A Note on Unanimity of Stockholders' Preferences among Alternative Production Plans: A Reformulation of the Ekern-Wilson Model

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A note on unanimity of stockholders’ preferences among alternative production plans: a reformulation of the Ekern-Wilson model

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This note, which was stimulated by Ekern and Wilson’s study of the theory of the firm in an economy with incomplete markets, gives conditions for ex ante and ex post stockholders to be unanimous in their respective preferences among alternative production plans. This modest extension of Ekern and Wilson’s analysis is facilitated by reinterpreting their model in standard Arrow-Debreu terms.

The situation described in the first Ekern-Wilson proposition on unanimity of stockholders’ preferences among production plans can be formally reduced to an Arrow-Debreu model with complete markets. This reformulation provides insight into the logical structure of their model, and permits one to draw an additional conclusion. Roughly speaking, one can paraphrase the results of this note as follows. Suppose that the production possibility sets of all producers span a linear subspace of distributions of returns across states of the world, and that an equilibrium of the stock market and choice of production plans has the property that the equilibrium production plans span that subspace; then the ex ante stockholders in each firm are unanimous in their preferences among alternative production plans, and the ex post stockholders of each firm are unanimous in their preferences among (local) directions of change from the equilibrium production plan.

Suppose, as in the Ekern-Wilson paper, that there are two dates (1 and 2), one commodity at each date, I consumer-stockholders, J firms, and K alternative states of the world at the second date. A plan $y_j$ for producer $j$ is a $(K+1)$-dimensional vector $(y_{jk})$, where $-y_{j0}$ is the firm’s input at date 1, and $y_{jk}$ is the

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1 See [2], hereafter referred to as Ekern-Wilson. Although the results in this note are formally self-contained, the note was intended to be read in conjunction with Ekern-Wilson, where the reader will also find more motivation, and also references to previous literature on the subject.

2 See [2], p. 175.
firm's net output at date 2 in state $k$ ($k = 1, \ldots, K$). Similarly, if $c_i = (c_{ik})$ is a consumption plan for consumer $i$, then $c_{i0}$ is interpreted as his consumption at date 1, and $c_{ik}$ is his consumption at date 2 in state $k$ ($k = 1, \ldots, K$).\(^3\)

Let $S$ be a given linear subspace of $\mathbb{R}^K$, and let $S' = \mathbb{R} \times S$, a subspace of $\mathbb{R}^{K+1}$. Here $S$ represents, as on page 176 of Ekern-Wilson, a set of “state-distributions of returns.” Consider now an “Arrow-Debreu” model of production and exchange,\(^4\) in which each consumer $i$ has a consumption set, say $C_i$, and each producer $j$ has a production set, say $Y_j$, and assume that the sets $C_i$ and $Y_j$ are all subsets of $S'$. Denote by $w_i$ the (commodity) endowment of consumer $i$, and assume that

$$w_{ik} = \begin{cases} e_i, & \text{for } k = 0, \\ 0, & \text{for } k = 1, \ldots, K. \end{cases}$$

As in Ekern-Wilson, let $\bar{z}^j_i$ denote the initial share of firm $j$ held by consumer $i$. Finally, let $q'$, a vector in $S'$, denote a “commodity price vector.” An Arrow-Debreu equilibrium is an $(I + J + 1)$-tuple, $[(c^*_i), (y^*_j), q']$ such that:

1. $q'$ is in $S'$;
2. for every $j$, $y^*_j$ maximizes $q' \cdot y_j$ on $Y_j$; and
3. for every $i$, $c^*_i$ is a preferred vector in the set of all $c_i$ in $C_i$ such that

$$q' \cdot c_i \leq q' \cdot w_i + \sum_j \bar{z}^j_i q' \cdot y^*_j. \quad (1)$$

4. Supply equals demand at each date and in each state, i.e.,

$$\sum_i c^*_i = \sum_i w_i + \sum_j y^*_j. \quad (2)$$

To relate an Arrow-Debreu equilibrium to an Ekern-Wilson equilibrium, write $q' = (q_0, q)$, with $q$ in $S$, write $c_i = (c_{i0}, z_i)$, with $z_i$ in $S$, and write $y_j = (y_{j0}, r_j)$, with $r_j$ in $S$. Consider an Arrow-Debreu equilibrium such that:

$$q_0 > 0, \quad (3)$$

$$r^*_{1j}, \ldots, r^*_{*j}, \text{ span } S. \quad (4)$$

Because of (3), we can normalize $q$ so that $q_0 = 1$. One can show (see the end of the note) that, because of (4), there exist numbers $s^*_{ij}$ such that

$$z^*_i = \sum_j s^*_{ij} r^*_{ij}, \quad \text{every } i, \quad (5)$$

$$\sum_i s^*_{ij} = 1, \quad \text{every } j. \quad (6)$$

Let $p_j = q' \cdot r^*_{ij}$. Then from the budget constraint (1), for every $i$,

$$c^*_{i0} + \sum_j s^*_{ij} p_j \leq e_i + \sum_j \bar{z}^j_i (y^*_{j0} + p_j). \quad (7)$$

