

# Self-enforcing climate-change treaties

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**In the absence of world government, an effective treaty to control the emissions of greenhouse gases should be self-enforcing. A self-enforcing treaty has the property that, if a country expects other countries to abide by the treaty, it will be in the self-interest of that country to abide by the treaty too. (A difficulty with the Kyoto Protocol is that it does not appear to lay the groundwork for a self-enforcing treaty). A self-enforcing treaty can be modeled as a Nash equilibrium of a suitably defined dynamic game among a large number of sovereign countries of diverse sizes and economic capabilities. We study such a game and characterize its equilibria (typically there are many) and the global-Pareto-optimal solutions. We identify one of the equilibria, which we call “business as usual,” with the current situation. The multiplicity of equilibria provides an opportunity to move from the inefficient business-as-usual equilibrium to one or more equilibria that are Pareto-superior. Using a calibrated model with 184 countries, we give numerical illustrations of business-as-usual and global-Pareto-optimal trajectories and estimate the potential welfare gains from a self-enforcing treaty.**

There is a growing scientific consensus that global warming and related climate changes have been occurring during the past three centuries, that a significant factor in this trend is the accumulation of atmospheric greenhouse gases (GHGs), and that human activity is largely responsible for this increased concentration (1, 2). Most important among those activities are the burning of fossil fuels and the deforestation of the earth's surface. Although this phenomenon is popularly referred to as “global warming,” in fact, the associated climate changes are more complex than just an increase in the temperature of the atmosphere near the earth's surface. In particular, it is expected that the volatility of the weather will also increase and that the changes will vary from one part of the earth to another. We shall therefore use the term “global climate change” to describe this process.

In many, if not most, parts of the world the process of global climate change, if left unchecked, will cause costly damage. In some parts, there may be benefits, such as increased agricultural productivity in some areas in the north central part of North America. On balance, it is estimated that the net cost of damage to the world will be significant, although less agreement exists on the quantitative estimates of this cost. However, efforts to reduce the rate of GHG emissions will also be costly. From a global point of view, the climate-change problem is that of discovering and implementing ways to reduce GHG emissions for which the global benefits exceed the cost.

At the root of the climate-change problem is a “tragedy of the common” (3). Emissions by any one country add to the ambient stock of GHGs for all countries, with its attendant effects on all countries. Thus, when considering how much to spend on controlling emissions, if each country only takes account of its own benefits and costs, then the resulting total emission rate for the world will be too high from a global point of view. In other words, starting from this “self-centered” regime, if every country reduced its emissions by some positive amount (and some appropriate transfers were made among the countries), then every country would be better off. In this article, we call the self-centered regime “business as usual” (BAU). (See *A Climate-Change Game* for a precise definition).

When a country faces an internal tragedy of the common, e.g., the creation of smog in a metropolitan area, the relevant jurisdiction (e.g., Los Angeles County) can pass laws and regulations to control

the activities of individual people and corporations that lead to the problem. Furthermore, the state or federal government can force different local areas to cooperate. For example, the South Coast Air Quality Management District, the agency responsible for regulating air quality in Los Angeles, was expanded beyond Los Angeles to include the five adjacent counties to deal with “most” of the region covered by the air pollution externality in the city.

In the absence of a world government, to escape from the BAU climate-change trajectory, countries must devise some self-enforcing international agreement or treaty. A self-enforcing treaty (SET) has the property that, if a country expects the other countries to abide by the treaty, then it will be in the self-interest of that country to abide by the treaty too. This perspective is central to our analysis. (One possible objection to the Kyoto Protocol is that it does not appear to lay the groundwork for a SET. See *Concluding Remarks*.)

In principle, there may be many SETs, which differ not only in their global efficiency but also in their distributions of costs and benefits among the countries. This article describes a methodology for identifying (characterizing) SETs in the context of the climate-change problem. The methodology is embodied in a mathematical model of dynamic strategic interaction among countries, which uses the theory of (dynamic) games (4). The model specifies the dynamic strategies available to the individual countries and the net benefit to each country that would result from every possible profile of countries' strategies. A Nash equilibrium of the game is a strategy profile such that no country can benefit by unilaterally changing its own strategy. We shall show that, in particular, BAU is one such equilibrium. If it were the only one, then the situation would be hopeless. Fortunately, we shall also show that, as is typical of dynamic games, many equilibria of the global climate-change game exist, and many are better for all countries concerned than BAU. Each such equilibrium is a candidate for a SET.

