The nominal interest rate satisfies $R_f = r + \pi - \xi'\lambda$, and for example, the real return on stocks is given by

$$ds/s = [r + \sigma_S(\lambda_S - \xi_S) - \xi'(\lambda - \rho\xi)] dt + \sigma_s dz_S - \xi'dz. \quad (14)$$
In equation (14) \( \lambda_S - \xi_S \) and \( \lambda - \rho \xi \) are the real prices of risk associated with the Brownian motions \( z_S \) and \( z \) respectively. The real equity premium is given by \( \sigma_S (\lambda_S - \xi_S) - \xi' (\lambda - \rho \xi) \).

2.3 Investment Opportunity Set

I denote real housing wealth by

\[ w^H = qH, \quad (15) \]

where the dynamics of the real house price, \( q \), are determined by the nominal house price dynamics, as given in (8), deflated by price level changes, as given in (9), and where \( H \) is the house size appearing in the lifetime utility equation (1). Real financial wealth is denoted by \( w^F \). The menu of available financial assets consists of stocks, 3-year bonds, 10-year bonds, and cash. The allocation to these four assets is denoted by \( x = (x^s, x^{b3}, x^{b10}, x^c) \). The two bonds are assumed to be zero-coupon bonds. We have

\[ w^F = x^s + x^{b3} + x^{b10} + x^c \quad (16) \]

The infinitesimal real financial return, \( r^F (x) \), for a given asset allocation \( x \) is given by

\[ r^F (x) = [r + (\sigma'_F (x) - \xi') (\lambda - \rho \xi)] dt + [\sigma'_F (x) - \xi'] dz, \quad (17) \]

where \( \sigma_F (x) \) is the vector of risk exposures for the nominal financial return. Using (3) and (11), it is given by

\[ \sigma_F (x) = \left( x^s \sigma_S, \left[-x^{b3} B_r (3) - x^{b10} B_r (10)\right] \sigma_r, \left[-x^{b3} B_\pi (3) - x^{b10} B_\pi (10)\right] \sigma_\pi, 0, 0 \right)' . \quad (18) \]

Notice that the real financial return is independent of the expected inflation rate, \( \pi \). As discussed above, the same holds for the real house price changes, which implies that the real investment opportunity set in my model is independent of the prevailing expected inflation rate.

I assume that renters cannot take short positions in any of the financial assets:

\[ x^s, x^{b3}, x^{b10}, x^c \geq 0 \text{ (for renters)} \quad (19) \]

Equations (16) and (19) imply that a renter cannot borrow against human capital, i.e.
\[ w^F \geq 0. \]

Homeowners can take a mortgage loan up to a fraction \( 1 - \delta \) of the market value of the house, where \( \delta \) is the minimum down payment fraction. They can use the proceeds to consume or to invest in stocks, bonds, and cash. I include the (negative) market value of the mortgage in my definition of financial wealth, which therefore can become negative. Total (financial plus housing) wealth, however, cannot be less than the minimum down payment of \( \delta \) times the value of the house.

A homeowner can choose between an adjustable-rate mortgage (ARM), a fixed-rate mortgage (FRM), and a hybrid mortgage which is a combination of an ARM and an FRM.\(^{12}\) I model an ARM (FRM) as a short position in cash (10-year bond). Doing so, I implicitly make two simplifying assumptions. First, I abstract from the prepayment option that is associated with FRMs in some countries, most notably the US.\(^{13}\) Second, I equate the borrowing and lending rate. Because defaults do not occur in my model, this assumption can be interpreted as implicitly equating a bank’s profit margin to the government subsidy on mortgage debt.\(^{14}\)

Following Cocco (2005), I assume that a homeowner can costlessly adjust the mortgage, as is typically the case for a home line of credit. Since I also allow for hybrid mortgages, the investor basically can take a negative cash and 10-year bond position, each and added up not to exceed \((1 - \delta)\) times the market value of the house. We have

\[
\begin{align*}
x^e, x^{k3} & \geq 0, \\
x^{b10}, x^c, x^{b10} + x^c & \geq -(1 - \delta) w^H. \quad \text{(for homeowners)}
\end{align*}
\]

Equations (16) and (20a)-(20b) imply that a homeowner can borrow up to market value of the house minus the down payment, i.e. \( w^F \geq -(1 - \delta) w^H \). Comparing the investment opportunity sets for renters and owners, owning alleviates the short-sale constraint on the

\(^{12}\) Practitioners use the term hybrid mortgage for a slightly different product: a mortgage with a fixed rate for the first few years and a floating rate thereafter.

\(^{13}\) Actual prepayment behavior by US investors is far from optimal, giving rise to a large literature on mortgage-backed securities pricing (see e.g. Schwartz and Torous (1989). Incorporating a realistic model for prepayment would make the numerical evaluation of the model intractable.

\(^{14}\) In the US, subsidies are not only provided directly through income tax deductions of mortgage interest rate payments, but also indirectly through the Government-Sponsored Enterprises (GSEs), like the Federal National Mortgage Association (FNMA) and the Federal Home Loan Mortgage Corporation (FMAC), see e.g. Frame and Wall (2002). Focusing on the subsidy through the deductibility of mortgage interest payment from taxable income, Amromin, Huang and Sialm (2006) argue that a significant number of US households can perform what they call a tax arbitrage by cutting back on their mortgage payments and investing the proceeds in tax-deferred accounts.
10-year bond and cash.

2.4 Housing Costs

Per-period housing expenses are a fraction $\zeta(I)$ of the market value of the house. It depends on the housing tenure indicator variable $I$, which is defined to be one for an investor who is currently owning and zero for renters. For homeowners, the housing expenses represent a maintenance cost, incurred to keep the house at a constant quality. The housing expenses are out-of-pocket costs, not reflected in the house price appreciation or mortgage interest payments. For renters the housing expenses represent the rental cost. For both homeowners and renters I assume a constant value over time, and denote it by $\zeta^{own}$ and $\zeta^{rent}$ respectively.\footnote{Assuming that the rent is a constant fraction of the market value of the house is for tractability reasons. It allows me to separate out the house price as a state variable, as formalized in equation (28).} We have

$$\zeta(I) = I\zeta^{own} + (1 - I)\zeta^{rent}.$$  

(21)

If the investor moves to another house, she pays (receives) the increase (decrease) in owner-occupied housing wealth. In addition, a one-time transaction cost is incurred. I consider it a move when the investor decides to change housing tenure, house size, or both. I define the new housing tenure indicator variable, $I^{new}$, as being 1 (zero) when the investor moves to an owner-occupied (a rental) house. The new house size is denoted by $H^{new}$ and the total costs are

$$m = I^{new}qH^{new} - IqH + qH^{new}\nu(I^{new})$$

(22)

The first two terms add up to the change in owner-occupied housing wealth. The third term represents the transaction cost. It equals a fraction $\nu^{own}$ ($\nu^{rent}$) of the market value of the new house when the investor moves to an owner-occupied (a rental) house, i.e.

$$\nu(I^{new}) = I^{new}\nu^{own} + (1 - I^{new})\nu^{rent}$$

(23)

Typically we have larger moving costs for the case the investor buys the new house, i.e. $\nu^{own} > \nu^{rent}$. When there is no move we have $m = 0$. 

\footnote{Assuming that the rent is a constant fraction of the market value of the house is for tractability reasons. It allows me to separate out the house price as a state variable, as formalized in equation (28).}
2.5 Labor Income

The sixth source of uncertainty captures labor income risk, which I assume is exogenous.\textsuperscript{16} Real labor income, \(l\), is subject to permanent shocks.\textsuperscript{17} In addition, real labor income has a deterministic component \(g(t)\ dt\) that captures the hump-shaped pattern of labor income. We have

\[
dl/l = g(t) dt + \sigma_l dz_l \quad \text{for } t \leq 45 \tag{24}
\]

\[
l = 0 \quad \text{for } t > 45 \tag{25}
\]

where time \(t = 45\) corresponds to the retirement age of 65. After retirement, labor income is assumed to be zero. That is, I study an investor who saves for her own retirement. Equivalently, I study the joint investment problem for an investor and a pension fund investing on the investor’s behalf, without separating these two parties explicitly.

