Chapter Four

Economic Planning Under Uncertainty: Recent Theoretical Developments

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INTRODUCTION

This is a survey of recent developments in mathematical theories of economic planning under uncertainty, and related research in mathematical economics. The presentation is essentially non-mathematical, but I cannot pretend that it will not be technical or abstract. I knew before I started preparing this chapter that the theory of national economic planning under uncertainty is in its infancy, and that it is still far from true applicability at the national level. This impression was confirmed as I reviewed the relevant literature. Nevertheless, I believe that the infant theory has some insights to offer the student of planning who is willing to make some investment in mathematical technique, and I hope this chapter will give some hints regarding what the literature has to offer.

The sources of uncertainty in planning include all of the uncertainties faced by the economy, as well as any uncertainties there may be about the role of the planners in the economy. It is customary to divide the uncertainties faced by the economy into two groups: (1) uncertainty about variables that are exogenous, i.e., not affected substantially by the actions of the economic agents within the country, such as weather, some conditions of world trade and politics, etc., and (2) uncertainty about the behavior of the economic agents themselves, including the behavior of some economic agents outside the country, such as important trading partners. The distinction is that variables in the second group may be significantly affected by the plan, whereas variables in the first group are not.

The original plan for this chapter was to concentrate on uncertainties of the first type, and in fact the title in the preliminary program for the conference was listed as "Planning under Conditions of Uncertainty about Economic Variables External to the Plan." As I shall argue below, it is doubtful whether one can in

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reality decouple the two types of uncertainty, even though the distinction may be valid from a conceptual point of view.

By far the bulk of the theoretical literature on economic planning under uncertainty deals with various problems of optimal allocation of resources, and this is reflected in the relative length of the section, "Characterization of Optimal Paths of Economic Growth Under Uncertainty," which is devoted to this class of problems. Upon reflection, I am not convinced that this emphasis (by present-day theorists) is merited, and I believe that the problems described in the other sections, although less finished and more speculative, will eventually attract as much, if not more, attention from mathematical economists. However, progress usually involves building on what has come before, and I hope I can give you a feeling for the progress that remains to be accomplished, without in any way minimizing the importance of what has been already achieved.

INTERACTION BETWEEN PLANNING AND IMPLEMENTATION

Mathematical theories of resource allocation planning at the national level have rarely considered explicitly the relations between planning and implementation. Mathematical planning theories have been primarily concerned with optimal decisions. First, we have theories that attempt to characterize optimal allocations of resources, e.g., in terms of shadow prices, present value, corresponding "profit" maximization, etc., or in terms of state valuation functions, as in dynamic programming, or even in terms of explicit formulas for optimal allocations in simple cases. Second, we have theories about the properties of alternative procedures (algorithms) for the calculation of optimal allocations (mathematical programming, linear and nonlinear decomposition, dynamic programming algorithms, etc.). These procedures could be interpreted as methods of preparing a plan.[1] Most of the literature on this topic does not consider uncertainty. The focus of attention is on properties of convergence, and sometimes on the interpretation of steps of the algorithm as activities of the various agents involved in the process of preparing the plan. A third group of theories deals with uncertainty explicitly, and focuses on the characterization or calculation of optimal decision rules for allocation. Such decision rules (as in the theory of optimal inventory control) typically require the decisionmaker to periodically take account of new information as it is observed, and could be interpreted as models of operating systems, that is to say, models of "implementation" of policy.

In practice, the activities of planning and implementation are typically carried on by different administrative units, especially in the case of planning at the national level. Plans are in the form of forecasts of what would be desirable and feasible trajectories of economic magnitudes (inputs, outputs, consumption...
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Prices, but they rarely take explicit account of uncertainty, and more rarely provide explicitly and formally for the revision of planned decisions during the period covered by the plan.

The gap between theory and practice has created tensions at both ends. Planners and decisionmakers are aware of the desirability, in principle, of more formal and explicit links between planning and implementation, and of increased interdependence and feedback between plans and decisions (e.g., "rolling plans"). But they are also aware of the overwhelming requirements for exchange and processing of information that are implied by serious, continuing feedbacks between planners and decisionmakers. Further, they are aware of the conflicts of interest and power that so often create obstacles in the way of effective exchange of information and the corresponding revision of plans and decision rules.

Theorists, too, are aware of these organizational problems. Indeed, the theories of games, teams, and statistical decision would appear to provide the tools for a more sophisticated approach, but thus far only the simplest examples have yielded to theoretical analysis.

These remarks are not intended to minimize the importance of the insights that planning theory has provided, nor the increased power provided by advances in methods of mathematical programming, computation, and information processing. Subsequent sections of this chapter will be devoted to describing some of these theoretical insights, and the models from which they arise. Nevertheless, it is my view that the theory of planning under uncertainty cannot make much further progress without an explicit attack on problems of organization. The incorporation of uncertainty into theoretical models of planning almost forces one, sooner or later, to face up to questions regarding the organization of the diverse activities of observation, communication, information processing, forecasting, and actual decisionmaking.

It may be that a continued exclusive reliance on a framework of optimal decision will not be fruitful for planning theory. On the one hand, a concern for economic planning seems to imply, almost by definition, a concern for a rational approach to the organization of economic activity. On the other hand, the optimal solution of economic decision problems on a national scale, taking account of uncertainty, seems totally beyond our capacities, now and in the indefinite future. A promising approach to the resolution of this dilemma is suggested by Simon's concept of "bounded rationality." As he defines it, "Theories that incorporate constraints on the information-processing capacities of the actor may be called theories of bounded rationality." [2] These constraints refer not only to what is often described as the cost of information, but also to the limited capacities of humans (and machines) for imagination, computation, and other aspects of internal information-processing. Thus far, this concept has not significantly penetrated the theoretical literature on national economic planning. [3]
CHARACTERIZATIONS OF OPTIMAL PATHS OF ECONOMIC GROWTH UNDER UNCERTAINTY

A Model of Economic Growth Under Uncertainty

Before describing various characterization of optimal economic growth under uncertainty, it will be useful to give a sketch of a theoretical framework within which one can discuss these results.

Consider an economy at successive dates (years, quarters, or months, etc.), starting from date zero. At each date there is a fixed list of conceivable commodities (goods and services), which we may, for convenience, take to be the same at all dates. The production possibilities at each date will be represented by an activity analysis model. At each date, each activity will be "operated" at some "level." The level of an activity at a given date determines the quantities of all the inputs at that date, and also determines the outputs, which are available for consumption or input into production at the next date. In other words, in this "period analysis," the production that results from the level of an activity at one date is assumed to take place during the period between that date and the next. If the period is short relative to the "period of production," then the list of commodities must include all unfinished goods, or goods-in-process, at different stages of production.

