Predatory Trading*

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Abstract

This paper studies predatory trading: trading that induces and/or exploits other investors’ need to reduce their positions. We show that if one trader needs to sell, others also sell and subsequently buy back the asset. This leads to price over-shooting and a reduced liquidation value for the distressed trader. Hence, the market is illiquid when liquidity is most needed. Further, a trader profits from triggering another trader’s crisis, and the crisis can spill over across traders and across assets.

Keywords: Predation, Valuation, Liquidity, Risk Management, Systemic Risk.

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1 Introduction

Risk managers for large traders such as hedge funds are understandably concerned that they may be forced to liquidate their position at significant cost. This forced liquidation may be triggered by a number of factors including collateral or margin calls, large unexpected fund redemptions, exchange regulations, and the institution’s own charters, VaR models, or pre-committed dynamic trading strategies.\(^1\) Downsizing a large position is especially costly if it causes a large price impact because the market liquidity at this time is low.

We provide a new framework for studying the strategic interaction between large traders in a setting in which some traders have to liquidate their position. Agents trade continuously and limit their trading intensity to minimize their price impact cost. Our analysis shows that other strategic traders trade in the same direction exactly when a distressed large investor is forced to unwind his position and needs liquidity the most. That is, they conduct predatory trading and withdraw liquidity instead of providing it. This predatory activity makes liquidation costly and typically leads to price overshooting. Moreover, predatory trading itself can drive out of the market a vulnerable trader who could have otherwise remained solvent.

These findings are in line with anecdotal evidence. A well-known example of predatory trading is the alleged trading against LTCM’s positions in the fall of 1998. Business Week wrote that “if lenders know that a hedge fund needs to sell something quickly, they will sell the same asset – driving the price down even faster. Goldman, Sachs & Co. and other counterparties to LTCM did exactly that in 1998.”\(^2\) Also, Cai (2002) finds that “locals” on the CBOE pits exploited knowledge of LTCM’s short positions in the treasury bond futures market. Another indication of the fear of predatory trading is evident in the opposition to UBS Warburg’s proposal to take over Enron’s traders without taking over its trading positions. This proposal was opposed on the grounds that “it would present a ‘predatory trading risk’ because Enron’s traders would effectively know the contents of the trading book.”\(^3\) Similarly, many institutional investors are forced by law or their own charter to sell bonds of companies which undergo debt restructuring procedures. Hradsky and Long (1989) document price overshooting in the bond market after the default announcements.

Furthermore, our model shows that an adverse wealth shock to one large institution, coupled with predation by other strategic traders, can have a ripple effect on other institutions and cause a widespread crisis in the financial sector. Consistently, the

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\(^1\)For example, Metallgesellschaft’s (MG) management had to close its short-term oil futures positions after the emergence of MG’s cash-flow problems caused Nymex to double their margin requirements and impose position limits. Also, traders who followed portfolio insurance trading strategies during the 1987 crash were pre-committed to sell as the market declined.


\(^3\)AFX News Limited, AFX – Asia, January 18th, 2002.
testimony of Alan Greenspan in the U.S. House of Representatives on Oct. 1st, 1998 indicates that the Federal Reserve Bank was worried that LTCM’s financial difficulties might destabilize the financial system as a whole:

“...the act of unwinding LTCM’s portfolio would not only have a significant distorting impact on market prices but also in the process could produce large losses, or worse, for a number of creditors and counterparties, and for other market participants who were not directly involved with LTCM.”

Our model provides guidance for the valuation of large security positions. We distinguish between three forms of value, with increasing emphasis on the position’s liquidity. Specifically, the “paper value” is the current mark-to-market value of a position; the “orderly liquidation value” reflects the revenue one could achieve by secretly liquidating the position; and the “distressed liquidation value” equals the amount which can be raised if one faces predation by other strategic traders, that is, with endogenous market liquidity. The analysis suggests that the latter value of a position depends on the holder’s general financial health and its exposure to predatory trading.

Our analysis also has normative implications for the design of hedge fund disclosure requirements. Large hedge funds that are active in illiquid markets should face less stringent disclosure standards than funds holding liquid assets or funds that are smaller in size. This is consistent with the disclosure guidelines described in the consensus statement by the IAFE Investor Risk Committee (IRC), which consists both of hedge funds and hedge fund investors. IRC (2001) states that “large hedge funds need to limit granularity of reporting to protect themselves against predatory trading against the fund’s position.” In the same vein, market makers at the London Stock Exchange prefer to delay the reporting of large transactions since it gives them “a chance to reduce a large exposure, rather than alerting the rest of the market and exposing them to predatory trading tactics from others.”

Our work is related to several strands of literature. Market impact due to asymmetric information is studied by Kyle (1985). Trading with exogenous market impact is studied theoretically by Cetin, Jarrow, and Protter (2002), and empirically by Chen, Stanzl, and Watanabe (2002). The notion of predatory trading partially overlaps with that of stock price manipulation, which is studied by Allen and Gale (1992) among others. One distinctive feature of predatory trading is that the predator derives profit from the price impact of the prey and not from his own price impact. The systemic risk component of our paper is related to the literature on financial crisis. Second generation currency crisis models with multiple equilibria were initiated by Obstfeld (1996) and refined using the global games framework by Morris and Shin (1998). Abreu and Brunnermeier (2001) can be viewed as the dynamic version of it. Bernardo and Welch

provide a simple model of “financial market runs.” Traders join the run out of fear of having to liquidate before the price recovers and to avail of the possibility of selling at an on average higher price today. Attari, Mello, and Ruckes (2002) is close in spirit to our paper, although their focus and (two-period) setting are different. They focus on a distressed trader’s incentive to buy to temporarily push up the price when facing a margin constraint, and a competitor’s incentive to profitably counter-trade and lend to this trader.

The remainder of the paper is organized as follows. Section 2 introduces the model, Section 3 provides some preliminary results that simplifies the analysis. Section 4 demonstrates the form of predatory trading in setting with a single and multiple predators. It also shows how predation can drive an otherwise solvent trader into financial distress and create a crisis for the whole market. Section 5 studies the difference between orderly and distressed liquidation value. Regulatory implications pertaining to disclosure and bail outs of hedge funds are discussed in 7. Detailed proofs are relegated to the Appendix.

2 Model

We assume that the economy has two assets: a riskless bond and a risky asset. For simplicity, we normalize the risk-free rate to zero. The risky asset has an aggregate supply of $S > 0$, and a final payoff $v$ at time $T$. Here, $v$ is a random variable with an expected value of $\mu$. One can also view the risky asset as the payoff associated with an arbitrage strategy consisting of multiple assets. The price of the risky asset at time $t$ is denoted by $p(t)$. The economy has two kinds of agents: large strategic traders (arbitrageurs), and long-term investors. We can think of the strategic traders as hedge funds and proprietary trading desks, and the long-term investors as pension funds and individual investors.

The long-term investors are price-takers and have, at each time, an aggregate demand of

$$Y(p) = \frac{1}{\lambda}(\mu - p),$$

depending on the current price $p$. We assume that the demand curve is downward sloping. This assumption is consistent with empirical findings of Shleifer (1986), Wurgler and Zhuravskaya (2002) and others. Theoretically, there are several mechanisms that can produce downward sloping demand curves including investor heterogeneity and risk aversion. For example, the demand curve is linearly downward sloping if each individual investor has constant absolute risk aversion, and if the payoff $v$ is normally distributed.