The model analyzed in this article represents the interactions among a large number of sovereign countries (~200) of diverse sizes and economic capabilities. To focus on the effects of changes in emissions, the model leaves out certain phenomena, like population change and economic growth. Some of these phenomena can be introduced without essential difficulty, especially if these factors are considered to be exogenous to the model. Also, to make the analysis computationally tractable, certain linear approximations are made. These and other assumptions are discussed further in *Concluding Remarks*. The reader is referred to working papers for the longer derivations of the mathematical results.

The next section sets forth the basic model, and *Equilibrium and Optimal Time Paths* describes some characteristics of the set of equilibrium time paths and the globally optimal time paths. *A Numerical Example* presents numerical examples of equilibrium and optimal trajectories (based on work by Sangwon Park<sup>§</sup>). Possible extensions of the model, concluding remarks, and a brief review of the related literature, are presented in *Concluding Remarks*.

Abbreviations: SET, self-enforcing treaty; GHG, greenhouse gas; BAU, business as usual; GPO, global Pareto optimum.

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§Park, S. (2003) Economics Department, Columbia University, New York, Working paper, *Strategic Analysis of Global Climate Change: A Calibration and Numerical Analysis*.

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### A Climate-Change Game

In this section we describe a mathematical model of a dynamic climate-change game. The players in the game are countries, and it is assumed that each country has the authority and political will to control its own rate of emission of GHGs, subject to technological and resource constraints.

There are  $I$  countries. The emission of (a scalar index of) GHGs during period  $t$  by country  $i$  is denoted by  $a_i(t)$ . [Time is discrete, with  $t = 0, 1, 2, \dots$ , ad infinitum, and the  $a_i(t)$  are nonnegative.] The emission of GHG in each country is related to its level of economic activity, notably the production and use of energy produced by burning fossil fuels, although other sources of GHGs exist (5). For simplicity we let  $e_i(t)$  denote a scalar index of inputs into production and consumption associated with the emission of GHGs during period  $t$  by country  $i$ . For brevity, we shall call  $e_i(t)$  the level of “energy input.” The output of the country is described by a scalar index, e.g., gross domestic product (GDP). This output depends on  $e_i(t)$  and other inputs according to the country’s current “production function.” In our model, the production function is in a “reduced form,” implicitly reflecting for each level of  $e_i(t)$  the corresponding levels of the other inputs and can be interpreted as holding constant in time, for each country, its stocks of capital and labor, and the technology of production, except for the production of energy. Thus country  $i$ ’s GDP in period  $t$  is denoted by  $Y_i[e_i(t)]$ . Given the country’s current technology, its emission of GHG during the period is assumed to be

$$a_i(t) = f_i(t)e_i(t), \quad i = 1, \dots, I. \quad [1]$$

The coefficient  $f_i(t)$  will be called the emission factor of country  $i$  in period  $t$ . [In an equivalent model, the emission factors are constant in time, but every unit of GDP may be produced (more efficiently) by successively smaller amounts of energy as time passes.]

Let  $A(t)$  denote the global (total) emission during period  $t$ .

$$A(t) = \sum_{i=1}^I a_i(t). \quad [2]$$

The total (global) stock of GHGs at the beginning of period  $t$  is denoted by  $g(t) + g_0$ , where  $g_0$  is what the “normal” steady-state stock of GHGs would be if there were negligible emissions from human sources (e.g., the level of GHGs in the year 1800). We might call  $g(t)$  the excess GHG, but we shall usually suppress the word “excess.” The law of motion for the GHG is assumed to follow the linear difference equation,

$$g(t + 1) = A(t) + \sigma g(t), \quad [3]$$

where  $\sigma$  is a given parameter ( $0 < \sigma < 1$ ). [This linear approximation is a gross simplification of GHG dynamics (6).] We may interpret  $(1 - \sigma)$  as the fraction of the beginning-of-period stock of GHG that is dissipated from the atmosphere during the period. The “surviving” stock,  $\sigma g(t)$ , is augmented by the quantity of global emissions,  $A(t)$ , during the same period.

We assume that for each country the cost of the damage due to climate change is linear in the global stock of GHG, i.e., equal to  $c_i g(t)$ , and is subtracted from the country’s GDP in that period.

Finally, each country can reduce its own emission factor,  $f_i(t)$ , but at a cost. We assume that this cost is proportional to the decrease in the emission factor, i.e., equal to  $k_i[f_i(t) - f_i(t + 1)]$ . Actions taken in one period to reduce its emission factor take effect in the next period. We assume that  $k_i > 0$  and that the changes in the emission factors are constrained by

$$m_i \leq f_i(t + 1) \leq f_i(t). \quad [4]$$

Thus, in each period a country can only reduce its emission factor, not increase it, and there is a lower bound on the (eventual) level of its emission factor.

