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INSIDER TRADING WITH A RANDOM DEADLINE

BY RENÉ CALDENTEY AND ENNIO STACCHETTI¹

We consider a model of strategic trading with asymmetric information of an asset whose value follows a Brownian motion. An insider continuously observes a signal that tracks the evolution of the asset's fundamental value. The value of the asset is publicly revealed at a random time. The equilibrium has two regimes separated by an endogenously determined time T . In $[0, T)$, the insider gradually transfers her information to the market. By time T , all her information has been transferred and the price agrees with the market value of the asset. In the interval $[T, \infty)$, the insider trades large volumes and reveals her information immediately, so market prices track the market value perfectly. Despite this market efficiency, the insider is able to collect strictly positive rents after T .

KEYWORDS: Insider trading, Kyle model, market microstructure, asset pricing.

1. INTRODUCTION

THIS PAPER STUDIES a model of strategic trading with asymmetric information of an asset whose value follows a Brownian motion. An insider receives a flow of (noisy) signals that tracks the evolution of the asset value. Other traders receive no signals and can only observe the total volume of trade. Since there is uncertainty about the value of the asset before the game starts, the first signal generates a lumpy informational asymmetry between the insider and the rest of the market participants. Subsequently, the insider receives a sequence of updates regarding the fundamental valuation of the asset. At an unpredictable time, a public announcement reveals the current value of the asset to all the traders. In equilibrium, the insider releases all her private information by a finite time T and keeps the market fully informed thereafter. Thus, she does not find it profitable to maintain informational asymmetry indefinitely.

Kyle (1985) introduced a dynamic model of insider trading² where an insider receives only one signal and the fundamental asset value does not change over time. Through trade, the insider progressively releases her private information to the market as she exploits her informational advantage. The market is also populated by many liquidity traders who are uninformed and trade randomly. At time 0, the insider observes the value of an asset. The same information is publicly released later, at time 1, to all market participants. In each trading period in the time interval $[0, 1]$, traders submit order quantities to a risk-neutral market maker who sets prices competitively and trades in his own account to clear the market. The market maker cannot observe individual trades, but can

¹We gratefully acknowledge the feedback of David Pearce. We also thank Markus Brunnermeier, Lasse Pedersen, and Debraj Ray and seminar participants at NYU.

²Glosten and Milgrom (1985) proposed an alternative formalization of Bagehot's (1971) informal model.

observe the total volume of trade in each trading period. The market maker also knows (in equilibrium) the strategy of the informed trader, and sets prices efficiently, conditional on past and present volumes of trade.

Kyle constructed a linear equilibrium where in each period the price adjustment is proportional to the volume of trade, and the insider's volume of trade is proportional to the gap between the asset value and the current market price. The market maker's estimate of the asset value, reflected in the current market price, improves over time. As the public announcement date approaches, this estimate converges to the value of the asset and the insider trades frantically in her desire to exploit any price differential.

Our model differs from Kyle's model in three important ways. First, the fundamental value of the asset follows a Brownian motion and, therefore, changes continuously over time. Second, in addition to the initial observation, the insider continuously receives a signal of the current fundamental value of the asset. Third, the public announcement date is unpredictable: it has an exponential distribution.

The first difference by itself is irrelevant. In Kyle's model it makes no difference whether at time 0 the insider observes the true value of the asset or just an unbiased signal. Moreover, the model where the insider observes the true value and the value of the asset follows a Brownian motion is formally equivalent to a model where the initial observation is an unbiased signal of the final value of the asset. But this feature of our model becomes important when it is combined with the second feature. Finally, the third feature removes the pressure in Kyle's model behind the trade frenzy that occurs as the announcement date approaches. In our model, where the announcement date is not deterministic, the insider has no urgency to exhaust all arbitrage opportunities, and release all her private information in the process, by a particular deadline. Thus, while it is evident that in Kyle's model the price will become efficient (in the sense that it incorporates all the available information) as time reaches the announcement date, it is unclear whether in our model the insider will ever fully reveal her private information.

It is exactly this feature of the equilibrium of the fixed horizon model that Back (1992) exploited to develop his elegant "backward programming" solution method. In a model with a random horizon, Back's method is not directly applicable.

Our model is not the first to introduce a public announcement with random time. Back and Baruch (2004) compared the models of Kyle (1985) and Glosten and Milgrom (1985). To facilitate the comparison, they adopted a Glosten and Milgrom model with a single long-lived insider (who times her transactions strategically) and a Kyle model with a random terminal time and a risky asset that takes only the values 0 or 1.

Kyle's original model is in discrete time. However, Kyle also showed that as the period length Δ converges to 0, the equilibrium converges to an equilibrium of the continuous-time limit model. He then interpreted the continuous-time

model as a good representation of a discrete-time model where the agents can trade frequently. We maintain this interpretation and view the continuous-time model as a mathematical convenience that affords us the powerful tools of stochastic calculus. The discrete-time version of our model has a unique equilibrium that converges to a well defined strategy profile as $\Delta \downarrow 0$. However, in our case, this limit strategy is not an equilibrium of the continuous-time model. The interpretation of the continuous-time model is therefore delicate and needs to be examined more carefully. The lack of “continuity” arises because in the limit the insider wants to trade at infinite rates after some time T . She still collects positive rents after T even though the price perfectly tracks the value of the asset. However, after T , her payoff function evaluated at the limit strategy is 0. Therefore, as we explain in Section 5, the limit strategy cannot be an equilibrium of the continuous-time model. Because our characterization of the equilibrium has a crisper form in the limit, our discussion below refers to the limit equilibrium.

Our model includes various special cases. The value of the asset remains constant over time if the variance of its Brownian motion is reduced to 0. Since in our model the insider observes the initial value without noise, the signals that track the value of the asset over time become superfluous. This version of our model is similar to Kyle’s model, where the insider is endowed only with an initial piece of private information, but with a random end time. Alternatively, we can specialize our model to give the insider no initial informational advantage. This is accomplished by informing *all* traders of the initial value of the asset. In this version of the model, the insider’s informational advantage arises exclusively from her ability to observe the evolution of the asset value. This is an important model in its own right. An interesting question in this model is how the insider “manages” the information asymmetry. For example, the insider could let the information asymmetry (the variance of the uninformed traders’ estimate of the current value) grow to reach asymptotically a certain limit or grow without bound. The larger is the information asymmetry, the more likely it is that the market will substantially misprice the asset and, therefore, the larger are the profitable arbitrage opportunities. Thus, in this model as well it is not evident how much of the insider’s information is incorporated in the market price and how quickly this happens. We study this special case in the process of constructing an equilibrium for our general model. It turns out that in equilibrium the insider fully reveals her information as soon as she receives it. Hence, the market price equals the asset value at all times. Yet, the insider makes strictly positive profits. In independent work, Chau and Vayanos (2008) reached the same conclusion (for this case without initial informational asymmetry) in a slightly different model. They assumed that the insider receives a flow of information, the asset pays a dividend, and there is no public announcement. In addition, they assumed that the market maker continuously observes a noisy signal of the value of the asset. In the absence of this noisy signal, their model would be formally equivalent to ours. Chau and Vayanos

(2008) limited attention to the steady state of their model and did not study how the equilibrium approaches the steady state. One implication of our results is that in the absence of an initial information asymmetry, the steady state is reached “immediately” (as the period length goes to 0), so although Chau and Vayanos (2008) assumed that trading had been taking place indefinitely, this is not needed.

The equilibrium of our general model has a remarkable feature. There is a time T , endogenously determined in equilibrium, by which the insider reveals all her information (if the public announcement has not yet occurred). Thus, even though there is no deterministic deadline, the price converges to the asset value at time T . Moreover, time T divides the equilibrium into two phases. As long as the public announcement does not occur, in the interval $[0, T)$ the insider gradually transfers her information to the market and the market’s uncertainty about the value of the asset decreases to 0 monotonically. In the interval $[T, \infty)$, the insider trades large volumes and reveals her information immediately, so market prices track the asset value perfectly. Nevertheless, as we explained above, after T , the insider collects strictly positive rents.

There is a vast literature on insider trading³ and many papers have extended Kyle’s model. Holden and Subrahmanyam (1992) and Foster and Viswanathan (1996) considered a market with multiple competing insiders. They showed that competition among insiders accelerates the release of their private information. In a one-period model with heterogeneous insiders, Spiegel and Subrahmanyam (1992) replaced Kyle’s uninformed liquidity traders with strategic utility-maximizing agents trading for hedging purposes. In a multiperiod setting, Mendelson and Tunca (2004) proposed an alternative endogenous liquidity trading model that allowed for various types of market information. Similar to our model, Back and Pedersen (1998) considered the case where the insider continuously observes private information. To prove that an equilibrium exists, they assumed that the insider’s initial amount of private information is relatively high compared to the flow of new information and that this flow decreases fast over time. We show that a similar condition is required in the continuous-time version of our model. Furthermore, we also show that it is precisely when this condition is violated—that is, when the insider’s initial private information is small compared to the inflow on new information—that our equilibrium reaches market efficiency at a fixed time T .

The rest of the paper is organized as follows. Section 2 introduces the discrete-time model and Section 3 constructs a Markovian equilibrium. In Section 4, we show that the Markovian equilibrium has a well defined limit equilibrium as the period length goes to 0 and we provide a full characterization of it. In Section 5, we formulate the continuous-time model and show that the

³For a comprehensive review of this literature and its connection to the broader market microstructure theory, we refer the reader to O’Hara (1997), Brunnermeier (2001), Biais, Glosten, and Spatt (2005), Amihud, Mendelson, and Pedersen (2006), and references therein.

limit equilibrium is *not* an equilibrium. Section 6 includes our concluding remarks.

2. MODEL DESCRIPTION

The market participants are the insider, the market maker, and a (large) number of liquidity traders. The market maker opens the floor for trading only at discrete times $\{t_n\}_{n \geq 0}$. These trading dates are evenly spaced over time (e.g., once a day) so that $t_n = n\Delta$ for some positive constant Δ . The interval of time $[t_n, t_{n+1})$ is called period n . During period n , the following sequence of events occurs. First, the insider (and only her) receives private information about the fundamental value V_n of the asset. Then the insider and the liquidity traders simultaneously place buy/sell orders x_n and y_n , respectively, for a quantity of the asset. An order is a binding contract to buy/sell a quantity of the asset (the “size of the order”) at a price determined by the market maker. Finally, after observing the total volume of trade $z_n = x_n + y_n$, the market maker sets the price p_n and trades the necessary quantity to close all orders. We assume that the market maker is not able to differentiate between insider and liquidity trading. He only observes the net volume of trade z_n .

This trading process continues until a random time τ , independent of the history of transactions and prices, when the fundamental value of the asset becomes public knowledge. At this time, the market price immediately matches the fundamental value and the insider loses her informational advantage. We assume that the public announcement occurs at the end of a period (after trading). That is, $\tau = \eta\Delta$, where $\eta \geq 0$ has a geometric distribution with probability of failure $\rho = e^{-\mu\Delta}$ for some fixed $\mu > 0$.

Liquidity traders are not strategic agents and they trade for idiosyncratic reasons. In particular, we assume that $\{y_n\}_{n \geq 0}$ is a sequence of independent and identically distributed (i.i.d.) normal random variables with mean 0 and variance $\Sigma_y = \sigma_y^2\Delta$. On the other hand, the insider trades strategically so as to maximize her expected net payoff during $[0, \tau]$. The insider’s payoff is driven by her informational advantage as she alone observes the evolution of the fundamental value of the asset V_n during $[0, \tau]$. We assume that V_n evolves as a random walk $\{V_n = V_{n-1} + v_n\}_{n \geq 1}$, where V_0 is normally distributed with mean \bar{V}_0 and variance Σ_0 , and $\{v_n\}_{n \geq 1}$ is a sequence of i.i.d. normal random variables with mean 0 and variance $\Sigma_v = \sigma_v^2\Delta$. The market maker and the rest of the market participants only know the distributions of V_0 and $\{v_n\}_{n \geq 1}$. Hence, V_0 represents a lumpy endowment of private information that the insider gets at time 0, while $\{v_n\}_{n \geq 1}$ represents the incremental private information that she receives over time.

At the beginning of each period n , before the fundamental value becomes public knowledge, the market maker commits to a pricing rule (that is legally binding). The rule specifies the price p_n for the current period’s transactions as a function of the total volume of trade z_n and the public history up to this time.

(For completeness, we define $p_{-1} = \mathbb{E}[V_0] = \bar{V}_0$.) The insider and the liquidity traders place their orders after the rule is announced. All orders are executed at the end of the period. The market maker observes the public history of prices and (total) volumes of trade. His information in period n is represented by the history $\mathcal{F}_n^M = (z_0, p_0, \dots, z_{n-1}, p_{n-1}, z_n)$. Similarly, the insider's information includes the public history of prices and trades, the private history of her orders, and the fundamental values she has observed. Her information in period n is represented by the history $\mathcal{F}_n^I = (v_0, x_0, z_0, p_0, \dots, x_{n-1}, z_{n-1}, p_{n-1}, v_n)$. The insider places her order at the beginning of the period, after observing the current value of the fundamental.

The insider and the market maker are risk-neutral agents. Given a trajectory $X = \{x_n\}$ for the insider's trading and $P = \{p_n\}$ for market prices, the insider's payoff is

$$\Pi(P, X) = \sum_{n=0}^{\eta} [V_n - p_n]x_n.$$

The insider maximizes the expected value of $\Pi(P, X)$. Since η has a geometric distribution,

$$\mathbb{E}[\Pi(P, X)] = \mathbb{E}\left[\sum_{n=0}^{\infty} \rho^n [V_n - p_n]x_n\right].$$

DEFINITION 1: A strategy for the market maker is an \mathcal{F}_n^M -adapted process $P = \{p_n\}_{0 \leq n \leq \eta}$, and a strategy for the insider is an \mathcal{F}_n^I -adapted process $X = \{x_n\}_{0 \leq n \leq \eta}$. The profile (P, X) is an equilibrium if (i) for any $n \geq 0$

$$p_n = \mathbb{E}[V_n | X, \mathcal{F}_n^M],$$

and (ii) given P, X maximizes $\mathbb{E}[\Pi(P, X)]$. Here $\mathbb{E}[V_n | X, \mathcal{F}_n^M]$ means the conditional expectation of V_n given the public history \mathcal{F}_n^M at time n and the insider's strategy X , which specifies how she trades every period as a function of her information.

We do not model explicitly competition among market makers, but we implicitly assume that our market maker competes in prices with other market makers.⁴ In equilibrium, this competition drives the market maker to set the

⁴The model is not exactly a game and our definition of an equilibrium differs from that of a Nash equilibrium. However, Kyle (1985) suggested that the two definitions would coincide in a game where two market makers simultaneously bid prices after observing the current volume of trade and the winner gets the right to clear the market at the winning price. To avoid collusion, we can assume that there is a large population of market makers and that each market maker participates in the bidding game only once.

price equal to the expected value of the asset's fundamental value given the history of information he has observed and the insider's trading strategy (this is condition (i) in Definition 1). The insider chooses her strategy so as to maximize her expected discounted profit, given that she knows how the market maker will choose prices (this is condition (ii) in Definition 1).

We will restrict attention to Markovian equilibria with a particular state space. At the beginning of period n , before the market maker observes the volume of trade, the state is (n, \bar{V}_n, Σ_n) , where $\bar{V}_n = \mathbb{E}[V_n | \mathcal{F}_{n-1}^M, X]$ is the market maker's estimate of V_n and $\Sigma_n = \mathbb{E}[(V_n - \bar{V}_n)^2 | \mathcal{F}_{n-1}^M, X]$ is the variance of this estimate, conditional on the insider's strategy X and market information available at the end of period $n - 1$. Note that since the market maker's estimate of V_n depends on the strategy X of the insider, the state and corresponding Markovian strategy profile need to be specified simultaneously.