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5 All random variables are defined on a probability space $(\Omega, \mathcal{F}, P)$.

6 These papers dispute Scholes (1972), who concludes that the demand curve is essentially flat.
Arbitrageurs $i \in \{1, 2, ..., I\}$ are risk-neutral and seek to maximize their expected profit. Each arbitrageur is large and hence his trading impacts the equilibrium price. He therefore acts strategically and takes his price impact into account when submitting his orders. Each arbitrageur $i$ has a given initial endowment, $x^i(0)$, of the risky asset. His holding at time $t$ is denoted by $x^i(t)$. The aggregate holding of risky assets by arbitrageurs is given by

$$X(t) = \sum_{i=1}^{I} x^i(t).$$

With this aggregate institutional holding, the market clearing price, $p(t)$, is seen, using (1), to be

$$p(t) = \mu - \lambda (S - X(t)).$$

We assume that each institution is restricted to hold

$$x^i(t) \in [-\bar{x}, \bar{x}],$$

and that $S > I\bar{x}$. These assumptions imply that even if all strategic investors combine their capital together they do not have enough to make the price equal to the expected value of the asset $E(v) = \mu$. We consider this case since unlimited institutional holdings imply the trivial outcome that $p = \mu$, while we are interested in the strategic interactions that arise if institutions may not have sufficient capital.

Furthermore, institutions are subject to a risk of financial crisis. If a crisis occurs at $t_0$, then all strategic traders learn who is affected by the crisis. If a strategic trader is in financial crisis, it must liquidate its position in the risky asset. We denote the set of “distressed” traders in crisis by $I^c$ and the set of unaffected traders, the “predators,” by $I^p$.

The assumption of forced liquidation can be explained by (external or internal) agency problems. Bolton and Scharfstein (1990) show that an optimal financial contract may leave an agent cash constrained even if the agent is subject to predation risk.\footnote{Bolton and Scharfstein (1990) focus on predation in the product markets, not the financial markets, but the argument is general.} Also, limited capital for a certain strategy and forced liquidation can be the result of a company’s risk management policy.

We assume that arbitrageurs can continuously trade infinitesimal amounts of assets. In particular, arbitrageur $i$ can choose his trading intensity $u^i$,

$$u^i(t) := \dot{x}^i = \frac{\partial x^i}{\partial t}.$$

In each instant, all orders are executed simultaneously with the same priority. As long as the total net trade of the strategic traders is not too large, that is as long as

$$\left| \sum_i u^i(t) \right| \leq U,$$

Bolton and Scharfstein (1990) focus on predation in the product markets, not the financial markets, but the argument is general.
all orders in this instant are executed at the current price \( p(t) \). If \( |\sum_i u^i(t)| > U \) there is a temporary price impact of \( \gamma \) for the orders beyond \( U \). More precisely, investor \( i \) incurs a cost of

\[
G \left( u^i(t), u^{-i}(t) \right) := \gamma \max \left\{ 0, u^i - \bar{u}, u - u^i \right\}
\]

where \( u^{-i}(t) := (u^1(t), \ldots, u^{i-1}(t), u^{i+1}(t), \ldots, u^I(t)) \) and where \( \bar{u} = \bar{u} (u^{-i}(t)) \) and \( u = u (u^{-i}(t)) \) are, respectively, the unique solutions to

\[
\bar{u} + \sum_{j \neq i} \min \left\{ u^j, \bar{u} \right\} = U;
\]

\[
u + \sum_{j \neq i} \max \left\{ u^j, \bar{u} \right\} = U.
\]

In words, \( \bar{u} (u) \) is the highest intensity with which trader \( i \) can buy (sell) without incurring the cost associated with a temporary price impact. Further, \( G \) is the product of the per-share cost, \( \gamma \), multiplied by the number of shares exceeding \( \bar{u} \) or \( u \). We assume for simplicity that the temporary price impact is large, \( \gamma \geq \lambda I \bar{u} \).

This market structure can be interpreted as follows.\(^8\) There is a limit order book consisting of orders made by long-term investors that is not infinitely deep. When the orders are hit at a moderate speed, new orders flow in, and the price walks down the demand curve. It takes time for new orders to enter the limit order book, though, and if the arbitrageurs trade too fast, they have a temporary impact cost associated with hitting orders far away from the equilibrium price. Another interpretation is that strategic traders must search for long-term investors in order to trade. Duffie, Gârleanu, and Pedersen (2002) provide a search framework in a different finance context.

The requirement that a trader in financial crises must liquidate its position is formalized as

\[
i \in I^{c} \quad \Rightarrow \quad \begin{cases} 
u^i(t) \leq -\frac{U}{T} & \text{if } x(t) > 0 \text{ and } t > t_0 \\ 
\nu^i(t) = 0 & \text{if } x(t) = 0 \text{ and } t > t_0 \\ 
\nu^i(t) \geq \frac{U}{T} & \text{if } x(t) < 0 \text{ and } t > t_0 
\end{cases}
\] (6)

This statement says that a trader in crisis must liquidate his position at least as fast as \( U/I \). This is the fastest that an agent can liquidate if all other traders are liquidating at the same time.\(^9\)

We note that our results do not depend qualitatively on the nature of the troubled agents’ liquidation strategy, nor do they depend on the assumption that such agents must liquidate the entire position. It suffices that a troubled institution must reduce its position before time \( T \) and that it cannot rebuild it immediately.

\(^8\)We could assume other market structures and get similar results. For instance, we could assume that each trader has a (personal) constraint on his trading intensity.

\(^9\)In equilibrium, to be discussed later, a troubled trader who must liquidate will maximize its profits by initially liquidating as fast as possible. Liquidating fast minimizes the costs of front-running by other traders.
Arbitrageur \( i \)'s objective is to maximize his expected wealth subject to the constraints described above. The arbitrageur’s wealth is the final value, \( x^i(T)v \), of his stock holdings reduced by the cost, \( u^i(t)p(t) + G \), of buying the shares, where \( G \) is the temporary impact cost. That is, the arbitrageur’s objective is:

\[
\max_{u^i(\cdot) \in \mathcal{U}^i} E \left( x^i(T)v - \int_0^T [u^i(t)p(t) + G \left( u^i(t), u^{-i}(t) \right)] dt \right),
\]

where \( \mathcal{U}^i \) is the set of \( \{ \mathcal{F}_t^i \} \)-adapted processes that satisfy (3) and (6). The filtration \( \{ \mathcal{F}_t^i \} \) represents trader \( i \)'s information. We assume that each strategic trader learns the extent of his own temporary price impact and, at time \( t_0 \), he also knows which traders must liquidate. We consider both the case in which the size of any distressed trader’s position is disclosed and the case in which it is not. Hence, \( \{ \mathcal{F}_t^i \} \) is generated by \( \mathcal{F}_{t \geq t_0} \) and by \( G(\left( u^i(t), u^{-i}(t) \right), \) ) and, in the case of disclosure, by \( x^i(t_0)1_{(t \geq t_0)}, i \in I^p \). Note that the filtration does not include the price process. This is because of (un-modelled) noise in the supply of assets. We abstract from supply uncertainty for ease of exposition in the main text of the paper. In Appendix B we show that our equilibrium is a Perfect Bayesian Nash equilibrium in a more general economy with supply uncertainty in which traders observe prices.