At each date, some quantities of resources are supplied exogenously to the economy. Negative resources would represent commitments undertaken before date zero to supply commodities to the outside world. Consumption at each date is what is left over from this date's resources, plus output from the previous period, after subtracting the inputs for current production. Consumption may be constrained in some fashion, e.g., to represent a minimum standard of living, or a required rate of growth, etc. A consumption program satisfying such constraints will be called acceptable.

Uncertainty about production is introduced by assuming that the inputs and outputs at each date depend not only on the levels of the corresponding activities, but also on certain random variables. Uncertainty about resources is similarly introduced by assuming that the resources at each date are random variables. The complete specification of all of the random variables that influence production and resources at any date will be called the state of the environment at that date. A partial history of the environment through date \( t \) specifies the states of the environment up through date \( t \). A complete history of the environment specifies the states of the environment at all dates.

The last date envisaged will be called the horizon. For "practical" purposes, one might suppose that the horizon could always be taken as finite. However, in many situations the horizon might be very distant, or difficult to specify, and in both these situations it might be mathematically convenient, and a good approximation, to take the horizon as infinite.

Thus uncertainty of resources and production is represented in terms of a...
(finite or infinite) sequence of random variables, the history of the environment. These random variables need not, of course, be mutually independent, in a statistical sense, nor need corresponding random variables at different dates have the same probability distribution. All that we assume is that (1) the joint probability distribution of the random variables can (in principle) be specified, and (2) the evolution of the history of the environment is not affected by any economic decisions (activity levels). Assumption (2) is really in the nature of a convention. All “variables” in the world are divided into three groups: (1) decisions (activity levels), (2) environment, and (3) all others, which are assumed to depend on decisions and the environment.

At each date, the activity levels are to be determined on the basis of information about the partial history of the environment up through that date. The rule according to which an activity level is determined is called a decision function at that date. A policy (or strategy) is a sequence of decision functions, one for each date and activity level at that date.

The more information the decisionmaker will have at any date about the partial history of the environment up through that date, the richer is the set of decision functions that he can use. A completely centralized economy is defined as one in which, at every date, every activity level may be determined on the basis of full knowledge of the partial history of the environment up through that date. An economy will be called decentralized to the extent that different activity levels are determined on the basis of different information about the environment. I should emphasize that I am speaking at this point of informational centralization and decentralization. The decentralization of authority will be discussed in the last section of this chapter.

For the remainder of this section, I shall confine my attention to the case of completely centralized economies, as just defined. In my view, a realistic approach to planning under uncertainty must inevitably deal with informational decentralization. However, theoretical research on this topic is relatively recent, and not at all as well-developed as is research on the model of informational centralization (see the section “Decentralization of Information”).

It follows from the definition of complete centralization that, at each date, the decisionmakers know the resources at that date, and know the inputs required by each activity at any level. However, the decisionmakers may still be uncertain about the outputs from activities at the time those activity levels are determined. This is expressed by assuming that the outputs from activities at one date may depend on the partial history of the environment up through the next date.

Once a particular policy is chosen, all of the variables in the economy are determined by the evolution of the environment, either directly, as in the case of the resources, or indirectly, as in the case of inputs, outputs, and consumption. Thus, for a given policy, all of the variables in the system are random variables, with a joint probability that is determined by (1) the initial stocks of the
economy, (2) the initial state of the environment, (3) the joint probability distribution of the successive states of the environment, and (4) the policy.

Thus far, nothing has been said about a criterion for choosing among alternative policies. Of course, the first requirement for a policy is that it lead to consumption sequences that are acceptable, whatever the state of the environment. Beyond that, theories of optimal decision assume that the centralized decisionmaker (planner) has preferences among alternative probability distributions of consumption sequences, or of final stocks of capital, or both. Such preferences imply preferences among alternative policies. For example, a popular assumption is that (1) there is a "utility function" ("social welfare function") that determines a "utility of consumption" at each date, (2) the utility of a consumption sequence is the sum of the (possibly discounted) utilities at the different dates, and (3) one policy is preferred to another if it leads to a larger expected total utility of consumption.

In this context, the grand decision problem of the planner is to choose a preferred policy among all those that lead to acceptable consumption sequences. Such a policy will be called optimal. The next three subsections deal with four topics in the characterization of optimal policies: (1) dynamic programming and the state valuation function, (2) certainty equivalence, (3) shadow prices, and (4) explicit solutions.

Note. The criterion of expected total (discounted) utility of consumption is so commonly used in theoretical work that some brief interpretive comments are in order concerning this criterion. First, the assumption that preferences can be represented in the form of expected utility implies assumptions of independence of tastes and beliefs (as well as certain technical assumptions of continuity, etc.)[6] Second, the assumption that preferences among sure consumption sequences can be represented in the form of a sum of (discounted or undiscounted) one-period utilities implies an assumption of independence of preferences as among consumptions at different dates. In particular, the discounting of future utilities implies a preference for present over future consumption, and thus (possibly) a "discrimination" against future generations.[7] In some problems it may make a difference whether at each date the utility of consumption is computed as (1) a utility of total consumption, (2) a utility of per capita consumption, or (3) a utility of per capita consumption multiplied by the number of consumers. Finally, the introduction of an infinite horizon may introduce special problems of convergence, continuity, etc., and the possibility of incompatibility among various properties that preferences among finite sequences of consumption naturally have. The limitation of space does not permit a more extended discussion of these questions here, but a full understanding of the difficulties inherent in a theory of planning under uncertainty requires some sophistication in these matters.[8]
the joint probability of the policy.

For choosing among policies, the planner is that it leads to state of the environment. For example, a popular welfare function is (1) state utility, or (2) the utility of a discounted utility at the if it leads to a larger planner is to choose a consumption sequence. Sections deal with four items: programming and shadow prices, and

Utility of consumption is explain that preferences can be functions of independence and continuity, among consumption and of independence of states. In particular, the present over future "against future generational" whether at each date the total consumption, (2) a per capita consumption introduction of an infinite, continuity, etc., properties that preferences . The limitation of space questions here, but a full eory of planning under the

Dynamic Programming and the State Valuation Function

Dynamic programming is a useful tool in many decision problems. For the purposes of this chapter, my interest in dynamic programming concerns the associated idea of the state valuation function, which provides insight into the theoretical structure of planning under uncertainty, and also may suggest useful heuristics for the solution of planning problems. As defined below, the state valuation function is a theoretical construct that imputes to each current state of the economy a "value," which is the maximum expected total discounted utility that can be achieved starting from that state. The importance of the state valuation function is that it enables the decision maker to transform a many-period decision problem into a sequence of two-period problems.