The utility of country  $i$  in period  $t$  (one-period payoff) is

$$v_i(t) = Y_i[e_i(t)] - c_i g(t) - k_i[f_i(t) - f_i(t + 1)]. \quad [5]$$

Let  $\delta$  denote the discount factor; then, the total discounted utility (total payoff) for country  $i$  is

$$v_i = (1 - \delta) \sum_{t=0}^{\infty} \delta^t v_i(t), \quad i = 1, \dots, I. \quad [6]$$

Note that each country’s current-period payoff depends directly on its current energy usage only through its production function, but it also depends on its own and others’ previous energy usage and emission factors through their effects on the current stock of GHG. Note, also, that the present value in the payoff function has been normalized, so that as the discount factor approaches unity, the normalized present value will typically approach as a limit the long-run average payoff. This finding does not have an impact on the analysis when the discount factor is fixed, but it does influence the interpretation of the numerical results when the discount factor is varied (*A Numerical Example*).

We assume that  $Y_i$  is strictly concave and twice differentiable, and it reaches a maximum at some finite level of energy use. The damage cost coefficients,  $c_i$ , are constant in time and strictly positive ( $c_i > 0$ ), although our method of analysis would allow them to have either sign. The discount factor,  $\delta$ , is the same for all countries, with  $0 < \delta < 1$ .

The state of the system at the beginning of period  $t$  is characterized by the  $(I + 1)$ -dimensional vector,  $s(t) = [f(t), g(t)]$ , where  $f(t) = [f_1(t), \dots, f_I(t)]$ . A strategy for a country determines for each period the country’s energy usage and emission factor as a function of the entire past history of the system, including the state variables up to the current period and the past actions of all the countries. A Nash Equilibrium is a profile of strategies such that no individual country can increase its payoff by unilaterally changing its strategy. A Nash equilibrium is the formal construct that corresponds to a self-enforcing treaty. Many Nash equilibria of the climate-change game will typically exist, and the set of Nash equilibria will depend on the initial state of the system.

It is important to note that no way exists for a country in any period to commit itself to follow a particular strategy in the future. In particular, because there is no world government, countries cannot sign binding contracts.

A stationary strategy for country  $i$  is a strategy that is history-independent and only depends on the current state,  $s$ , which is then mapped into a current action,  $a_i$ . A Markov Nash Equilibrium (MNE) is a Nash Equilibrium in which every country’s strategy is stationary. In a Markov Nash Equilibrium, no matter which period and history of emissions we consider, a country’s best option from that point on is to follow through on the remainder of its Markov Nash strategy.

Finally, it is useful to have as a benchmark the concept of a global Pareto optimum (GPO). Let  $x = (x_i)$  be a vector of positive numbers, one for each country. A GPO corresponding to  $x$  is a profile of strategies that maximizes the weighted sum of country payoffs,

$$v = \sum_i x_i v_i, \quad [7]$$

which we shall call the global welfare. One interpretation of a GPO is that it is what a “world government” would like to do for the world if it could force the national governments to act in the way that it deemed fit.

Without loss of generality, we may take the weights,  $x_i$ , to sum to  $I$ . We emphasize that a different global welfare function corresponds to each vector of weights and, hence, in general, a different GPO.

### Equilibrium and Optimal Time Paths

**BAU Equilibrium.** In this section we characterize the set of equilibrium trajectories and the set of GPO trajectories. We start by describing a particular equilibrium, which we call BAU. This benchmark equilibrium appears to correspond to what we currently observe in the world.

We shall show that BAU equilibrium strategies have the form:

$$e_i(t) = E_i[f_i(t)], \quad f_i(t+1) = F_i[f_i(t)], \quad t \geq 0, i = 1, \dots, I. \quad [8]$$

Furthermore,  $f_i(t)$  will be constant after period 0 and will equal either  $m_i$  or  $f_i(0)$ . Note that such a strategy is stationary, as defined in the preceding section. In fact, the argument of a BAU strategy is the country's own current emission factor only and does not include the current stock of GHG or the emission factors of the other countries.