**Definition 1** An equilibrium is a set of processes \((u^1, \ldots, u^I)\) such that, for each \( i \), \( u^i \) solves (7), taking \( u^{-i} = (u^1, \ldots, u^{i-1}, u^{i+1}, \ldots, u^I) \) as given.

### 3 Preliminary Analysis

In this section, we show how to solve a trader’s problem. For this, we re-write trader \( i \)'s problem (7) as a constant (which depends on \( x(0) \)) plus

\[
\lambda S x^i(T) - \frac{1}{2} \lambda \left[ x^i(T) \right]^2 - \lambda \int_0^T \left[ u^i(t) X^{-i}(t) + G(\left(u^i(t), u^{-i}(t)\right) \right] dt,
\]

where we use \( E(v) = \mu \), the expression (2) for the price, the relation \( \int_0^T \dot{x}^i(t) x^i(t) dt = \frac{1}{2} \left[ x(t) \right]^2 \bigg|_0^T \), and where we define

\[
X^{-i}(t) := \sum_{j=1, \ldots, I, j \neq i} x^j(t).
\]

Under our standing assumptions, it can be shown that any optimal trading strategy satisfies \( x^i(T) = \bar{x} \) if trader \( i \) is not in distress. That is, the trader ends up with the maximum capital in the arbitrage position. Furthermore, it is not optimal to incur the temporary impact cost, that is, each trader optimally keeps his trading intensity within his bounds \( \underline{u} \) and \( \bar{u} \). Hence, we have the following result, which is useful in solving each trader’s optimization problem and hence is useful for deriving equilibrium.
Lemma 1 A trader’s problem can be written as

$$\min_{u^i(\cdot) \in \mathcal{U}} \int_0^T u^i(t) X^{-i}(t) \, dt$$

s.t. $$\bar{x} = x^i(T) = x^i(0) + \int_0^T u^i(t) \, dt \quad \text{if } i \in I^p$$

$$u^i(t) \in [u(u^{-i}(t)), \pi(u^{-i}(t))] .$$

The lemma shows that the trader’s problem is to minimize $$\int u^i(t) X^{-i}(t) \, dt$$, that is to minimize his trading cost, not taking into account how his own trading affects prices. This is because the model is set up such that the trader cannot make or lose money based on the way his own trades affect prices. Rather, the trader makes money by exploiting the way in which the other traders affect prices. This distinguishes predatory trading from price manipulation (see, for instance, Allen and Gale (1992)).

4 The Predatory Phase ($t \in [t_0, T]$)

We consider first, the “predatory phase,” that is, the period $[t_0, T]$ in which some strategic traders face financial crisis. In Section 6, we analyze the full game including the “investment phase” $[0, t_0)$ in which traders decide how large (arbitrage) positions to take. We assume that each institution has a given position in the risky asset of $x(t_0) \in (0, \bar{x}]$ at time $t_0$. Furthermore, we assume for simplicity that there is “sufficient” time to trade, $t_0 + 2\bar{x}I/U < T$.

We proceed in two stages. Section 4.1 analyzes how strategic traders exploit the fact that others must liquidate their position, and the implied price effects. Section 4.2 endogenizes agents’ default and studies how traders may force others out of the market, possibly leading to a wide-spread crisis.

4.1 Exogenous Default

Here, we take as given the set, $I^c$, of distressed traders. A distressed trader $j$ sells, in equilibrium, his shares at constant speed $u^j = -U/I$ from $t_0$ until $t_0 + x(t_0)I/U$, and thereafter $u^j = 0$. This behavior is optimal, as we later confirm. This liquidation strategy is known by all the strategic traders.

The predators’ strategies are more interesting. We first consider the simplest case in which there is a single predator, and subsequently we consider the case with multiple competing surviving traders.

4.1.1 Single Predator ($I^p = 1$)

In the case with a single “surviving” trader, the strategic interaction is simple: the predator, say $i$, is merely choosing his optimal trading strategy given the known liq-
uidation strategy of the trader in crisis. Specifically, $X^{-i}$ is the total position of the distressed traders, which is decreasing to 0, and staying constant thereafter. Hence, using Lemma 1, we get the following equilibrium.

**Proposition 1** With $I^p = 1$, the following is the unique equilibrium: Each distressed trader sells with constant speed $U/I$ for $\tau = \frac{x(t_0)}{U/I}$ periods. The predator sells as fast as he can without causing a temporary price impact, for $\tau$ time periods and then buys back for $\frac{\tau}{U}$ periods. That is,

$$u^i(t) = \begin{cases} -\frac{U}{I} & \text{for } t \in [t_0, t_0 + \tau) \\ U & \text{for } t \in [t_0 + \tau, t_0 + \tau + \frac{\tau}{U}) \\ 0 & \text{for } t \geq t_0 + \tau + \frac{\tau}{U} \end{cases}$$

The price overshoots; the price dynamics is

$$p^i(t) = \begin{cases} p(t_0) - \lambda U [t - t_0] & \text{for } t \in [t_0, t_0 + \tau) \\ p(t_0) - \lambda I x(t_0) + \lambda U [t - (t_0 + \tau)] & \text{for } t \in [t_0 + \tau, t_0 + \tau + \frac{\tau}{U}) \\ \mu + \lambda [\bar{x} - S] & \text{for } t \geq t_0 + \tau + \frac{\tau}{U} \end{cases}$$

where $p(t_0) = \mu + \lambda (Ix(t_0) - S)$.

We see that although the surviving arbitrageur wants to end up with all his capital invested in the arbitrage position ($x^i(T) = \bar{x}$), he is selling as long as the liquidating trader is selling. He is selling to profit from the (temporary) price swings that occur in the wake of the liquidation. The predatory trader would like to “front run” the distressed trader by selling before him and buying back shares after the distressed trader has pushed down the price further. Since both traders can sell at the same speed, the equilibrium is that they sell simultaneously, and the predator buys back in the end.

The selling by the predatory trader leads to price “over-shooting.” The price falls not only because the distressed trader is liquidating, but also because the predatory trader is selling as well. After the distressed trader is done selling, the predatory trader starts buying until he is at his capacity, and this pushes the price up towards its new equilibrium level.

The predatory trader is profiting from the distressed trader’s liquidation for two reasons. First, the predator can sell his assets for an average price that is higher than the price at which he can buy them back after the distressed trader has left the market. Second, the predator can buy the additional units cheaply until he reaches his capacity. Hence, the predator has an incentive to try to cause the distressed trader’s liquidation. We discuss in Section 4.2 how a predator may try to kill a vulnerable trader by selling shares and causing a price drop.

The predatory behavior by the surviving agent makes liquidation excessively costly for the distressed agent. To see this, suppose a trader estimates the liquidity in “normal
times,” that is, when no trader is in distress. The liquidity as given by the price sensitivity to demand changes is given by $\lambda$ as is seen in (2). When liquidity is needed by the distressed trader, however, the liquidity is halved due to the fact that the predator is selling as well. Specifically, the price moves by $2\lambda$ for each unit the distressed trader is selling.