Labor income is assumed to be correlated with real house price innovations, but not with stocks, bonds, and the price level,\textsuperscript{18} i.e.

\[
dl/l = g(t) dt + \rho_{ql} \sigma_l \left[ \frac{\xi_u}{\sqrt{\xi_u^2 + \xi_u^2}} dz_v - \frac{\xi_v}{\sqrt{\xi_u^2 + \xi_u^2}} dz_u \right] + \sqrt{1 - \rho_{ql}^2} \sigma_l dz_k \quad \text{for } t \leq 45, \tag{26}
\]

with \(dz_k\) is the component of labor income shocks orthogonal to \(dz\), and with \(\rho_{ql}\) the correlation of labor income with real house price innovations. Notice that the term in square brackets is chosen such that the correlation with realized inflation, as given by equation (9), is zero.

2.6 Investor’s Problem

The investor maximizes lifetime utility, see equation (1), over real consumption, \(c\), portfolio choice, \(x\), and the new housing tenure and size, \(I^{\text{new}}\) and \(H^{\text{new}}\) (when she decides to move).

\textsuperscript{16}Bodie, Merton and Samuelson (1992) show that endogenous labor income may increase the optimal risk taking in the financial portfolio.

\textsuperscript{17}Viceira (2001), Yao and Zhang (2005a) and Munk and Sørensen (2005) assume only permanent shocks to labor income as well. Cocco, Gomes and Maenhout (2005), Campbell and Cocco (2003) and Cocco (2005) also allow for transitory, individual labor income shocks.

\textsuperscript{18}The validity of this assumption depends on the specific investor at hand. See e.g. Benzoni, Collin-Dufresne, and Goldstein (2006) and Lynch and Tan (2006) for an analysis with correlated labor income and stock returns.
This is subject to the following real financial wealth dynamics

\[
dw^F = w^F \rho^F (x) + ld t - w^H \zeta (I) dt - m - cd t. \tag{27}
\]

The five right-hand-side terms in (27) represent the financial portfolio return (as defined in (17) and (18)), labor income (as defined in (24), (25), and (26)), per-period housing expenses (as defined in (21)), moving costs (as defined in (22), and (23)), and consumption, respectively. Real housing wealth, \( w^H \) is defined in (15).

At any time, the investor’s asset allocation is subject to restriction (19) when currently renting and (20a)-(20b) when currently owning. Finally, through identity (16), the restrictions on asset positions put a lower bound on real financial wealth, which puts an upper bound on real consumption, \( c \), in (27). By moving to a smaller house, investors can always attain strictly positive consumption, and hence the problem is well defined.

### 2.7 Solution Method

The state variables for the investor’s investment problem are given by the current housing tenure, \( I \), financial wealth, \( w^F \), housing wealth, \( w^H \), labor income, \( l \), real house price, \( q \), real interest rate, \( r \), and time \( t \). From the financial wealth dynamics provided above, it is clear that a strategy for other goods and housing consumption \( \{c_t, H_t\}_{t=1}^{T} \) is sustainable starting in state \( (I, w^F, w^H, l, q, r, t) \) if and only if the consumption strategy \( \{uc_t, vH_t\}_{t=1}^{T} \) is sustainable starting in state \( (I, uw^F, vw^H, vl, q, r, t) \) for any \( v > 0 \). Similarly, a consumption strategy \( \{c_t, H_t\}_{t=1}^{T} \) is sustainable starting in state \( (I, w^F, w^H, l, q, r, t) \) if and only if the consumption strategy \( \{ct, vH_t\}_{t=1}^{T} \) is sustainable starting in state \( (I, w^F, w^H, l, q/v, r, t) \) for any \( v > 0 \). Exploiting the fact that lifetime utility, as given in equation (1), is homogeneous of degree \( 1 - \gamma \) in \( \{c_t, H_t\}_{t=1}^{T} \) and homogeneous of degree \( (1 - \gamma) \psi \) in \( \{H_t\}_{t=1}^{T} \), we can write the indirect utility function as,

\[
\max_{\{c\},\{x\},\{H^{new}\},\{I^{new}\}} U_t = \left( \frac{w}{q^w} \right)^{1-\gamma} J (I, y, h, r, t), \tag{28}
\]

\[
w = w^F + w^H, \tag{29}
\]

\[
h = w^H / w, \tag{30}
\]

\[
y = w / l. \tag{31}
\]
where \( w \) is total real wealth, \( h \) is the housing-to-wealth ratio, \( y \) is the wealth-to-income ratio, and \( J \) is the part of the indirect utility function that cannot be determined in closed form. So, I am able to separate out two state variables, which makes the model tractable.

Terminal utility \( J(I_T, y_T, h_T, r_T, T) \) is known. It is zero in the baseline case where the investor derives no utility from leaving a bequest. To determine indirect utility, \( J \), for \( t < T \), I choose a grid over \( y, h, r, \) and \( t \), and use backward induction. I denote the time step size by \( \Delta t \). For every point on the grid, I solve

\[
\left( \frac{w_t}{q_t} \right)^{1-\gamma} J(I_t, y_t, h_t, r_t, t) = \max_{c_t, x_t, I_t^{new}, H_t^{new}} \left( \frac{c_t^{1-\psi} (H_t^{new})^\psi}{1-\gamma} \right) \Delta t \]

subject to the above-mentioned constraints on the choice variables and the dynamics of the state variables. Without loss of generality I can normalize the separable state variables \( w_t \) and \( q_t \) to 1. Further details on the solution method are provided in appendix A.

2.8 Simulation Exercise

The non-separable state variables for the problem are: the current housing tenure indicator variable, \( I \), the wealth-to-income ratio, \( y \), the housing-to-wealth ratio, \( h \), the real interest rate, \( r \), and time. The number of non-separable state variables exceeds the dimension of the world we live in, which makes it impossible to show the full solution in one graph. Instead, I will illustrate the model’s implications in several graphs and tables. For the graphs I simulate paths for the non-separable state variables using derived optimal choices. Along with values for the non-separable state variables on a particular path, I obtain the values for the choice variables and separable state variables. I will show results from age 20 to 80 for the mean investor, determined by averaging the state and choice variables over 10,000 (simulated) investors.

3 Calibration

This section presents and motivates the calibrated values for the model parameters.
3.1 Term Structure

The calibrated parameter values for the asset price dynamics and the labor income process are presented in table 1. The values for the real interest, expected inflation, and unexpected inflation rate are taken from Van Hemert, De Jong and Driessen (2005). In that paper we used quarterly US data on nominal interest rates and inflation from 1973Q1 to 2003Q4 and back out the real interest and expected inflation rate with a Kalman filter technique. The half-life of real interest rate shocks, as implied by $\kappa_r$, is around 1.1 years. This is much shorter than the half-life in expected inflation rate shocks, as implied by $\kappa_\pi$, which is around 12.6 years. This is consistent with the findings of Brennan and Xia (2002) and Campbell and Viceira (2001). I set the nominal unexpected inflation premium, $\lambda_u$, equal to zero.

3.2 Stock and House Price Dynamics

I calibrate the parameters governing stock and house price dynamics to quarterly US data from 1980Q2 to 2003Q4. For the stock data I use an index comprising all NYSE, AMEX and NASDAQ firms. For house price data I use a repeated-sales index for houses in Atlanta, Boston, Chicago and San Francisco. I have no data on market-imputed rent, but for the financial asset allocation $\theta_v\lambda_v - r^{imp}$ and not $\lambda_v$ and $r^{imp}$ separately is relevant. I estimate $\theta_v\lambda_v - r^{imp}$ from the data and without loss of generality set $r^{imp}$ equal to the mean real interest rate $\bar{r}$. The nominal risk premium on the housing investment is slightly negative, $\sigma_Q\lambda_Q - r^{imp} = -0.53\%$, implying the average house price appreciation is slightly below the risk free rate. The negative excess return on housing is at odds with the current popular perception that the housing investment is a sure bet with a high return. Himmelberg, Mayer, and Sinai (2005) discuss the up- and downswings in US house prices and argue that about one-third of the recent run-up in house prices merely reflects a return to the house price levels prevalent before the downswing in the early 1990s. Notice that the average total excess return on the house, $\sigma_Q\lambda_Q$, is still positive as long as the imputed rent is larger than 0.53%.