Here is a precise characterization of BAU strategies. Define

$$w_i = \frac{c_i}{1 - \delta\sigma}. \quad [9]$$

Assume that, for each country,

$$Y_i'(0) > \delta w_i f_i(0).$$

Define the function  $E_i$  implicitly by the equation,

$$Y_i'[E_i(y)] = \delta w_i y. \quad [10]$$

Define the function  $Z_i$  by

$$Z_i(y) = k_i y + \left( \frac{\delta}{1 - \delta} \right) \{ Y_i'[E_i(y)] - \delta w_i y E_i(y) \}, \quad [11]$$

and let  $F_i(f_i)$  be a value of  $y$  that maximizes  $Z_i(y)$  subject to the constraint corresponding to Eq. 4, i.e.,

$$F_i(f_i) = \arg \max_y \{ Z_i(y) : m_i \leq y \leq f_i \}. \quad [12]$$

(If there is more than one maximizing value of  $y$ , pick any one.) Note that, although the function  $Z_i$  does not depend on  $f_i$ , the function  $F_i$  does, because of the constraint. We shall call

$$(E, F) = \{ (E_i, F_i) : i = 1, \dots, I \}$$

a BAU strategy profile.

Observe that, since  $Y_i$  is concave,  $E_i(y)$  is decreasing in  $y$  (use Eq. 11). Using Eq. 11 again, one verifies that

$$Z_i'(y) = k - \left( \frac{\delta^2 w_i}{1 - \delta} \right) E_i(y), \quad [13]$$

and hence  $Z_i(y)$  is a convex function of  $y$ . It follows that  $Z_i(y)$  is maximized in  $y$  at one of the endpoints of the interval  $[m_i, f_i]$ . There are two cases to consider. If  $Z_i'(m_i) \geq 0$ , then  $F_i(f_i) = f_i$  for all  $f_i \geq m_i$ . If  $Z_i'(m_i) < 0$ , then there is some  $y_i^0 > m_i$  such that

$$\begin{aligned} F_i(f_i) &= m_i \text{ for } f_i < y_i^0, \\ F_i(f_i) &= f_i \text{ for } f_i > y_i^0. \end{aligned} \quad [14]$$

(Thus,  $F_i$  is a "bang-bang" policy.) Note that each emission factor is constant after period 1.

Let  $V_i(f, g)$  denote country  $i$ 's (total discounted) payoff when each country uses its BAU strategy, and the initial state is  $(f, g) = [f(0), g(0)]$ . The function  $V_i$  is called country  $i$ 's value function.

**Theorem 1.** A BAU strategy profile is a Markov equilibrium, called a BAU equilibrium. Along the equilibrium path, each country  $i$ 's emission and emission factor are constant after the first period, and the emission factor is equal either to  $m_i$  or  $f_i(0)$ . The value function for country  $i$  is

$$V_i(f, g) = Y_i[E_i(f_i)] - c_i g - k_i [f_i - F_i(f_i)] + u_i - \delta w_i g', \quad [15]$$

where

$$u_i = \left( \frac{\delta}{1 - \delta} \right) \{ Y_i[E_i(F_i(f_i))] - \delta w_i \sum_j F_j(f_j) E_j[F_j(f_j)] \}, \quad [16]$$

$$g' = \sigma g + \sum_j f_j E_j(f_j). \quad [17]$$

The proof of this theorem uses a standard dynamic programming method.<sup>11</sup>

**GPO Strategy Profiles.** We now characterize the GPO strategy profiles for the same underlying model. As in Eq. 7, let  $x_i > 0$  be the weight given to country  $i$  in the global welfare function. Define the emission policy function,  $\hat{E}_i$ , implicitly by the equation,

$$Y_i'[\hat{E}_i(y)] = \frac{\delta w y}{x_i}, \quad [18]$$

where

$$w = \frac{\sum_i x_i c_i}{1 - \delta\sigma} = \sum_i x_i w_i. \quad [19]$$

Define the functions  $\hat{Z}_i$  by

$$\hat{Z}_i(y) = k_i y + \left( \frac{\delta}{1 - \delta} \right) \left\{ Y_i'[\hat{E}_i(y)] - \delta \left( \frac{w}{x_i} \right) y \hat{E}_i(y) \right\}, \quad [20]$$

and let  $\hat{F}_i(f_i)$  maximize  $\hat{Z}_i(y)$  subject to the constraint

$$m_i \leq y \leq f_i.$$

Comparing Eq. 20 with Eq. 11 reveals that the coefficient  $w_i$  in Eq. 11 has been replaced by  $(w/x_i)$ . Again, one can show that  $\hat{Z}_i(y)$  is convex in  $y$ , and hence,  $\hat{F}_i(f_i)$  equals either  $m_i$  or  $f_i$ , accordingly as  $\hat{Z}_i'(m_i)$  is  $>$  or  $<$   $\hat{Z}_i'(f_i)$ .