DEFINITION 2: A strategy profile (P, X) is Markovian if for each n , the insider's order x_n and the market maker's price p_n depend only on the current state (n, \bar{V}_n, Σ_n) and the signals they receive in period n : v_n for the insider and z_n for the market maker. In this case, we write $x_n = X(n, \bar{V}_n, \Sigma_n, V_n)$ and $p_n = P(n, \bar{V}_n, \Sigma_n, z_n)$. If (P, X) is a Markovian strategy profile, let

$$\Pi_n(\bar{V}_n, \Sigma_n, V_n) = \mathbb{E} \left[\sum_{k=n}^{\eta} (V_k - p_k) x_k \mid \bar{V}_n, \Sigma_n, V_n, (P, X) \right]$$

be the insider's expected payoff-to-go for the transactions made from period n until the fundamental value is publicly revealed, when the current state is (n, \bar{V}_n, Σ_n) and the insider observes V_n . When (P, X) is a Markovian equilibrium, $p_{n-1} = \bar{V}_n$ for all n .

3. MARKOVIAN EQUILIBRIUM

In this section, we construct a *linear Markovian equilibrium* (P, X) , that is, a Markovian equilibrium such that

$$(1) \quad \begin{aligned} P(n, \bar{V}_n, \Sigma_n, z_n) &= \bar{V}_n + \lambda_n(\Sigma_n) z_n, \\ X(n, \bar{V}_n, \Sigma_n, V_n) &= \beta_n(\Sigma_n)(V_n - \bar{V}_n), \end{aligned}$$

where $\{\lambda_n\}$ and $\{\beta_n\}$ are sequences of functions $\lambda_n, \beta_n: \mathbb{R}_{++} \rightarrow \mathbb{R}_+$. The construction exploits the key property that the trajectory $\{\Sigma_n\}$ is *deterministic* and independent of the history of trades. As a result, the sequences $\{\lambda_n\}$ and $\{\beta_n\}$ are also deterministic and hereafter we drop the arguments Σ_n (we also drop this argument in the function Π_n). Moreover, since in equilibrium $\bar{V}_n = p_{n-1}$ for all n , hereafter we do not differentiate these two variables and write, for

example, $\Pi_n(p_{n-1}, V_n)$ instead of $\Pi_n(\bar{V}_n, \Sigma_n, V_n)$.

THEOREM 1: *There exist unique sequences $\{\lambda_n\}, \{\beta_n\} \in \mathbb{R}_{++}$ such that the linear strategy profile (P, X) defined by (1) is a Markovian equilibrium. In equilibrium, $\{\Sigma_n\}$ is a deterministic trajectory that is not affected by the (stochastic) choices of the insider and the market maker. Furthermore, there exist deterministic sequences $\{\alpha_n\}, \{\gamma_n\} \subset \mathbb{R}_{++}$ such that the insider's expected payoff-to-go for (P, X) satisfies*

$$(2) \quad \Pi_n(p, V) = \alpha_n(V - p)^2 + \gamma_n \quad \text{for all } n \geq 0.$$

Given $\Sigma_0 > 0$, there is a unique nonnegative value β_0 that generates the equilibrium profile $\{(\Sigma_n, \beta_n, \lambda_n, \alpha_n, \gamma_n)\}_{n \geq 0}$ recursively through the systems of equations

$$(3) \quad \Sigma_{n+1} = \Sigma_v + \frac{\Sigma_n \Sigma_y}{\beta_n^2 \Sigma_n + \Sigma_y}, \quad \beta_{n+1} \Sigma_{n+1} = \rho \beta_n \Sigma_n \left[\frac{\Sigma_y^2}{\Sigma_y^2 - \beta_n^4 \Sigma_n^2} \right],$$

$$\lambda_n = \frac{\beta_n \Sigma_n}{\beta_n^2 \Sigma_n + \Sigma_y},$$

$$(4) \quad \alpha_n = \frac{1 - \lambda_n \beta_n}{2\lambda_n}, \quad \rho \gamma_{n+1} = \gamma_n - \frac{1 - 2\lambda_n \beta_n}{2\lambda_n(1 - \lambda_n \beta_n)} (\Sigma_v + \lambda_n^2 \Sigma_y),$$

where

$$(5) \quad \gamma_0 = \sum_{k=0}^{\infty} \rho^k \left(\frac{1 - 2\lambda_k \beta_k}{2\lambda_k(1 - \lambda_k \beta_k)} \right) (\Sigma_v + \lambda_k^2 \Sigma_y).$$

PROOF: Let us consider first the market maker's equilibrium condition (i.e., condition (i) in Definition 1). If the insider uses the trading strategy $x_n = \beta_n(V_n - p_{n-1})$ for some deterministic sequence $\{\beta_n\}$, then the market price in period n satisfies

$$p_n = \mathbb{E}[V_n | z_n = y_n + \beta_n(V_n - p_{n-1}), \mathcal{F}_{n-1}^M].$$

Conditional on the available market information \mathcal{F}_{n-1}^M , the pair (V_n, z_n) is a normally distributed two-dimensional random vector. Hence, by the projection theorem for normal random variables, we get that

$$p_n = \mathbb{E}[V_n | \mathcal{F}_{n-1}^M] + \frac{\text{Cov}[V_n, z_n | \mathcal{F}_{n-1}^M]}{\text{Var}[z_n | \mathcal{F}_{n-1}^M]} (z_n - \mathbb{E}[z_n | \mathcal{F}_{n-1}^M])$$

$$= p_{n-1} + \frac{\beta_n \Sigma_n}{\beta_n^2 \Sigma_n + \Sigma_y} z_n.$$

Note that this pricing rule satisfies condition (1) with λ_n as in equation (3). In addition,

$$\begin{aligned}\Sigma_{n+1} &= \text{Var}[V_{n+1}|z_n, \mathcal{F}_{n-1}^M] = \Sigma_v + \text{Var}[V_n|z_n, \mathcal{F}_{n-1}^M] \\ &= \Sigma_v + \text{Var}[V_n|\mathcal{F}_{n-1}^M] \left(1 - \frac{\text{Cov}[V_n, z_n|\mathcal{F}_{n-1}^M]^2}{\text{Var}[V_n|\mathcal{F}_{n-1}^M] \text{Var}[z_n|\mathcal{F}_{n-1}^M]}\right) \\ &= \Sigma_v + \frac{\Sigma_n \Sigma_y}{\beta_n^2 \Sigma_n + \Sigma_y}.\end{aligned}$$

Since Σ_{n+1} is independent of z_n , it follows that the sequence $\{\Sigma_n\}$ is indeed deterministic.

Let us now turn to the insider optimization problem in period n . Assume that the market maker uses the pricing rule $p_n = p_{n-1} + \lambda_n z_n$ for some constant λ_n . Furthermore, suppose that there exist two constants α_{n+1} and γ_{n+1} such that $\Pi_{n+1}(p, V) = \alpha_{n+1}(V - p)^2 + \gamma_{n+1}$. Then, the insider's expected payoff-to-go in period n , $\Pi_n(p_{n-1}, V_n)$, is

$$\begin{aligned}\max_{x_n} \mathbb{E}[(V_n - p_{n-1} - \lambda_n(x_n + y_n))x_n \\ + \rho(\alpha_{n+1}[V_n + v_n - p_{n-1} - \lambda_n(x_n + y_n)]^2 + \gamma_{n+1})].\end{aligned}$$

Under the condition $\rho\lambda_n\alpha_{n+1} < 1$ (otherwise the insider's payoff would be unbounded), the optimization problem above is concave in x and the optimal solution is obtained from the first-order condition

$$(6) \quad x_n = \beta_n(V_n - p_{n-1}), \quad \text{where} \quad \beta_n = \frac{1 - 2\rho\lambda_n\alpha_{n+1}}{2\lambda_n(1 - \rho\lambda_n\alpha_{n+1})}.$$

Thus X_n defined by (1) is indeed the insider's best reply function. Plugging back the optimal value of x_n into the optimization above, we get that

$$\Pi_n(p_{n-1}, V_n) = \frac{(V_n - p_{n-1})^2}{4\lambda_n(1 - \rho\lambda_n\alpha_{n+1})} + \rho(\alpha_{n+1}(\Sigma_v + \lambda_n^2 \Sigma_y) + \gamma_{n+1}).$$

That is, $\Pi_n(p_{n-1}, V_n)$ is a quadratic function of the price gap $V_n - p_{n-1}$, as required, and the coefficients of $\Pi_n(p, V)$ satisfy the recursive equations $\alpha_n = [4\lambda_n(1 - \rho\lambda_n\alpha_{n+1})]^{-1}$ and $\gamma_n = \rho(\gamma_{n+1} + \alpha_{n+1}(\Sigma_v + \lambda_n^2 \Sigma_y))$. Use (6) and then replace the expression for λ_n in (3) to obtain

$$\alpha_n = \frac{1}{4\lambda_n(1 - \rho\lambda_n\alpha_{n+1})} = \frac{1 - \lambda_n\beta_n}{2\lambda_n} = \frac{\Sigma_y}{2\beta_n\Sigma_n}.$$

Invert (6) and then replace the expression for λ_n in (3) to obtain

$$\alpha_{n+1} = \frac{1 - 2\lambda_n\beta_n}{2\rho\lambda_n(1 - \lambda_n\beta_n)} = \frac{\Sigma_y^2 - \beta_n^4 \Sigma_n^2}{2\rho\beta_n\Sigma_n\Sigma_y} = \frac{\Sigma_y}{2\beta_{n+1}\Sigma_{n+1}}.$$

The last equality produces the equation for $\beta_{n+1}\Sigma_{n+1}$ in (3).

Note that the first two equations in (3) and (4) form an independent difference equation in (Σ_n, β_n) alone. For any initial value vector (Σ_0, β_0) , this difference equation has a unique solution. The value of Σ_0 is given, but the value of β_0 is “free.” The next two equations are static: the variables (λ_n, α_n) can be computed independently once the sequence $\{(\Sigma_k, \beta_k)\}$ has been constructed. Similarly, given $\{(\Sigma_k, \beta_k)\}$, $\{\gamma_k\}$ is uniquely defined by the initial value γ_0 and its linear dynamic equation in (4).

To complete the proof, we need to show that there exists a unique value for β_0 that generates—through the recursions (3) and (4)—a sequence $\{(\Sigma_n, \beta_n, \lambda_n, \alpha_n, \gamma_n)\}_{n \geq 0}$ that specifies an equilibrium. In the [Appendix](#), we show that there exists a function $\Psi: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ such that (i) $\beta_0 > \Psi(\Sigma_0)$ leads to bluffing and (ii) $\beta_0 < \Psi(\Sigma_0)$ leads to unbounded payoffs. As we discuss below, both bluffing and unbounded payoffs are not consistent with equilibrium. Hence, for each $\Sigma_0 > 0$, $\beta_0 = \Psi(\Sigma_0)$ is the only feasible choice.

Bluffing: If $\beta_0 > \Psi(\Sigma_0)$, then eventually $\beta_n < 0$ for some n (see the [Appendix](#)). That is, the insider bluffs trading on the wrong side of the spread. Moreover, when $\beta_n < 0$, (3) and (4) imply that $\lambda_n < 0$ and $\alpha_n < 0$, which is a contradiction.

Unbounded Payoffs: If $\beta_0 < \Psi(\Sigma_0)$, the sequence $\{(\Sigma_n, \beta_n)\}$ converges to $(\infty, 0)$. Then the market maker’s strategy specified by the corresponding sequence $\{\lambda_n\}$ allows the insider to extract unbounded payoffs (see Lemmas 1 and 3 in the [Appendix](#)). But the sequence $\{\beta_n\}$ generated by (3) and (4) generates a bounded payoff (see Lemma 2). Therefore, $\{\beta_n\}$ is not optimal for the insider, despite the fact that it was constructed to satisfy local optimality conditions. Thus, in this case, $(\{\lambda_n\}, \{\beta_n\})$ is not an equilibrium.

Iterate the recursive equation for γ_n in (4) to get

$$\begin{aligned} \rho^n \gamma_n &= \gamma_0 - \sum_{k=0}^{n-1} \rho^k \left(\frac{1 - 2\lambda_k \beta_k}{2\lambda_k(1 - \lambda_k \beta_k)} \right) (\Sigma_v + \lambda_k^2 \Sigma_y) \\ &= \gamma_0 - \sum_{k=1}^n \rho^k \alpha_k (\Sigma_v + \lambda_{k-1}^2 \Sigma_y). \end{aligned}$$

When $\beta_0 = \Psi(\Sigma_0)$, the sequence $\{(\Sigma_k, \beta_k, \lambda_k, \alpha_k)\}$ converges (and is independent of γ_0). Therefore, if we choose γ_0 as in (5), then $\lim_{n \rightarrow \infty} \rho^n \gamma_n = 0$. This implies that

$$\lim_{n \rightarrow \infty} \rho^n \mathbb{E}[II_n(p_{n-1}, V_n)] = \lim_{n \rightarrow \infty} \rho^n (\alpha_n \Sigma_n + \gamma_n) = 0.$$

That is, the transversality condition is satisfied. Thus, the sequence $\{\beta_n\}$ is optimal against $\{\lambda_n\}$ and generates the continuation value functions $II_n(p, V) = \alpha_n(V - p)^2 + \gamma_n$.

The optimality conditions included in (3) assume that the value function is quadratic. Lemmas 1 and 2 in the Appendix imply that for any deterministic equilibrium, the value function is quadratic. Thus, we conclude that there is a unique deterministic equilibrium $(\{\lambda_n\}, \{\beta_n\})$, determined by equations (3) and (4) and the initial condition $\beta_0 = \Psi(\Sigma_0)$. Q.E.D.

In general, we can characterize the set of linear Markovian equilibria as those sequences $\{(\Sigma_n, \beta_n, \lambda_n, \alpha_n, \gamma_n)\}_{n \geq 0}$ that satisfy the recursions in (3) and (4) and converge to the (unique) stationary equilibrium of the game $(\hat{\Sigma}, \hat{\beta}, \hat{\lambda}, \hat{\alpha}, \hat{\gamma})$ given by

$$\begin{aligned}
 (7) \quad \hat{\Sigma} &= \frac{1 + \sqrt{1 - \rho}}{\sqrt{1 - \rho}} \Sigma_v, & \hat{\beta} &= \left(\frac{\Sigma_y(1 - \rho)}{\Sigma_v(1 + \sqrt{1 - \rho})} \right)^{1/2}, \\
 \hat{\lambda} &= \left(\frac{\Sigma_v(1 - \sqrt{1 - \rho})}{\Sigma_y \rho} \right)^{1/2}, \\
 \hat{\alpha} &= \frac{1}{2} \left(\frac{\Sigma_y}{\Sigma_v(1 + \sqrt{1 - \rho})} \right)^{1/2}, & \hat{\gamma} &= \frac{\rho \hat{\alpha} (\Sigma_v + \hat{\lambda}^2 \Sigma_y)}{1 - \rho}.
 \end{aligned}$$

The following proposition highlights some additional properties of an equilibrium profile $\{(\Sigma_n, \beta_n)\}_{n \geq 0}$ and its proof follows directly from the properties of Ψ discussed in the proof of Theorem 1 in the Appendix. Some of these properties are used in the proof of Theorem 2 that characterizes the limiting continuous-time profile as Δ goes to 0.