It is not my intention to review the theory of dynamic programming, but a few basic concepts are needed for an understanding of the state valuation function. Suppose that the sequence of successive states of the environment forms a Markov chain, with an infinite horizon. Suppose also that preference among policies is represented by the expected total discounted one-period utilities of consumption (see the above section on a model of economic growth under uncertainty). Assume that the transition probability law is the same at each date, and that the one-period utility function is the same for all dates, with a constant discount factor. In this situation, with sufficient regularity conditions, one can show that there is an optimal policy that is stationary, in the sense that, at each date, the optimal activity levels are a function of the current state of the environment and the current stocks of commodities, and this function is the same for each date. The state of the economy at each date can therefore be adequately represented by the pair consisting of the state of the environment and the vector of stock levels. Denote this state, at date $t$, by $e_t$. For any policy $\pi$, let $V_\pi(e_t)$ denote the expected total discounted utility of consumption from date $t$ on that is implied by using the policy $\pi$, if the state of the economy at date $t$ is $e_t$. The function $V_\pi$ is called the state valuation function associated with the policy $\pi$. The function $V_\pi$ satisfies an interesting and important recursive equation. Starting from the state $e_t$ at date $t$, and using the policy $\pi$, let $c_t$ denote the resulting consumption at date $t$, and let $e_{t+1}$ denote the next state of the economy. At date $t$, the consumption $c_t$ is known with certainty, but the planner is uncertain about the next state of the economy. The expected total discounted utility from date $(t+1)$ on is $EV_\pi(e_{t+1})$, where the symbol $E$ denotes mathematical expectation with respect to the (uncertain) next state $e_{t+1}$, and the discounting starts only at date $(t+1)$. Let $u(c_t)$ denote the one period utility of the consumption $c_t$ at date $t$, and let $\delta$ denote the planner's discount factor. Then the expected total discounted utility starting from date $t$ is a sum of two terms: the utility $u(c_t)$ in the current period, and the discounted expected future utility, $EV_\pi(e_{t+1})$. Hence we have

...
the recursive equation,

$$V_\pi(e_t) = u(c_t) + \delta E V_\pi(e_{t+1}).$$ \hspace{1cm} (4-1)

The state valuation function corresponding to an optimal policy plays a special role in the theory of dynamic programming, and is simply called the state valuation function. I shall denote it here by $V^\ast$. Corresponding to (4-1), there is the so-called functional equation of dynamic programming, which characterizes $V^\ast$. This functional equation can be motivated as follows. At date $t$, the current consumption is determined by the current state of the economy, and the current vector of activity levels, say $\xi_t$. The choice of a vector of current activity levels will also lead to a vector of (uncertain) outputs at the next date, and thus to a new (uncertain) state of the economy at the next date. If the planner follows an optimal policy from date $(t+1)$ on, the expected utility, discounted from $(t+1)$ on, will be $EV^\ast(e_{t+1})$. Hence the optimal choice of activity levels at date $t$ must maximize the sum of the utility of current consumption and the discounted expected optimum future utility, i.e.,

$$V^\ast(e_t) = \max_{\xi_t} \left[ u(c_t) + \delta E V^\ast(e_{t+1}) \right],$$ \hspace{1cm} (4-2)

which is the functional equation of dynamic programming. Under appropriate conditions, $V^\ast$ is the unique solution of the functional equation (4-2). \cite{13}

Furthermore, knowledge of $V^\ast$ is equivalent to knowledge of an optimal policy, in the sense that if one knows $V^\ast$, then one can determine the best activity levels for any state of the economy by choosing the activity levels that maximize the expression in square brackets on the right-hand side of (4-2). Thus, knowledge of the state valuation function would enable the planner to reduce the problem of finding an optimal policy to a sequence of two-period problems, in each of which it is required only to find an optimal vector of activity levels.

In a practical planning problem of any size, the apparent simplification of computation provided by the state valuation function may be illusory, since it may be just as much work to calculate the state valuation function as it is to calculate the entire optimal policy. Nevertheless, the functional equation yields an important insight into the structure of the problem. Implicit in any optimal policy is a valuation of states of the economy that reflects completely the repercussions of current action on future utilities, and this valuation completely characterizes an optimal policy. Indeed, implicit in any policy, optimal or not, is a corresponding valuation, and higher valuations correspond to better policies.

The functional equation corresponds intuitively to many heuristic attempts to solve dynamic optimization problems. For example, suppose that one makes a guess as to the correct valuation of states, and this guess is represented by a function, say $U^0$. One can then calculate what would be the optimal current actions if the next state valuation of current state, the function $U^0$ by the third function, $U^2$. Induction approximation to $V^\ast$ may be evaluated the final stock planned actions will typically planning period, unless horizon at each revision.

The state valuation function as will be indicated in the

Shadow Prices

The concept of shadow price (Lagrange-Kuhn-Tucker) and, in particular, theory of shadow prices to a straightforward, although mathematical difficulties, known about the theory of growth under uncertainty planning.

A price system for associates to each commodity in the environment at that date, stochastic processes of prices may be conditional on the partial derivatives. The cost of the vector of the history of the environment value of the outputs of the conditional probability distribution of history of the environment is the difference between

A price system will be
actions if the next state were valued according to $U^0$; this would imply a valuation of current states according to some new function, say $U^1$. Now replace the function $U^0$ by the function $U^1$, and repeat the above procedure, obtaining a third function, $U^2$. Indeed, one can show that, under suitable conditions, the sequence of functions obtained in this way converges to the (true) state valuation function, $V^*$. If the first guess, $U^0$, is not too bad, a good approximation to $V^*$ may be obtained after only a small number of iterations.

Following up this idea, the activity levels planned for the beginning of a long-term plan may be quite good, even if the planners have not correctly evaluated the final stocks at the end of the planning period. Of course, the planned actions will typically become worse and worse as one proceeds into the planning period, unless one constantly revises the plan, with a new planning horizon at each revision ("rolling plans" again!).

The state valuation function is also intimately connected with shadow prices, as will be indicated in the next subsection.

**Shadow Prices**

The concept of shadow price is familiar from mathematical programming (Lagrange-Kuhn-Tucker multipliers), theories of optimal allocation of resources, and, in particular, theories of optimal growth.\[14\] The extension of the theory of shadow prices to optimization under uncertainty is, in some sense, straightforward, although a general treatment can give rise to considerable mathematical difficulties.\[15\] In this subsection I shall briefly review what is known about the theory of shadow prices for our particular problem of optimal growth under uncertainty, and interpret the implications of this theory for planning.

A price system for our model of economic growth under uncertainty associates to each commodity, at each date, for each partial history of the environment at that date, a number, or price. Thus a price system determines a stochastic process of prices, i.e., a sequence of random prices, whose joint probability distribution is determined by the joint probability distribution of the states of the environment. These prices are to be interpreted as discounted prices, with the discounting taking place from date zero. Given a price system, one can calculate the expected (discounted) profit for an activity at that date, conditional on the partial history of the environment up through that date, as follows. The cost of the activity is the cost of the current inputs, calculated according to the vector of prices associated with the current date and partial history of the environment. The expected revenue of the activity is the expected value of the outputs of the activity at the next date, calculated according to the conditional probability distribution of prices at the next date, given the partial history of the environment at the current date. The expected discounted profit is the difference between expected revenue and cost.