**Theorem 2.** Given the weights  $(x_i)$ , the following strategy profile is globally Pareto optimal: for each country  $i$ ,

$$\begin{aligned} e_i(t) &= \hat{E}_i[f_i(t)], \\ f_i(t+1) &= \hat{F}_i[f_i(t)]. \end{aligned} \quad [21]$$

In particular, both the emission factor and the "energy input" are constant after the initial date.

The proof is similar to that of Theorem 1.<sup>11</sup>

**A Comparison of BAU and GPO Strategy Profiles.** As one might expect, a BAU equilibrium need not be optimal, not only because "energy" inputs are too high but also because a country that should reduce its emission factor in a Pareto optimum may not do so in the BAU. However, the comparison of the BAU with the set of GPOs is not as straightforward as one might at first expect.

<sup>11</sup>Radner, R. (1998) *A Game-Theoretic Approach to Global Warming*, Woytinsky Lecture, University of Michigan, Ann Arbor, MI, March 11, 1998.

We start with a useful lemma, which implies that if, for a given country  $i$  in a given period, the country uses the same emission factor in the BAU and GPO profiles, then its BAU energy use (and hence emission) exceeds its GPO energy use (and hence emission). Define

$$f_i = f_i(0), \quad [22]$$

$$e_i = E_i(f_i), \quad a_i = f_i e_i, \quad [23]$$

$$\hat{e}_i = \hat{E}_i(f_i), \quad \hat{a}_i = f_i \hat{e}_i \quad [24]$$

The lemma states that, for this case of fixed emission factors, each country's BAU emission exceeds its GPO emission.

**Lemma 3.** Let  $E_i$  and  $\hat{E}_i$  be the BAU and GPO emission functions defined in Eqs. 10 and 18, respectively. For all vectors  $x = (x_i)$  of strictly positive weights, and every emission factor  $f_i > 0$ ,

$$E_i(f_i) > \hat{E}_i(f_i). \quad [25]$$

**Proof:** Let

$$e_i = E_i(f_i), \quad \hat{e}_i = \hat{E}_i(f_i).$$

From Eqs. 10 and 18,

$$Y_i(e_i) = \delta w_i f_i,$$

$$Y_i(\hat{e}_i) = \frac{\delta w f_i}{x_i}.$$

From

$$x_i c_i < \sum_j x_j c_j,$$

it follows that

$$\frac{\delta c_i}{1 - \delta \sigma} < \frac{\delta \sum_j x_j c_j}{x_i (1 - \delta \sigma)}.$$

The conclusion of the lemma now follows from the assumption that  $Y_i$  is strictly concave.

**Theorem 4.** A BAU equilibrium is not globally Pareto optimal.

**Proof:** Consider a BAU equilibrium with initial GHG level  $g$ , and let  $\mathbf{f}$  denote the corresponding time path of emission factors for all the countries, i.e.,

$$\mathbf{f} = (\mathbf{f}_i),$$

where

$$\mathbf{f}_i = [f_i(0), f_i(1), \dots, \text{ad infinitum}]. \quad [26]$$

For any given vector  $x$  of strictly positive weights define the constrained GPO profile, given  $(\mathbf{f}, g)$ , [CGPO( $\mathbf{f}, g$ )] to be the profile that maximizes the weighted sum of payoffs among all profiles that have the initial GHG level,  $g$ , and the same time path,  $\mathbf{f}$ , of emission factors. By the Lemma, at every period every country's emission in the BAU equilibrium exceeds the corresponding emission in CGPO ( $\mathbf{f}, g$ ). Hence, the BAU cannot be a CGPO ( $\mathbf{f}, g$ ) for any weight vector  $x$ . It follows that the BAU equilibrium cannot be a (non-constrained) GPO with the same initial state, which completes the proof.

Does moving from a BAU to a corresponding nonconstrained GPO necessarily reduce the emissions of all countries? The answer to this question is more complex. Recall that we have assumed that emission factors can be decreased but not increased. (See *Concluding Remarks* for remarks on this assumption.) Somewhat paradoxically, a decrease in a country's emission factor need not lead to a decrease in GPO emissions for that country. Hence, a decrease in one country's emission factor could lead to a loss in welfare for

other countries. To see this, consider a GPO profile. If at a given point of time the emission factor of a country is  $f_i$ , then the GPO emission is

$$\hat{a}_i = f_i \hat{E}_i(f_i), \quad [27]$$

so that

$$\frac{d\hat{a}_i}{df_i} = \hat{E}_i(f_i) + f_i \hat{E}'_i(f_i). \quad [28]$$

Of course, if the energy input were held constant, then a decrease in the emission factor would result in a decrease in the emissions. However, we have already seen that

$$\hat{E}'_i(f_i) < 0. \quad [29]$$

Hence, a decrease in the emission factor for a given country has two opposing effects. From Eq. 28,

$$\frac{d\hat{a}_i}{df_i} > 0 \quad [30]$$

if and only if

$$\left( \frac{d \log \hat{E}_i(f_i)}{d \log f_i} \right) = \hat{E}'_i(f_i) \left[ \frac{f_i}{\hat{E}_i(f_i)} \right] > -1. \quad [31]$$

Note that the absolute value of the left-hand side of Eq. 31 is what economists call the elasticity of  $\hat{E}_i(f_i)$  with respect to  $f_i$ .