PROPOSITION 1: *Let $\{(\Sigma_n, \beta_n)\}_{n \geq 0}$ be the discrete-time equilibrium of Theorem 1. Suppose Σ_0 is greater (less) than or equal to $\hat{\Sigma}$. Then the sequences $\{\Sigma_n\}$ and $\{\beta_n \Sigma_n\}$ are decreasing (increasing) in n .*

According to Proposition 1, despite the fact that the insider’s trades are informative and reduce the market uncertainty, when the initial variance $\Sigma_0 < \hat{\Sigma}$, Σ_n increases with n . In this case, the variance reduction induced by insider trading is insufficient to compensate for the additional uncertainty generated by the evolution of $\{V_n\}$, and market prices are less informative over time.

4. CONTINUOUS-TIME APPROXIMATION

In this section, we analyze the limit of the discrete-time linear equilibrium of Theorem 1 as Δ goes to 0. In particular, we will show that the discrete-time linear Markovian equilibrium $\{(\Sigma_n, \beta_n, \lambda_n, \alpha_n, \gamma_n) : n \geq 0\}$ converges point-

wise to a continuous-time profile $\{(\Sigma_t, \beta_t, \lambda_t, \alpha_t, \gamma_t) : t \in \mathbb{R}_+\}$ in an appropriate sense.

First, let us explicitly rewrite the discrete-time model in terms of the calendar time t . For any time $t \geq 0$, the corresponding trading period is denoted by $n_t = \lfloor t/\Delta \rfloor$. We would like to express the insider's strategy $x_n = \beta_n(V_n - p_{n-1})$ in terms of her trading rate per unit of time. For this, we define $\beta^\Delta(t) = \beta_{n_t}/\Delta$. For any $t \geq 0$, define the continuous time extensions

$$\begin{aligned} p^\Delta(t) &= p_{n_t-1}, & V^\Delta(t) &= V_{n_t}, & \Sigma^\Delta(t) &= \Sigma_{n_t}, \\ \lambda^\Delta(t) &= \lambda_{n_t}, & \alpha^\Delta(t) &= \alpha_{n_t}, & \gamma^\Delta(t) &= \gamma_{n_t}, \\ \Pi^\Delta(t) &= \Pi_{n_t}(p^\Delta(t), V^\Delta(t)) = \alpha^\Delta(t)(V^\Delta(t) - p^\Delta(t))^2 + \gamma^\Delta(t), \end{aligned}$$

and the cumulative trading processes

$$X^\Delta(t) = \sum_{k=0}^{n_t} x_k, \quad Y^\Delta(t) = \sum_{k=0}^{n_t} y_k, \quad Z^\Delta(t) = X^\Delta(t) + Y^\Delta(t).$$

For ease of exposition, we assume that there exist two independent Brownian motions B_t^y and B_t^v such that $y_n = \sigma_y(B_{(n+1)\Delta}^y - B_{n\Delta}^y)$ and $v_n = \sigma_v(B_{(n+1)\Delta}^v - B_{n\Delta}^v)$. It follows that $Y^\Delta(t)$ and $V^\Delta(t)$ converge uniformly over compact sets to $Y_t = \sigma_y B_t^y$ and $V_t = \sigma_v B_t^v$, respectively. Also, in the limit, as $\Delta \downarrow 0$, τ is exponentially distributed with rate μ . Finally, recall that $\Sigma_y = \sigma_y^2 \Delta$, $\Sigma_v = \sigma_v^2 \Delta$, and $\rho = e^{-\mu \Delta}$.

THEOREM 2: *Let T be the unique nonnegative root of the equation*

$$\Sigma_0 + \sigma_v^2 T = \sigma_v^2 \left[\frac{e^{2\mu T} - 1}{2\mu} \right]$$

and define, for all $t \geq 0$,

$$\begin{aligned} (8) \quad \Sigma_t &= \frac{\sigma_v^2}{2\mu} \left[e^{2\mu(T-t)^+} - 2\mu(T-t)^+ - 1 \right], & \beta_t &= \frac{\sigma_v \sigma_y e^{\mu(T-t)^+}}{\Sigma_t}, \\ \lambda_t &= \frac{\sigma_v e^{\mu(T-t)^+}}{\sigma_y}, & \alpha_t &= \frac{\sigma_y e^{-\mu(T-t)^+}}{2\sigma_v}, \\ \gamma_t &= \frac{\sigma_y \sigma_v e^{-\mu(T-t)^+}}{4\mu} \left[e^{2\mu(T-t)^+} + 2\mu(T-t)^+ + 3 \right]. \end{aligned}$$

Then $(\Sigma^\Delta(t), \beta^\Delta(t), \lambda^\Delta(t), \alpha^\Delta(t), \gamma^\Delta(t))$ converges pointwise to $(\Sigma_t, \beta_t, \lambda_t, \alpha_t, \gamma_t)$ as $\Delta \downarrow 0$ for all $t \geq 0$. The market price $P^\Delta(t)$, the insider cumulative trading process $X^\Delta(t)$, and the market trading process $Z^\Delta(t)$ converge weakly to P_t ,

X_t , and Z_t , respectively, solutions of the system of stochastic differential equations (SDEs)

$$dZ_t = dX_t + dY_t, \quad dP_t = \lambda_t dZ_t,$$

$$dX_t = \begin{cases} \beta_t(V_t - P_t) dt, & \text{if } t < T, \\ \sigma_y dB_t^y + \sigma_v dB_t^v, & \text{if } t \geq T, \end{cases}$$

with border conditions $Z_0 = X_0 = Y_0 = 0$ and $P_0 = \mathbb{E}[V_0]$. The insider expected payoff converges to $\Pi_t = \alpha_t(V_t - p_t)^2 + \gamma_t$.

As in Kyle’s (1985) model, we could be tempted to argue that the limiting profile $(\Sigma_t, \beta_t, \lambda_t, \alpha_t, \gamma_t)$ is an equilibrium of a continuous-time model in which trades and prices change continuously. In Section 5, however, we will show that this (continuity) property does not hold in our model. Hence, we can only interpret the continuous-time profile $(\Sigma_t, \beta_t, \lambda_t, \alpha_t, \gamma_t)$ as an asymptotically good approximation of the discrete-time equilibrium of Theorem 1 when agents trade frequently. With this interpretation, we will refer to $(\Sigma_t, \beta_t, \lambda_t, \alpha_t, \gamma_t)$ as the *limit equilibrium*.

Theorem 2 reveals a number of important features of the limit equilibrium. A remarkable property is the existence of a finite time T , endogenously determined, such that $\Sigma_t = 0$ for $t \geq T$. That is, for Δ sufficiently small, the insider essentially reveals all her private information by time T . After T , the price always matches the fundamental value of the asset. Despite this market efficiency the insider is still able to collect positive rents ($\Pi(t, 0) = \gamma_T > 0$) in $t \in [T, \infty)$. The source of these rents is the continuous inflow of new information that the insiders gets by privately observing the evolution of the fundamental value. Indeed, one can show (see Theorem 3 below) that in the absence of these rents, either because V_t is constant or because the insider loses her capacity to track V_t , the insider would have no incentive to speed up her trading and market efficiency would only be reached asymptotically ($T = \infty$).

To get a sense of how likely it is that market efficiency is reached in the limit equilibrium, let us compare T and the average time $1/\mu$ at which the announcement date occurs. From the definition of T in Theorem 2, we can show that

$$T \leq \frac{1}{\mu} \quad \text{if} \quad \Sigma_0 \leq \left(\frac{e^2 - 3}{2}\right) \frac{\sigma_v^2}{\mu} \sim 2 \frac{\sigma_v^2}{\mu}.$$

Roughly speaking, the previous inequalities suggest that, on average, market efficiency is reached when the insider’s initial (lumpy) private information Σ_0 is less than twice her average cumulative inflow of new private information σ_v^2/μ . Furthermore, one can show that as $\sigma_v \rightarrow \infty$, the switching time T converges to 0 and market efficiency is reached instantaneously. On the other hand, as $\sigma_v \downarrow 0$, the switching time T diverges to $+\infty$ and efficiency is only reached

asymptotically. The volatility coefficient σ_v determines the amount of information asymmetry. The following proposition shows that the higher is σ_v , the faster the insider reveals her information, but also the larger is her profit. Let $\mathbb{E}[II_t]$ be the market's best estimate of the insider's expected continuation payoff from time t on, that is, $\mathbb{E}[II_t] = \alpha_t \Sigma_t + \gamma_t$. Because of the deterministic evolution of Σ_t , α_t , and γ_t , $\mathbb{E}[II_t]$ is also the insider's ex ante (at time 0, before observing any signals) expected payoff-to-go from t onward.

PROPOSITION 2: *In the limit equilibrium, the value of Σ_t weakly decreases with σ_v for all $t \geq 0$. On the other hand, $\mathbb{E}[II_t]$ is equal to*

$$\mathbb{E}[II_t] = \frac{\sigma_v \sigma_y}{\mu} \cosh(\mu(T - t)^+),$$

which is increasing in σ_v for all $t \geq 0$.

The more volatile is the fundamental value, the faster the price adjusts to the current intrinsic value. However, this efficiency comes at a cost. Indeed, the insider is willing to trade away her private information faster because the market maker compensates her for doing so. Hence, we expect market prices to be more informative when the volatility of the fundamental value is higher. For example, when there is no volatility ($\sigma_v = 0$), market efficiency ($\Sigma_t = 0$) is reached only asymptotically as $t \rightarrow \infty$ and the insider's ex ante payoff is minimized.

In a discrete-time equilibrium, the market maker's expected payoff is 0. This property is preserved in the limit equilibrium of Theorem 2. Thus, the liquidity traders' expected loss must equal the insider's expected profit, $\mathbb{E}[II_t]$, which according to Proposition 2 decreases monotonically with time in $[0, T)$ and stays constant after T . Thus, liquidity traders who place their orders late in the game expect to make smaller losses.

Theorem 2 also shows that the market maker fulfills his obligation in a rather strong sense after T . He is concerned with setting prices so that $p_t = \mathbb{E}[V_t | \mathcal{F}_t^M]$. Theorem 2 implies that p_t converges uniformly on compact sets to V_t in $[T, \infty)$.⁵ As a result, after T , the insider trading volume X_t behaves as a Brownian motion and has unbounded variation. It is also interesting to note that $X_t - X_T$ is independent of σ_v .

Finally, we note that the limit equilibrium satisfies the smooth-pasting condition

$$\lim_{t \uparrow T} \dot{\Sigma}_t = 0.$$

This is in contrast to the equilibria obtained in models that assume a fixed announcement date (e.g., Kyle (1985)), where Σ_t does not approach 0 smoothly.

⁵This follows from the Skorohod representation theorem and the fact that $M_t = V_t - p_t$ converges weakly to (the continuous process) 0 for $t \geq T$.

5. CONTINUOUS-TIME EQUILIBRIUM

In this section, we formulate the continuous-time counterpart of the discrete-time model of Section 3 and show that the limit equilibrium of Theorem 2 is not an equilibrium of this model. However, we also show that in a modified continuous-time model where the insider’s flow of new information is bounded, the limit equilibrium is an equilibrium.

Similar to the discrete-time model, we denote by V_t the fundamental value of the asset at time t which evolves as an arithmetic Brownian motion, $V_t = \sigma_v B_t^v$.

A strategy profile is a pair of processes (X, P) , where $X_t \in \mathcal{F}_t^I$ is the insider’s cumulative trading volume up to time t , and $P_t \in \mathcal{F}_t^M$ is the price set by the market maker at time t . Following the formulation of the continuous-time model in Back (1992), we restrict the trading process X to the class \mathcal{S} of continuous, \mathcal{F}_t^I -adapted square-integrable semimartingales. This is a technical requirement that allows us to write the insider payoff as a stochastic integral of the market price with respect to her trading strategy. More precisely, for a given profile (X, P) , the insider’s expected discounted payoff, $\mathbb{E}[\Pi(P, X)]$, is defined as

$$\mathbb{E}[\Pi(P, X)] = \mathbb{E}\left[V_\tau X_\tau - \int_0^\tau P_t dX_t - [X, P]_\tau\right],$$

where $[X, P]_t$ is the quadratic covariation between X_t and P_t .⁶ A continuous-time equilibrium is a profile (X, P) with the following properties: (i) given P , $X \in \mathcal{S}$ maximizes $\mathbb{E}[\Pi(X, P)]$, and (ii) the price process P satisfies the equilibrium condition

$$P_t = \mathbb{E}[V_t | \mathcal{F}_t^M, X], \quad 0 \leq t < \tau.$$

For the analysis that follows, we find it convenient to rewrite the insider’s payoff using the following identity

$$V_\tau X_\tau = \int_0^\tau V_t dX_t + \int_0^\tau X_t dV_t + \int_0^\tau d[X, V]_t,$$

where $[X, V]_t$ is the quadratic covariation between X_t and V_t . Plugging this identity back into Π , taking expectation, and canceling the stochastic integral with respect to the martingale V_t , we get

$$\begin{aligned} \mathbb{E}[\Pi(P, X)] = \mathbb{E}\left[\int_0^\infty e^{-\mu t} (V_t - P_t) dX_t \right. \\ \left. + \int_0^\infty e^{-\mu t} d[X, V]_t - \int_0^\infty e^{-\mu t} d[X, P]_t \right], \end{aligned}$$

⁶Intuitively, this term arises because the price paid by the insider is computed at the end of the period, and, therefore, it includes the effect of the insider’s last trade dX_t . For a formal derivation, see equation (11) in Back (1992).

since τ is exponentially distributed with rate μ and is independent of \mathcal{F}_t^I .

Now we show that the strategy profile (P, X) associated to the limit equilibrium of Theorem 2 cannot be an equilibrium of the continuous-time model. Consider the insider’s expected payoff-to-go from time T onward,

$$\begin{aligned} \Pi_T(P, X) = \mathbb{E} & \left[\int_T^\infty e^{-\mu(t-T)} (V_t - P_t) dX_t \right. \\ & \left. + \int_T^\infty e^{-\mu(t-T)} d[X, V]_t - \int_T^\infty e^{-\mu(t-T)} d[X, P]_t \right]. \end{aligned}$$

After time T , the market maker’s pricing strategy P is given by $dP_t = \lambda_T dZ_t$, where $\lambda_T = \sigma_v/\sigma_y$, and the insider’s cumulative volume of trade is a martingale process such that $dX_t = \sigma_y[dB_t^v - dB_t^y]$. Thus, $V_t - P_t \equiv 0$, the first stochastic integral with respect to X_t has 0 expectation, and the quadratic covariations between X_t and V_t and between X_t and P_t satisfy $d[X, V]_t = \sigma_y \sigma_v dt$ and $d[X, P]_t = \lambda_t \sigma_y^2 dt = \sigma_y \sigma_v dt$, respectively. It follows that $\Pi_T(P, X) = 0$ and so X cannot be a best reply to P . This shows that there is a discontinuity in the insider’s payoff function as we move from discrete time to continuous time. Indeed, recall that the insider’s payoff Π^A of the discrete-time equilibrium of Theorem 2 satisfies $\lim_{\Delta \downarrow 0} \Pi_t^A = (\sigma_y \sigma_v)/\mu > 0$ for all $t \geq T$. This discontinuity is the result of the divergence of the insider’s trading rate β_t^A to infinity for $t \geq T$ as $\Delta \downarrow 0$.⁷ In turn, this divergence is due to the existence of an unbounded flow of future private information. When the inflow of new information is small (for example, when $\sigma_v = 0$ because V_t is constant or the insider cannot track V_t after $t = 0$), the insider would collect small rents after the market reaches full efficiency. Therefore, the insider instead spends her private information slowly and market efficiency is reached only asymptotically ($T = \infty$). In this case the limit equilibrium of Theorem 2 is effectively an equilibrium of the continuous-time game. This and other properties of the equilibrium are summarized in the following theorem.