A price system will be said to sustain an optimal policy if, for each partial
history up through that date, and each activity, the optimal activity level maximizes expected discounted profit. In this case, I shall also say that the price system is a system of shadow prices associated with the given optimal policy.

Notice that, in the calculation of expected profit for an activity at a given date, all inputs and outputs must be priced. In particular, one must calculate the costs and revenues corresponding to used capital goods and goods-in-process. However, an alternative calculation is possible, extending over the entire period of the plan (or any shorter period), if the goods-in-process and used capital at a given date are actually used by the activity, and not disposed of elsewhere. In this calculation, the expected discounted profit of an activity over the period of the plan is equal to the sum of the expected values of the net outputs at each date. The net output of each commodity at each date is, of course, the difference between the output of that commodity (from the previous period) and the current input. Note that, in the case of goods-in-process and used capital goods that are actually used, the net output is zero, and hence shadow prices for these commodities are not needed in order to calculate the expected discounted profit for the entire planning period.

One can show that, under the usual assumptions of convexity, etc., to every optimal policy there corresponds a system of shadow prices, provided that (1) the planning horizon is finite, and (2) there are only finitely many states of the environment at each date. [16] In principle, these last two assumptions are not restrictive, since an infinite horizon and an infinite set of states are, in some sense, only mathematical idealizations of a very large finite horizon, and a very large set of states, respectively.

As a technical digression, it might be noted that infinite horizons are often analytically convenient in the theoretical treatment of growth, especially for examining the “long-run” properties of growth paths. Also, infinite sets of states are common in even elementary probability models (e.g., the Gaussian and Poisson distributions). On the other hand, the introduction of such infinities into the theory of optimal growth leads to special mathematical difficulties, as already noted, and the theory of shadow prices in such models is still incomplete.

I turn now to some implications of the theory of shadow prices for planning under uncertainty.

Expected profit maximization. It should be emphasized that the appropriate criterion for each activity is expected profit. In other words, with the correct probability distribution of shadow prices, no adjustment for risk aversion is needed.

The rate of interest is stochastic. As already noted, the shadow prices are to be interpreted as discounted prices. In other words, the system of shadow prices reflects, not only relative prices at each date, but rates of interest between different dates. In general, proportional (except by interest rates. Corresponding price index, there will be more precise on this po discounted price of comm variables. The own-interest

\[ r_{ht} = \frac{P_{ht}}{P_{h,t+1}} - 1 \]

Analogously, for any price corresponding average rate

\[ r_t = \frac{\sum_t v_t P_{ht}}{\sum_t v_t P_{h,t+1}} \]

Thus, the sequence of inter-

Current relative prices a current shadow price of com-

\[ P_{ht} = \frac{P_{ht}}{\sum_k v_k P_{kt}} \]

The current price of corn commodity relative to the sense that the weighted sum of current shadow prices als-

Price flexibility and pr- how the sequence of sto-

See, however, note 17 with.
different dates. In general, since the shadow prices at difference dates will not be proportional (except by accident), there is no unique natural definition of interest rates. Corresponding to each choice of numeraire, or each choice of price index, there will be a different system of (shadow) interest rates. To be more precise on this point, we need a little notation. Let \( p_{ht} \) denote the discounted price of commodity \( h \) at date \( t \); the prices \( p_{ht} \) are, of course, random variables. The **own-interest rate for commodity \( h \) at date \( t \)** is defined as

\[
\rho_{ht} = \left( \frac{p_{ht}}{p_{h,t+1}} \right) - 1. \tag{4-3}
\]

Analogously, for any price index with strictly positive weights \( v_h \), there is a corresponding **average rate of interest at \( t \)**, defined by

\[
r_t = \left( \frac{\sum_t v_h p_{ht}}{\sum_t v_h p_{h,t+1}} \right) - 1. \tag{4-4}
\]

Thus, the sequence of interest rates, however defined, forms a stochastic process.

**Current relative prices are stochastic.** Using again the weights \( v_h \), define the **current shadow price of commodity \( h \) at date \( t \)** by

\[
p_{ht} = \frac{p_{ht}}{\sum_k v_k p_{kt}}. \tag{4-5}
\]

The current price of commodity \( h \) at date \( t \) reflects only the price of that commodity relative to the prices of other commodities at the same date, in the sense that the weighted sum of the current prices is always unity. The sequence of current shadow prices also forms a stochastic process. [17]

**Price flexibility and probabilistic forecasts.** The preceding remarks show how the sequence of stochastic discounted shadow prices determines corresponding sequences of interest rates and current prices, given a set of weights. Indeed, it is easy to see that, for a fixed set of weights, the interest rates and current prices completely characterize the discounted shadow prices, i.e., one can calculate the sequence of shadow prices from the sequences of interest rates and current prices. It follows from the preceding remarks that, if the planners or

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\( ^a \)See, however, note 17 with regard to stationary stochastic programs.
decisionmakers are to use shadow prices as guides to planning and allocation, then they must be prepared to (1) change current prices and interest rates from one date to the next, according to observations of the state of the environment, and (2) make probabilistic forecasts of future current prices and interest rates, i.e., forecasts of these variables must be in terms of probability distributions, or at least characteristics of such distributions, such as means, variances, correlations, etc. [18] As far as I am aware of actual practice, price "flexibility" and forecasting of this type would be unusual in planning, and would even be regarded with some skepticism by planners. Some pros and cons of such use of shadow prices will be discussed below, especially in the section "Decentralization of Information." In any case, one may question the practicality of calculating shadow prices, a point to which I shall now turn.

Calculation of shadow prices. The shadow prices associated with an optimal policy give information about that policy, and are closely connected with the state valuation function discussed in the previous subsection. Indeed, whereas the state valuation function characterizes the optimal policy globally, the shadow prices characterize it locally. To that extent, the remarks in the previous subsection concerning the calculation of the state valuation function are applicable as well to the shadow prices. Since the shadow prices contain less information than does the state valuation function, they may in some cases be easier to calculate or estimate. These problems are not different, in principle, from the corresponding problems in the case of certainty, although the sheer size of the calculation problem may be orders of magnitude greater! Problems and pitfalls in the estimation of shadow prices are, of course, familiar from studies of cost-benefit analysis. [19]

Certainty-Equivalents

Because of the difficulty of calculating optimal policies (or the associated state valuation functions, or the shadow prices) in dynamic planning models of any complexity, there is a great incentive for discovering classes of problems that permit simplified calculations, or at least simplified approximations. One intuitively attractive method consists in the replacement of the various random variables in the decision problem by corresponding nonrandom variables, thus converting a problem of decisionmaking under uncertainty into one under certainty. (For example, each random value might be replaced by its expected value.) In such a procedure, the nonrandom variables are called the certainty-equivalents of the corresponding random variables; I shall call the corresponding decision problem (under certainty), the certainty-equivalent problem.