Based on the house price index, the annual house price change has a standard deviation of 2.67%. Case and Shiller (1989) argue that the standard deviation of individual house price changes is close to 15.00%, like individual stocks. Because price changes of individual

\[ I thank the Case-Shiller-Weiss company for providing us with this data. \]

\[ Brunnermeier and Julliard (2006) argue that houses are subject to mispricing caused by investors suffering from money illusion. \]
houses are far from perfectly correlated, aggregation leads to a considerable reduction of the variability. Since I am interested in the dynamics of an individual house, I scale aggregate house price changes by a factor 15.00%/2.67% = 5.6 around its mean. I calculate correlations with the house price innovation on a yearly instead of a quarterly basis to account for the tendency of house prices to adjust more slowly to news than financial assets. Extending the calibration horizon beyond one year makes little difference. Nominal house price changes are found to be negatively correlated with real interest rate shocks and positively correlated with expected inflation shocks. The scaling of house prices changes implicitly assumes that the correlation between an individual house price change and financial asset returns equals the correlation between the aggregate house price change and financial asset returns. The calibrated coefficients of correlation are therefore likely to be biased upward in size. As an alternative parameterization, in section 6 I consider zero correlations between the housing return and financial assets returns; presented as alternative parameters in table 1.

### 3.3 Labor Income

In accordance with Munk and Sørensen (2005), I adapt the estimated labor income profile of Cocco, Gomes and Maenhout (2005) to a continuous-time setting. The deterministic part of the change in labor income is given by

\[ g(t) = b + 2c(t + 20) + 3d(t + 20)^2. \]  

(33)

where \( t + 20 \) is the age of the investor. Cocco, Gomes and Maenhout (2005) estimate \( b \), \( c \) and \( d \) for three groups characterized by the highest level of education achieved: no high school, high school, and college. I focus on the high school group. As in Munk and Sørensen (2005), I set the income rate volatility at \( \sigma_l = 0.10 \). Recall that post-retirement income is assumed to be zero. Cocco (2005) and Ortalo-Magné and Rady (2006) document a positive correlation between house price and labor income shocks. In his model, Cocco (2005) uses a perfect correlation between housing and aggregate labor income shocks (for tractability reasons) and an imperfect correlation between housing and temporary labor income shocks. Yao and Zhang (2005a) use a correlation of 0.2, which I adopt.\(^{21}\)

\(^{21}\)Spiegel (2001) illustrates in a general equilibrium model how house prices and local economic growth can be linked.
3.4 Other Parameters

Table 2 provides the other parameter values. For the risk aversion parameter I examine two values: $\gamma = 3$ for a more risk-tolerant investor and $\gamma = 9$ for a more risk-averse investor. For the parameter governing housing preferences I choose $\psi = 0.2$, which is the same as in Yao and Zhang (2005a). Cocco (2005) chooses $\psi = 0.1$. The subjective discount rate is set at $\beta = 0.96$. Following Yao and Zhang (2005a), the rent rate is $\zeta_{\text{rent}} = 6\%$, maintenance costs are $\zeta_{\text{own}} = 1.5\%$, transaction costs when moving to an owner-occupied house are $\nu_{\text{own}} = 6\%$ and the down payment on the house is $\delta = 20\%$, all expressed as percentages of the market value of the house. Cocco (2005) chooses $1\%$, $8\%$ and $15\%$ for $m$, $\nu_{\text{own}}$ and $\delta$ respectively. Yao and Zhang (2005a) assume a zero transaction cost for moving to a rental house. Taking into account the cost of moving I choose $\nu_{\text{rent}} = 1\%$.

The implied per-period cost of owning, assumed to be the housing expenses minus the expected excess return on the housing investment, $\zeta_{\text{own}} - (\sigma Q \lambda Q - r^{\text{imp}}) = 2.03\%$, is much less than the per-period cost of renting, $\zeta_{\text{rent}} = 6.00\%$. In this comparison I use the risk free rate as the opportunity cost of investing in housing. It is a priori unclear whether owning or renting is less risky, because housing is both an investment and consumption good.

3.5 Starting Values Simulation Exercise

The starting values for the simulation exercise are presented in table 3. The investor starts with $7,500 wealth. She rents a house worth $45,000 (wealth, $w$, times housing-to-wealth ratio, $h$). Her wage is $15,000 per year (wealth, $w$, divided by the wealth-to-income ratio, $y$). The real interest rate is at its long-term mean value of $\bar{r} = 2.26\%$. Time and the the real house price are normalised at 0 and 1 respectively.

4 Results

In this section, I present the solution to the baseline case model presented in section 2 using the calibrated parameter values presented in section 3. The solution comprises the optimal housing tenure, house size, financial portfolio, and consumption choice, all conditional on the state of the world.
4.1 Consumption

The top panel of figure 1 shows consumption and house size for the mean investor with risk aversion parameter $\gamma = 3$. These are the two variables that directly enter the investor’s lifetime utility function, as presented in equation (1). The bottom panel of figure 1 shows labor income and accumulated wealth for the mean $\gamma = 3$ investor. Young investors have large human capital and little wealth, which makes them borrowing constrained. Over time the investor’s labor income increases and we see in the top panel of figure 1 that both housing and other goods consumption rises between age 20 and 25. Figure 2 shows the annualized move rate for the mean investor and the fraction of investors owning the house they live in.\footnote{In some phases of life, the move rate in figure 2 is somewhat jumpy. At the start and end of the life cycle this is due to limited heterogeneity in wealth across the 10,000 simulated investors, i.e. all investors have very little wealth at in these phases of life. However, for example around age 67 the hump in the move down rate is more likely due to the solution technique that involves approximation by choosing a finite grid for the state variables. A finer grid would be pose computational difficulties; the current computation time is 10 hours on 60 parallel-connected computers.} No investor owns in the earliest phase of life. Investors have too little wealth saved to pay the down payment on a reasonably sized house. Some investors move to a larger rental house in this period however. Recall that moves are generated only by endogenous factors in my model.

Between ages 25 and 35 investors buy their first home. In most cases, this home is smaller home than the rental they are vacating. This is evident from the decline in house size in the top panel of figure 1 or the many moves down around this age in figure 2. Owning involves lower per-period housing costs than does renting. This makes investors eager to buy, even if they do not have enough wealth for the down payment on a house as big as the one they are renting.

By age 40 most investors own the house they live in, as can be seen from the solid line in figure 2 equaling 1. The mean house size rises until age 60 because only later in life do investors have enough wealth for the down payment on bigger house. In this age category, other goods consumption decreases slightly, reflecting a decreasing desire to substitute for the initially small house. See equation (1) and the text below for a discussion on this phenomenon. House size is then fairly constant until age 70, and decreasing from then on.

In the baseline case we assume no bequest motive. Consequently, investors want to consume all their wealth before they die. Because of the compulsory down payment on the house, the investor optimally decreases house size (and therefore down payment) and eventually returns to renting towards the end of her life. In figure 2 the move from an
owner-occupied to a rental house is evident from the large moving rate around age 78. Due to lower per-period housing costs, housing wealth is released fairly late in life. This causes consumption to be high in the last period of life. An additional reason for other goods consumption being high late in life is again the substitution motive, i.e. compensating for a smaller house in that phase of life.

4.2 Portfolio Choice and Asset Allocation

Figures 3 and 4 show the portfolio choice and wealth accumulated (top panel) and the asset allocation (bottom panel) for the mean investor with risk aversion parameters \( \gamma = 3 \) and \( \gamma = 9 \) respectively. Portfolio shares equal the asset allocations divided by (total) wealth. Portfolio shares add up to 1. The mortgage constitutes of the negative portfolio shares (top panel) and the negative asset allocation (bottom panel). The mortgage position in the top panel exactly equals the part of housing wealth that exceeds the dashed, horizontal line for total portfolio share equal to 1. Consequently, net housing wealth is exactly the part of housing wealth that is underneath this dashed, horizontal line.