This “budget constraint” is to be compared with (4) of Ekern-\(^3\) I have denoted here by $c_{i0}$ what Ekern and Wilson denote by $c_i$, and my $c_{ik}$ is their $\sum_j p_{ij} f_{jk} (x_j)$.

\(^4\) As in, say, Debreu [1].
Wilson. Furthermore, for any \( c_i = (c_{i0}, z_i) \) in \( C_i \), there exist numbers \( s_{ij} \) such that \( z_i = \Sigma s_{ij} r^*_j \) (since \( z_i \) is in \( S \)). Hence \( (c^*_i, 0, \Sigma s_{ij} r^*_j) \) is a preferred vector in the set of all vectors \( (c_{i0}, \Sigma s_{ij} r^*_j) \) in \( C_i \) such that
\[
c_{i0} + \Sigma s_{ij} p_j \leq e_i + \Sigma \delta_{ij} (y^*_{j0} + p_j). \tag{8}
\]

Compare this last budget constraint with (7). Hence \( \{(c_{i0}, [z^*]), (p_i)\} \) is an Ekern-Wilson equilibrium, given the plans \( y^* = (y^*_{j0}, r^*_j) \).

In the context of the Arrow-Debreu formulation of the model, given the price vector \( q' = (1, q) \), all of the ex ante shareholders of firm \( j \) (i.e., all consumers \( i \) for which \( s^*_{ij} > 0 \)) would agree that the firm should maximize the value \( y_{j0} + q \cdot r_j \). Hence all of the ex ante stockholders of firm \( j \) are unanimous in their preferences among production plans of the firm.

With regard to the ex post stockholders, an argument similar to the one given in Ekern-Wilson can be used to show that they are unanimous in their preferences among feasible (local) directions of change from the equilibrium production plan, provided that the consumers' preferences are sufficiently regular (smoothness, convexity, etc.). To be more precise, for any firm \( h \), let \( f_h = (g^*_h, g_h) \) be a vector in \( R^h+1 \) such that, for all real numbers \( t \) in some interval \([0, t^*]\),
\[
\begin{align*}
(1) & \quad y^* + tf_h \text{ is in } Y_h; \\
(2) & \quad \text{for every } i, \gamma_i(t) = (c^*_{i0}, z^*_i + ts^*_{ih} g_h) \text{ is in } C_i.
\end{align*}
\]
The vector \( f_h \) is a feasible direction of change from the production plan \( y^* \), and \( g_h \) is the corresponding direction of change from the return vector \( r^*_h \). If \( q \cdot g_h > 0 \), then there is some sufficiently small \( t \) such that each ex post stockholder \( i \) for which \( s^*_{ih} > 0 \) prefers the consumption vector \( \gamma_i(t) \) to the consumption vector \( c^*_i \). An analogous statement (in reverse) follows if \( q \cdot g_h < 0 \). The logic behind this proposition is, of course, the same argument as that used to justify the standard real income test (in the small) for a single consumer.

Note that each ex ante stockholder with positive shares wants the firm to maximize \( y_{j0} + p_j \), whereas each ex post stockholder with positive shares wants the firm to maximize \( p_j \), which could be interpreted as the stock market value of the firm. Hence these two groups agree with each other after the net input of the firm at the first date has been fixed.\(^5\)

It remains to justify the representation (5)-(6). First consider the special case in which \( r^*_1, \ldots, r^*_j \) are linearly independent. Since they span \( S \), there is a unique representation of each \( z^*_i \) in the form (5). Adding (5) over all consumers gives
\[
\Sigma z^*_i = \Sigma_j (\Sigma s^*_{ij}) r^*_j. \tag{9}
\]
But by (2), the condition that supply equal demand,
\[
\Sigma z^*_i = \Sigma_j r^*_j. \tag{10}
\]
If one compares (9) and (10), and recalls that the $r^*_j$ are linearly independent, then condition (6) follows. In the case in which the $r^*_j$ are not linearly independent, one chooses a linearly independent subset that spans $S$, and then proceeds with an obvious variation on the above argument.

References