The solution of a certainty-equivalent problem will yield specific decisions for both the present and future dates, rather than decision rules or policies. The decisions for date zero will be implemented; the calculated decisions for future dates may be called the planned decisions. All of these calculated decisions will reflect the information available to the decisionmaker at date zero. When one arrives at date one (the second date), new information will typically become available. The decisionmaker, however, can then extend the certainty-equivalent solution to a new certainty-equivalent problem, and so forth. For example, one could formulate a sequence of successive revisions of the information at each date, and thereby define a policy, which however, that each successive problem is an implicit assumption that a

The certainty-equivalent problem which I shall now illustrate is one in which the decisionmaker observes the partial history of the environment at any date further, that at each date t, the decisionmaker observes the partial history of the environment from the beginning through date t. In other words, in order to specify the decision problem, the decisionmaker must specify

$$
\sum_{t=0}^{T} K^t = \frac{K^{t+1} - 1}{K - 1}
$$

unknowns. Notice that this

In the certainty-equivalent problem, the decision problem has (T+1)K unknowns, which increases only like T.

Under what conditions
available. The decisionmaker is not constrained to implement, at date one, the planned decisions for date one that were calculated at date zero. A natural extension of the certainty-equivalent procedure at date one would be to reformulate a new certainty-equivalent problem, based upon the new information. (For example, one might replace the random variables at future dates by their conditional expected values, given the partial history of the environment through date one.) The solution to this certainty-equivalent problem at date one could be interpreted as a revision of the plan that was adopted at date zero. Proceeding in this manner from one date to the next, the decisionmaker formulates a sequence of certainty-equivalent problems, the solutions of which represent successive revisions of the original plan in the light of newly available information at each date. Thus the successive decisions do indeed reflect new information; in other words, the sequence of certainty-equivalent problems defines a policy, which I shall call the certainty-equivalent policy. Notice, however, that each successive certainty-equivalent problem is solved under the implicit assumption that no new information will become available in the future!

The certainty-equivalent approach has obvious computational attractions, which I shall now illustrate. For example, suppose that at each date \( t \) the decisionmaker observes the state of the environment at that date, and also remembers the partial history of the environment up to that date, and suppose further that at each date there are \( K \) possible states of the environment. There are therefore \( K^t \) partial histories of the environment up through date \( t \), assuming that the state of the environment at date \( t \) is fixed. A policy must determine what decision to take at date \( t \) for each of the \( K^t \) alternative partial histories up through date \( t \). In other words, for each date \( t \), there are \( K^t \) "unknowns" to determine, in order to specify a policy. If the horizon is \( T \), then in total a policy must specify

\[
\sum_{t=0}^{T} K^t = \frac{K^{t+1} - 1}{K - 1}
\]

(4-6)

unknowns. Notice that this number grows geometrically with the horizon, \( T \).

In the certainty-equivalent approach, however, the first certainty-equivalent problem has \( (T+1)K \) unknowns, the second has \( TK \) unknowns (\( K \) for each remaining date), the third has \( (T-1)K \) unknowns, etc. The total number of unknowns is therefore

\[
\sum_{t=0}^{T} (T + 1 - t)K = \frac{(T+1)(T+2)}{2} K
\]

(4-7)

which increases only like \( T^2 \).

Under what conditions is the certainty-equivalent policy optimal? Under
what conditions is it approximately optimal? More generally, what are the directions of the errors made when one uses a certainty-equivalent policy rather than an optimal policy?

An answer to the first question is provided by the Simon-Theil theorem.[20] If (1) the criterion function whose expected value is to be maximized is a quadratic function of the decision variables, for every history of the environment, (2) the coefficients of the second degree terms do not depend on the environment, (3) the usual second-order conditions are satisfied, and (4) the certainty-equivalents at each date are taken to be the conditional expected values of the coefficients of the zero and first degree terms of the criterion function, given the partial history of the environment up through that date, then the certainty-equivalent policy is optimal. Furthermore, the planned decisions at each date (i.e., the calculated values of future decisions in the certainty-equivalent problem) are equal to the conditional expected values of the corresponding decisions as determined by the optimal policy, given the partial history of the environment up through that date. In other words, at each date, the “revised plan” gives the expected decisions, in the precise sense of mathematical expectation.

The quadratic case covered by the Simon-Theil theorem is, of course, quite special, but the same method could yield good approximations in more general cases. Indeed, if the criterion function were twice-differentiable in the decision variables, then it would be approximated by a quadratic function in the “neighborhood” of any sequence of decisions, and if the uncertainty about the coefficients were sufficiently small, then one would expect this approximation to be adequate. This idea can be made precise, as was shown by Theil in the static case and by Malinvaud in the dynamic case.[21]

Unfortunately, it is difficult to obtain information about the directions of the errors caused by using the Simon-Theil certainty-equivalent policy instead of the optimal policy. This question is equivalent to the question: how are the optimal decisions affected if uncertainty about the coefficients in the criterion function increases in a way that leaves the expected values of these coefficients unchanged? The answers to these questions appear to depend on the third and higher derivatives of the criterion function with respect to the decision variables, and these are parameters of a planning problem that are rarely known with precision. Malinvaud gives some interesting examples of this in the paper just cited.

Explicit Solutions

I am not aware of explicit (closed form) solutions of classes of problems of economic planning under certainty in any generality. Three special classes of problems might be mentioned here, however. First, some examples of optimal savings and portfolio choice might be interpreted as one-sector, i.e., macro-economic planning models. These examples generally involve utility functions that exhibit constant risk aversion.[22]
Explicit solutions for a multi-sector dynamic linear-logarithmic model with uncertainty can easily be adapted from the solutions in the certainty case, provided that the uncertainty in production is confined to multiplicative coefficients of the production functions in each sector. (These solutions have not, however, been published.)

Third, the vast literature on optimal inventory policies may be interpreted as a special subfield of economic planning under uncertainty. Nevertheless, the examples for which closed form explicit solutions are available are still much simpler than what would be needed to plan large-scale inventory systems, with many economic agents.

DECENTRALIZATION OF INFORMATION

Plans as Information Signals
I have defined an economy to be informationally decentralized to the extent that different activity levels are determined on the basis of different information about the environment. According to this broad definition, every economy is informationally decentralized to some extent. From this viewpoint, a national plan provides a set of information signals to economic agents, but these signals complement other information that is available only at the levels of the ministry, enterprise, plant, shop, etc. No national plan can hope to be based on complete information about the environment, even at the time it is drawn up, nor can one hope to develop procedures that will keep plans perfectly up-to-date. Therefore, when we think seriously about planning under uncertainty, we are forced to consider the nature of the information transmitted in the plan to the economic decisionmakers, the delays in this information, and the feedbacks (if any) from decisionmakers to planners at the national level. Unfortunately, theoretical research along these lines is recent and fragmentary, and does not typically make much of a distinction between planning and implementation.