In their first years, investors have very little wealth compared to the value of their human capital. This creates a desire to leverage risk taking in the financial portfolio in order to reap the associated risk premium. Stocks have the highest risk premium in the model presented. Both the more risk-tolerant \( \gamma = 3 \) and the more risk-averse \( \gamma = 9 \) investor predominantly hold stocks when they are young. The financial portfolio best offsetting changes in the nominal house price consists of short stocks, short 3-year bonds, long 10-year bonds and short cash.\(^{23}\) For an investor expecting to downsize her housing position this is the appropriate hedge portfolio. For an investor who is expecting to buy a (bigger) house in the near future the opposite portfolio positions are needed. Young investors cannot hedge effectively against rent price increases (which are tied to house price increases) because this would involve a negative 10-year bond position. By assumption, labor income shocks also cannot be hedged with financial assets.

Between age 25 and 35 a house is purchased. Both the \( \gamma = 3 \) and the \( \gamma = 9 \) investor choose an ARM in this phase of life, as can be seen by the negative cash position in figures 3 and 4. Doing so, the young homeowner saves on the bond risk premium paid on an FRM. Moreover she chooses the maximum allowed loan size to leverage up the riskiness

\(^{23}\)This can easily be obtained by solving \( \sigma_F(x) = -\theta \) for \( x^* \), \( x^{3} \), \( x^{10} \), where \( \sigma_F(x) \) is given in equation (18).
of the financial portfolio and exploit the associated risk premium. The financial portfolio still consists mainly of stocks, but there is also a small holding of 10-year bonds. This 10-year bond position hedges against real interest rate changes. The expected real return on housing and financial wealth equals the real interest rate plus a constant risk premium, hence the real interest rate summarizes the investment opportunities. The investor prefers the 10-year bond to the 3-year bond for its larger risk premium associated with the larger exposure to expected inflation shocks; table 4 presents the exposures of these two bonds to real interest rate and expected inflation risk. Notice that the hedging demand is greater for the more risk-averse $\gamma = 9$ investor (figure 4).

As more wealth is accumulated between age 40 and 65 and human capital is capitalized, the hedge demand against falling real interest rates increases and the desire for leveraged stock exposure decreases. For the $\gamma = 3$ investor, this results in increasing 10-year bond holdings. In contrast, the $\gamma = 9$ investor gradually switches to 3-year bonds between age 55 and 65. The reason for this difference is twofold. First the more risk-tolerant $\gamma = 3$ investor is more willing to bear the expected inflation risk of the 10-year bond and thereby reap the associated risk premium. Second, the $\gamma = 3$ investor has larger stock holdings, leaving her with less financial wealth to construct a hedge portfolio against falling interest rates, which in turn induces her to invest in the bond with the largest exposure to real interest rate, which is the 10-year bond (see table 4). Interestingly, the 10-year bond position for the $\gamma = 9$ investor becomes negative around the retirement age of 65. Since both the cash and 10-year bond position are negative, no longer a pure adjustable-rate mortgage is optimal, but the investor prefers a hybrid mortgage. The speed of mean reversion in the real interest rate and expected inflation rate determines the exposure of bond prices to shocks in these factors. The exposures presented in table 4 imply that a portfolio consisting of a $1 position in a 3-year bond and a $-\$2.8/7.7 position in a 10-year bond has the property that it has a negative exposure to real interest and a zero exposure to expected inflation rate shocks. It is therefore the portfolio that hedges against negative shocks to the real interest rate without incurring expected inflation risk. This explains the negative 10-year bond holdings, by holding the hybrid mortgage, for the $\gamma = 9$ investor around the retirement age.24

The composition of the bond portfolio before the sale of the house around age 78 clearly differs from the composition of the bond portfolio after the sale. This shows that part of the bond portfolio toward the end of the life cycle is motivated by hedging house price falls in anticipation of selling the house. Since I abstract from longevity risk, the investor is able

---

24 The optimal mortgage choice at retirement is consistent with results presented in Van Hemert, De Jong, and Driessen (2005), who abstract from labor income.
to exhaust her savings fully, as can be seen by the zero wealth at age 80.

The above analysis provides interesting insights into two empirical stylized facts in the US that are sometimes considered puzzling. First it rationalizes why on average more young investors in the US take ARMs.\(^{25}\) Young homeowners have large human capital and therefore a desire for leveraged risk taking in their financial wealth, thereby exploiting the associated risk premium. The ARM provides this leverage. Older homeowners are more concerned with adverse shifts in the real interest rate they earn on their accumulated capital. An FRM, in conjunction with a position in short term bonds, allows them to hedge against falling real interest rates. Second, many investors simultaneously hold both a long position in fixed-income securities, e.g. by holding bonds in their pension account, and a short position in fixed-income securities by having an FRM on their house.\(^{26}\) The above analysis, and the discussion of table 4, shows that such a long-short position helps in hedging real interest rate risk without incurring much inflation rate risk, provided that the maturity of the FRM is larger than that of the long position in bonds.

The stock allocation in figures 3 and 4 decreases with age. The empirical evidence for this is debated. Agnew, Balduzzi, and Sunden (2003) argue that the age pattern in the share allocated to stocks is hump shaped, peaking around age 50. Ameriks and Zeldes (2004) argue that this pattern could be due to a cohort effect, and stress the impossibility of separately identifying age, time, and cohort effects. Moreover, Ameriks and Zeldes argue that the pattern is flat conditional on stock market participation. Most at odds with the empirical evidence on stock allocation is the near 100% stock allocation at the start of life in figures 3 and 4. Gomes and Michaelides (2005) show that a fixed stock market participation, a feature absent in my model, can bring their model’s predictions on stock holdings close to observed values.

4.3 Effect Illiquidity of the House on Optimal Portfolio Choice

In this subsection I further study how the illiquidity of the house, i.e. the presence of moving cost, affects the optimal portfolio choice. Table 5 shows the optimal portfolio choice for a homeowner at the retirement age of 65 years for different housing-to-wealth ratios. Again I consider both a more risk-tolerant \(\gamma = 3\) (panel A) and a more risk-averse \(\gamma = 9\)

\(^{25}\)In the 18-24 age category, 32% prefers an ARM, while in the 45-64 age category only 19% prefer an ARM; see the opinion survey commissioned by the Consumer Federation of America (2004).

\(^{26}\)As of June 30th, 2000, TIAA-CREF, the largest private pension provider in the U.S., held 38.4% of its assets in the TIAA traditional annuity investment account, which consists mainly of fixed-income investments (Ameriks and Zeldes (2004)).
(panel B) investor. The real interest rate and wealth-to-income ratio are fixed and set at the mean value of the previous simulation exercise. That is, the real interest rate is set equal to its long-term mean, \( r = \bar{r} \), and the wealth-to-income ratio is set equal to \( y = 18 \) and \( 20 \) for the \( \gamma = 3 \) and \( \gamma = 9 \) investor respectively. I also report the utility loss \( UL \), measured as the certainty-equivalent loss in (housing plus other goods) consumption. In this case the utility loss compares the utility in the optimal housing-to-wealth ratio, \( h_{optimal} \), to the utility in a particular housing-to-wealth ratio, \( h \), keeping the values of the other state variables constant, i.e.

\[
UL = \left( \frac{J(I, h, y, r, t)}{J(I, h_{optimal}, y, r, t)} \right)^{\frac{1}{1-\gamma}} - 1. \tag{34}
\]

At any time, an investor has the ability to move to the house size that is optimal given the values of the other state variables. However, because moving involves transaction costs, there is range for the housing-to-wealth ratio where the investor optimally chooses not to move. When the housing-to-wealth ratio is outside this range, she optimally chooses to move to a house that brings her inside the range again. Notice that the range is narrower for the more risk-averse \( \gamma = 9 \) investor compared to the more risk-tolerant \( \gamma = 3 \) investor. The optimal housing-to-wealth ratio is lower for the \( \gamma = 9 \) investor. At retirement, the investor is typically overexposed to house price risk, in the sense that the value of the house exceeds the present value of future housing consumption. This makes a more risk-averse investor less willing to own a large house.