The next theorem states that, if Eq. 31 is satisfied, then the switch from a BAU to a corresponding GPO will not increase any country's emission factor, but it may decrease it.

**Theorem 5.** Suppose that Eq. 32 is satisfied, then

$$\begin{aligned} \text{if } F_i(f_i) = m_i, \text{ then } \hat{F}(f_i) = m_i, \text{ whereas} \\ \text{if } F_i(f_i) = f_i, \text{ then } \hat{F}(f_i) = m_i \text{ or } f_i. \end{aligned} \quad [32]$$

Thus, the switch from the BAU to a GPO with the same initial state will not increase any country's emission factor in any period, but may decrease it.

(Proof omitted.<sup>9</sup>)

As an immediate corollary of the preceding lemma and theorem, we have:

**Corollary 6.** If Eq. 31 is satisfied, then the switch from the BAU to a GPO with the same initial state will decrease every country's emissions in every period.

**The Set of Equilibrium Outcomes.** The preceding subsection described one particular equilibrium, "business as usual," which we think of as characterizing the current world situation. Infinitely many equilibria exist, however, to which we now turn our attention. Although we do not have a complete characterization of the set of all equilibria, we can give some useful information about them. In particular, we can show that there are cases (sets of parameter values) for which equilibria exist that are Pareto-superior to the BAU, i.e., better for every country, provided that the countries' discount factor is not too low. On the other hand, there are cases for which no GPO can be sustained by an equilibrium. Our treatment here is informal.<sup>10</sup> The basic message is that, even if GPOs cannot be sustained by equilibria, typically equilibria will exist that are Pareto superior to the BAU and that the set of equilibria will be larger, the closer the discount factor is to unity.\*\*

<sup>9</sup>Dutta, P. K. & Radner, R. (2001) Columbia University and New York University, Working paper, *A Strategic Analysis of Global Warming*.

\*\*Dutta, P. K. & Radner, R. (2002) Columbia University and New York University, Working paper, *Choosing Cleaner Technologies*.

We start with a negative result. To sustain a GPO, the countries must credibly be able to threaten to increase their emissions above the GPO level in the event of a “defection.” Suppose that in Eq. 4,  $m_i = 0$  for all  $i$  and that in a GPO every country reduces its emission factor to zero. If all the countries but one follow the GPO, then it will not be possible for them to punish a defector, because their emission factors will already be zero. Hence, it will not be possible to sustain the GPO as an equilibrium. This result is still valid with a small amount of convexity in the technical-change cost function.

Here is a sketch of a positive result. Suppose that the minimum attainable emission factors are all strictly positive. (In Eq. 4,  $m_i > 0$  for all  $i$ .) For any vector of weights, there is a discount factor sufficiently close to unity such that the corresponding GPO requires that every country reduce its emission factor to the minimum. For some set of weights, the GPO is Pareto-superior to the BAU when the emission factors are at their minimum. Hence, for  $\delta$  sufficiently close to 1, the GPO can be sustained by a “trigger strategy” in which the players threaten to revert to the BAU (given the then current state) in the case of a defection.

More information about the set of equilibrium outcomes is given in working papers cited above.<sup>11\*\*</sup>

### A Numerical Example

We illustrate the preceding results with some numerical examples, which are taken from a much more extensive analysis by Sangwon Park.<sup>8</sup> In his model, 184 countries are grouped for calibration purposes into eight regions: the United States, Western Europe, other high income, Eastern Europe, middle income, lower middle income, China, and lower income. [This grouping enables Park to utilize the data and estimates provided by Nordhaus and Boyer (7).] The base year is 1998 ( $t = 0$ ). Referring to Eq. 5, the output of country  $i$ , after subtracting the amount of investment needed to maintain the capital stock at its 1998 level, is given by:

$$Y_i(e_i(t)) = \phi_i K_i^{\gamma_i} \gamma_i^{1-\gamma_i} e_i(t)^{\beta_i} - p_i e_i(t),$$