The few studies on which I shall report below may be interpreted in terms of a simplified "model" of planning and information flows. The planners sit in an office and receive information about the environment, production, demand, etc. They make periodic computations, and periodically send out to various economic decisionmakers (or other planners at intermediate levels) information signals in the form of production targets, shadow prices, forecasts of demand, etc. They also periodically receive signals from the decisionmakers concerning estimates of current or past values of these and other variables (or perhaps also forecasts of them). The lags involved in these signals may be anything from weeks to years. The resulting economic decisions (e.g., activity levels) form a stochastic process. Two different planning procedures are to be compared in terms of the character of the two stochastic processes of decisions that they generate, and in terms of the corresponding costs of observation, computation, and communication.
Price Signals
Much of the literature on "market socialism," decentralized planning procedures, and decomposition methods for mathematical programming has emphasized the possibility of using prices as information signals from planners to decisionmakers (in a context of certainty). [24] In the typical procedure of this type, at each date the planners send price signals to the production managers (and possibly to the consumers), and receive in turn information about supplies and demands. If such procedures were to be used in real time, under conditions of uncertainty, the environment would change stochastically during the period of time required for each iteration. In other words, the environment would not stand still while the procedure converged to an "optimal" solution. Hence we are led to study the performance of such a system under conditions in which decisions about allocation and production are taken before the shadow prices have reached their "optimal" or "equilibrium" values. Since the performance of the system is stochastic, we may ask whether the resulting process is stochastically stable, and if so, calculate such measures of performance as (steady state) average output, variance of output, etc.

Once the problem is formulated in this way, it is natural to study the optimal decision rules, given the system that generates the information. If the price signals are not equilibrium prices, i.e., if supply does not equal demand at those prices, then decisions must be taken regarding how the existing supply is to be allocated, if it is less than demand, or what to do with excess supply, if supply exceeds demand. Furthermore, enterprise managers may have to make various local decisions before they know what their supplies will be for the current period.

The performance of such price signalling procedures, and the characterization of the corresponding optimal decision-rules, have been studied by Groves and Radner, under quite restrictive assumptions, using the methods of the theory of teams. The pattern of information exchange is one in which a central "resource manager" sends out price signals at each date to the "enterprise managers," who respond with "demands" that would maximize a suitably defined "shadow profit" criterion. At this stage, the enterprise managers must also make decisions concerning some "local" variables, before knowing the quantities of all the resources that will be allocated to them by the central manager. On the basis of the information he has received up to that point, the central manager determines the allocations of the resources to the enterprise managers. New observations of their respective parts of the environment are then made by all the agents, and the cycle is repeated. The aspects of the environment that vary from date to date are the quantities of centrally allocated resources, and the technical conditions of production in the enterprises. The criterion is long-run average output.

Groves and Radner actually studied a "static" version of this model, in which the agents have no "memory," and start afresh each date, but with a different environment, of course. [25] This is equivalent to studying the average or expected performance of a single iteration is allowed bef chosen optimally, given the iteration. Under an assumption that it is quadratic, it is shown that prices yield as large an exchange of information between Groves studies a dynamic economic model, and showed that prices are partially adjusted differences between supply Groves-Radner "static" case able to show that if the er demand, as well as estimates again as good as complete ex the central model of the percentage loss as computational centralization (approx. Groves-Radner model.)

The results just described and appear to depend heavily on validity. However, Groves[27] analogous to the approximate and Malinvaud in the case of approximation results are robust: strikingly efficient in convey decisions, even out of equilibria.

Production Targets and Decisions
If production functions generate large changes in demand, adjustment processes in enterprises may oscillate. This unpleasant property has at some iterations the enterprise current experience, and also process eventually converges programming literature, to the center of production target.

--The precise assumptions are: see the section "Characterizing Uncertainty."
expected performance of an adjustment process (algorithm) in which only a single iteration is allowed before decisions must be made, and these decisions are chosen optimally, given the limited information provided by that single iteration. Under an assumption that the production functions of the enterprises are quadratic,\(^b\) it is shown that *optimal decisions based on the price and demand signals yield as large an expected output as could be achieved by a complete exchange of information between the enterprises and the center.*

Groves studies a dynamic version of this model,\(^{[26]}\) as described in the next to last paragraph, and showed that when the decisionmakers have memories, and prices are partially adjusted from date to date according to the perceived differences between supply and demand, then the simple result of the Groves-Radner "static" case no longer holds. However, for his model, he was able to show that if the enterprises send to the center forecasts of future demand, as well as estimates of current demand, then the resulting system is again as good as complete exchange of information between the enterprises and the center. A further result was that, *as the number of enterprises grows large, the percentage loss as compared with complete information (complete informational centralization) approaches zero.* (This result also was valid in the Groves-Radner model.)

The results just described have been derived for a highly simplified model, and appear to depend heavily on the quadratic assumption for their exact validity. However, Groves\(^{[27]}\) has proved for the Groves-Radner model a result analogous to the approximate-certainty-equivalence theorems derived by Theil and Malinvaud in the case of a single decisionmaker. To the extent that these approximation results are robust, they suggest that *price and demand signals are strikingly efficient in conveying the information needed for good allocation decisions, even out of equilibrium.*

**Production Targets and Decomposition Algorithms**

If production functions are close to linear, then small changes in prices may generate large changes in demand. Under these conditions, in the case of certainty, adjustment processes that rely exclusively on price signals to the enterprises may oscillate excessively during convergence to the optimal solution. This unpleasant property has the consequence (among other disadvantages) that at some iterations the enterprises will be proposing plans that are far from their current experience, and also far from the final inputs and outputs to which the process eventually converges. This difficulty has led, in the mathematical programming literature, to the study of algorithms that involve computation at the center of production targets for the enterprises, possibly in addition to price signals.\(^{[28]}\)

\(^b\)The precise assumptions are similar to those that permit the use of certainty equivalents; see the section "Characterization of Optimal Paths of Economic Growth Under Uncertainty."
The appropriate mathematical model for the study of such problems would seem to be the linear programming (linear activity analysis) model. In this model, the inputs and outputs are linear functions of the activity levels. In addition, there are constraints corresponding to factors or inputs whose supplies are fixed. Unfortunately, the solutions of linear programming problems can rarely be expressed in closed form as functions of the parameters. This makes difficult the analytical study of problems of uncertainty in which the parameters are themselves random variables. For such problems, methods of simulation and numerical analysis seem indispensable, at least given the theoretical techniques that are currently available.