For larger housing-to-wealth ratios, the investor can take a larger mortgage. However, because the size of the mortgage may not exceed \( 1 - \delta \) times the housing wealth at any time, she has less wealth available to take long positions in financial assets. As a result, for larger housing-to-wealth ratios, the investor tends to shift her bond portfolio to long-term bonds, which have a larger absolute loading on real interest rate risk (see table 4). By doing so, she can maintain the appropriate hedge against changing interest rates. This results in the general tendency to increase the maturity of the bond portfolio with the housing-to-wealth ratio. Stock allocation tends to be crowded out more for larger housing to wealth ratios, consistent with the empirical evidence.\(^{27}\) However, there is an additional effect that blurs the picture somewhat. Investors act less risk averse close to the border of the no-move region than when they are close to the optimal housing to wealth ratio.\(^{28}\) In panel A this is

\(^{27}\)See e.g. Heaton and Lucas (2000) and Kullmann and Siegel (2003).
\(^{28}\)See e.g. Grossman and Laroque (1990) for a study on optimal behavior in the presence of an illiquid asset like a house.
best understood by noting that the 10-year bond allocation at the left border of the no-move region \((h = 0.2)\) is larger than at \(h = 0.3\) and \(0.4\). In panel B, it is more clearly understood by noting the slightly rising stock allocation near the right border of the no-move region \((h = 0.4\) vs. \(h = 0.5)\).

5 Sensitivity analyses and extensions

The modeling framework presented above allows for many possible sensitivity analyses and extensions. In this section I choose the following three: idiosyncratic house price risk, bequest motive, and suboptimal mortgage choice.

5.1 Idiosyncratic House Price Risk

In this subsection, I investigate portfolio choice when house price risk is idiosyncratic. There are three motivations to do so. First, there is likely to be considerable heterogeneity among investors with respect to the co-movement of their house price with financial assets and labor income. For example, Davidoff (2006) determined the coefficient of correlation between house prices and labor income in the US and finds a wide variation in this coefficient across industries and regions. The analysis in this subsection helps us to identify to what extent and in what phase of life this heterogeneity in house price co-movement with other assets leads to heterogeneity in the optimal portfolio choice. Second, as discussed in section 3, the calibrated values for the correlation between house price and financial asset price changes are likely to be biased upward in size because of the scaling technique used. The analysis here allows us to check the consequences this bias. Third, comparing the portfolio choice for the \(\gamma = 3\) investor from the previous section (figure 3) with the portfolio choice with fully idiosyncratic house price risk here, allows us to better identify which part of the portfolio choice in figure 3 was driven by a motive to hedge house price risk.

Figure 5 shows the portfolio shares and accumulated wealth (top panel) and asset allocation (bottom panel) for the mean \(\gamma = 3\) investor with idiosyncratic house price risk. Comparing this figure with figure 3, which shows the mean \(\gamma = 3\) investor with a house price risk that can be partially hedged, we see that investors takes into account that they will downsize their housing wealth and eventually return to renting at the end of their life cycle. This creates a motive to hedge against falling house prices. As discussed in section 3, this calls for a long 10-year bond and a short 3-year bond position. Indeed the 10-year bond
portfolio share is lower and the 3-year bond portfolio share larger in figure 5 (top panel) where there is no additional hedge demand because of the idiosyncratic risk of the housing investment. This hedge demand is also detectable in figures 3 and 4 (top panels), where the 10-year bond portfolio share decreases and the 3-year bond portfolio share increases once the house is sold. Further comparing figures 3 and 5 (top panels), we see little differences between the two graphs in the early stages of life, indicating that hedging house price risk with financial assets does not play a very important role in the accumulation phase of life.\footnote{As Sinai and Souleles (2005) note, ownership in itself may provide a hedge against future housing cost risk, which in turn might influence the tenure decision. Pelizzon and Weber (2006) estimate the difference between the value of a house and the present value of future housing consumption costs using Italian household portfolio data.}

### 5.2 Bequest Motive

In this subsection, I study optimal portfolio choice for an investor who derives utility from leaving terminal wealth $w_T$ as a bequest. Denoting the utility derived from leaving a bequest as $V$, lifetime utility is then given by

$$U_t = \int_t^T \beta^{s-t} \left( \frac{c_s}{1-Hs} \right)^{1-\gamma} ds + \beta^{T-t} V(w_T, q_T, r_T).$$

(35)

In accordance with Yao and Zhang (2005a), I assume that the beneficiaries purchase a real annuity, which in turn is used to pay for 15 years of other-goods consumption and housing rental costs. The payout rate for a $\$1$ annuitized wealth will depend on the real interest rate at time $T$, and is denoted by $K(r_T)$. An analytical expression for the payout rate is provided in appendix B.

Assuming the same Cobb-Douglas preferences over housing and other goods for the bequest function, a fraction $\psi$ is spent on rental costs and a fraction $1 - \psi$ is spent on other-goods consumption. The utility from a bequest is given by

$$V(w_T, q_T, r_T) = \int_T^{T+15} \beta^{s-T} \left( \frac{[1-\psi] K(r_T) w_T^{1-\psi} \Zeta^{1-\gamma} (\Zeta^{\psi} q_T)^{1-\gamma}}{\ln \beta} \right)^{1-\gamma} ds$$

(36)

The same homogeneity properties apply so that the indirect utility is again of the functional
Figure 6 shows the portfolio shares and accumulated wealth (top panel) and asset allocation (bottom panel) for the mean $\gamma = 3$ investor. Comparing it with figure 3, which shows the mean $\gamma = 3$ investor without bequest motive, we notice three main differences. First, with a bequest motive the investor accumulates considerably more wealth. Moreover, the investor does not run down her accumulated wealth after the retirement age of 65. Second, the investor does not return to renting toward the end of the life cycle. The main motivation to return to renting without a bequest motive is that this enables the investor to consume all of the housing equity. This motivation is clearly less relevant in the presence of a bequest motive. Third, with a bequest motive, the portfolio shares after retirement remain fairly constant, in contrast to a situation without a bequest motive. Before retirement, however, the portfolio shares with and without bequest motive are very similar.

5.3 Suboptimal Mortgage Choice

The share of newly originated mortgages that is of the FRM type in the US has varied roughly between 30% and 90% over the last two decades. In some other countries, such as the UK, ARMs are far more common. In the above analysis investors more often choose the ARM, see figures 3 and 4. The ARM allows investors to leverage the risk taking in the financial portfolio and exploit the associated risk premium. With an FRM the investor pays a risk premium; the risk premium on long-term bonds. In a speech to the Credit Union National Association, Alan Greenspan (2004), former chairman of the Federal Reserve, criticized the preference of US households for FRMs: "... the traditional fixed-rate mortgage might be an expensive method of financing a home.", and "Fixed-rate mortgages seem unduly expensive to households in other countries.". The prepayment option, abstracted from in the analysis above, is not likely to make the FRM cheaper than the ARM. First, an investor pays in advance a premium to compensate the lender for writing the prepayment option. Second, households behave suboptimally in exercising this prepayment option, see e.g. Schwartz and Torous (1989).

