

Analysis of a Decentralized Production-Inventory System

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We model an isolated portion of a competitive supply chain as a $M/M/1$ make-to-stock queue. The retailer carries finished goods inventory to service a Poisson demand process, and specifies a policy for replenishing his inventory from an upstream supplier. The supplier chooses the service rate, i.e., the capacity of his manufacturing facility, which behaves as a single-server queue with exponential service times. Demand is backlogged and both agents share the backorder cost. In addition, a linear inventory holding cost is charged to the retailer, and a linear cost for building production capacity is incurred by the supplier. The inventory level, demand rate, and cost parameters are common knowledge to both agents. Under the continuous-state approximation where the $M/M/1$ queue has an exponential rather than geometric steady-state distribution, we characterize the optimal centralized and Nash solutions, and show that a contract with linear transfer payments replicates a cost-sharing agreement and coordinates the system. We also compare the total system costs, the agents' decision variables, and the customer service levels of the centralized versus Nash versus Stackelberg solutions.

(Make-to-Stock Queue; Game Theory)

1. Introduction

Within many supply chains, a devoted upstream agent, referred to here as the *supplier*, produces goods for a downstream agent, called the *retailer*, in a make-to-stock manner. Broadly speaking, the performance (e.g., service levels, cost to produce and hold items) of this isolated portion of the supply chain is dictated by three factors: (i) retailer demand, which is largely exogenous but can in some cases be manipulated via pricing and advertising, (ii) the effectiveness of the supplier's production process and the subsequent transportation of goods, and (iii) the inventory replenishment policy, by which retailer demand is mapped into orders placed with the supplier. If the supplier and retailer are under different ownership or are independent entities within the same firm, then their competing objectives can lead to severe coordination problems: The supplier typically wants the retailer to

hold as much inventory as possible, while the retailer prefers to hold very little inventory and desires rapid response from the supplier. These tensions may deteriorate overall system performance.

The recent explosion in the academic supply chain management literature is aimed at this type of multi-agent problem. Almost without exception, the papers that incorporate stochastic demand employ variants of one of two prototypical operations management models: the newsvendor model, or the Clark-Scarf (1960) multiechelon inventory model. One-period and two-period versions of newsvendor supply chain models have been studied intensively to address the three factors above; see Agrawal et al. (1999), Cachon (1999), and Lariviere (1999), for recent reviews. Although many valuable insights have been generated by this work, these models are primarily useful for style goods and products with very short