Hogan [29] studied the performance of several decomposition algorithms in the same spirit as in the Groves-Radner investigation, but using numerical methods. Between iterations of the algorithm, the environment changes according to some probabilistic law (possibly incorporating serial correlation). As in our previous discussion, each iteration of the algorithm in question is interpreted as an exchange of information between planners and enterprises, as well as the results of decentralized computation within the planning unit and the enterprises. Performance of the procedures was tested using ranges of parameters suggested by input-output tables for five different countries.

The results of Hogan's study are too complicated to summarize fully here. He was able to confirm the stochastic stability of the algorithms, and to demonstrate that such numerical simulation is feasible and sufficiently accurate with moderately long histories of simulated iterations. The most surprising results were that

as the variance of both the technology and the resource supplies increases, and as the accuracy of the starting information (at each iteration) becomes poorer, the performance of the procedures becomes better. This result is due to the fact that the decomposition procedure tends to converge to the optimal plan quicker when the starting vectors are farther from their optimal configuration. This results (in turn) from the fact that the prospective indices shift more under these conditions, so that the range of vectors contained in the memory of the central board is greater and the known portion of the production sets is also greater. This allows quick convergence in the early iterations. [30]

UNCERTAINTY AND DECENTRALIZATION OF AUTHORITY

The theories of planning under uncertainty that have been considered thus far in this chapter have explicitly or implicitly assumed the existence of a single overall objective or criterion to be maximized, even though there may be a number of different decisionmakers with different information. In this respect, these theories fall within the framework of the theory of teams rather than the theory of games, the latter among the decisionmakers, optimal, given the limitation agents in the economic system, the incentives for the agents required information. Indeed, victim to, or be manipulated organization. [31]

Students of bureaucracymay go with limited information, it will to determine whether a part to is following the pre- require the supervisor to: subordinate decisionmaker, i to de facto decentralization o

If the behavior of decision perfectly predictable, this will the organization. In fact, the individual decisionmakers in games has not even been able behavior in conflict situation an organization, uncertainty about the behavior (decision two classes of uncertainty m in practice.

If informational decentralization, then it follows complete, or even near one hand, this is a platitude position of many large economies of the "command economy"

This proposition poses a economy. An effective syste the benefits that derive from decision-rules. But the accu income, power, etc. Such i contrary to the spirit of a capitalist economies. To pro one must be willing to le

This does not deny that the the among different economic sy
theory of games, the latter of which deals explicitly with conflicts of interest among the decisionmakers. The theorist may invent decision rules that are optimal, given the limitations on the information available to the different agents in the economic system, but one must still face the problem of providing the incentives for the agents to follow these decision rules and provide the required information. Indeed, the postulated information system may itself fall victim to, or be manipulated by, the power struggles of the members of the organization. [31]

Students of bureaucracy—and bureaucrats—are well aware of the power that may go with limited information. [32] Under conditions of uncertainty and limited information, it will typically be difficult for a supervisor (or organizer) to determine whether a particular decisionmaker is providing correct information or is following the prescribed decision-rules, since to achieve this would require the supervisor to have all the information that is available to the subordinate decisionmaker. In other words, informational decentralization leads to de facto decentralization of authority.

If the behavior of decisionmakers in situations of conflict of interest were perfectly predictable, this would not introduce any additional uncertainty into the organization. In fact, there is considerable uncertainty about the behavior of individual decisionmakers in conflict situations. Furthermore, the theory of games has not even been able to produce an agreed-upon definition of “rational” behavior in conflict situations. These considerations lead me to conclude that, in an organization, uncertainty about the environment brings with it uncertainty about the behavior (decision-rules) of the members of the organization. These two classes of uncertainty may be distinguished conceptually, but not separated in practice.

If informational decentralization generates decentralization of authority, and if modern economies are too complex to be operated with information centralization, then it follows that no modern economic system can be operated with complete, or even nearly complete centralization of authority. [6] (On the one hand, this is a truism, but on the other hand, it contradicts the official position of many large economic organizations, although by this date the myth of the “command economy” has perhaps at last been laid to rest.) This proposition poses a dilemma that is particularly acute for a socialist economy. An effective system of incentives must allow decisionmakers to share the benefits that derive from the improvement of information, decisions, and decision-rules. But the accumulation of these benefits leads to inequalities in income, power, etc. Such inequalities are typically self-perpetuating, and are contrary to the spirit of socialism. An analogous phenomenon is familiar in capitalist economies. To provide the incentives for firms to make improvements, one must be willing to let them reap some monopoly profits from such

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6This does not deny that there may be significant differences in centralization of authority among different economic systems.
improvements, if only in the short run. Thus, under conditions of uncertainty, the effective operation of a capitalist economy is inconsistent with “perfect competition.”

The proposition may also be paraphrased as follows. Under conditions of uncertainty, information is a valuable input into production, i.e., it is one of the “means of production.” Since the decentralization of information is unavoidable, there will inevitably be decisionmakers who “own” information that is not owned by others. In this sense, under conditions of uncertainty, pure public ownership of the means of production is impossible, at least with the currently available technology of observation, communication, and computation.

CONCLUDING REFLECTIONS

We have seen in this brief survey that most of the theoretical literature on economic planning under uncertainty deals with various problems of optimal allocation of resources and optimal growth (investment policy). This literature can be viewed in perspective as an extension of “mathematical programming” models to the case of uncertainty. As some of the other chapters in this volume show, the use of mathematical programming models is not unknown in applied work on economic planning, and one can anticipate that such applications will gradually be extended to cover cases of uncertainty. This development will be facilitated by the dramatic rate of increase in computational power that continues to characterize the development of the computing industry, in terms of both hardware and software.

Although I have questioned here the relevance, in some deep sense, of optimization models to planning, two observations in defense of optimization models should be emphasized. These observations are based upon the relatively long history of the use of such models in operations research, and the shorter history in national economic planning. First, the application of mathematical programming models stimulates more intense empirical research on the quantitative relations in the economic system. In these applications, econometric, mathematical, and computational progress must go hand-in-hand. Second, in responsible applications of optimization models, the optimization is done “parametrically”: a family of solutions is calculated corresponding to a range of parameter values in the objective function (the optimization criterion). In this way, policymakers can see how the optimal policy depends on the crucial parameters of the objective function, and are provided with a family of alternative policies whose consequences can be studied quantitatively and qualitatively in terms of various criteria that are not explicitly incorporated into the objective function.

The theory of optimal planning also has qualitative implications for the structure of the planning process, especially in the formulation described in the section “Characterizations of Optimal Paths of Economic Growth Under Uncertainty” as “dynamic programming.” Optimal planning under uncertainty calls for “rolling plans,” i.e., planned actions as system becomes aware of how updating should, of computational costs.

A particular form of the “certainty-equivalent” strategy, and under optimal strategy. It corresponds in some ways to a not prepared to document.

In spite of the fact that the best sense of the processing requirements of real-time computer time, the computer is the concept of “fully optimized” or the most significant on the organizational decentralization.