The distressed trader’s excess liquidation cost equals the predator’s profit from preying. The long-term investors are not being exploited by the predator. The price overshooting implies that long-term investors are buying shares and selling them at the same price. Hence, it does not matter for the long-term investors whether the predator preys or not.\(^{10}\)

**Numerical Example**

We illustrate this predatory behavior with a numerical example. We let $\lambda = 10$, $\mu = 140$, $S = 4$, $t_0 = 5$, $T = 8$, $c = 1$, $x(t_0) = 0.8$, and $\bar{x} = 1$. Figures 1 and 2 illustrate the holding of the two traders and Figure 3 shows the price dynamics.

**4.1.2 Multiple Predators ($P \geq 2$)**

We saw in the previous example how a single predatory trader has an incentive to “front run” the distressed trader by selling as long as the distressed trader is selling. With multiple surviving traders this incentive remains, but another effect is introduced: these traders want to end up with all their capital in the arbitrage position and they want to buy their shares sooner than the other arbitrageurs do.

\(^{10}\)Long-term investors could profit from using a predatory strategy. We assume, however, that these investors do not have sufficient skills and information to do that.
The proposition below shows that in equilibrium predatory traders sell for a while, and then start buying back before the distressed traders have finished their liquidation.

**Proposition 2** With $I^p \geq 2$ and $x(t_0) \geq \frac{I^p - 1}{I^p - 1} \bar{x}$, in the unique equilibrium, each distressed trader sells with constant speed $U/I$ for $\frac{x(t_0)}{U/I}$ periods. Furthermore, each predator $i \in I^p$ sells at trading intensity $U/I$ for $\tau := \frac{x(t_0) - \frac{I^p - 1}{I^p} \bar{x}}{U/I}$ periods and buys back shares at a trading intensity of $\frac{U}{I(I^p - 1)}$ until $t_0 + \frac{x(t_0)}{U/I}$. That is,

$$u^i_s(t) = \begin{cases} 
-U/I & \text{for } t \in [t_0, t_0 + \tau) \\
\frac{U}{I(I^p - 1)} & \text{for } t \in [t_0 + \tau, t_0 + \frac{x(t_0)}{U/I}) \\
0 & \text{for } t \geq t_0 + \frac{x(t_0)}{U/I} 
\end{cases}$$

(11)

The price overshoots; the price dynamics is

$$p^*(t) = \begin{cases} 
p(t_0) - \lambda U [t - t_0] & \text{for } t \in [t_0, t_0 + \tau) \\
p(t_0) - \lambda U \tau + \lambda \frac{U}{I(I^p - 1)} [t - (t_0 + \tau)] & \text{for } t \in [t_0 + \tau, t_0 + \frac{x(t_0)}{U/I}) \\
\mu + \lambda [\bar{x} I^p - S] & \text{for } t \geq t_0 + \frac{x(t_0)}{U/I}, 
\end{cases}$$

(12)

where $p(t_0) = \mu + \lambda I x(t_0) - \lambda S$.

The proposition shows that price overshooting also occurs in the case of multiple predators as long as their initial holding, $x(t_0)$, is large enough. Section 6 shows that traders optimally choose a large position that makes the price overshooting relevant if the risk of default is not too large. Proposition 2’ (in Appendix A) shows that there
is no price overshooting when traders have small positions relative to their limits, that is, if $x(t_0) < \frac{I_p-I_c}{I_p-I_c} \bar{x}$.

The price overshooting is lower if there are more predators, since more predators behave more competitively:

**Proposition 3** Keeping constant the fraction, $I_p/I$, of predators, total stock holdings at $t_0$, $Ix(t_0)$, and total capacity, $Ix(T)$, the price overshooting is decreasing in the number of predators, $I_p$. As $I_p$ approaches infinity, the price overshooting disappears.

**Numerical Example**

We consider the cases with a total number of traders $I = 3, 9, \text{and} 27$. For each case, we assume that a third of the traders are in crisis, that is, $I_c/I = 1/3$. As in the previous example, we let $\lambda = 10$, $\mu = 140$, $S = 4$, $t_0 = 5$, $T = 8$, the total trading speed be $U = 2$, the total initial holding be $x(t_0) \cdot I = 1.6$, and the total trader holding capacity be $\bar{x} \cdot I = 2$. Figure 4 illustrates the holdings of predatory traders, and Figure 5 shows the price dynamics.

We see that there is a substantial price overshooting when the number of predators is small, and that the overshooting is decreasing as the number of predators increases. More predators increase the competitive pressure to buy shares early. Hence, the liquidation cost for a distressed trader is decreasing in the number of predators (even holding fixed the total trading capacity).

We note a striking difference between the case of one predator and the case with several predators. With a single predator, the predator keeps selling until the distressed traders have finished their liquidation, whereas with several predators the competitive pressure makes the predators buy back earlier and finish buying back at the same time as the distressed traders have finished their selling.

**Collusion.** The predators can profit from collusion. In particular, they could increase their revenue from predation by selling until the troubled traders were finished liquidating, and only then start rebuilding their positions. Hence, through collusion, the predators could jointly act like a single predator (with the slight modification that multiple predators have more capital). Collusive and non-collusive outcomes are qualitatively different. A collusive outcome is characterized by predators buying shares only after the troubled traders have left the market, and by a large price overshooting. In contrast, a non-collusive outcome is characterized by predators buying all the shares they need by the time the troubled traders have finished liquidating and by a relatively smaller price over-shooting.

Collusion could potentially occur through an explicit arranged agreement or through an implicit arrangement called “tacit” collusion. Tacit collusion means that the collusive outcome is the equilibrium in a non-cooperative game. In our model, tacit collusion cannot occur. However, if strategic traders could observe each others’ trading activity,
then tacit collusion might arise since, in that case, predators could “punish” a predator that deviates from the collusive strategy.\footnote{If traders could observe each others’ trades, then we would have to change accordingly our definition of strategies and equilibrium. A rigorous analysis of such a model is beyond the scope of this paper.}

### 4.2 Endogenous Default and Systemic Risk

So far we have assumed that one or a certain fraction of the institutional traders exogenously fall into financial distress, without specifying the underlying cause. In this section, we study how predatory activity can lead to contagious default events. We assume that a trader is forced to sell when his wealth drops to a threshold level $W$.\footnote{The forced liquidation may be caused, for instance, by margin constraints or a risk-management strategy.} Trader $i$’s wealth at $t$ consists of his position $x_i(t)$ of the asset that our analysis focuses on, as well as wealth held in other assets $O_i$. That is, his “paper wealth” is 

$$W_i(t) = x_i(t)p(t) + O_i(t).$$

The value of the other holdings $O_i(t)$ is subject to an exogenous shock at time $t_0$, which can be observed by all traders. Thereafter $O_i(t)$ stays constant. We assume that after a negative shock, trader $i$ is cash constrained and is unable to purchase more shares.

Obviously, if the wealth shock $\Delta O_i$ at $t_0$ is so large that $W_i(t_0) \leq W$, the trader is immediately in distress and must liquidate. Smaller negative shocks with $W_i(t_0) > W$ can, however, also lead to an “endogenous default,” since the potential selling behavior of predators and other distressed traders may erode the wealth of trader $i$ even further.
That is, predation can drive other traders into bankruptcy.