In this subsection, I study an investor who, suboptimally, always chooses an FRM to finance her house. Figure 7 shows the portfolio share and wealth (top panel) and the asset

\[\text{form given in (28).}\]

\[\text{figure 6 shows the portfolio shares and accumulated wealth (top panel) and asset allocation (bottom panel) for the mean } \gamma = 3 \text{ investor. Comparing it with figure 3, which shows the mean } \gamma = 3 \text{ investor without bequest motive, we notice three main differences. First, with a bequest motive the investor accumulates considerably more wealth. Moreover, the investor does not run down her accumulated wealth after the retirement age of 65. Second, the investor does not return to renting toward the end of the life cycle. The main motivation to return to renting without a bequest motive is that this enables the investor to consume all of the housing equity. This motivation is clearly less relevant in the presence of a bequest motive. Third, with a bequest motive, the portfolio shares after retirement remain fairly constant, in contrast to a situation without a bequest motive. Before retirement, however, the portfolio shares with and without bequest motive are very similar.}\]

\[5.3 \text{ Suboptimal Mortgage Choice}\]

\[\text{The share of newly originated mortgages that is of the FRM type in the US has varied roughly between 30\% and 90\% over the last two decades.}^{30}\ \text{In some other countries, such as the UK, ARMs are far more common. In the above analysis investors more often choose the ARM, see figures 3 and 4. The ARM allows investors to leverage the risk taking in the financial portfolio and exploit the associated risk premium. With an FRM the investor pays a risk premium; the risk premium on long-term bonds.}^{31}\ \text{In a speech to the Credit Union National Association, Alan Greenspan (2004), former chairman of the Federal Reserve, criticized the preference of US households for FRMs: "... the traditional fixed-rate mortgage might be an expensive method of financing a home."}, \text{and "Fixed-rate mortgages seem unduly expensive to households in other countries.". The prepayment option, abstracted from in the analysis above, is not likely to make the FRM cheaper than the ARM. First, an investor pays in advance a premium to compensate the lender for writing the prepayment option. Second, households behave suboptimally in exercising this prepayment option, see e.g. Schwartz and Torous (1989).}\]

\[\text{In this subsection, I study an investor who, suboptimally, always chooses an FRM to finance her house. Figure 7 shows the portfolio share and wealth (top panel) and the asset}\]

\[\text{\footnote{\textit{30\text{See e.g. Campbell (2006) or Kojien, Van Hemert, and Van Nieuwerburgh (2006).}}}}\]

\[\text{\footnote{\textit{31\text{Kojien, Van Hemert, and Van Nieuwerburgh (2006) show for both the US and UK that the empirical time variation in the share of newly-originated mortgages that is of the FRM type can to a large extent be explained by time variation in the long-term bond risk premium. When the premium is high investors shy away from the FRM contract.}}}\]
allocation (bottom panel) for the mean $\gamma = 3$ investor. Comparing this figure to the case of optimal mortgage choice presented in figure 3, we notice substantial differences. Investors start owner-occupying their house only later in life, they pay down the mortgage more quickly, and accumulate less wealth during their working life. The utility loss, evaluated at age 20, expressed as certainty-equivalent loss in (housing plus other goods) consumption, is a substantial $-2.84\%$.

6 Conclusion

This study investigates housing and portfolio choice under stochastic inflation and real interest rates. Investors can finance the purchase of a house with an adjustable-rate mortgage (ARM) or a fixed-rate mortgage (FRM). I show that the mortgage choice is an integral part of the overall household allocation problem. In particular, the household interest rate risk management is determined jointly by the mortgage choice and bond portfolio choice.

Consistent with empirical evidence, I find that (i) many investors hold both a long and a short position in fixed income securities, and (ii) young investors are more likely to take an ARM. I further find that in most phases of life the ARM is the preferred mortgage type, and that suboptimally taking an FRM is associated with large utility losses. Given a positive bond risk premium, an FRM is a more expensive way of financing your home. The preference for ARMs is consistent with the reality in the UK, but inconsistent with the US, where FRMs are more popular. My findings are supportive of the critical comments expressed by Alan Greenspan, former chairman of the Federal Reserve, that an FRM might be an expensive method of financing a home.\footnote{See the speech to the Credit Union National Association, Greenspan (2004).}

The choice of mortgage type is first and foremost a choice between different interest rate products and, as was shown, should therefore be analyzed in conjunction with other financial decisions. However, there might be additional effects that must be left for future research. First, the payments on an FRM are higher than on an ARM for a normal, upward-sloping, nominal interest rate curve. In countries where mortgage payments are tax-deductible this might result in larger tax benefits for homeowners financing their house with an FRM. Second, in some countries FRMs have an embedded prepayment option, which in turn gives rise to a option premium on the mortgage rate. Third, some homeowners default on their mortgage.
Appendix A: Solution Method

As a timing convention, I assume that state variables in period $t$ are defined after the labor income of period $t$ (equaling $l_t \Delta t$) is received, but before period-$t$ consumption, per-period out-of-pocket housing, and moving costs are incurred. This puts an attainable lower bound on the wealth-to-income ratio of $y_t \geq l_t \Delta t / l_t = \Delta t$. I choose a 60-point grid on $[\Delta t, 50]$ for $y$. By choosing the lower limit of the grid for $y$ equal to the attainable lower bound $\Delta t$, I don’t need to extrapolate on the lower end of the grid, which greatly enhances the precision of the numerical procedure. The upper limit of the grid for $y$ is set large enough so that it is never reached when simulating paths for the state variables. The grid for $h$ is chosen differently for homeowner and for renters, i.e. depends on $I$. For homeowner I choose $[0, 1/\delta]$ with step size 0.1. The upper limit $1/\delta$ is the largest attainable value considering the down payment requirement. We have $\delta = 0.2$, hence $1/\delta = 5$. For homeowners I choose $[0, 25]$ with step size 0.5. For the real interest rate $r$ I choose a 5-point grid around the unconditional mean, $\bar{r}$. I choose an interval length of one month, i.e. $\Delta t = 1/12$.

For the optimization in (32), I combine two methods: (i) the Direction Set (Powell’s) Method in multidimensions, which uses a numerically-determined derivative of the objective function to determine the search direction, and (ii) the Downhill Simplex Method in multidimensions (Nelder and Mead (1965)), which does not use any derivative information. See Press et al. (1992) for an excellent introduction to these methods. I find that these methods are complementary in their ability to find the global maximum in different states of the world, and combining them works very well. I also consider many different starting values to initiate the optimization algorithm, including the optimal values for the choice variables one period ahead.

The conditional expectation in (32) is approximated using a 3-point Gaussian quadrature for each of the six sources of uncertainty represented by the six Brownian motions (Tauchen and Hussey (1991)). To determine $J(I_{t+\Delta t}, y_{t+\Delta t}, h_{t+\Delta t}, r_{t+\Delta t}, t + \Delta t)$ for values of $y_{t+\Delta t}$, $h_{t+\Delta t}$ and $r_{t+\Delta t}$ not on the grid, I use linear interpolation on log($J$). The possibility to move causes the indirect utility to be kinked on the boundaries of the no-move region, making cubic spline interpolation techniques less suitable. At a given point in time, the solution to (32) for different values of $I$, $y$, $h$, and $r$ can be determined simultaneously, making parallel computation techniques feasible. The calculations are performed on 60 parallel-connected computers with two 3.4 GHz processors each. I use the C programming language. One run takes approximately 10 hours.
Appendix B: Payout Rate Real Annuity

Denoting the real price at time $T$ for an inflation-indexed bond with a $1$ (real) payoff at time $T + \tau$ by $i_{T,T+\tau}$, the total payout rate of a $1$ annuitized is given by

$$K (r_T) = \left( \int_T^{T+R} i_{T,T+\tau} d\tau \right)^{-1}$$

(37)

The real pricing kernel that prices financial securities, $M_t$, has dynamics

$$\frac{dM}{M} = -rdt + (\xi - \rho^{-1} \lambda)'dz$$

(38)

Now

$$i_{T,T+\tau} = E_T \left[ \frac{M_{T+\tau}}{M_T} \right]$$

$$= \exp \left\{ E_T \left[ \ln \left( \frac{M_{T+\tau}}{M_T} \right) \right] + \frac{1}{2} \text{Var}_T \left[ \ln \left( \frac{M_{T+\tau}}{M_T} \right) \right] \right\},$$

(39)

where the second step follows from the lognormality of $M_{T+\tau}/M_T$. Expressions for both the expectation and variance of the log pricing kernel are straightforwardly derived (see e.g. Brennan and Xia (2002)). We have that

$$i_{T,T+\tau} = \exp \left( A(\tau) - B_r(\tau) (r_T - \bar{r}) \right),$$

(40)

where $B_r(\tau) = \kappa_r^{-1} (1 - e^{-\kappa_r \tau})$, as before, and

$$A(\tau) = -\bar{r} \tau - \frac{\sigma_r^2}{4\kappa_r^3} \left[ 2\kappa_r (B_r(\tau) - \tau) + \kappa_r^2 B_r^2(\tau) \right] - \frac{\sigma_r}{\kappa_r} (\xi - \rho^{-1} \lambda)' e_{2}(\tau - B_r(\tau)), \quad (41)$$

with $e_2$ the unit vector with a 1 as its second element and zeros elsewhere.
References


Amromin, Gene, Jennifer Huang, and Clemens Sialm (2006), The tradeoff between mortgage prepayments and tax-deferred retirement savings, NBER Working paper 12502.