Some recent research was described in Behavioral approaches primarily used the to a certain extent. They are not as amenable to more traditional approach. In the field of economic

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3. See, however,
for "rolling plans," i.e., sequential planning strategies that involve the updating of planned actions as new information about the environment and the economic system becomes available. The frequency and comprehensiveness of this updating should, of course, depend upon the associated informational and computational costs.

A particular form of sequential updating was described in the section "Characterizations of Optimal Paths of Economic Growth Under Uncertainty" as the "certainty-equivalent" approach. This approach will usually be less demanding in terms of information and computation than a fully optimal sequential strategy, and under certain conditions will provide a good approximation to an optimal strategy. It is my impression that the certainty-equivalent approach corresponds in some sense to the informal practice of many planners, but I am not prepared to document that impression in this chapter.

In spite of the rapid progress of information processing technology (in the broadest sense of the term), a large gap remains between the information processing requirements of fully optimal planning strategies and the capabilities of real man-machine planning systems, now and in the foreseeable future. Indeed, I support the view that, from a deeper methodological point of view, the concept of "fully optimal planning" is meaningless. In any case, I speculate that the most significant next advances in the theory of economic planning will focus on the organizational and behavioral aspects of information processing and decentralization.

Some recent research on the mathematical theory of informational decentralization was described in the section "Decentralization of Information." To date, behavioral approaches to the theory of planning and resource allocation have primarily used the tools of computer simulation,[33] and computer programs will probably continue to constitute an important form for the expression of behavioral theories of decisionmaking. In addition, reports of recent research (largely unpublished) suggest that theories of bounded rationality are also amenable to more traditional forms of mathematical analysis.[34] If so, then we are on the brink of a new stage of rapprochement between theory and practice in the field of economic planning under uncertainty.

NOTES


3. See, however, J. Kornai, Anti-Equilibrium (Amsterdam: North-Holland
Publishing Co., 1971); his criticisms of general equilibrium theory are at some points in the spirit of bounded rationality.

4. This is the now familiar model of statistical decision theory; see, e.g., L.J. Savage, The Foundations of Statistics (New York: John Wiley and Sons, Inc., 1954).


6. See, for example, Savage, Foundations of Statistics; Marschak and Radner, Theory of Teams, Ch. 1; and K.J. Arrow, “Exposition of the Theory of Choice Under Uncertainty,” Ch. 2 in Decision and Organization, ed. McGuire and Radner.


9. The term “state valuation function” is to be distinguished from the term “state preference function” that one finds in some literature on the economic theory of socialism; see, e.g., J. Drewnowski, “The Economic Theory of Socialism,” Journal of Political Economy 69, 4 (August 1961): 547–58. In the latter term, the reference is to the (political) state, whereas in the former it is to the state of the economy at a particular date. The “state preference function” corresponds to what has been described above as the preferences of the planner (social welfare function).


11. In a Markov chain, at any date, the conditional distribution of the future, given the partial history of the environment up through the present date, depends on the partial history only through the present state of the environment. See, e.g., W. Fellner, An Introduction to Probability Theory and Its Applications, Vol 1, third edition (New York: John Wiley and Sons, Inc. 1968), or any other introduction to stochastic processes.

12. For this and other facts about optimal policies for the present case, see P. Jeanjean, “Optimal Growth with Stochastic Technology in a Multisector Economy,” Technica Science, University of

13. Again, see Jeanjean’s functional equation to

14. For an introd.

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13. Again, see Jeanjean, "Optimal Growth," for sufficient conditions for the functional equation to characterize optimal policies.


16. A result of this type follows immediately from Chs. 5 and 7 of Debreu, Theory of Value.

17. Such price processes can be quite general, even in simple cases. Even if the states of the environment are independent and identically distributed, the sequence of shadow prices will typically not have this property (see J. Schechtman, "Some Applications of Competitive Prices to Dynamic Programming Problems Under Uncertainty," Operations Research Report No. 73–5 and Ph.D. dissertation, University of California, Berkeley, March 1973). If the states of the environment are Markovian, then the sequence of prices (state, price vector) will in general also be Markovian, but the price process by itself will not. If the environment is stationary, then under certain conditions it is known that the price process will be stationary, or asymptotically stationary (see R. Radner, "Optimal Stationary Consumption with Stochastic Production and Resources," Journal of Economic Theory 6, 1 [February 1973], pp. 68–90; and R.A. Dana, "Evaluation of Development Programs in a Stationary Stochastic Economy with Bounded Primary Resources," Technical Report No. 19, Center for Research in Management Science, University of California, Berkeley, February 1973; to appear in Proceedings of a Symposium on Mathematical Methods in Economics, ed. J. Los [Amsterdam: North-Holland Publishing Co., 1974]). In this case, the time-average of the rate of interest is zero, for all price indices.

18. In the pure form of "indicative planning," the planners attempt to make forecasts of prices and quantities such that, if the various economic decision-
makers believe these forecasts, then they will behave in such a way as to confirm the forecasts for the history of the environment that is actually realized. The theoretical possibility of making such “self-fulfilling” price forecasts (which must, of course, be probabilistic) has been demonstrated by Radner, “Existence of Equilibrium of Plans, Prices, and Price Expectations in a Sequence of Markets,” *Econometrica* 40, 2 (March 1972): 289–304, for an economy with incomplete markets for future and contingent delivery.


21. H. Theil, *Optimal Decision Rules for Government and Industry* (Amsterdam: North-Holland Publishing Co., 1964); E. Malinvaud, “First-Order Certainty Equivalence,” *Econometrica* 37, 4 (October 1969): 706–18. A precise statement of their results would require more space and technical apparatus than are feasible for this chapter. I should also point out that neither the exact nor the approximate certainty-equivalent results require that the criterion function be separable in time, as was assumed in the model we have been considering in this chapter.


29. T.M. Hogan, “A Properties of Several N Report No. 10, Center California, Berkeley, May 30. Ibid., p. 8; the Teams”) and Hogan (“C deal with the dy procedures for economic a recent seminar at the announced that research Related models of the Ch. 7 of Marschak and references to other (mod-

31. For a comparison for an introduction to th Radner, “Teams,” Ch. Radner, pp. 189–215; or Teams. See, also, T.F. G 1973): 617–32, for a prov-

32. See, for example bureaucratique [The B 1963].

33. Simon, “Border Behavioral Theory of the


30. Ibid., p. 8; the emphasis is mine. The papers by Groves ("Incentives in Teams") and Hogan ("Comparison") are the only ones with which I am familiar that deal with the dynamic "real-time" properties of models of iterative procedures for economic planning under conditions of uncertainty. However, at a recent seminar at the University of California, Berkeley, Professor Kornai announced that research on this subject was currently being done in Hungary. Related models of the dynamics of decisionmaking in teams were analyzed in Chapter 7 of Marschak and Radner, *Theory of Teams*, where one can also find references to other (moderately) relevant literature.