where  $K_i$  and  $L_i$  are the capital and labor inputs, respectively;  $e_i(t)$  is the input in year  $t$  of “energy” (a proxy for a scalar index of emissions-producing inputs, measured in coal-equivalent metric tons);  $p_i$  is the price of “energy”; and the Greek letters are parameters. Capital, labor, and energy inputs are specific to each country, as is  $\phi_i$ . The parameters  $\beta_i$ ,  $\gamma_i$ , and  $p_i$  are the same for all countries in the same region. Referring to the rest of the model as specified in Eqs. 1–5,  $c_i$ ,  $k_i$ , and  $m_i$  are based on a combination of country- and group-specific data.

All countries have the same discount factor,  $\delta$ , which for sensitivity analysis has been varied between 0.97 and 0.995. These would seem to bracket the values commonly discussed in the literature, motivated by different interpretations of  $\delta$  as reflecting “social values” or the returns on investment.

Finally, in computing GPO trajectories, we have taken the weights  $x_i$  in Eq. 7 to be equal to 1. The consequences of using other weights have been explored elsewhere.<sup>8</sup> In what follows, welfare is measured in 1990 U.S. dollars, and emissions are measured in gigatons of carbon.

The benchmark case has  $\delta = 0.97$  and damage coefficients based on the study by Fankhauser (8). This case has been calibrated so that the BAU matches available data and estimates for 1998. The benchmark GPO uses the same parameter values and initial conditions. Recall that in the BAU and the GPO the emissions are constant after the first period. Globally, moving from the BAU to the GPO reduces annual emissions by 68% and increases global welfare (normalized total discounted present value) by 0.63%. Table 1 shows a breakdown of these figures into eight groups of countries. The reductions in emissions range from 66% (United States, Western Europe, and other high-income countries) to 72% (Eastern Europe). The corresponding increases in welfare range from 3.04% (China) to 0.32% (United States).

**Table 1. Benchmark case**

Region	BAU emissions, GtC	GPO emissions, % decrease	GPO value, % increase
United States	1.50	66	0.32
Western Europe	0.86	66	0.35
OHI	0.59	66	0.41
Eastern Europe	0.74	72	0.54
MI	0.41	67	1.47
LMI	0.58	69	1.31
China	0.85	69	3.04
LI	0.66	69	1.37
Total	6.18	68	0.63

OHI, other high income; MI, middle income; LMI, lower middle income; LI, lower income; GtC, gigatons carbon.

Table 2 shows the results of a sensitivity analysis in which  $\delta$  is increased to 0.995 and each Fankhauser estimate of  $c_i$  is multiplied by 5. In these ranges, the increase in the damage coefficients is the most powerful, and the combined effect of increasing the damage coefficients and  $\delta$  is to change the percent decrease in global emissions from 68% to 76%, the percent increase in global welfare from 0.63% to 12.79%, which are quite significant changes.

Park also shows that the results are quite sensitive to the minimum attainable values of the emission factor, i.e., the limits of technical change, about which considerable uncertainty exists.

A comparison of a GPO with the BAU gives an upper bound on the improvement that can be attained by moving to another equilibrium. As noted in *Equilibrium and Optimal Time Paths*, the actual attainable such improvement may be less than this upper bound (the “second best” may be less than the “first best”), although the shortfall may be small if  $\delta$  is close to 1.

Finally, an increase in the present value of welfare of, say, 2% (as in Table 2) may seem small. However, recall that our present value (Eq. 6) is normalized, so that when  $\delta$  is close to 1 the normalized present value is close to the long-term yearly average. If 2% of a country’s GDP were invested every year, the rate of growth of its economy could be increased significantly.

### Concluding Remarks

We conclude with some remarks on the limitations of the research reported here, on possible directions for future research, and a discussion of the existing literature and the Kyoto Protocol.

**Population Change, Investment, Capital Accumulation, and Technical Progress.** To the extent that population change is exogenous to the model of climate change, it is straightforward to generalize the model and results presented here. The calculations are slightly more complicated but still quite tractable.<sup>11</sup> The same is true of changing capital stocks and technical progress. However, although a weak case could be made for treating population as exogenous, not even that can be said for capital and technical progress.

**Table 2. Sensitivity analysis**

Discount factor	Damage cost coefficients	
	Fankhauser $c_i$	5× Fankhauser $c_i$
$\delta = 0.97$	68	76
	0.63	3.95
$\delta = 0.995$	72	83
	1.98	12.79

GPO percent emissions decreases (upper) and percent value increases (lower).