Kullmann, Cornelia, and Stephan Siegel (2003), Real estate and its role in household portfolio choice, Working paper.


Munk, Claus and Carsten Sørensen (2005), Dynamic asset allocation with stochastic income and interest rates, Working paper.


Piazessi, Monika, Martin Schneider, and Selale Tuzel (2006), Housing, consumption, and asset pricing, Forthcoming in the *Journal of Financial Economics*.


Table 1: Calibrated parameter values for asset price dynamics.

This table presents the calibrated parameter values for the asset price dynamics. Parameter values for the real interest, expected inflation, and unexpected inflation rate are taken from Van Hemert, De Jong, and Driessen (2005), who use quarterly US data on nominal interest rates and inflation from 1973Q1 to 2003Q4. The house price dynamics are calibrated to Case-Shiller-Weiss repeated-sales data for Atlanta, Boston, Chicago, and San Francisco, from 1980Q2 to 2003Q4. For stocks an index comprising all NYSE, AMEX, and NASDAQ firms over the same sample period as the house price data is used.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>(Alternative)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock return process: $dS/S = (R_f + \sigma_S \lambda_S) dt + \sigma_S dz_S$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>0.1748</td>
<td></td>
</tr>
<tr>
<td>$\lambda_S$</td>
<td>0.2288</td>
<td></td>
</tr>
<tr>
<td>Real riskless interest rate process: $dr = \kappa_r (\bar{r} - r) dt + \sigma_r dz_r$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>0.0226</td>
<td></td>
</tr>
<tr>
<td>$\kappa_r$</td>
<td>0.6501</td>
<td></td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.0183</td>
<td></td>
</tr>
<tr>
<td>$\lambda_r$</td>
<td>-0.3035</td>
<td></td>
</tr>
<tr>
<td>Expected inflation process: $d\pi = \kappa_\pi (\bar{\pi} - \pi) dt + \sigma_\pi dz_\pi$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>0.0351</td>
<td></td>
</tr>
<tr>
<td>$\kappa_\pi$</td>
<td>0.0548</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>0.0191</td>
<td></td>
</tr>
<tr>
<td>$\lambda_\pi$</td>
<td>-0.1674</td>
<td></td>
</tr>
<tr>
<td>House price process: $dQ/Q = (R_f + \sigma_Q \lambda_Q - r^{imp}) dt + \sigma_Q dz_Q = (R_f + \theta' \lambda - r^{imp}) dt + \theta' dz$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_S$</td>
<td>0.0079 (0.0000)</td>
<td></td>
</tr>
<tr>
<td>$\theta_r$</td>
<td>-0.0129 (0.0000)</td>
<td></td>
</tr>
<tr>
<td>$\theta_\pi$</td>
<td>0.0427 (0.0000)</td>
<td></td>
</tr>
<tr>
<td>$\theta_v$</td>
<td>0.1418 (0.1500)</td>
<td></td>
</tr>
<tr>
<td>$\lambda_v$</td>
<td>0.1315 (0.1150)</td>
<td></td>
</tr>
<tr>
<td>$r^{imp}$</td>
<td>0.0226</td>
<td></td>
</tr>
<tr>
<td>$\sigma_Q$</td>
<td>0.1500</td>
<td></td>
</tr>
<tr>
<td>$\lambda_Q$</td>
<td>0.1150</td>
<td></td>
</tr>
<tr>
<td>Realized inflation process: $d\Pi/\Pi = \pi dt + \sigma_\Pi dz_\Pi = \pi dt + \xi' dz$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\xi_S$</td>
<td>-0.0033</td>
<td></td>
</tr>
<tr>
<td>$\xi_r$</td>
<td>0.0067</td>
<td></td>
</tr>
<tr>
<td>$\xi_\pi$</td>
<td>0.0012</td>
<td></td>
</tr>
<tr>
<td>$\xi_v$</td>
<td>-0.0236 (0.0000)</td>
<td></td>
</tr>
<tr>
<td>$\xi_u$</td>
<td>0.0474 (0.0530)</td>
<td></td>
</tr>
<tr>
<td>$\lambda_u$</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\Pi$</td>
<td>0.0535</td>
<td></td>
</tr>
<tr>
<td>Real labor income process: $dl/l = g(t) dt + \sigma_1 dz_1$, where $g(t) = b + c (t + 20) + 3d (t + 20)^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.1000</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>0.1682</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>-0.00323</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>0.000020</td>
<td></td>
</tr>
<tr>
<td>Correlations: $\rho_Sr$</td>
<td>-0.1643</td>
<td></td>
</tr>
<tr>
<td>$\rho_S\pi$</td>
<td>0.0544</td>
<td></td>
</tr>
<tr>
<td>$\rho_r\pi$</td>
<td>-0.2323</td>
<td></td>
</tr>
<tr>
<td>$\rho_{ql}$</td>
<td>0.2000 (0.0000)</td>
<td></td>
</tr>
</tbody>
</table>

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### Table 2: Choice of other parameters.

This table reports the parameter values that need to be set in addition to the calibrated parameters governing the asset price dynamics.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion</td>
<td>$\gamma$</td>
<td>3 or 9</td>
</tr>
<tr>
<td>Housing preferences</td>
<td>$\psi$</td>
<td>0.20</td>
</tr>
<tr>
<td>Subjective discount rate</td>
<td>$\beta$</td>
<td>0.96</td>
</tr>
<tr>
<td>Rental rate</td>
<td>$\zeta_{rent}$</td>
<td>6.0%</td>
</tr>
<tr>
<td>Maintenance rate</td>
<td>$\zeta_{own}$</td>
<td>1.5%</td>
</tr>
<tr>
<td>Move to rent cost</td>
<td>$\nu_{rent}$</td>
<td>1.0%</td>
</tr>
<tr>
<td>Move to own cost</td>
<td>$\nu_{own}$</td>
<td>6.0%</td>
</tr>
<tr>
<td>Minimum down payment</td>
<td>$\delta$</td>
<td>0.20</td>
</tr>
</tbody>
</table>

### Table 3: Starting values for simulation exercise.

This table presents the starting values at age 20 used for the simulation exercise. The separable state variables do not affect the solution up to a multiplicative factor. Total wealth is included in order to interpret the variables in dollar terms. The zero value for the housing tenure indicator means that the investor is renting at the starting age of 20.

<table>
<thead>
<tr>
<th>State variable</th>
<th>Symbol</th>
<th>Value</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing tenure indicator</td>
<td>$I$</td>
<td>0</td>
<td>Non-separable, binary</td>
</tr>
<tr>
<td>Wealth-to-labor income ratio</td>
<td>$y$</td>
<td>0.5</td>
<td>Non-separable</td>
</tr>
<tr>
<td>Housing-to-wealth ratio</td>
<td>$h$</td>
<td>6.0</td>
<td>Non-separable</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>$r$</td>
<td>$\bar{\pi} = 2.26%$</td>
<td>Non-separable</td>
</tr>
<tr>
<td>Time</td>
<td>$t$</td>
<td>0</td>
<td>Non-separable, normalization</td>
</tr>
<tr>
<td>Real total wealth</td>
<td>$w$</td>
<td>$$7,500</td>
<td>Separable</td>
</tr>
<tr>
<td>Real house price</td>
<td>$q$</td>
<td>1.0</td>
<td>Separable, normalization</td>
</tr>
</tbody>
</table>

### Table 4: Bond price exposure to real interest and expected inflation rate shocks.

This table shows the exposure of the 3- and 10-year nominal bond price to shocks in the real interest and expected inflation rate. The exposure to these shocks for a bond with maturity $\tau$ is given by $B_r(\tau)$ and $B_\pi(\tau)$ respectively (see equations (11)-(13)). The value for the mean reversion is taken from table 1. Notice that a portfolio consisting of a $\$1$ position in a 3-year bond and a $-\$2.8/7.7$ position in a 10-year bond has the property that it has a negative exposure to real interest rate shocks and a zero exposure to expected inflation rate shocks.