We let $W(I^c)$ be the maximum wealth at $t_0$ such that a trader $i$ cannot avoid financial crisis if $I^c$ traders are expected to be in crisis. More precisely, for $I^c > 0$, it is the maximum wealth such that for any strategy, $u^i \leq 0$, it holds that $\min_{t \in [t_0, T]} W^i(t, u^i, u^{-i}) \leq W$, where $u^{-i}$ has $(I^c - 1)$ strategies of liquidating and $I^p$ strategies of preying. Further, for $I^c = 0$, it is $W(0) = W$.

A trader who knows that he must liquidate in the future, finds it optimal to start selling already at time $t_0$.

**Proposition 4** Consider an equilibrium in which $I^c$ traders initially liquidate and $I^p = I - I^c$ traders prey as described in Propositions 1, 2 and 2'. Then, for every $i \in I^c$, $W^i(t_0) < W(I^c)$ and for every $i \in I^p$, $W^i(t_0) > W(I^c)$.

Interestingly, the higher is the expected number, $I^c$, of defaulting traders, the higher is the “survival hurdle” $W(I^c)$.

**Proposition 5** The more traders are expected to be in crisis, the harder it is to survive. That is, $W(I^c)$ is increasing in $I^c$.

This insight follows from that fact that a higher number of defaulting traders make predation more fierce and lowers the price at all times, making survival more difficult.

Proposition 5 proves useful in understanding systemic risk. Financial regulators are concerned that the financial difficulty of one or two institutions can drag down many more investors, thereby destabilizing the whole economy. Our framework explains why this spillover effect occurs. To see this, consider the economy depicted in Figure 6. Trader 1’s wealth is in the range of $(W(1), W(2)]$, trader 2’s wealth is in $(W(2), W(3)]$, and trader 3’s is in $(W(3), W(4)]$. The three remaining traders (4, 5, and 6) have enough reserves to fight off any crisis, that is, their wealth, is above $W(I)$.

If no trader faces any wealth shock at $t_0$, there is a unique equilibrium. No strategic trader is in distress and all of them immediately start increasing their position from $x(t_0)$ to $\hat{x}$. On the other hand, if trader 4 faces a wealth shock at $t_0$ such that $W^4(t_0) < W$, he can also drag strategic traders 1, 2 and 3 down. The fact that trader 4 is forced to liquidate his position encourages predation, and the price is depressed. This, in turn, brings three other traders into financial difficulty. As shown in Figure 7, if everybody expects that four traders will be in distress, traders 1, 2, 3 and 4 have to liquidate their position since their wealth level is below $W(4)$. This situation highlights the systemic risk component mentioned above. The financial difficulty of one trader endangers the financial stability of three other traders.

However, there is also a second equilibrium in which only trader 4 liquidates. If other traders do not panic and everybody expects that only trader 4 will go under, traders 1, 2, 3, and 5 prey and buy back after a short while. The predation is, however,
less fierce in this equilibrium in the sense that predators start re-purchasing shares earlier (i.e. smaller turning point, \( t_0 + \tau \)).

It is important to notice that the multiplicity in our example does not arise when trader 5 also faces a wealth shock at \( t_0 \) such that \( W^5(t_0) < W(1) \). In this case, the equilibrium in which traders 4 and 5 together drive traders 1, 2 and 3 into ruin is unique.

In our perfect information setting, all traders know the instant after \( t_0 \); how the equilibrium will play out. That is, they know the entire future price path as well as the number of predators \( I^p \) and victims \( I^c \). In a more complex setting in which traders’ wealth shocks are not perfectly observable and the price process is noisy, this need not be the case. A trader might start selling shares not knowing when the price decline stops. He might expect to act as a predator but may actually end up as prey.

In the case of multiple equilibria, coordination on the side of predators might lead to more predation, while coordination on the part of the potential victims might prevent a financial crisis. For example, coordination among predators is required if two or more predators are needed to push the price sufficiently down to drive a third trader into financial distress. On the other hand, coordination among vulnerable traders might prevent predation. For example one can imagine a situation, where the wealth level of two of three strategic traders drops into the range \([W, W(2)]\) such that if both these traders stay put, their wealth level never fall below \( W \). Thus, the predator will not attack and all traders survive. On the other hand, if the vulnerable traders start selling along with the predator, they drive each other into distress since the price decline erodes their wealth to a level below \( W \). In other words, their panic selling behavior helps the predator to exploit the situation.

The dangers of and the possibility of remedying the systemic risk in financial markets provides a strong argument for intervention by regulatory bodies, such as central
banks. A bailout of one or two traders or even only a coordination effort can stabilize prices and ensure the survival of numerous other vulnerable traders. However, it also spoils the profit opportunity for the remaining predators who would otherwise benefit from the financial crisis. From an ex-ante perspective, the anticipation of crisis preventive action by the central bank reduces the systemic risk of the financial sector and hence, traders are more willing to exploit arbitrage opportunities. This reduces the initial mispricing.

Finally, while in our equilibrium all vulnerable traders start liquidating their position from \( t_0 \) onwards, one sometimes observes that these traders miss the opportunity to reduce their position early. This exacerbates the predation problem, since a delayed reaction on the part of the distressed traders allows the predators to front-run. The phenomenon of delayed reaction by vulnerable traders may be explained in an enriched version of our framework. First, if prices are fluctuating, the trader might "gamble for resurrection" by not selling early, in the hope that a positive price shock will liberate him from financial distress. Second, if selling activity cannot be kept secret, a desire to appear solvent might prevent a troubled trader from selling early.

5 Valuation with Endogenous Liquidity

Predatory trading has implications for valuation of large positions. We consider three levels of valuation with increasing emphasis on the position’s liquidity.

**Definition 2** (i) The “paper value” of a position of \( x \) at time \( t \) is \( V^{\text{paper}} = xp(t) \); (ii) the “orderly liquidation value” is \( V^{\text{orderly}} = x \left( p(t) - \frac{1}{2} \lambda x \right) \); and (iii) the “distressed liquidation value”, \( V^{\text{distressed}} \), is the revenue raised in equilibrium when predators are preying.

The paper value is the simple mark-to-market value of the position. The orderly liquidation value is the revenue raised in a secret liquidation, taking into account the fact that the demand curve is downward sloping. The downward sloping demand curve implies that liquidation makes the price drop by \( \lambda x \), resulting in an average liquidation price of \( p(t) - \frac{1}{2} \lambda x \).

The distressed liquidation value takes into account not only the downward sloping demand curve, but also the strategic interaction between arbitrageurs and, specifically, the costs of predation. We note that \( V^{\text{distressed}} \) depends on the characteristics of the market such as the number of predators, the number of troubled traders, and their initial holdings. For instance, the distressed valuation of a position declines if other traders also face financial difficulty.

Clearly, the orderly liquidation value is lower than the paper value. The distressed liquidation value is even lower if the predators have initially large positions.\(^{13}\)

\(^{13}\)If the predators initial position is low, then the distressed liquidation value can be greater than the orderly liquidation value. This is because the public announcement of a distressed liquidation can,
Proposition 6  If \( x(t_0) \geq \sqrt[1-p/(p-1)]{\bar{x}} \) then the paper value of the position is greater than the orderly liquidation value, which in turn is greater than the distressed liquidation value. That is, \( V^{\text{paper}} > V^{\text{orderly}} > V^{\text{distressed}} \).