We now assume that the insider’s strategy belongs to the space \mathcal{B} of trade rates β such that

$$(9) \quad \mathbb{E} \left[\int_0^\infty e^{-\mu t} |\beta_t| M_t^2 dt \right] < \infty.$$

Condition (9) rules out some bluffing schemes where the insider trades in the “wrong” direction and accumulates unbounded losses before accumulating unbounded gains.

⁷In a modified model with quadratic transactional costs, the insider’s trading strategy would be bounded (as a referee suggested) and we expect the limit of discrete-time equilibria to be itself an equilibrium of the continuous-time model.

THEOREM 3: *Suppose the asset's volatility $\sigma_v(t)$ is a function of time, and let Γ_t be the insider's cumulative inflow of private information from time t onward, that is,*

$$\Gamma_t = \int_t^\infty \sigma_v^2(t) dt.$$

Assume that $\Gamma_0 < \infty$ and $(\Sigma_0 + \Gamma_0)e^{-2\mu t} > \Gamma_t$ for all t . When the insider's strategy space is constrained by (9), there exists a continuous-time linear Markovian equilibrium that satisfies

$$(10) \quad \Sigma_t = (\Sigma_0 + \Gamma_0)e^{-2\mu t} - \Gamma_t, \quad \lambda_t = \sqrt{\frac{2\mu(\Sigma_0 + \Gamma_0)}{\sigma_y^2}} e^{-\mu t}, \quad \beta_t = \frac{\sigma_y^2 \lambda_t}{\Sigma_t},$$

$$(11) \quad \alpha_t = \frac{e^{\mu t}}{2} \sqrt{\frac{\sigma_y^2}{2\mu(\Sigma_0 + \Gamma_0)}}, \quad \gamma_t = \alpha_t \Gamma_t + \frac{\sigma_y^2 \lambda_t}{4\mu}.$$

Under the conditions of Theorem 3, in equilibrium $\Sigma_t \downarrow 0$ as $t \rightarrow \infty$, but $\Sigma_t > 0$ for all $t \geq 0$. More importantly, the trading rate β_t remains bounded for all $t \geq 0$, so the insider's strategy is a process of bounded variation. When the flow of new information is substantial, the insider is happy to trade intensely to exploit current arbitrage opportunities. Even though in the process she "informs" the market about what she knows now, new arbitrage opportunities will develop soon. In the limit equilibrium, she transfers all her information (initial + flow) by time T , but when this flow is relatively low, she is not willing to trade that fast.

6. CONCLUSIONS

The paper introduces a model that combines a random announcement time with an insider who receives a flow of information. The new model produces a (limit) equilibrium with novel features. Two distinct regimes emerge. Before the endogenous time T , the insider is indifferent about how to consume her information stock, which includes the initial signal and the flow information she receives in the interval $(0, T]$. Nevertheless, in equilibrium she exhausts all this stock by time T , so that the market reaches full efficiency at time T . After T , she is eager to exhaust any additional piece of information immediately. As she does, she keeps the market fully informed until the public announcement, which reveals no further information.

The flow of new information, that in principle exacerbates the informational asymmetry, in equilibrium induces the insider to release her information faster. Interestingly, the market is uniformly better informed and reaches full efficiency earlier when this source of informational asymmetry (the variation of

the innovation process) is larger. However, the larger the asymmetry, the larger are the rents extracted by the insider.

The analysis also exposes a potential difficulty with continuous-time models. The natural discrete-time model has an equilibrium that, albeit difficult to construct explicitly, has a well defined limit as the period length decreases to 0. However, this limit equilibrium *is not* an equilibrium of the corresponding continuous-time model.

APPENDIX

DEFINITION OF Ψ : To characterize the function $\Psi(z)$, we find it convenient to introduce the change of variables

$$A_n = \frac{\Sigma_n}{\Sigma_v}, \quad B_n = \frac{\beta_n \Sigma_n}{\sqrt{\Sigma_y \Sigma_v}}.$$

Then equation (3) implies that $(A_{n+1}, B_{n+1}) = F(A_n, B_n)$, where

$$F_A(A_n, B_n) = 1 + \frac{A_n^2}{A_n + B_n^2}, \quad F_B(A_n, B_n) = \rho \left[\frac{A_n^2 B_n}{A_n^2 - B_n^4} \right].$$

Let

$$G_1(A) = \sqrt{\frac{A}{A-1}}, \quad G_2(A) = \sqrt{A}[1-\rho]^{1/4}, \quad G_3(A) = \sqrt{A}.$$

Since in equilibrium β_n must be positive for all n , a point (A, B) is *feasible* only if $F_B(A, B) \geq 0$, that is, only if $B \leq G_3(A)$. The function G_1 is defined so that $F_A(A, G_1(A)) = A$. If $B > G_1(A)$, then $F_A(A, B) < A$, and if $B < G_1(A)$, then $F_A(A, B) > A$. Similarly, the function G_2 is defined so that $F_B(A, G_2(A)) = B$. If $B > G_2(A)$, then $F_B(A, B) > B$, and if $B < G_1(A)$, then $F_B(A, B) < B$. As Figure 1 shows, the graphs of these functions partition the (A, B) space into five regions. In R_1 , $F(A, B)$ is always to the left and higher than (A, B) , and any sequence $\{(A_n, B_n)\}$ with initial point (A_0, B_0) in this region eventually crosses the graph of G_3 and becomes infeasible. In R_2 , $F(A, B)$ is always to the left and lower than (A, B) . In R_3 , $F(A, B)$ is always to the right and lower than (A, B) . In R_4 , $F(A, B)$ is always to the right and higher than (A, B) . R_5 is the region of infeasible points. In Figure 1 we have also plotted four sequences, each starting in a different region. A sequence that remains feasible must start in R_2 , R_3 , or R_4 , and any sequence that starts in R_3 always remain feasible, but not all sequences that start in R_2 or R_4 remain feasible. Sequences that start in R_1 always become infeasible.

By definition, the intersection of the graphs of G_1 and G_2 defines a stationary point (\hat{A}, \hat{B}) such that $(\hat{A}, \hat{B}) = F(\hat{A}, \hat{B})$ (see equation (7) for details of this stationary point in terms of the original state variables).

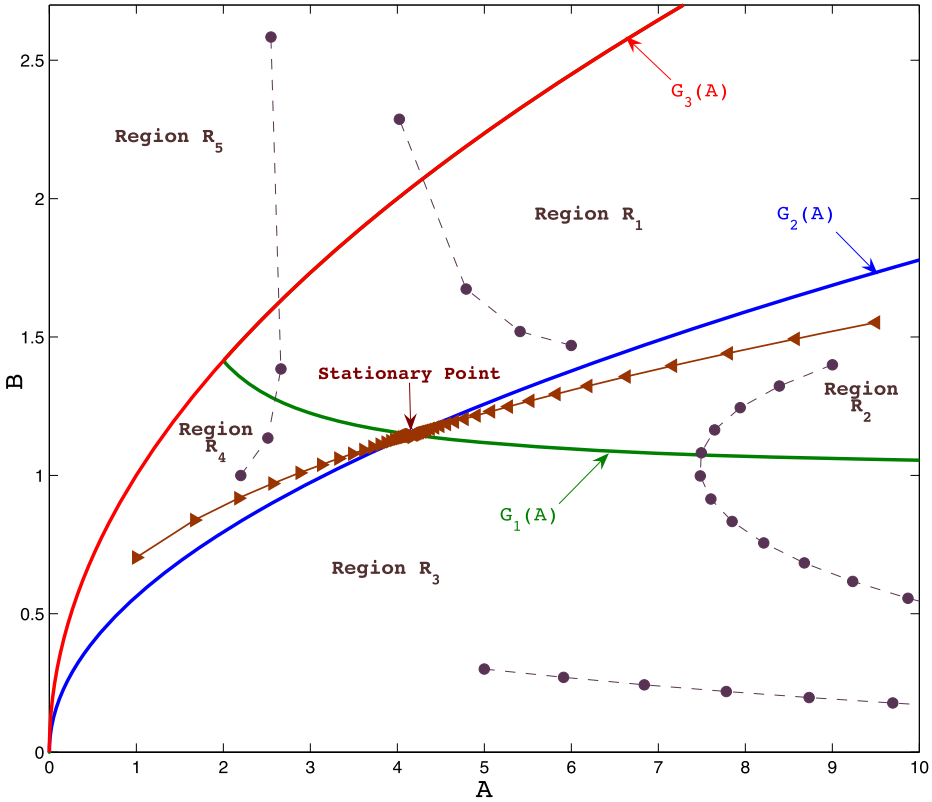


FIGURE 1.—Partition induced by the functions G_1 , G_2 , and G_3 .

Now, by continuity of the vector field F , there exists a curve \mathcal{C} , contained in $R_2 \cup R_4$ and passing through (\hat{A}, \hat{B}) , such that $F(A, B) \in \mathcal{C}$ for all $(A, B) \in \mathcal{C}$. That is, \mathcal{C} is the largest subset of \mathbb{R}^2 such that $F(\mathcal{C}) \subset \mathcal{C}$ and $(\hat{A}, \hat{B}) \in \mathcal{C}$. We do not have an analytical representation for \mathcal{C} , but we can approximate it numerically. This curve is strictly increasing and it approaches the origin to the left (but it does not contain it). Therefore, there exists a strictly increasing function $\psi: (0, \infty) \rightarrow (0, \infty)$, such that $(A, B) \in \mathcal{C}$ if and only if $B = \psi(A)$. For any initial $A_0 > 0$, let $B_0 = \psi(A_0)$. Then the sequence $\{(A_n, B_n)\}$, where $(A_{n+1}, B_{n+1}) = F(A_n, B_n)$ for each n , is contained in \mathcal{C} (that is, $B_n = \psi(A_n)$ for all $n \geq 0$) and, therefore, remains feasible forever. Moreover, $(A_n, B_n) \rightarrow (\hat{A}, \hat{B})$ as $n \rightarrow \infty$. When $A_0 < \hat{A}$ (respectively, $A_0 > \hat{A}$), $B_0 < \hat{B}$ ($B_0 > \hat{B}$) and $\{(A_n, B_n)\}$ is monotonically increasing (decreasing). In summary, for any given $\Sigma_0 > 0$, if we initialize

$$\beta_0 = \Psi(\Sigma_0), \quad \text{where} \quad \Psi(\Sigma_0) = \frac{\sqrt{\Sigma_y \Sigma_v}}{\Sigma_0} \psi\left(\frac{\Sigma_0}{\Sigma_v}\right),$$

we obtain a feasible sequence $\{(\Sigma_n, \beta_n, \lambda_n, \alpha_n)\}$. Moreover, this sequence converges. In particular, $\{\lambda_n\}$ is decreasing and converges to $\hat{\lambda} > 0$. Therefore, there exists $M > 0$ such that $\lambda_n \leq M$ for all n and

$$\sum_{n=1}^{\infty} \frac{\rho^n}{\lambda_n} < +\infty.$$

LEMMA 1: *Assume that the market maker's strategy $\{P_n\}$ is specified by a deterministic sequence $\{\lambda_n\} \subset \mathbb{R}_{++}$. Let*

$$S = \sum_{n=1}^{\infty} \frac{\rho^n}{\lambda_n}.$$

If $S = \infty$, then $\Pi_n(p, V) = \infty$ for all $n \geq 0$ and $(p, V) \in \mathbb{R}^2$. If $S < \infty$ and there is $M > 0$ such that $\lambda_n < M$ and $\rho\lambda_n/\lambda_{n+1} \leq 1$ for all $n \geq 0$, then there exist positive sequences $\{\alpha_n\}$ and $\{\gamma_n\}$ such that $\lambda_n\alpha_{n+1} \leq 1/2$ and $\rho\Pi_n(p, V) = \alpha_n(p - V)^2 + \gamma_n$ for all $n \geq 0$.

PROOF: For each $n \geq 0$ and each $k \geq 0$, consider the finite horizon problem for the insider where the fundamental value is made public at the end of period $n + k$ if it has not been publicly revealed before. Let $\Pi_{k,n}(p, V)$ be the insider's optimal discounted value from period n onward in this problem, when the price and fundamental value in period $n - 1$ are (p, V) . Obviously, $\Pi_{k,n}(p, V) \leq \Pi_n(p, V)$ (because the insider can always choose $x_s = 0$ for all $s > n + k$) and $\lim_{k \rightarrow \infty} \Pi_{k,n}(p, V) = \Pi_n(p, V)$ for all $n \geq 0$ and all $(p, V) \in \mathbb{R}^2$.

We first show inductively in k that for each n , either

$$(12) \quad \Pi_{k,n}(p, V) = \frac{a_{k,n}}{\lambda_n}(V - p)^2 + \frac{b_{k,n}}{\lambda_n}\Sigma_v + c_{k,n}\lambda_n\Sigma_y$$

for some constants $(a_{k,n}, b_{k,n}, c_{k,n})$ or $\Pi_{k,n} \equiv \infty$. When $k = 0$,

$$\begin{aligned} \Pi_{0,n}(p, V) &= \max(V - p - \lambda_n x)x \\ &= \frac{(V - p)^2}{4\lambda_n}, \end{aligned}$$

so $\Pi_{0,n}$ satisfies (12) with $a_{0,n} = 1/4$ and $b_{0,n} = c_{0,n} = 0$ for all $n \geq 0$. By induction, assume first that $\Pi_{k,n+1}$ satisfies (12) for a given (k, n) . We then show that either $\Pi_{k+1,n}$ also satisfies (12) or $\Pi_{k+1,n} \equiv \infty$. We have that

$$\begin{aligned} \Pi_{k+1,n}(p, V) &= \max_{x \in \mathbb{R}}(V - p - \lambda_n x)x \\ &\quad + \rho\mathbb{E}[\Pi_{k,n+1}(V + W_n, p + \lambda_n(x + Y_n))] \end{aligned}$$

$$\begin{aligned}
 &= \max_{x \in \mathbb{R}} (V - p - \lambda_n x)x \\
 &\quad + \rho \left[\frac{a_{k,n+1}}{\lambda_{n+1}} [(V - p - \lambda_n x)^2 + \Sigma_v + \lambda_n^2 \Sigma_y] \right. \\
 &\quad \left. + \frac{b_{k,n+1}}{\lambda_{n+1}} \Sigma_v + c_{k,n+1} \lambda_{n+1} \Sigma_y \right].
 \end{aligned}$$

When $\rho a_{k,n+1} \lambda_n / \lambda_{n+1} < 1$, the quadratic objective function is concave and $\Pi_{k+1,n}$ satisfies (12) with

$$(13) \quad a_{k+1,n} = \frac{1}{4} \left[1 - a_{k,n+1} \frac{\rho \lambda_n}{\lambda_{n+1}} \right]^{-1},$$

$$(14) \quad b_{k+1,n} = \frac{\rho \lambda_n}{\lambda_{n+1}} [a_{k,n+1} + b_{k,n+1}], \quad c_{k+1,n} = \rho \left[a_{k,n+1} \frac{\lambda_n}{\lambda_{n+1}} + c_{k,n+1} \frac{\lambda_{n+1}}{\lambda_n} \right].$$

When $\rho a_{k,n+1} \lambda_n / \lambda_{n+1} \geq 1$, the quadratic objective function is convex and $\Pi_{k+1,n} \equiv \infty$. By induction, now assume instead that $\Pi_{k,n+1} \equiv \infty$. Then $\Pi_{k+n+1-s,s} \equiv \infty$ for all $s = 0, \dots, n$. This concludes the proof by induction.