<table>
<thead>
<tr>
<th>Bond maturity, $\tau$</th>
<th>Price exposure to shocks in</th>
<th>Real interest rate, $B_r(\tau)$</th>
<th>Expected inflation, $B_\pi(\tau)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 years</td>
<td></td>
<td>$-1.3$</td>
<td>$-2.8$</td>
</tr>
<tr>
<td>10 years</td>
<td></td>
<td>$-1.5$</td>
<td>$-7.7$</td>
</tr>
</tbody>
</table>
Table 5: Portfolio choice for different housing-to-wealth ratios.

This table reports the optimal portfolio choice for a homeowner of age 65 at different housing-to-wealth ratios. The variables $x^s$, $x^{b10}$, $x^{b3}$, and $x^c$ denote the proportional allocation to stocks, 3-year bonds, 10-year bonds, and cash. Negative positions in the 10-year bond (cash) reflect a position in a fixed-rate (adjustable-rate) mortgage. The labor-to-income ratio is set equal to the mean of the simulation analysis, i.e. $y = 16$ and $18$ for the $\gamma = 3$ and $\gamma = 9$ investor respectively. The real interest rate is set equal to the long run mean $\bar{r}$. In addition this table presents the utility loss $UL$, measured as the certainty equivalent loss of having a house size different from the optimal house size (recognizable by a zero value for $UL$). Move indicates an investor optimally chooses to change house size in a particular state of the world.

**Panel A: the investor has risk aversion $\gamma = 3$**

<table>
<thead>
<tr>
<th>Variable</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^s$</td>
<td>move</td>
<td>0.38</td>
<td>0.35</td>
<td>0.33</td>
<td>0.33</td>
<td>0.32</td>
<td>0.30</td>
<td>0.27</td>
<td>move</td>
</tr>
<tr>
<td>$x^{b3}$</td>
<td>move</td>
<td>0.13</td>
<td>0.16</td>
<td>0.15</td>
<td>0.10</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>move</td>
</tr>
<tr>
<td>$x^{b10}$</td>
<td>move</td>
<td>0.44</td>
<td>0.42</td>
<td>0.43</td>
<td>0.46</td>
<td>0.53</td>
<td>0.55</td>
<td>0.56</td>
<td>move</td>
</tr>
<tr>
<td>$x^c$</td>
<td>move</td>
<td>−0.16</td>
<td>−0.24</td>
<td>−0.32</td>
<td>−0.40</td>
<td>−0.48</td>
<td>−0.56</td>
<td>−0.64</td>
<td>move</td>
</tr>
<tr>
<td>$UL$</td>
<td>move</td>
<td>−1.72</td>
<td>−0.53%</td>
<td>0.00%</td>
<td>−0.19%</td>
<td>−0.85%</td>
<td>−1.62%</td>
<td>−2.20%</td>
<td>move</td>
</tr>
</tbody>
</table>

**Panel B: the investor has risk aversion $\gamma = 9$**

<table>
<thead>
<tr>
<th>Variable</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^s$</td>
<td>move</td>
<td>0.11</td>
<td>0.11</td>
<td>0.10</td>
<td>0.11</td>
<td>move</td>
<td>move</td>
<td>move</td>
<td>move</td>
</tr>
<tr>
<td>$x^{b3}$</td>
<td>move</td>
<td>0.84</td>
<td>0.82</td>
<td>0.80</td>
<td>0.78</td>
<td>move</td>
<td>move</td>
<td>move</td>
<td>move</td>
</tr>
<tr>
<td>$x^{b10}$</td>
<td>move</td>
<td>−0.12</td>
<td>−0.09</td>
<td>−0.07</td>
<td>−0.01</td>
<td>move</td>
<td>move</td>
<td>move</td>
<td>move</td>
</tr>
<tr>
<td>$x^c$</td>
<td>move</td>
<td>−0.04</td>
<td>−0.15</td>
<td>−0.25</td>
<td>−0.39</td>
<td>move</td>
<td>move</td>
<td>move</td>
<td>move</td>
</tr>
<tr>
<td>$UL$</td>
<td>move</td>
<td>−0.49%</td>
<td>0.00%</td>
<td>−0.53%</td>
<td>−1.43%</td>
<td>move</td>
<td>move</td>
<td>move</td>
<td>move</td>
</tr>
</tbody>
</table>
This figure shows housing and other goods consumption (top panel) and labor income and wealth (bottom panel) for an investor with a coefficient of relative risk aversion equal to 3. Results are calculated by taking the average value over 10,000 investors, using the derived optimal policy functions and for each investor simulating a different path for the exogenous stochastic processes. Starting values at age 20 are given in table 3.
Figure 2: Move rate and fraction owning for the mean $\gamma = 3$ investor.
This figure shows the move rate and fraction owning for an investor with a coefficient of relative risk aversion equal to 3. The move rate is annualized and split into house size increases (move up) and decreases (move down). Results are calculated by taking the average value over 10,000 investors, using the derived optimal policy functions and for each investor simulating a different path for the exogenous stochastic processes. Starting values at age 20 are given in table 3.
Figure 3: Portfolio shares and wealth (top panel) and asset allocation (bottom panel) for the mean $\gamma = 3$ investor.

This figure shows the portfolio shares and accumulated wealth (top panel) and asset allocation (bottom panel) for an investor with a coefficient of relative risk aversion equal to 3. Results are calculated by taking the average value over 10,000 investors, using the derived optimal policy functions and for each investor simulating a different path for the exogenous stochastic processes. Starting values at age 20 are given in table 3.
Figure 4: Portfolio shares and wealth (top panel) and asset allocation (bottom panel) for the mean $\gamma = 9$ investor.

This figure shows the portfolio shares and accumulated wealth (top panel) and asset allocation (bottom panel) for an investor with a coefficient of relative risk aversion equal to 9. Results are calculated by taking the average value over 10,000 investors, using the derived optimal policy functions and for each investor simulating a different path for the exogenous stochastic processes. Starting values at age 20 are given in table 3.
Figure 5: Portfolio shares and wealth (top panel) and asset allocation (bottom panel) for the mean \( \gamma = 3 \) investor with idiosyncratic house price risk.

This figure shows the portfolio shares and accumulated wealth (top panel) and asset allocation (bottom panel) for the case of fully idiosyncratic house price risk (alternative calibrated parameters in table 1). The investor has a coefficient of relative risk aversion equal to 3. Results are calculated by taking the average value over 10,000 investors, using the derived optimal policy functions and for each investor simulating a different path for the exogenous stochastic processes. Starting values at age 20 are given in table 3.
Figure 6: Portfolio shares and wealth (top panel) and asset allocation (bottom panel) for the mean $\gamma = 3$ investor with bequest motive. This figure shows the portfolio shares and accumulated wealth (top panel) and asset allocation (bottom panel) for an investor with a bequest motive. The investor has a coefficient of relative risk aversion equal to 3. Results are calculated by taking the average value over 10,000 investors, using the derived optimal policy functions and for each investor simulating a different path for the exogenous stochastic processes. Starting values at age 20 are given in table 3.
Figure 7: Portfolio shares and wealth (top panel) and asset allocation (bottom panel) for the mean $\gamma = 3$ investor who suboptimally always chooses an FRM. This figure shows the portfolio shares and accumulated wealth (top panel) and asset allocation (bottom panel) for an investor who suboptimally always chooses an FRM. The investor has a coefficient of relative risk aversion equal to 3. Results are calculated by taking the average value over 10,000 investors, using the derived optimal policy functions and for each investor simulating a different path for the exogenous stochastic processes. Starting values at age 20 are given in table 3.