The low distressed liquidation value is a consequence of predation. In particular, predation causes the price to initially drop much faster than what is warranted by the distressed trader’s own sales. Hence, the market is endogenously more illiquid when a distressed trader needs liquidity the most.

Consider an arbitrageur estimating the liquidity of the market in “normal” times. This liquidity estimate leads to an estimate of the liquidation value of \( V^{\text{orderly}} \). The actual liquidation value in the case of distress, however, may be much lower.

It is interesting to consider what happens as the number of predators grow, keeping constant their total predation capacity. More predators implies that their behavior is more similar to that of a price-taking agent. This more competitive behavior makes predation less fierce, and increases the distressed liquidation value.

Importantly, even in the limit with infinitely many predators, the distressed liquidation value is strictly lower than the orderly liquidation value. This is because the price drops faster with preying predators, and this leads to a reduction in the liquidation value. Recall, in contrast, that Proposition 3 shows that the price overshooting disappears as the number of predators grow. It is, however, not only the price overshooting that reduces the distressed liquidation value.

Proposition 7  Suppose \( x(t_0) \geq \sqrt[1-p/(p-1)]{\bar{x}} \) and keep constant the fraction, \( I^p/I \), of predators, the total arbitrage capital, \( I\bar{x} \), and the total initial holding, \( Ix(t_0) \). Then, the total distressed liquidation value, \( I^cV^{\text{distressed}} \) is increasing in the number of predators, \( I^p \). In the limit, as \( I^p \) approaches infinity, the total distressed liquidation revenue remains strictly smaller than the total orderly liquidation value, \( I^cV^{\text{orderly}} > \lim_{I^p \to \infty} (I^cV^{\text{distressed}}) \).

One could further apply our framework to study the \textit{ex-ante} value of a large position (and the expertise in trading in this market), taking into account the risk of predation against oneself and the possible rewards from predation against others. This would be relevant, for instance, when considering a take-over of a hedge fund.

6  The Investment Phase \( (t \in [0, t_0]) \)

So far, we have taken as given the number, \( x(t_0) \), of shares that traders hold before predation starts. In this section, we analyze the process of acquisition of the initial
position prior to $t_0$. The strategic traders’ initial position at time 0 — when they learn of the arbitrage opportunity — is assumed to be zero. To separate the investment phase from the predatory phase, we assume that $t_0$ occurs sufficiently late, that is $t_0 > \frac{x}{U/I}$. This ensures that traders can acquire any position $x \in [-\bar{x}, \bar{x}]$ prior to $t_0$. We focus, in this section, on the case in which at most one institution faces financial crisis. Specifically, we assume\(^{14}\) that with probability $\pi$, $I^c$ is a singleton with all traders having equal probability of being in crisis, and with probability $1 - \pi$, no trader is in crisis, that is, $I^c = \emptyset$. Proposition 8 describes the initial trading by institutional investors.

**Proposition 8** First, all traders buy at the rate $U/I$ until they have accumulated a position of $x(t_0)$. If $I > 2$ and a distressed trader’s position is not disclosed then

$$x(t_0) = \left(1 - \frac{\pi}{I}\right) \bar{x}.$$ 

If $I = 2$ or if a distressed trader’s position is disclosed then

$$x(t_0) = \left(1 - \frac{\pi}{I-1}\right) \bar{x}.$$ 

After time $t_0$, traders use the (predatory) strategies described in Propositions 1, 2, and 2’.

All traders try to acquire their desired position $x(t_0)$ as quickly as possible. This is because delay is costly, since other traders’ acquisitions increase the price. Importantly, it is this desire of the traders to quickly acquire a large position that later leaves them vulnerable to predation.

The optimal pre-$t_0$ position, $x(t_0)$, balances the costs and benefits associated with the three possible outcomes: (i) no trader faces crisis, (ii) another trader faces crisis, and (iii) the trader himself is in distress. First, with probability $(1 - \pi)$, no strategic trader faces financial distress before time $T$. In this case, all traders try to acquire additional shares up to their maximum position $\bar{x}$ after time $t_0$. A trader who acquired a larger position prior to $t_0$ has the advantage that he purchased it at a lower price.

Second, with probability $\pi \frac{I-1}{I}$, one of the other traders faces financial difficulty and must liquidate his position. In this case, it is advantageous for a surviving trader to have a smaller position $x(t_0)$ since it is cheaper to acquire the shares after $t_0$.

Finally, with probability $\frac{\pi}{I}$, the trader himself must liquidate his position. In this case, the effect of having bought — out of equilibrium — a large position initially depends on whether this position is disclosed. If the distressed trader’s position is made

\(^{14}\)Here, we make (implicitly) the simplifying assumption that the default risk does not depend on the size of the position.
public then it is always worse, in case of distress, to have bought a larger position. This is because the predation makes the liquidation very costly.

If a distressed trader’s position is not disclosed and if \( I > 2 \), then the effect of initially buying some extra shares may be reversed because the initial purchase is secret. This explains why the initial position is larger in the case of no disclosure \((1 - \frac{\pi}{T} > 1 - \frac{\pi}{T+1})\). We discuss the effect of disclosure further in Section 7.2.

The risk default limits the initial aggregate acquisition and hence \( x(t_0) < \bar{x} \). It is interesting to compare the total initial arbitrage position, \( Ix(t_0) \) with the expected surviving arbitrage capital. The expected number of survivors is \( I - \pi \) so the expected surviving arbitrage capital is \( \bar{x}(I - \pi) \).

If \( I = 2 \) or if there is disclosure, the initial total arbitrage position, \( Ix(t_0) = \bar{x}(I - \pi \frac{1}{T-1}) \), is lower than the expected surviving arbitrage capital. This is because of the large cost associated with selling (an extra unit) to a monopolist predator or to predators who know that you bought the extra shares.

Hence, the risk of costly predation limits the arbitrage positions, and this makes the price depart initially further away from the expected value of the asset.

With \( I > 2 \) and no disclosure, the initial total arbitrage position is equal to the expected surviving arbitrage capital, that is, \( Ix(t_0) = \bar{x}(I - \pi) \), leading to a “martingale-like” property of the price. This is because, in case one defaults, the marginal share is sold at a “fair” price when all the predators have bought back their position. If the risk of default was increasing in the size of the position, however, then arbitrage positions would initially be smaller.

7 Further Implications of Predatory Trading

7.1 Contagion

Predatory trading suggests a novel mechanism for financial contagion. Suppose that the arbitrageurs have large positions in several markets. Further suppose that a large arbitrageur incurs a loss in one market, bringing this trader in financial trouble. Then, this arbitrageur must downsize its operations and hence reduces all his positions. As explained by our model, other traders have an incentive to undertake predatory trading, thereby enhancing the price impact in all the affected markets.

This type of contagion is not driven by a correlation in economic fundamentals or by information spillovers, but, rather by the composition of the holdings of large traders who must significantly reduce their positions. This insight has the following empirical implication: a shock to one security, which is held by large vulnerable traders, may be contagious to other securities that are also held by the vulnerable traders.
7.2 Regulation and Risk Management

Predatory trading may have implications for the appropriateness of the regulation of securities trading and of arbitrageurs such as hedge funds. We discuss in turn the implications for disclosure regulation, VaR risk management, circuit breakers, and the up-tick rule.