**Nonlinearities.** As already noted, the model makes use of various linear approximations for the sake of analytical and computational tractability.

**The cost of reducing emissions factors.** Our model assumes a linear cost of reducing a country's emission factor with a fixed lower bound. It can be shown that a small amount of convexity in the technical change cost function will not remove the bang-bang nature of optimal technical change. It is probably more realistic to assume a convex cost function with sharply increasing marginal cost after some point. A preliminary analysis suggests that, in this case, for optimal and equilibrium strategies, emission-factor reduction typically takes place in stages over a period of years.

**The damage cost of climate change.** It has been argued that some of the costs of climate change could be highly nonlinear in the temperature. For example, it is feared that a breakup and melting of polar ice, with its release into the ocean, could significantly accelerate the rise in the sea level. Other nonlinearities arise in the relationship between the level of GHGs and the atmospheric temperature. (S. Bose has taken a first step toward a model with discontinuous jumps in the cost function.<sup>††</sup>)

**Other Topics.** The present model is deterministic and should be expanded to take account of uncertainty. First, realistic models of climate change typically include a stochastic component in the explanation of climate dynamics. Second, scientific uncertainty exists about the actual processes of climate change. An appropriate model would use the theory of stochastic dynamic games (9) and would pose new theoretical challenges.

Some writers have advocated the use of trade sanctions to help enforce a climate-change treaty, which would expand the set of instruments available for reaching an equilibrium. However, use of trade sanctions would merge two "international games" that have hitherto remained rather separate, and achieving an equilibrium in the larger game would require negotiating a new treaty in a larger and more complex context.

Another approach would be to create an "International Bank for the Environment." Such a bank could provide a mechanism for some countries to aid other countries in the reduction of emissions and emission factors, with such aid being placed in escrow with the Bank and paid to the recipients in stages on completion of the required actions. Analysis of such a model is needed.

**Literature and the Kyoto Protocol.** A large volume of literature on the economics of climate change and the costs and benefits at various levels of GHG emissions exists. A central question there is to determine the level of emissions that is globally Pareto-optimal. An excellent example of this is (ref. 7). A smaller volume of literature emphasizes the need for treaties to be self-enforcing, as does this article. (See refs. 10 and 11.) Ref. 11 also describes many environmental treaties. Where we depart from the existing game-theoretic literature is in the dynamic modeling; we allow GHGs to accumulate and stay in the environment for a (possibly long) period of time. By contrast, all of the previous literature makes the

assumption that GHGs decay within a short time span, the single period of the model. (Technically, those analyses use static one-shot games or repeated games in which identical one-shot games are played repeatedly, which implies that the state variables, i.e., the GHG stock and emission factors, remain constant over time.)

In terms of conclusions, our findings in terms of welfare loss are similar to those of other authors, including those who focus on the first-best issue. Nordhaus and Boyer (7) calculate a United States loss from global climate change of the order of 0.5% of GDP (for doubling of preindustrial levels); other estimates have been in the 1–2% range (12). This agrees with our findings of an increase in global welfare (respectively, U.S. welfare) in the range of 0.6–2% (respectively, 0.3–1%) for different discount rates with our benchmark damage cost coefficients. Where we differ is in the size of the emission cuts that our model predicts. Whereas Kyoto had called for cuts in the 10% range relative to 1990 levels and some of the literature has proposed that even that is too much to be globally optimal (see ref. 7), our numerical computations ask for much deeper emission cuts. We conjecture that the dynamic element of our model, that gases can persist in the atmosphere for a hundred years or more, which is in line with the known evidence, makes the deeper cuts optimal. Furthermore, our results for both potential welfare increases and emission cuts are quite sensitive to increases in the damage cost coefficients.

The Kyoto Protocol appears not to have been the basis of a self-enforcing treaty, principally because it did not build in any effective sanctions that would be applied if countries failed to meet their targets. This contrasts with General Agreement of Tariffs and Trade and its attendant institutional structure, the World Trade Organization. All that is said in the Kyoto protocol, in Article 18, is that procedures and mechanisms for compliance should be determined by the parties at their first meeting and should include "an indicative list of consequences." Subsequently, at the Hague November 2000 meeting, the most popular proposal (coming from the Dutch Environment Minister Jan Pronk) was that countries would face an escalating series of target reductions in the future if they failed to comply in the current stage. A watered-down version of this proposal was adopted in Bonn in March 2001, after the United States had pulled out of the treaty. Yet even this version had several problems; in principle, countries could postpone retribution indefinitely, and the base from which the enhanced reduction would be required was to be worked out in the future, etc. (See ref. 10, chapter 15).

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