Let us now assume that $\sum \rho^n / \lambda_n = \infty$. In this case, we will show that $\Pi_{k,n}(p, v) \rightarrow \infty$ as $k \rightarrow \infty$, for all n and (p, v) .

Since $a_{0,n} = 1/4$ and $\rho \lambda_n / \lambda_{n+1} > 0$ for all $n \geq 0$, it is easy to see (by induction) that (13) implies $a_{k,n} > 1/4$ for all $k \geq 1$ and $n \geq 0$. Fix $n \geq 0$. For any $k \geq 1$, if there exists $j \in \{1, \dots, k\}$ such that $1 \leq a_{k-j,t+j} \rho \lambda_{n+j} / \lambda_{n+j+1}$, then $\Pi_{k-j+1,n+j-1} \equiv \infty$, which implies that $\Pi_{k,n} \equiv \infty$ and $\Pi_n \equiv \infty$. Conversely, if $1 > a_{k-j,t+j} \rho \lambda_{n+j} / \lambda_{n+j+1}$ for all $j \in \{1, \dots, k\}$, then (14) implies that

$$\begin{aligned}
 b_{k,n} &\geq \frac{\rho \lambda_n}{\lambda_{n+1}} \left[\frac{1}{4} + b_{k-1,n+1} \right] \geq \frac{\rho \lambda_n}{\lambda_{n+1}} \left[\frac{1}{4} + \frac{\rho \lambda_{n+1}}{\lambda_{n+2}} \left[\frac{1}{4} + b_{k-2,n+2} \right] \right] \geq \dots \\
 &\geq \frac{\lambda_n}{4} \left[\frac{\rho}{\lambda_{n+1}} + \dots + \frac{\rho^k}{\lambda_{n+k}} \right].
 \end{aligned}$$

Note that

$$\sum_{j=1}^{\infty} \frac{\rho^j}{\lambda_{n+j}} = \frac{1}{\rho^n} \sum_{j=t+1}^{\infty} \frac{\rho^j}{\lambda_j} = \frac{1}{\rho^n} \left[\sum_{j=1}^{\infty} \frac{\rho^j}{\lambda_j} - \sum_{j=1}^n \frac{\rho^j}{\lambda_j} \right] = \infty.$$

Therefore,

$$\Pi_n(p, v) \geq \Pi_{k,n}(p, v) \geq \frac{\Sigma_v}{\lambda_n} b_{k,n} \geq \frac{\Sigma_v}{4} \sum_{j=1}^k \frac{\rho^j}{\lambda_{n+j}} \quad \text{for all } k \geq 1,$$

which implies again that $\Pi_n \equiv \infty$ since the last term converges to ∞ as $k \rightarrow \infty$. Thus, either way, $\Pi_n \equiv \infty$.

Finally, assume that $\sum \rho^n/\lambda_n < \infty$ and $\rho\lambda_n/\lambda_{n+1} \leq 1$ for all $n \geq 1$. In this case, we show that each $\Pi_n(p, V)$ is a quadratic function of $(V - p)$.

Since $a_{0,n} = 1/4$, it is easy to show by induction that $1/4 < a_{k,n} < 1/2$ for all $k \geq 1$ and $n \geq 0$. The function $f(a, d) = [4(1 - ad)]^{-1}$ is increasing in a and d when $ad < 1$. Since $a_{1,n+1} > 1/4 = a_{0,n+1}$ for all $n \geq 0$, $a_{2,n} = f(a_{1,n+1}, d_n) > f(a_{0,n+1}, d_n) = a_{1,n}$ for all $n \geq 0$. Repeating this argument forward, we conclude that $\{a_{k,n}\}_{k=1}^\infty$ is an increasing sequence and it must converge. Let $\alpha_n = \lim_{k \rightarrow \infty} a_{k,n}/\lambda_n$. Now $a_{k,n+1} < 1/2$ for all k and $\rho\lambda_n/\lambda_{n+1} \leq 1$ imply that $\lambda_n\alpha_{n+1} \leq 1/2$.

Again, $a_{k,n} < 1/2$ for all $k \geq 0$ and $n \geq 0$ imply that

$$b_{k,n} \leq \frac{\rho\lambda_n}{\lambda_{n+1}} \left[\frac{1}{2} + b_{k-1,n+1} \right] \leq \dots \leq \frac{\lambda_n}{2} \left[\frac{\rho}{\lambda_{n+1}} + \dots + \frac{\rho^k}{\lambda_{n+k}} \right] < \frac{\lambda_n}{2\rho^n} \sum_{j=t+1}^\infty \frac{\rho^j}{\lambda_j} < \infty.$$

By induction in k , we now show that $b_{k,n} < b_{k+1,n}$ for all $k \geq 0$ and $n \geq 1$. Clearly $b_{0,n} = 0 < b_{1,n}$ for all $n \geq 0$. Since $a_{k,n+1} < a_{k+1,n+1}$, if the inequality holds for (k, n) , then

$$b_{k+1,n} = d_n[a_{k,n+1} + b_{k,n+1}] < d_n[a_{k+1,n+1} + b_{k+1,n+1}] = b_{k+2,n}.$$

That is, for each $n \geq 0$, the sequence $\{b_{k,n}\}_{k=0}^\infty$ is increasing and hence it must converge. Solving (14), we obtain

$$c_{k,n} = \frac{1}{\lambda_n} \sum_{j=1}^k \rho^j \frac{\lambda_{n+j-1}^2}{\lambda_{n+j}} a_{k-j,n+j}.$$

One can show that for each $n \geq 0$, the sequence $\{c_{k,n}\}_{k=1}^\infty$ is increasing, and since $\lambda_s \leq M$ and $a_{j,s} < 1/2$ for all j and s ,

$$c_{k,n} \leq \frac{M^2}{2\lambda_n} \sum_{j=1}^k \frac{\rho^j}{\lambda_{n+j}} < \frac{M^2}{2\lambda_n\rho^n} \sum_{j=t+1}^\infty \frac{\rho^j}{\lambda_j} < \infty$$

and the sequence must converge. Let $\gamma_n = \lim_{k \rightarrow \infty} [b_{k,n}\Sigma_v/\lambda_n + c_{k,n}\lambda_n\Sigma_y]$. Then $\Pi_n(p, v) = \alpha_n(v - p)^2 + \gamma_n$. Q.E.D.

LEMMA 2: *Let $\{\beta_n\}$ be an arbitrary deterministic strategy for the insider. Assume that $\{\lambda_n\}$ satisfies the equilibrium condition $p_n = \mathbb{E}[V|\{\beta_n\}, \mathcal{F}_n^M]$. Then the insider's expected payoff when she follows $\{\beta_n\}$ is finite.*

PROOF: The insider's payoff satisfies

$$\Pi = \sum_{n=0}^{\eta} (V_{\eta} - p_n)x_n = \sum_{n=0}^{\eta} \beta_n (V_{\eta} - p_n)(V_n - p_{n-1}).$$

Let us write p_n and V_n in terms of the primitive stochastic sequences $\{v_n\}$ and $\{y_n\}$ with $v_0 = V_0$. We have that $V_n = \sum_{n=0}^n v_n$. In addition,

$$\begin{aligned} p_n &= p_{n-1} + \lambda_n (\beta_n (V_n - p_{n-1}) + y_n) \\ &= (1 - \lambda_n \beta_n) p_{n-1} + \lambda_n \beta_n \sum_{n=0}^n v_n + \lambda_n y_n. \end{aligned}$$

Suppose that there exist sequences $A(n)$, $B(k, n)$, and $C(k, n)$ such that $A(0) = 1$, $B(0, 0) = C(0, 0) = B(k, n) = C(k, n) = 0$ for $k > n$, and

$$p_n = A(n)p_{-1} + \sum_{k=0}^n [B(k, n)v_k + C(k, n)y_k].$$

It follows that

$$\begin{aligned} A(n) &= (1 - \lambda_n \beta_n) A(n-1), \\ B(k, n) &= \lambda_n \beta_n + (1 - \lambda_n \beta_n) B(k, n-1) \quad \text{for } 0 \leq k \leq n, \\ C(k, n) &= (1 - \lambda_n \beta_n) C(k, n-1) \quad \text{for } 0 \leq k < n \quad \text{and} \\ C(n, n) &= \lambda_n. \end{aligned}$$

For $j \geq i$, let $\psi(i, j) := \prod_{k=i}^j (1 - \lambda_k \beta_k)$. Iterating the recursions above, we get

$$\begin{aligned} A(n) &= \psi(0, n), \quad B(k, n) = \sum_{j=k}^n \psi(j+1, n) \lambda_j \beta_j, \\ C(k, n) &= \psi(k+1, n) \lambda_k. \end{aligned}$$

The insider's conditional expected payoff given η is

$$\begin{aligned} \mathbb{E}[\Pi | \eta] &= \sum_{n=0}^{\eta} \beta_n \mathbb{E}[(V_{\eta} - p_n)(V_n - p_{n-1}) | \eta] \\ &= \sum_{n=0}^{\eta} \beta_n \mathbb{E}[(V_n - p_n)(V_n - p_{n-1})] = \sum_{n=0}^{\eta} [D_n^1 + D_n^2 + D_n^3], \end{aligned}$$

where

$$\begin{aligned}
 D_n^1 &= \beta_n A(n) A(n-1) p_{-1}, \\
 D_n^2 &= \beta_n \sum_{k=0}^n (1 - B(k, n-1))(1 - B(k, n)) \Sigma_v(n), \\
 D_n^3 &= \beta_n \sum_{k=0}^n C(k, n-1) C(k, n) \Sigma_y,
 \end{aligned}$$

$\Sigma_v(0) = \Sigma_0$, and $\Sigma_v(n) = \Sigma_v$ for $n > 0$. We now bound these terms. We have that

$$\begin{aligned}
 \lambda_n &= \frac{\beta_n \Sigma_n}{\beta_n^2 \Sigma_n + \Sigma_y} \\
 \Rightarrow 1 - \lambda_n \beta_n &= \frac{\Sigma_y}{\beta_n^2 \Sigma_n + \Sigma_y} \in (0, 1) \quad \text{for all } n \geq 0.
 \end{aligned}$$

Hence, $A(n)$ is a nonnegative decreasing sequence with $0 \leq A(n) \leq A(0) \leq 1$. Moreover, $0 \leq B(k, n) \leq B(0, n)$ for all $0 \leq k \leq n$. The sequence $B(0, n)$ satisfies the recursion

$$B(0, n) = \lambda_n \beta_n + (1 - \lambda_n \beta_n) B(0, n-1), \quad B(0, 0) = 0.$$

Therefore, $0 \leq B(k, n) \leq B(0, n) \leq 1$. Also, the function $f(x) = x/(ax^2 + 1)$ reaches its global maximum at $x = \pm\sqrt{1/a}$. Therefore,

$$|\lambda_n| \leq \frac{\sqrt{\Sigma_n}}{1 + \Sigma_y} \leq \frac{1}{2} \sqrt{\frac{\Sigma_n}{\Sigma_y}}, \quad |\beta_n| (1 - \lambda_n \beta_n) = \lambda_n \frac{\Sigma_y}{\Sigma_n} \leq \frac{1}{2} \sqrt{\frac{\Sigma_y}{\Sigma_n}}.$$

Thus we obtain the bounds

$$|D_n^1| = |\beta_n| (1 - \lambda_n \beta_n) \prod_{k=0}^{n-1} (1 - \lambda_k \beta_k)^2 \leq \frac{1}{2} \sqrt{\frac{\Sigma_y}{\Sigma_n}} \leq \frac{1}{2} \sqrt{\frac{\Sigma_y}{\Sigma_v}},$$

since $1 - \lambda_k \beta_k \in (0, 1)$ and $\Sigma_n \geq \Sigma_v$ for all $n \geq 1$. For the second term we have that

$$\begin{aligned}
 |D_n^2| &= |\beta_n| (1 - \lambda_n \beta_n) \sum_{k=0}^n (1 - B(k, n-1))^2 \Sigma_v(n) \\
 &\leq \frac{n+1}{2} \sqrt{\frac{\Sigma_y}{\Sigma_v}} \max\{\Sigma_0, \Sigma_v\}.
 \end{aligned}$$

Finally, for the third term we have that

$$|D_n^3| = |\beta_n|(1 - \lambda_n\beta_n) \sum_{k=0}^n C(k, n - 1)^2 \Sigma_y \leq \frac{\Sigma_y}{2} \sqrt{\frac{\Sigma_y}{\Sigma_v}} \sum_{k=0}^n \lambda_k^2,$$

where the last inequality uses the fact that $C(k, n - 1) \leq \lambda_k$. The recursion for Σ_{n+1} implies that $\Sigma_{n+1} \leq \Sigma_v + \Sigma_n$. Therefore, $\Sigma_n \leq n\Sigma_v + \Sigma_0$. We conclude that

$$|D_n^3| \leq \frac{\Sigma_y}{2} \sqrt{\frac{\Sigma_y}{\Sigma_v}} \sum_{k=0}^n \frac{\Sigma_n}{4\Sigma_y} \leq \frac{1}{8} \sqrt{\frac{\Sigma_y}{\Sigma_v}} \frac{(n + 1)(n + 2)}{2} \max\{\Sigma_0, \Sigma_v\}.$$

Combining all the pieces together we get that

$$|D_n| \leq \frac{1}{2} \sqrt{\frac{\Sigma_y}{\Sigma_v}} \left[1 + \left(1 + \frac{n + 2}{8} \right) (n + 1) \max\{\Sigma_0, \Sigma_v\} \right] \leq Kn^2$$

for some constant K . Therefore,

$$\mathbb{E}[II] = \mathbb{E}[\mathbb{E}[II|\eta]] = \sum_{n=0}^{\infty} \rho^n D_n < \infty. \tag{Q.E.D.}$$

LEMMA 3: Choose $\beta_0 < \Psi(\Sigma_0)$, and let $\{\lambda_n\}$ and $\{\beta_n\}$ be the corresponding strategies generated by (3) and (4) for the market maker and the insider. Then $\sum \rho^n/\lambda_n = \infty$ and the insider can make infinite profits. Moreover, $\{\beta_n\}$ is not a best reply against $\{\lambda_n\}$.

PROOF: We show that if $\beta_0 < \Psi(\Sigma_0)$, then $\sum \rho^n/\lambda_n = \infty$. Lemma 1 above then implies that the insider’s expected payoff is unbounded. However, by Lemma 2, $\{\beta_n\}$ generates finite profits. Therefore, $\{\beta_n\}$ is not optimal against $\{\lambda_n\}$.

