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Robert W. Rosenthal

Robert Rosenthal died on February 27, 2002, of a sudden heart attack. He was just 57, in the prime of his professional life. He is missed and loved by the many friends, colleagues and students who knew him. His incisive, constructive views helped develop many new ideas, and even inspired directions of research that do not explicitly bear his name. The fact that he never took received wisdom for granted was profoundly nurturing, and he applied this ability not just to the research of others but also to his own work. In examining his distinguished list of publications, one is struck by a constant sense of unease with economic theory, and game theory in particular, which led him to visit and revisit many of the outstanding problems in the area—particularly those dealing with the question of rational behavior—in different, eclectic ways.

Bob Rosenthal made fundamental contributions to the way in which we think about game theory, some better known than others. To take an example of a little-known contribution, Theorem 3 in [9] is a formal statement of what is now widely known as the Revelation Principle. Roger Myerson refers to this theorem in his own first paper on this subject [5], indicating Bob's priority for this result. (Of course, the Revelation Principle has been independently "discovered" more than once, and Bob himself, seeing the result formally re-stated some years after his own rendition of it, reportedly remarked, "Why on earth would anybody want to write that down?")

What follows is a (necessarily) selective account of some of Bob's contributions to economic theory.

In [10] Bob introduced a now-classical framework: random matching. To our understanding, this is the first explicit and rigorous analysis of games played across "a sequence of varying opponents". The paper reads as a reaction to the theory of repeated games, observing correctly that many problems involve not repeated play, but changing partners. One immediate and basic implication of this structure is (or should be) that histories are imperfectly observed. Bob assumed that only the most recent action taken by a freshly matched partner is observable. With this assumption in hand, one may describe a (Markov) strategy by a rule that maps one's immediate past actions, and that of the opponent, to an action to be taken today. Together, these strategies induce a Markov process on societal actions as a whole. Bob considered the steady states of such processes, against which each individual Markov strategy forms a best response. [He recognized that out-of-steady-state individual best responses are no longer Markov, thus raising the very difficult question of convergence in this framework.]

Random matching is, of course, widely used today. Indeed, Bob applied the idea himself in joint work with Henry Landau [12]. This beautiful paper (one of the few

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for which Bob professed a particular fondness) uses random matching to study the evolution of individual reputations in bargaining, and the social influence of such reputations on individual bargaining behavior. Call this influence a *custom*. If asymmetric outcomes (one yields, the other is firm) are efficient in the sense of maximizing additive surplus, as they often may be, then customs are potentially useful as a coordination device. But there are many potential customs. Rosenthal and Landau study and compare different customs that might persist in steady state, and compare the efficiency gains they yield. Those interested in the evolution of social norms, or more broadly in the social determinants of individual behavior, will benefit from studying the methodology introduced in this paper.

In [11] Bob addressed the paradoxical nature of backward induction in perfect information games. Of course, he was not the only one to do so around this time, but he was possibly unique (at the time) in refusing to budge from the complete information setup. In contrast to incomplete information about types, Bob stressed the possibility that actions may conform only in some probabilistic way to the best choice. There are many provocative ideas raised in this paper. Among them is the introduction of the beautiful and puzzling centipede game: not that it raised any new paradoxes but that it displayed, in a minimal, elegant and forceful way, the nature of the backward-induction problem. Little wonder that the centipede game is used as a basic parable in textbooks, philosophical musings on the meaning of rationality, as well as experiments.

A fresh reading of this paper would be rewarding to many. In it, Bob advocates an approach which is very close indeed to the popular quantal-choice model, in which the probability of making a choice varies directly with the relative payoff advantage of that choice, but may not be zero or one. His beautiful insight is that such "trembles" echo back, in an amplified way, as one recurses backwards through the game tree. For instance, in the centipede game, a slight error at the very last stage actually *increases* the relative payoff advantage of "passing" at the penultimate stage, even though it may still be better to stop. This creates a slightly larger tendency to pass at the penultimate stage. Now at the stage just anterior to the penultimate stage it is even more profitable, relatively speaking, to pass. As one inducts up the tree this must ultimately make "passing" the best strategy, if the tree is long enough.

On another front, very early in Bob's professional career, he expressed dissatisfaction with the way in which cooperative game theory approaches the problems it seeks to tackle. (Of course, Bob was not alone in making these points (for example, see [4] and W. Lucas, "On solutions to n-person games in partition function form", Ph.D. dissertation, University of Michigan, Ann Arbor, 1963). The attack [7,8] was not along the popular lines that cooperative game theory relies too heavily on binding agreements, which is no real criticism at all, but focused on the very fundamentals of the theory: characteristic functions. Bob observed that the construction of the characteristic function "implicitly assumes that players expect other players actually to carry out threats which are injurious to their own welfare". Of course, such a criticism has no value if there are no externalities across agents, for then coalitions are effectively in isolation anyway. But Bob's focus *was* on cases with externalities, and these early papers realized, very clearly, that this shortcoming

would be fundamental to cooperative game theory. Only in recent literature has this issue been resurrected in a fresh way.

In [2,3] Bob, with Douglas Gale, investigated the consequences of types of bounded rationality exhibited by "experimenters" and "imitators". In the context of an example with one experimenter and a number of imitators, they explored in detail the varied system dynamics that can be generated by the interaction of these two behaviors. This example provides an interesting and elegant illustration of how simple strategic "rules of thumb" can interact to produce complicated dynamics, and also of the roles of substitutability and complementarity of actions in games.

Motivated by the observation that mixed strategies in games seem to have limited appeal in many practical situations, in [6] Bob, with Roy Radner, explored the question of how far one can get without using mixed-strategy equilibria. They proved that in games with a finite number of players and moves, if each player observes a private signal with an atomless distribution, which is independent of the observations and payoffs of the other players, then for every mixed strategy equilibrium there is a corresponding pure strategy equilibrium with the same expected payoffs for each player. (In particular, this implies that every such game has a pure-strategy equilibrium.) On the other hand, they gave examples of games that illustrate the importance of the "independence assumption", as well as the importance of the condition that each player have only finitely many moves. Hence the conditions needed for the "purification" of mixed-strategy equilibria are surprisingly strong. However, in [1] it is shown that conditions sufficient for the existence of pure-strategy approximations to mixed-strategy equilibria are significantly weaker.

We hope that this incomplete account of Bob's research gives some idea of the range of his interests, critical insights, and contributions. What this academic discussion cannot illustrate, however, is Bob's many-layered web of rich and productive interactions with researchers and students from all over the world. In the ultimate analysis, and notwithstanding the deep nature of his own contributions, this ability to interact was possibly Bob's defining feature as a scholar. His untimely death will deprive economic theory of the important contributions that we would have expected from him in the future, but more significantly, it has deprived us—his friends, colleagues, and students—of Bob's insight, humor, and great intelligence.

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