*To be written.*

7.3 Bailouts

*To be written.*

8 Conclusion

*To be written.*

A Proofs

A.1 Proof of Proposition 1

The surviving trader, $i$, wants to minimize $\int_{t_0}^{T} u^i(t) X^{-i}(t) \, dt$ subject to the constraints that $x^i(T) = \bar{x}$ and $|u^i(t)| \leq U/I$. Here, $X^{-i}(t)$ is the position of the trader in financial trouble so

$$X^{-i}(t) = \begin{cases} 
        x(t_0) - U/I t & \text{for } t \in [t_0, t_0 + x(t_0)I/U] \\
        0 & \text{for } t > t_0 + x(t_0)I/U
\end{cases}$$

Since $X^{-i}(t)$ is decreasing, $\int_{t_0}^{T} u^i(t) X^{-i}(t) \, dt$ is minimized by choosing the control variable as stated in the proposition.

A.2 Proof of Proposition 2

Suppose that $x(t_0) \geq \frac{p}{T-1} \bar{x}$ and, without loss of generality, that trader $i$ is not the trader in financial crisis, and that all other traders are using the proposed equilibrium strategies. Then, the total position, $X^{-i}(t)$, of the other traders is

$$X^{-i}(t) = \begin{cases} 
        (I - 1) \left( x(t_0) - \frac{U}{T} t \right) & \text{for } t \in [t_0, t_0 + \tau] \\
        (I^p - 1) \bar{x} & \text{for } t > t_0 + \tau
\end{cases}$$

Trader $i$ wants to minimize $\int_{t_0}^{T} u^i(t) X^{-i}(t) \, dt$ subject to the constraints that $x^i(T) = \bar{x}$ and $u^i(t) \in [u, \bar{u}]$. Since $X^{-i}(t)$ is decreasing and then constant, $\int_{t_0}^{T} u^i(t) X^{-i}(t) \, dt$
is minimized by choosing \( u^i = u \) as long as \( X^{-i}(t) \) is decreasing, and by choosing a positive \( u^i \) thereafter. Hence, the \( u^i \) that is described in the proposition is optimal. We note that trader \( i \)'s objective function does not depend on the speed with which he buys back after time \( \tau \). There is a single speed, however, which is consistent with equilibrium.

To prove the uniqueness of this equilibrium, we show that, in any symmetric equilibrium, \( X^{-i} \) must satisfy the following: (i) \( X^{-i}(t) > (I^p - 1) \bar{x} \) implies \( \dot{X}^{-i}(t) = -U \frac{I - 1}{T} \). (ii) \( X^{-i}(t) < (I^p - 1) \bar{x} \) implies \( \dot{X}^{-i}(t) = U \left( I - \frac{\bar{x}}{I^p T} \right) > 0 \). It is easily seen that there is a unique \( X^{-i}(t) \) satisfying these two conditions.

### A.3 Proposition 2’ and its Proof

**Proposition 2’** Suppose that \( x(t_0) < \frac{I^p - 1}{T - 1} \bar{x} \) and let \( \tau := -\frac{x(t_0) - \frac{I^p - 1}{T - 1} \bar{x}}{U \left( 1 - \frac{\bar{x}}{I^p T} \right)} \). The unique symmetric equilibrium strategy is for each predator \( i \in I^p \) to buy fast for \( \tau \) periods and keep buying at a lower trading intensity until \( t_0 + \frac{x(t_0)}{U^{p/T}} \). More precisely,

\[
\begin{align*}
  u^i^*(t) &= \begin{cases} 
    \frac{U(t + I^p)}{U^p T} & \text{for } t \in [t_0, t_0 + \tau), \\
    \frac{U^p T}{I^p - 1} & \text{for } t \in [t_0 + \tau, t_0 + \frac{x(t_0)}{U^{p/T}}), \\
    0 & \text{for } t \geq t_0 + \frac{x(t_0)}{U^{p/T}}.
  \end{cases}
\end{align*}
\]  

\[(13)\]

**Proof**

Analogous to the proof of Proposition 2.

### A.4 Proof of Proposition 8

We give a sketch of the proof. To see the optimality of trader \( i \)'s strategy, we first note that, for any value of \( x^i(t_0) \), it is optimal to use the equilibrium strategy after time \( t_0 \). The argument for this follows from the proofs of Propositions 1 and 2. Further, prior to \( t_0 \), it is optimal to acquire shares at a rate of \( \bar{x} \) until the trader has reached his pre-\( t_0 \) target. This follows from the incentive to acquire the position before other traders drive up the price.

The equilibrium level of \( x(t_0) \) is derived in the remainder of the proof. We consider trader \( i \)'s expected profit in connection with buying \( x(t_0) + \Delta \) shares, given that other traders buy \( x(t_0) \) shares. More precisely, we use Lemma 1 and consider how the marginal \( \Delta \) infinitesimal shares affect the “trading cost” \( \int u^i(t)X^{-i}(t)dt \). First, buying \( \Delta \) infinitesimal extra shares prior to time \( t_0 \) costs \( \Delta(I - 1)x(t_0) \) since the shares are optimally bought when the other traders have finished buying and \( X^{-i} = (I - 1)x(t_0) \).

The benefit, after \( t_0 \), of having bought the \( \Delta \) shares depends on whether (i) no trader is in distress, (ii) another trader is in distress, or (iii) the trader himself is in distress:
(i) If no trader is in distress then having the extra $\Delta$ shares saves a purchase at the per-share cost of $X^{-i} = (I - 1)\bar{x}$. This is because the marginal shares are bought in the end when the other $(I - 1)$ traders each have acquired a position of $\bar{x}$.

(ii) If another trader is in financial distress then having the extra $\Delta$ shares saves a purchase at the per-share cost of $X^{-i} = (I - 2)\bar{x}$. This is the total position of the other $I - 2$ predators when the defaulting trader has liquidated his entire position.

(iii a) Suppose $I > 2$ and the position of the distressed trader is not disclosed at time $t_0$. Then, if the trader himself is in financial distress, the extra $\Delta$ shares can be sold when $X^{-i} = (I - 1)\bar{x}$. This is the position of the predators when one has just finished liquidating. At that time the predators have preyed and re-purchased their position.

(iii b) Suppose $I = 2$ or that the position of the distressed trader is disclosed at time $t_0$. Then, if the trader himself is in financial distress, the extra $\Delta$ shares can be sold when $X^{-i} = (I - 2)\bar{x}$. To see this, note that the extra shares imply that the predators prey longer ($\tau$ larger) because they know that one must liquidate a larger position. Hence, the marginal shares are effectively sold at the worst time when $X^{-i}$ is at its lowest point.

We can now derive the equilibrium $x(t_0)$ by imposing the requirement that the marginal cost of buying the extra shares equals the marginal benefit. In the case in which $I > 2$ and the position of the distressed trader is not disclosed at time $t_0$, we have

$$(I - 1)x(t_0) = (1 - \pi)(I - 1)\bar{x} + \pi \frac{I - 1}{I}(I - 2)\bar{x} + \pi \frac{1}{I}(I - 1)\bar{x},$$

implying that

$$x(t_0) = (1 - \frac{\pi}{I})\bar{x}.$$  

In the case in which $I = 2$ or the position of the distressed trader is disclosed at time $t_0$, we have

$$(I - 1)x(t_0) = (1 - \pi)(I - 1)\bar{x} + \pi \frac{I - 1}{I}(I - 2)\bar{x} + \pi \frac{1}{I}(I - 2)\bar{x},$$

implying that

$$x(t_0) = (1 - \frac{\pi}{I - 1})\bar{x}.$$  

The global optimality of buying $x(t_0)$ shares is seen as follows. First, buying fewer shares than $x(t_0)$ is not optimal since the infra-marginal shares are bought cheaper (in term of $X^{-i}$) than $(I - 1)\bar{x}$ and their expected benefits are at least $(I - 1)\bar{x}$. Second, buying more shares than $x(t_0)$ costs $(I - 1)\bar{x}$ per share, and the expected benefit of these additional shares is at most $(I - 1)\bar{x}$.  