When $\beta_0 < \Psi(\Sigma_0)$, the sequence $\{(A_n, B_n)\}$ lies below \mathcal{C} and remains feasible forever. Moreover, for some finite N , $(A_n, B_n) \in R_3$ for all $n \geq N$. Therefore, $A_n < A_{n+1}$ for all $n \geq N$ and $A_n \rightarrow \infty$. Recall that the graphs of G_1 and G_2 intersect at (\hat{A}, \hat{B}) , and that $(A, B) \in R_3$ and $A \geq \hat{A}$ imply that $B \leq G_1(A)$. The function $h(A) = (A - 1)^2/[A(A - 2)]$ is decreasing for all $A > 2$, and $h(A) \rightarrow 1$ as $A \rightarrow \infty$. Let $\omega \in (\rho, 1)$. Without loss of generality, assume that N is such that $A_N \geq \hat{A}$ and $h(A_N) \leq \omega/\rho$. Then $B_n \leq G_1(A_n)$ for all $n \geq N$ and, therefore, for all $n \geq N$,

$$\begin{aligned} (15) \quad B_{n+1} &= F_B(A_n, B_n) = \rho \left[\frac{A_n^2 B_n}{A_n^2 - B_n^4} \right] \\ &\leq \rho \left[\frac{A_n^2 B_n}{A_n^2 - [G_1(A_n)]^4} \right] = \rho h(A_n) B_n \leq \omega B_n. \end{aligned}$$

Since $B_N \leq \hat{B}$, this implies that $B_n \leq \hat{B}\omega^{n-N}$ for all $n \geq N$. From equation (3),

$$\lambda_n = \frac{\beta_n \Sigma_n}{\beta^2 \Sigma_n + \Sigma_y} = \frac{A_n B_n}{A_n + B_n^2} \sqrt{\frac{\Sigma_v}{\Sigma_y}} < B_n \sqrt{\frac{\Sigma_v}{\Sigma_y}}.$$

Since we would like to show that $\sum \rho^n / \lambda_n = \infty$, we need a tighter upper bound on B_n . Note, however, that

$$B_{n+1} = F_B(A_n, B_n) = \rho \left[\frac{A_n^2 B_n}{A_n^2 - B_n^4} \right] \geq \rho B_n \quad \text{for all } n \geq 0,$$

so there is not a lot of slack in the previous upper bound (15) for B_{n+1} .

For any $\varepsilon > 0$, let $N^* > N$ be such that $\hat{B}\omega^{N^*-N} < \varepsilon$. Then, for all $n \geq N^*$,

$$\begin{aligned} A_{n+1} &= F_A(A_n, B_n) = 1 + \frac{A_n^2}{A_n + B_n^2} \geq 1 + \frac{A_n}{1 + \varepsilon^2 / A_n} \\ &\geq 1 + A_n \left[1 - \frac{\varepsilon^2}{A_n} \right] = A_n + (1 - \varepsilon^2). \end{aligned}$$

Let $e = 1 - \varepsilon^2$. Then $A_{N^*+n} \geq A_{N^*} + ne > ne$ for all $n \geq N^*$. Feeding this bound back into (15), we obtain that

$$\begin{aligned} B_{N^*+n+3} &\leq \rho h((n+2)e) B_{N^*+2+n} \leq \dots \\ &\leq \rho^n h((n+2)e) h((n+1)e) \dots h(3e) B_{N^*+3}. \end{aligned}$$

Choose $\varepsilon < 1/4$ so that $\varepsilon^2 < 1/16$. Then, for all $k \geq 3$,

$$\begin{aligned} h(ke) &= \frac{[k - 1 - k\varepsilon^2]^2}{[k - k\varepsilon^2][k - 2 - k\varepsilon^2]} \\ &= 1 + \frac{1}{k(k-2) - 2k(k-1)\varepsilon^2 + k^2\varepsilon^4} \\ &< 1 + \frac{1}{k[k-2-2(k-1)\varepsilon^2]} < 1 + \frac{8}{k[7k-15]} \leq 1 + \frac{4}{k^2}. \end{aligned}$$

Let

$$\begin{aligned} H_n &= \left[1 + \frac{4}{1^2} \right] \left[1 + \frac{4}{2^2} \right] \dots \left[1 + \frac{4}{n^2} \right], \\ a_n &= \frac{1}{H_n} = \left[\frac{1^2}{1^2 + 4} \right] \dots \left[\frac{n^2}{n^2 + 4} \right]. \end{aligned}$$

Note that $[1 + 4/1^2][1 + 4/2^2] = 10$. Hence, $B_{N^*+n+3} < \rho^n B_{N^*+3} H_{n+2}/10$. Therefore,

$$\sqrt{\frac{\sum_v}{\sum_y}} \sum_{n \geq 1} \frac{\rho^n}{\lambda_n} > \sum_{n \geq 1} \frac{\rho^n}{B_n} > \sum_{n \geq 1} \frac{10\rho^{N^*+3+n}}{\rho^n H_{n+2} B_{N^*+3}} = \frac{10}{B_{N^*+3}} \rho^{N^*+3} \sum_{n \geq 3} a_n.$$

Gauss’s test (see, for example, Knopp (1990)) states that if

$$\frac{a_{n+1}}{a_n} = 1 - \frac{c}{n} - \frac{g_n}{n^\varepsilon},$$

where $\varepsilon > 1$ and $\{g_n\}$ is bounded, then $\sum a_n$ converges when $c > 1$ and diverges when $c \leq 1$. In our case,

$$\frac{a_{n+1}}{a_n} = \frac{(n + 1)^2}{(n + 1)^2 + 4} = 1 - \left[\frac{4n^2}{(n + 1)^2 + 4} \right] \frac{1}{n^2},$$

so $c = 0$ and $\varepsilon = 2$. Therefore $\sum a_n = \infty$, so $\sum \rho^n / \lambda_n = \infty$ and the insider makes infinite profits. Q.E.D.

The following lemma is used in the proof of Theorem 2.

LEMMA 4: *Let $\{f_n\}$ be a sequence of convex functions on $[T, \infty)$ (where $T \in \mathbb{R}$ is arbitrary). Assume that f_n converges pointwise to 0. That is, for all $t \geq T$, $f_n(t) \rightarrow 0$ as $n \rightarrow \infty$. Then, for each $t > T$, $\partial f_n(t) \rightarrow \{0\}$.*

PROOF: By contradiction, assume that there exists $t^* > T$ and a subgradient $s_n \in \partial f_n(t^*)$ for each n , such that $\{s_n\}$ does not converge to 0. Without loss of generality, assume that $s_n \rightarrow \bar{s} < 0$. Then, for each $t \in [T, t^*)$, $f_n(t) \geq f_n(t^*) + s_n(t - t^*)$, and taking limits as $n \rightarrow \infty$, we obtain $0 \geq -\bar{s}(t^* - t) > 0$, which is a contradiction. Q.E.D.

PROOF OF THEOREM 2: To emphasize the dependence of the discrete-time equilibrium on the length of a period, let us denote by $\{(\Sigma_n^\Delta, \beta_n^\Delta, \lambda_n^\Delta, \alpha_n^\Delta, \gamma_n^\Delta)\}$ the discrete-time equilibrium of Theorem 1 and denote by $(\widehat{\Sigma}^\Delta, \widehat{\beta}^\Delta, \widehat{\lambda}^\Delta, \widehat{\alpha}^\Delta, \widehat{\gamma}^\Delta)$ the corresponding stationary equilibrium for an arbitrary $\Delta > 0$. Since $\Sigma_0 > 0$, it follows that $\Sigma_0 > \widehat{\Sigma}^\Delta \sim O(\sqrt{\Delta})$ for Δ sufficiently small. As a result, the sequences $\{\Sigma_n^\Delta\}$ and $\{\lambda_n^\Delta\}$ are monotonically decreasing, while the sequences $\{\beta_n^\Delta\}$ and $\{\alpha_n^\Delta\}$ are monotonically increasing in n for Δ sufficiently small. Also, recall that the profile $(\Sigma^\Delta(t), \beta^\Delta(t), \lambda^\Delta(t), \alpha^\Delta(t), \gamma^\Delta(t))$ is a continuous-time piecewise-linear approximation of the discrete-time equilibrium such that $(\Sigma^\Delta(t), \beta^\Delta(t), \lambda^\Delta(t), \alpha^\Delta(t), \gamma^\Delta(t)) = (\Sigma_n^\Delta, \beta_n^\Delta/\Delta, \lambda_n^\Delta, \alpha_n^\Delta, \gamma_n^\Delta)$ for all $t \in [n\Delta, (n + 1)\Delta)$.

The remaining proof is divided into two parts. In Part I, we show the pointwise convergence of $(\Sigma^\Delta(t), \beta^\Delta(t), \lambda^\Delta(t), \alpha^\Delta(t), \gamma^\Delta(t))$ to $(\Sigma_t, \beta_t, \lambda_t, \alpha_t, \gamma_t)$ as $\Delta \downarrow 0$. In Part II, we prove the weak convergence of $(P^\Delta(t), X^\Delta(t), Z^\Delta(t))$ to (P_t, X_t, Z_t) .

Part I. The proof of this part is organized as follows. First, we show that equations (3) and (4), which characterize the evolution of $\{(\Sigma_n^\Delta, \beta_n^\Delta, \lambda_n^\Delta, \alpha_n^\Delta, \gamma_n^\Delta)\}$, converge to a system of ordinary differential equations as $\Delta \downarrow 0$. Then we show that the solution of these ordinary differential equations (ODEs) defines a continuous-time profile $(\Sigma(t), \beta(t), \lambda(t), \alpha(t), \gamma(t))$ that is arbitrarily closed (as $\Delta \downarrow 0$) to $(\Sigma^\Delta(t), \beta^\Delta(t), \lambda^\Delta(t), \alpha^\Delta(t), \gamma^\Delta(t))$ for all $t < T$, where $T = \sup\{t > 0: \Sigma(s) > 0, \forall s < t\}$. As in the discrete-time case, $\beta(0)$ is a free parameter for this continuous-time profile. Finally, we show that $\beta(0)$ is uniquely determined using two properties of the discrete-time equilibrium: (i) Σ_n^Δ is decreasing in n , which provides a lower bound on $\beta(0)$, and (ii) $\beta_n^\Delta \Sigma_n^\Delta$ is decreasing in n , which provides an upper bound on $\beta(0)$. We conclude the first part of the proof, showing that these upper and lower bounds coincide.

For notational convenience, let us define $q^\Delta(t) := \beta^\Delta(t)\Sigma^\Delta(t)$ and $r^\Delta(t) := \Delta/\Sigma^\Delta(t)$. The recursive equations (3) and (4) imply that

$$\begin{aligned}
 (16) \quad & \frac{\Sigma^\Delta(t + \Delta) - \Sigma^\Delta(t)}{\Delta} = \sigma_v^2 - \frac{(q^\Delta(t))^2}{\sigma_y^2 + (q^\Delta(t))^2 r^\Delta(t)}, \\
 & \frac{q^\Delta(t + \Delta) - q^\Delta(t)}{\Delta} = \left[\frac{\sigma_y^4(e^{-\mu\Delta} - 1)/\Delta + (q^\Delta(t))^4 r^\Delta(t)/\Sigma^\Delta(t)}{\sigma_y^4 - (q^\Delta(t))^4 (r^\Delta(t))^2} \right] q^\Delta(t), \\
 & \lambda^\Delta(t) = \frac{q^\Delta(t)}{\sigma_y^2 + (q^\Delta(t))^2 r^\Delta(t)}, \quad \alpha^\Delta(t) = \frac{1 - \lambda^\Delta(t)\beta^\Delta(t)\Delta}{2\lambda^\Delta(t)}, \\
 & \frac{\gamma^\Delta(t + \Delta) - \gamma^\Delta(t)}{\Delta} \\
 & = \frac{e^{\mu\Delta} - 1}{\Delta} \gamma^\Delta(t) - \frac{1 - 2\lambda^\Delta(t)\beta^\Delta(t)\Delta}{2\lambda^\Delta(t)(1 - \lambda^\Delta(t)\beta^\Delta(t)\Delta)} (\sigma_v^2 + (\lambda^\Delta(t))^2 \sigma_y^2) e^{\mu\Delta}.
 \end{aligned}$$

For a given t , suppose that $\limsup_{\Delta \downarrow 0} (\beta^\Delta(t))^2 \Sigma^\Delta(t) < \infty$. Then $(q^\Delta(t))^2 r^\Delta(t)/\sqrt{\Delta}$ is negligible for Δ sufficiently small and, as $\Delta \downarrow 0$, the system of equations (16) converges to

$$(17) \quad \dot{\Sigma}(t) = \sigma_v^2 - \frac{q(t)^2}{\sigma_y^2}, \quad \dot{q}(t) = -\mu q(t),$$

$$(18) \quad \lambda(t) = \frac{1}{2\alpha(t)} = \frac{q(t)}{\sigma_y^2}, \quad \dot{\gamma}(t) = \mu \gamma(t) - \frac{\sigma_v^2 + \lambda(t)^2 \sigma_y^2}{2\lambda(t)}.$$

Since $\Sigma_0 > 0$, the condition $\limsup_{\Delta \downarrow 0} (q^\Delta(t))^2 r^\Delta(t) = 0$ is satisfied at $t = 0$ (otherwise $\liminf_{\Delta \downarrow 0} \lambda_\Delta^\Delta = \infty$). It follows (by continuity) that the convergence above

holds for all $t \in [0, T)$ for some positive $T > 0$. Then by integrating (17) in this range, we obtain a continuous-time profile $(\Sigma(t), \beta(t), \lambda(t), \alpha(t), \gamma(t))$ given by

$$(19) \quad \begin{aligned} \Sigma(t) &= \Sigma_0 + \sigma_v^2 t - \frac{(\beta(0)\Sigma_0)^2}{2\mu\sigma_y^2}(1 - e^{-2\mu t}), \\ q(t) &= \beta(0)\Sigma_0 e^{-\mu t} \quad \text{for } t < T, \end{aligned}$$

for some constant of integration $\beta(0)$. Note that for this continuous-time solution, the condition $\limsup_{\Delta \downarrow 0} (q^\Delta(t))^2 r^\Delta(t) = 0$ reduces to $\Sigma(t) > 0$. Hence, given $\beta(0)$, T is uniquely determined as the smallest (positive) solution of the equation $\Sigma(t) = 0$. We denote by $T(\beta(0))$ this value which solves

$$0 = \Sigma_0 + \sigma_v^2 T - \frac{(\beta(0)\Sigma_0)^2}{2\mu\sigma_y^2}(1 - e^{-2\mu T}).$$

Suppose $\beta(0)$ is small enough so that $T(\beta(0)) = \infty$. Then $\Sigma(t) > 0$ for all $t \geq 0$ and $\lim_{\Delta \downarrow 0} (\Sigma^\Delta(t), \beta^\Delta(t)) = (\Sigma(t), \beta(t))$ for all $t \geq 0$. But in this case $\lim_{t \rightarrow \infty} \Sigma(t) = \infty$, which implies that $\lim_{t \rightarrow \infty} \Sigma^\Delta(t) = \infty$ for Δ sufficiently small, contradicting the monotonicity of Σ_n^Δ . As a result, $T(\beta(0)) < \infty$ and so $\beta(0)$ is bounded below by $\beta^L(0)$ such that $\Sigma(t) = \dot{\Sigma}(t) = 0$ at $t = T(\beta^L(0))$. That is, $\beta^L(0)$ satisfies

$$\sigma_v^2 - \frac{(\beta^L(0)\Sigma_0)^2}{\sigma_y^2} e^{-2\mu T} = 0$$

where T solves

$$0 = \Sigma_0 + \sigma_v^2 T - \sigma_v^2 \left(\frac{e^{2\mu T} - 1}{2\mu} \right).$$

Suppose now that $\beta(0) > \beta^L(0)$ so that $T(\beta(0)) < \infty$. Then the fact that Σ_n^Δ is monotonically decreasing in n for all Δ and equation (19) lead to

$$\begin{aligned} \Sigma_t &= \lim_{\Delta \downarrow 0} \Sigma^\Delta(t) \\ &= \begin{cases} \Sigma_0 + \sigma_v^2 t - \frac{(\beta(0)\Sigma_0)^2}{2\mu\sigma_y^2}(1 - e^{-2\mu t}), & \text{if } t < T(\beta(0)), \\ 0, & \text{if } t \geq T(\beta(0)). \end{cases} \end{aligned}$$

In what follows, we show that for $t \geq T(\beta(0))$,

$$(20) \quad \lim_{\Delta \downarrow 0} \frac{\Sigma^\Delta(t + \Delta) - \Sigma^\Delta(t)}{\Delta} = 0.$$

Equation (16) and the fact that $\Sigma^\Delta(t)$ and $q^\Delta(t)$ are decreasing functions of t imply that $\Sigma^\Delta(t)$ is convex for all $t \geq T(\beta(0))$. Hence, by Lemma 4, $\Sigma^\Delta(t)$ satisfies equation (20). We can use this result together with the first equation in (16) to show that for $t \geq T(\beta(0))$,

$$\lim_{\Delta \downarrow 0} q^\Delta(t) = \lim_{\Delta \downarrow 0} \widehat{\beta}^\Delta \widehat{\Sigma}^\Delta = \sigma_v \sigma_y \quad \text{a.e.}$$

This follows from the fact that $q^\Delta(t)$ is decreasing in t for all Δ and, therefore, is bounded, which implies

$$\lim_{\Delta \downarrow 0} (q^\Delta(t))^2 r^\Delta(t) = \lim_{\Delta \downarrow 0} (\beta^\Delta(t) \Sigma^\Delta(t))^2 \frac{\Delta}{\Sigma^\Delta(t)} = 0$$

since $\Sigma^\Delta(t) \geq \widehat{\Sigma}^\Delta \geq \sigma_v(\Delta + \sqrt{\Delta/\mu})$. On the other hand, from equation (19) we get that

$$\lim_{t \uparrow T(\beta_0)} \lim_{\Delta \downarrow 0} q^\Delta(t) = \beta(0) \Sigma_0 e^{-\mu T(\beta(0))}.$$

Hence, unless $\beta(0) \Sigma_0 e^{-\mu T(\beta(0))} = \sigma_v \sigma_y$ (or equivalently $\beta(0) = \beta^L(0)$), the limit function $q(t)$ would have a discontinuity at $t = T(\beta(0))$. But from the second equation in (16), such a discontinuity is not possible because the term

$$\left[\frac{\sigma_y^4 (e^{-\mu\Delta} - 1) / \Delta + (q^\Delta(t))^4 r^\Delta(t) / \Sigma^\Delta(t)}{\sigma_y^4 - (q^\Delta(t))^4 (r^\Delta(t))^2} \right] q^\Delta(t)$$

is uniformly bounded in t and Δ^8 , which by the Arzelà–Ascoli theorem implies that the limit function $q(t)$ is continuous.

Part II. To prove the weak convergence of $(P^\Delta(t), X^\Delta(t), Z^\Delta(t))$ to (P_t, X_t, Z_t) , we introduce the price gap processes $M_t^\Delta := V_t^\Delta - P_t^\Delta$ and $M_t := V_t - P_t$, and show that M_t^Δ converges weakly to M_t . Specifically, we will invoke Theorem 2.1 in Prokhorov (1956) and prove the convergence of the finite-dimensional distributions of M_t^Δ to those of M_t , and then show the compactness of M_t^Δ in Δ .

For any $\Delta > 0$, the corresponding discrete-time equilibrium characterizes the values of M_t^Δ , β_t^Δ , and λ_t^Δ only at the discrete sequence of times $\{i\Delta\}_{i \geq 0}$. To extend these functions to \mathbb{R}_+ , we introduce the following notation: for any $t > 0$, we define, $n_t^\Delta := \lim_{s \uparrow t} \lfloor s/\Delta \rfloor$ and $\underline{t}^\Delta := n_t^\Delta \Delta$, and for any function f_t^Δ , we define $f_{\underline{t}^\Delta}^\Delta := f_{n_t^\Delta \Delta}^\Delta$. Using a slight abuse of notation, we redefine the continuous piecewise linear version of M_t^Δ for any $t > 0$ as

$$M_t^\Delta = M_{\underline{t}^\Delta}^\Delta (1 - \lambda_{\underline{t}^\Delta}^\Delta \beta_{\underline{t}^\Delta}^\Delta (t - \underline{t}^\Delta)) + \sigma_v (B_t^v - B_{\underline{t}^\Delta}^v) - \lambda_{\underline{t}^\Delta}^\Delta \sigma_y (B_t^y - B_{\underline{t}^\Delta}^y),$$

⁸This follows from the fact that in equilibrium, $q^\Delta(t)$ is nonnegative and decreasing in t , and $(q^\Delta(t))^4 (r^\Delta(t))^2 \leq \sigma_y^4 (1 - \rho)$ (see Figure 1).

with border condition $M_0^\Delta = V_0 - \mathbb{E}[V_0]$. Since we are only concerned with the weak convergence of M_t^Δ , we will simplify the notation, replacing the term $\sigma_v(B_t^v - B_{\underline{t}^\Delta}^v) - \lambda_{\underline{t}^\Delta}^\Delta \sigma_y(B_t^y - B_{\underline{t}^\Delta}^y)$ by $\sigma_{\underline{t}^\Delta}^\Delta(B_t - B_{\underline{t}^\Delta})$, where B_t is a Wiener process and $(\sigma_{\underline{t}^\Delta}^\Delta)^2 = \sigma_v^2 + \sigma_y^2(\lambda_{\underline{t}^\Delta}^\Delta)^2$. Iterating the recursion for M_t^Δ above, we get that

$$M_t^\Delta = M_0^\Delta A(0, n_t^\Delta) + \sum_{k=0}^{n_t^\Delta} A(k+1, n_t^\Delta) \sigma_{k\Delta}^\Delta (B_{(k+1)\Delta \wedge t^\Delta} - B_{k\Delta}),$$

where

$$A(j, n_t^\Delta) := \prod_{k=j}^{n_t^\Delta} (1 - \lambda_{k\Delta}^\Delta \beta_{k\Delta}^\Delta (\min\{(k+1)\Delta, t^\Delta\} - k\Delta)).$$

Since both λ_t^Δ and β_t^Δ are deterministic processes, it follows that M_t^Δ is a Gaussian process. Hence, its finite-dimensional distribution is fully characterized by its mean and variance–covariance processes. For $t > 0$, we have

$$\mu_t^\Delta := \mathbb{E}[M_t^\Delta] = M_0^\Delta A(0, n_t^\Delta).$$

From Part I, we know that (i) λ_t^Δ converges pointwise to a smooth, strictly positive, and bounded function λ_t for all $t \geq 0$, and (ii) the function β_t^Δ is nondecreasing in t for all $t \geq 0$ and converges pointwise to a smooth nondecreasing function β_t in $[0, T)$ and to infinity in $t \geq T$. We conclude that as $\Delta \downarrow 0$, then

$$\lim_{\Delta \downarrow 0} \mu_t^\Delta = \mathbb{1}(t < T) e^{-\int_0^t \lambda(s)\beta(s) ds},$$

where $\mathbb{1}(t < T)$ is the indicator function equal to 1 if $t < T$ and equal to 0 otherwise. Similarly, if we define $\Gamma^\Delta(t, t') := \mathbb{E}[(M_t^\Delta - \mu_t^\Delta)(M_{t'}^\Delta - \mu_{t'}^\Delta)]$ to be the variance–covariance process of M_t^Δ , then as $\Delta \downarrow 0$, we get (for $t < t'$)

$$\lim_{\Delta \downarrow 0} \Gamma^\Delta(t, t') = \mathbb{1}(t' < T) e^{-\int_t^{t'} \lambda_s \beta_s ds} \int_0^t e^{-2\int_s^t \lambda_u \beta_u du} (\sigma_v^2 + \lambda_s^2 \sigma_y^2) ds.$$

Consider now the limiting gap process $M_t = V_t - P_t$. It follows from the system of SDEs in Theorem 2 that M_t satisfies the SDE

$$dM_t = -\lambda_t \beta_t M_t + \sigma_y dB_t^y - \lambda_t \sigma_y dB_t^v \quad \text{for all } t < T$$

and $M_t = 0$ for $t \geq T$. As a result, M_t is also a Gaussian process. Furthermore, for $t < T$, we can integrate the SDE above to get

$$M_t = M_0 e^{-\int_0^t \lambda_s \beta_s ds} + \int_0^t e^{-\int_s^t \lambda_u \beta_u du} (\sigma_v dB_s^v - \lambda_s \sigma_y dB_s^y).$$

It is a matter of simple calculations to show that the mean process $\mathbb{E}[M_t]$ and the variance–covariance process $\Gamma(t, t') = \mathbb{E}[(M_t - \mathbb{E}[M_t])(M_{t'} - \mathbb{E}[M_{t'}])]$ coincide with $\lim_{\Delta \downarrow 0} \mu_t^\Delta$ and $\lim_{\Delta \downarrow 0} \Gamma^\Delta(t, t')$ computed above. We conclude that the finite-dimensional distribution of M_t^Δ converges to the finite-dimensional distribution of M_t for all $t \geq 0$. In particular, it is worth noticing that M_T^Δ converges weakly to 0 as $\Delta \downarrow 0$.

We now prove that $\{M_t^\Delta : \Delta > 0\}$ is tight in $[0, T]$ for an arbitrary $T > 0$. For this we show that for every $\varepsilon > 0$

$$\lim_{\delta \downarrow 0} \limsup_{\Delta \downarrow 0} \mathbb{P}\left(\sup_{|t-s| \leq \delta} |M_t^\Delta - M_s^\Delta| \geq \varepsilon\right) = 0.$$

For $0 \leq s < t \leq T$ such that $i\Delta \leq s \leq (i+1)\Delta$ and $j\Delta \leq t \leq (j+1)\Delta$, it follows that

$$|M_t^\Delta - M_s^\Delta| \leq |M_{j\Delta}^\Delta - M_{i\Delta}^\Delta| + |M_{(i+1)\Delta}^\Delta - M_{i\Delta}^\Delta| + |M_{(j+1)\Delta}^\Delta - M_{j\Delta}^\Delta|.$$

For notational convenience, let us introduce the notation $M_i^\Delta = M_{i\Delta}^\Delta$ (a similar notation is used for $\beta_i^\Delta, \lambda_i^\Delta, \sigma_i^\Delta, \Sigma_i^\Delta$, and B_i). Defining $\delta^\Delta := \lfloor \delta/\Delta \rfloor + 1$, $T^\Delta := \lfloor T/\Delta \rfloor$, and $\mathcal{T}^\Delta := \lfloor \mathcal{T}/\Delta \rfloor$, for Δ sufficiently small, the previous inequality implies that

$$\mathbb{P}\left(\sup_{|t-s| \leq \delta} |M_t^\Delta - M_s^\Delta| \geq \varepsilon\right) \leq 3\mathbb{P}\left(\sup_{|j-i| \leq \delta^\Delta} |M_j^\Delta - M_i^\Delta| \geq \varepsilon/3\right).$$

Using the recursion for M_i^Δ , we get that

$$|M_j^\Delta - M_i^\Delta| \leq \sum_{k=i}^{j-1} \lambda_k^\Delta \beta_k^\Delta \Delta |M_k^\Delta| + \left| \sum_{k=i}^{j-1} \sigma_k^\Delta (B_{k+1} - B_k) \right|$$

and so

$$\begin{aligned} &\mathbb{P}\left(\sup_{|j-i| \leq \delta^\Delta} |M_j^\Delta - M_i^\Delta| \geq \varepsilon\right) \\ &\leq \mathbb{P}\left(\sup_{|j-i| \leq \delta^\Delta} \sum_{k=i}^{j-1} \lambda_k^\Delta \beta_k^\Delta |M_k^\Delta| \Delta \geq \varepsilon/2\right) \\ &\quad + \mathbb{P}\left(\sup_{|j-i| \leq \delta^\Delta} \left| \sum_{k=i}^{j-1} \sigma_k^\Delta (B_{k+1} - B_k) \right| \geq \varepsilon/2\right). \end{aligned}$$

From Part I we know that $\lambda^\Delta(t)$ is uniformly bounded in Δ and t and strictly positive. So, for the purpose of the result that we need to prove, we can conve-

niently assume that without loss of generality (w.l.o.g.), $\lambda_k^\Delta = 1$ (and $\sigma_k^\Delta = 1$). It follows that for the last term on the right that

$$\lim_{\delta \downarrow 0} \limsup_{\Delta \downarrow 0} \mathbb{P} \left(\sup_{|j-i| \leq \delta^\Delta} \left| \sum_{k=i}^{j-1} \sigma_k^\Delta (B_{k+1} - B_k) \right| \geq \varepsilon/2 \right) = 0$$

(e.g., by invoking Lévy’s theorem on the modulus of continuity for Brownian motion). Hence, to complete the proof, it is now sufficient to show that

$$(21) \quad \lim_{\delta \downarrow 0} \limsup_{\Delta \downarrow 0} \mathbb{P} \left(\sup_{|j-i| \leq \delta^\Delta} \sum_{k=i}^{j-1} \beta_k^\Delta |M_k^\Delta| \Delta \geq \varepsilon \right) = 0.$$

From the definition of M_k^Δ (and the assumption $\lambda_k^\Delta = \sigma_k^\Delta = 1$), we get that

$$\begin{aligned} \beta_k^\Delta |M_k^\Delta| &\leq |M_0^\Delta| \beta_k^\Delta \prod_{j=0}^{k-1} (1 - \beta_j^\Delta \Delta) \\ &\quad + \left| \sum_{j=0}^{k-1} \beta_k^\Delta \left(\prod_{n=j+1}^{k-1} (1 - \beta_n^\Delta \Delta) \right) (B_{j+1} - B_j) \right|. \end{aligned}$$

Suppose $\mathcal{T} < T$. Then in the region $t \in [0, \mathcal{T}]$, the function β_t^Δ is uniformly bounded in Δ and t , and the condition in equation (21) will follow. So, let us assume that $\mathcal{T} \geq T$. From Part I, it follows that there exist positive constants

$$\frac{K_1}{\Sigma_t^\Delta} \leq \beta_t^\Delta \leq \frac{K_2}{\Sigma_t^\Delta} \quad \text{for } t \in [0, \mathcal{T}].$$

This follows from the fact that $\beta_t^\Delta \Sigma_t^\Delta$ converges to a positive bounded function as Δ goes to zero. Furthermore, given the limiting behavior of Σ_t^Δ as $\Delta \downarrow 0$, one can show that for any $t \in [T, \mathcal{T}]$, there exists a positive constant K_t (independent of Δ) such that $\Sigma_s^\Delta \leq \Sigma_t^\Delta + K_t(t - s)^2$ for all $s \in [0, t]$. As a result, in $[T, \mathcal{T}]$ we get that

$$\begin{aligned} &\sup_{|j-i| \leq \delta^\Delta} |M_0^\Delta| \Delta \sum_{k=i}^{j-1} \beta_k^\Delta \prod_{j=0}^{k-1} (1 - \beta_j^\Delta \Delta) \\ &\leq |M_0^\Delta| (\delta + \Delta) \max_{T^\Delta \leq k \leq \mathcal{T}^\Delta} \left\{ \beta_k^\Delta \exp \left(- \sum_{j=0}^{k-1} \beta_j^\Delta \Delta \right) \right\} \\ &\leq |M_0^\Delta| (\delta + \Delta) \max_{T^\Delta \leq k \leq \mathcal{T}^\Delta} \left\{ \frac{K_2}{\Sigma_k^\Delta} \exp \left(- \int_0^{k\Delta} \frac{K_1 dt}{\Sigma_k^\Delta + K_{k\Delta}(k\Delta - s)^2} \right) \right\} \end{aligned}$$

$$= |M_0^\Delta|(\delta + \Delta) \max_{T^\Delta \leq k \leq \mathcal{T}^\Delta} \left\{ \frac{K_2}{\Sigma_k^\Delta} \exp\left(-\frac{K_1}{\sqrt{K_{k\Delta}\Sigma_k^\Delta}} \arctan\left(\frac{k\Delta}{\sqrt{K_{k\Delta}\Sigma_k^\Delta}}\right)\right) \right\}$$

$$\xrightarrow{\Delta \downarrow 0} 0,$$

where the convergence follows from the fact that $\Sigma_t^\Delta \rightarrow 0$ as $\Delta \downarrow 0$ for any $t \geq T$. It follows from the previous derivation that there exists a constant \bar{K} independent of Δ such that

$$\max_{T^\Delta \leq k \leq \mathcal{T}^\Delta} \left\{ \beta_k^\Delta \prod_{n=0}^{k-1} (1 - \beta_n^\Delta \Delta) \right\} \leq \bar{K}.$$