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B Noisy Asset Supply

In this section we consider an economy in which (i) agents can condition on prices and (ii) supply of assets is noisy. In a pure-strategy equilibrium, an agent cannot learn from prices if another trader deviated from his equilibrium strategy. This is because, in equilibrium, any price change is ascribed to supply uncertainty. This feature of the noisy-supply equilibrium is the justification for the equilibrium concept described in Definition 1. We show that the equilibrium strategies for the non-noisy economy also constitutes an equilibrium in a corresponding game with noisy supply.

We assume that the supply, $S_t$, is a Brownian motion with volatility $\sigma$, that is,

$$dS_t = \sigma dW_t,$$

where $W$ is a standard Brownian motion. Agent $i$ maximizes his expected wealth

$$\max_{u(t) \in U} E \left(- \int_0^T u^i(t)p(t)dt + x^i(T)v \right),$$

where $U$ is the set of $\{F_t\}$-adapted processes, and $\{F_t\}$ is generated by the price process $\{p_t\}$ the crisis indicator $1_{t \geq t_0}$ and by $G(u^i(t), u^{j-i}(t))$. The price is defined as before by $p(t) = \mu - \lambda(S_t - X(t))$, where $S_0 > I$. We use the definition $\bar{p}(t) = \mu - \lambda(S_0 - X(t))$. With this definition, the agent’s objective function can be written as

$$E \left(x^i(T)v - \int_0^T [u^i(t)p(t) + G(u^i(t), u^{j-i}(t))]dt + \right) =$$

$$E \left(x^i(T)v - \int_0^T [u^i(t)\bar{p}(t) + G(u^i(t), u^{j-i}(t))]dt + E \int_0^T u^i(t)[\bar{p}(t) - p(t)]dt.\right)$$

The first term on the right hand side is the same as the objective function with a constant supply of $S_0$. Hence, this term is maximized by the equilibrium strategy if all other agents use the equilibrium strategy. The second term is, as we show next, zero under an additional assumption. For any $\{F_t\}$-adapted process, $u$, it holds that

$$E \int_0^T u^i(t)[\bar{p}(t) - p(t)]dt$$

$$= \lambda E \int_0^T u^i(t)[S_0 - S_1]dt$$

$$= \lambda E \int_0^T u^i(t)[S_T - S_1]dt - \lambda E \left( \int_0^T u^i(t)dt [S_T - S_0] \right)$$

$$= \lambda E \int_0^T u^i(t)E_t [S_T - S_1]dt - \lambda E \left( \int_0^T u^i(t)dt [S_T - S_0] \right)$$

$$= -\lambda E \left( \int_0^T u^i(t)dt [S_T - S_0] \right).$$

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If we assume that the agent must choose a strategy with \( \tilde{x} = x_T = x_0 + \int_0^T u^i(t)dt \), then the last term is zero. This assumption means that the agent must end up fully invested in the asset. We note that the agent would optimally choose \( x_T = 1 \) as long as \( S \) is not too small, and \( S \) is close to \( S_0 \) with large probability if \( \sigma \) is small.

Even if we do not impose the additional assumption that \( x_T = \tilde{x} \), we can show that the equilibrium in the model without supply uncertainty is an \( \varepsilon \)-equilibrium in the model with noisy supply. (See Radner (1980) for a discussion of \( \varepsilon \)-equilibria.) This property of the strategies follows from the fact that the latter term above can be bounded as follows

\[
\left| E \left( \int_0^T u^i(t) dt \ [S_T - S_0] \right) \right| \leq E \left( \int_0^T |u^i(t)| dt \ |S_T - S_0| \right) \\
\leq E \left( \int_0^T |u^i(t)| dt \ |S_T - S_0| \right) \\
\leq E \left( \int_0^T cd dt \ |S_T - S_0| \right) \\
\leq cT E |S_T - S_0| \\
= cT \sigma E |W_T - W_0| \\
\rightarrow 0 \quad \text{as} \ \sigma \rightarrow 0.
\]

Hence, agent \( i \)'s maximal gain from deviating from the strategy of the non-noisy game approaches 0 as the supply uncertainty vanishes (\( \sigma \rightarrow 0 \)).
Table 1: Examples of risks associated with predatory trading.

<table>
<thead>
<tr>
<th>Issue</th>
<th>Source</th>
<th>Quotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systemic risk</td>
<td>Testimony of Alan Greenspan, U.S. House of Representatives, 10/1/98</td>
<td>“It was the judgment of officials at the Federal Reserve Bank of New York, who were monitoring the situation on an ongoing basis, that the act of unwinding LTCM’s portfolio in a forced liquidation would not only have a significant distorting impact on market prices but also in the process could produce large losses, or worse, for a number of creditors and counterparties, and for other market participants who were not directly involved with LTCM.”</td>
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<td>Predation, LTCM</td>
<td>Business Week, 2/26/01</td>
<td>“If lenders know that a hedge fund needs to sell something quickly, they will sell the same asset – driving the price down even faster. Goldman Sachs and counterparties to LTCM did exactly that in 1998. (Goldman admits it was a seller but says it acted honorably and had no confidential information.)”</td>
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<td>LTCM</td>
<td>New York Times Magazine, 1/24/99</td>
<td>Meriwether quoting another LTCM principal: “the hurricane is not more or less likely to hit because more hurricane insurance has been written. In financial markets this is not true ... because the people who know you have sold the insurance can make it happen.”</td>
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<td>Enron, UBS Warburg</td>
<td>AFX News Limited, AFX - Asia, January 18, 2002.</td>
<td>UBS Warburg’s proposal to take over Eron’s traders without taking over the trading book was opposed on the ground that “it would present a ‘predatory trading risk’, as Enron traders effectively know the contents of the trading book.”</td>
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<td>Disclosure</td>
<td>IAFE Investor Risk Committee (IRC) (2001)</td>
<td>“For large portfolios, granular disclosure is far from costless and can be ruinous. Large funds need to limit granularity of reporting sufficiently to protect against predatory trading.”</td>
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<td>Market making</td>
<td>Financial Times (London), 6/5/1990, section I, page 12.</td>
<td>UK market makers wanted to keep the right to delay reporting of large transactions since this would give them “a chance to reduce a large exposure, rather than alerting the rest of the market and exposing them to predatory trading tactics from others.”</td>
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<td>Predation</td>
<td>Securities Week, 11/11/91</td>
<td>“knowledge that other customers intend to sell large amounts of stock should a specified decline occur in the stock’s price... the dishonest customer’s stock selling will trigger ... selling done by other customers ... result in a large decline in the stock’s price.”</td>
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References