Finally, we have that

$$\mathbb{P}\left(\sup_{|j-i| \leq \delta^\Delta} \sum_{k=i}^{j-1} \Delta \left| \sum_{j=0}^{k-1} \beta_k^\Delta \left(\prod_{n=j+1}^{k-1} (1 - \beta_n^\Delta \Delta) \right) (B_{j+1} - B_j) \right| \geq \varepsilon\right)$$

$$\leq \mathbb{P}\left((\delta + \Delta) \times \max_{T^\Delta \leq k \leq \mathcal{T}^\Delta} \left\{ \left| \sum_{j=0}^{k-1} \beta_k^\Delta \left(\prod_{n=j+1}^{k-1} (1 - \beta_n^\Delta \Delta) \right) (B_{j+1} - B_j) \right| \geq \varepsilon \right\}\right)$$

$$\leq \mathbb{P}\left((\delta + \Delta) \bar{K} \max_{T^\Delta \leq k \leq \mathcal{T}^\Delta} \left\{ \left| \sum_{j=0}^{k-1} \prod_{n=0}^j (1 - \beta_n^\Delta \Delta)^{-1} (B_{j+1} - B_j) \right| \geq \varepsilon \right\}\right)$$

$$\leq \frac{(\delta + \Delta)^2 \bar{K}^2}{\varepsilon^2} \mathbb{E} \left[\sum_{j=0}^{\mathcal{T}^\Delta - 1} \prod_{n=0}^j (1 - \beta_n^\Delta \Delta)^{-2} \Delta \right]$$

$$\leq \frac{(\delta + \Delta)^2 \bar{K}^2 \mathcal{T}}{\varepsilon^2} \max_{0 \leq j \leq \mathcal{T}^\Delta} \prod_{n=0}^j (1 - \beta_n^\Delta \Delta)^{-2}.$$

The third inequality uses Doob’s inequality. From Part I we know that β_n^Δ is an increasing function of n and that it converges to $\beta_\infty^\Delta = \hat{K}/\sqrt{\Delta}$ for a fixed constant \hat{K} independent of Δ . As a result, $\max_{0 \leq j \leq \mathcal{T}^\Delta} \prod_{n=0}^j (1 - \beta_n^\Delta \Delta)^{-2}$ is uniformly bounded. We conclude that the $\lim_{\delta \downarrow 0} \limsup_{\Delta \downarrow 0}$ of the probability above is equal to zero as required. *Q.E.D.*

PROOF OF PROPOSITION 2: Recall from Theorem 2 that Σ_t satisfies

$$\Sigma_t = \Sigma_0 + \sigma_v^2 t - \sigma_v^2 e^{2\mu T} \left[\frac{1 - e^{-2\mu t}}{2\mu} \right] \quad \text{for } t < T$$

and $\Sigma_T = 0$ for all $t \geq T$, where $T \geq 0$ is the unique solution to

$$\Sigma_0 + \sigma_v^2 T = \sigma_v^2 \left[\frac{e^{2\mu T} - 1}{2\mu} \right].$$

Since T decreases with σ_v , it suffices to prove that Σ_t decreases with σ_v for $t < T$.

In what follows, and without loss of generality, we will normalize the value of μ such that $2\mu = 1$ (this is equivalent to rescaling time). With this normalization, the derivative of Σ_t ($t < T$) with respect to σ_v^2 is equal to

$$\frac{\partial \Sigma_t}{\partial \sigma_v^2} = t - e^T(1 - e^{-t}) - \sigma_v^2 e^T(1 - e^{-t}) \frac{\partial T}{\partial \sigma_v^2} \quad \text{for } t < T.$$

In addition, from the definition of T , it follows that

$$\frac{\partial T}{\partial \sigma_v^2} = \frac{1}{\sigma_v^2} \left[\frac{1 + T - e^T}{e^T - 1} \right].$$

Plugging this value back onto $\partial \Sigma_t / \partial \sigma_v^2$, we get that for $t < T$,

$$\frac{\partial \Sigma_t}{\partial \sigma_v^2} = t - (1 - e^{-t}) \left[\frac{T}{1 - e^{-T}} \right] \leq 0.$$

The inequality follows from the fact that $t/(1 - e^{-t})$ is an increasing function of t .

Let us now prove the monotonicity of the insider’s ex ante expected payoff. First of all, from the expressions for Σ_t , α_t , and γ_t in Theorem 2, it follows that $\mathbb{E}[\Pi_t] = \alpha_t \Sigma_t + \gamma_t$ is equal to (under the normalization $2\mu = 1$)

$$\mathbb{E}[\Pi_t] = 2\sigma_y \sigma_v \cosh\left(\frac{1}{2}(T - t)^+\right) \quad \text{for } t \geq 0.$$

Note that to prove the monotonicity of $\mathbb{E}[\Pi_t]$ with respect to σ_v , it is enough to focus on the case $t \leq T$. The derivative with respect to σ_v is given by

$$\begin{aligned} \frac{\partial \mathbb{E}[\Pi_t]}{\partial \sigma_v} &= 2\sigma_y \cosh\left(\frac{1}{2}(T - t)\right) + \sigma_y \sigma_v \sinh\left(\frac{1}{2}(T - t)\right) \frac{\partial T}{\partial \sigma_v} \\ &= 2\sigma_y \cosh\left(\frac{1}{2}(T - t)\right) + 2\sigma_y \sinh\left(\frac{1}{2}(T - t)\right) \left[\frac{1 + T - e^T}{e^T - 1} \right] \end{aligned}$$

$$\begin{aligned}
 &= 2\sigma_y \sinh\left(\frac{1}{2}(T-t)\right) \left[\frac{T}{e^T - 1} \right] + 2\sigma_y \exp\left(\frac{T-t}{2}\right) \\
 &\geq 0. \qquad \qquad \qquad \text{Q.E.D.}
 \end{aligned}$$

PROOF OF THEOREM 3: The insider’s Hamilton–Jacobi–Bellman (HJB) optimality condition are given by

$$\begin{aligned}
 0 &= \max_{\beta} \left\{ -\lambda_t \beta M \Pi_M + \frac{1}{2} \lambda_t^2 \sigma_y^2 \Pi_{MM} + \Pi_t - \mu \Pi + M^2 \beta \right\} \\
 &\text{for } t \in [0, \infty).
 \end{aligned}$$

Suppose, we guess a quadratic value function of the form $\Pi(t, M) = \alpha_t M^2 + \gamma_t$ for deterministic functions α_t and γ_t . The HJB equation is satisfied if and only if $\dot{\alpha}_t - \mu \alpha_t = 0$, $1 - 2\lambda_t \alpha_t = 0$, and $\alpha_t(\sigma_v^2(t) + \lambda_t^2 \sigma_y^2) + \dot{\gamma}_t - \mu \gamma_t = 0$. The first two conditions lead to $\lambda_t = \lambda_0 e^{-\mu t}$ and $\alpha_t = e^{\mu t} / [2\lambda_0]$ for some constant $\lambda_0 > 0$. Replacing these two functions, the solution of the last differential equation is

$$\gamma_t = \frac{1}{2\lambda_0} (C + \Gamma_t) e^{\mu t} + \frac{\sigma_y^2 \lambda_t}{4\mu}$$

for some constant $C \geq 0$ (since $\Gamma_t \downarrow 0$ as $t \rightarrow \infty$, $C \geq 0$ is required to ensure that $\gamma_t \geq 0$ for all t).

Note that the HJB condition does not provide any information about how to select the insider’s strategy β_t . (Effectively, we have solved the HJB equation using the fact that the insider is indeed indifferent.) To determine the value of β_t , we must turn to the market maker’s filtering conditions. The condition $P_t = \mathbb{E}[V_t | \mathcal{F}_t^M]$ implies that P_t is the orthogonal projection V_t on \mathcal{F}_t^M in L^2 , and we can interpret the equilibrium market price as the solution to a classical Kalman–Bucy filtering problem. Let the signal process be the value of the fundamental V_t , with dynamics $dV_t = \sigma_v dB_t^v$ and the observation process be the price process P_t , with dynamics $dP_t = \lambda_t dZ_t = \beta_t \lambda_t (V_t - P_t) dt + \sigma_y \lambda_t dB_t^y$. Let v_t be the corresponding optimal (in mean square sense) filtering estimate of V_t and let Σ_t be the filtering error. Then the equilibrium condition is $P_t = v_t$. The generalized Kalman filter conditions for the pair (V_t, P_t) are given by

$$dv_t = \frac{\Sigma_t \beta_t}{\lambda_t \sigma_y^2} [dP_t - \lambda_t \beta_t (v_t - P_t) dt], \quad \dot{\Sigma}_t = \sigma_v^2 - \frac{(\Sigma_t \beta_t)^2}{\sigma_y^2}.$$

To recover the identity $P_t = v_t$, we need to impose that $\Sigma_t \beta_t = \lambda_t \sigma_y^2$. This equality together with the border condition $v_0 = P_0$ imply that $v_t = P_t$ for all $t > 0$. This equality also implies that $(\Sigma_t \beta_t)^2 = \lambda_t^2 \sigma_y^4$. Therefore, the market maker’s filtering conditions are

$$\Sigma_t \beta_t = \lambda_t \sigma_y^2, \quad \dot{\Sigma}_t = \sigma_v^2(t) - \sigma_y^2 \lambda_t^2,$$

which guarantee that the market maker equilibrium condition $P_t = \mathbb{E}[V_t | \mathcal{F}_t^M]$ is satisfied. Since $\lambda_t = \lambda_0 e^{-\mu t}$, it follows that

$$\Sigma_t = \Sigma_0 + \Gamma_0 - \Gamma_t - \frac{\sigma_y^2 \lambda_0^2}{2\mu} (1 - e^{-\mu t}), \quad \beta_t = \frac{\sigma_y^2 \lambda_0 e^{-\mu t}}{\Sigma_t}.$$

To complete the proof, we need to specify the values of the two constant λ_0 and C and verify that the proposed value function $\Pi(t, M) = \alpha_t M^2 + \gamma_t$ and trading strategy β_t effectively solve the insider's problem. This final step is achieved by imposing the transversality condition $\lim_{t \rightarrow \infty} e^{-\mu t} \mathbb{E}[\Pi(t, M_t)] = 0$ for β_t .

To avoid confusion, we now use β_t to denote the trading strategy in equation (10) use and $\tilde{\beta}_t$ to denote an arbitrary policy in \mathcal{B} . To explicate the dependence of M_t on a trading strategy $\{\tilde{\beta}_s : 0 \leq s \leq t\}$, we will use the notation $M_t(\tilde{\beta})$.

Since $\Pi(t, M) = \alpha_t M^2 + \gamma_t$ satisfies the HJB equation for any strategy $\tilde{\beta} \in \mathcal{B}$, it follows that

$$(22) \quad \begin{aligned} \Pi(0, M_0) &= \mathbb{E} \left[\int_0^t e^{-\mu s} \tilde{\beta}_s M_s^2(\tilde{\beta}) ds + e^{-\mu t} \Pi(t, M_t(\tilde{\beta})) \right] \\ &\geq \mathbb{E} \left[\int_0^t e^{-\mu s} \tilde{\beta}_s M_s^2(\tilde{\beta}) ds \right]. \end{aligned}$$

In addition,

$$e^{-\mu t} \mathbb{E}[\Pi(t, M_t(\beta_t))] = \frac{1}{2\lambda_0} \left(\mathbb{E}[M_t^2(\beta)] + C + \Gamma_t + \frac{\sigma_y^2 \lambda_0^2 e^{-2\mu t}}{2\mu} \right).$$

Thus, the transversality condition holds only if $C = 0$ and $\lim_{t \rightarrow \infty} \mathbb{E}[M_t^2(\beta)] = 0$. We can show that

$$\begin{aligned} \mathbb{E}[M_t^2(\beta)] &= M_0^2 e^{-2\lambda_0 \int_0^t e^{-\mu s} \beta_s ds} \\ &\quad + \int_0^t e^{-2\lambda_0 \int_s^t e^{-\mu u} \beta_u du} (\sigma_v^2(s) + \sigma_y^2 \lambda_0^2 e^{-2\mu s}) ds. \end{aligned}$$

Hence, $\lim_{t \rightarrow \infty} \mathbb{E}[M_t^2(\beta)] = 0$ if $\int_0^\infty e^{-\mu s} \beta_s ds = \infty$. This last requirement together with the fact that $\beta_t = \sigma_y^2 \lambda_0 e^{-\mu t} / \Sigma_t$ imply that $\lim_{t \rightarrow \infty} \Sigma_t = 0$. Therefore,

$$\lambda_0 = \sqrt{\frac{2\mu(\Sigma_0 + \Gamma_0)}{\sigma_y^2}}.$$

With these choices of λ_0 and C , the transversality condition is satisfied for β_t , and taking limits in equation (22), we get that

$$\begin{aligned} \Pi(0, M_0) &= \mathbb{E} \left[\int_0^\infty e^{-\mu s} \beta_s M_s^2(\beta) ds \right] \\ &\geq \mathbb{E} \left[\int_0^\infty e^{-\mu s} \tilde{\beta}_s M_s^2(\tilde{\beta}) ds \right] \quad \text{for all } \tilde{\beta} \in \mathcal{B}. \end{aligned}$$

In the last step, we used (9) and $\beta_t > 0$ for all t , and invoked the Lebesgue convergence theorem to interchange limits and expectations.

We conclude the proof by showing that the equilibrium strategy satisfies condition (9). Given the expression for $\mathbb{E}[M_t]$ above, this condition is equivalent to

$$M_0^2 \int_0^\infty \dot{f}_t e^{-f_t} dt + \int_0^\infty f_t e^{-f_t} \left[\int_0^t (\sigma_v^2(s) + \sigma_y^2 \lambda_0^2 e^{-2\mu s}) e^{f_s} ds \right] dt < \infty,$$

where $f_t := 2\lambda_0 \int_0^t e^{-\mu s} \beta_s ds$ and \dot{f}_t is its first derivative with respect to t . Note that the first integral is equal to 1. Using the Fubini theorem to reverse the order of integration, the second integral is equal to

$$\begin{aligned} &\int_0^\infty (\sigma_v^2(s) + \sigma_y^2 \lambda_0^2 e^{-2\mu s}) e^{f_s} \left[\int_s^\infty f_t e^{-f_t} dt \right] ds \\ &= \int_0^\infty (\sigma_v^2(s) + \sigma_y^2 \lambda_0^2 e^{-2\mu s}) ds = \Gamma_0 + \sigma_y^2 \lambda_0^2 / (2\mu) < \infty. \quad \text{Q.E.D.} \end{aligned}$$

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