Introduction: Symposium on Noncooperative Bargaining

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This Symposium includes 12 models of bargaining. Each of the following characteristics is shared by most of them: (1) bargaining is noncooperative within a set of rules governing the process of negotiation (the “Nash Program”); (2) bargaining is bilateral; (3) some or all of the traders are incompletely informed. The authors explore the existence, multiplicity, and efficiency of equilibria and the plausibility of equilibria as descriptions of observed behavior. In addition, some authors discuss the effects of nonbinding preplay communication (“cheap talk”), the effects of increasing the number of players, and alternatives to the Nash equilibrium framework. Journal of Economic Literature Classification Numbers: 022, 026.

INTRODUCTION

According to a dictionary [2], to bargain is “to negotiate the terms of a sale, exchange, or other agreement.” Understanding of bargaining is fundamental to economics. Without bargaining, transactions cannot occur, and without transactions, economics is limited to the study of Robinson Crusoe economies. Any complete theory of the functioning and efficiency of markets must have at its base a theory of how bargaining determines who trades what at what prices.

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Generally, adequate theories of bargaining exist only for the degenerate, polar cases of perfect competition and monopoly, in which respectively no agent has any bargaining power and all bargaining power is concentrated with a single agent. If each agent has significant bargaining power in the sense that his rejection of a proposed agreement substantially reduces the other agents' utilities, then the available theories, with a few notable exceptions, are generally not fully satisfactory. Bilateral bargaining is the classic example of this: each agent has bargaining power, and adequate theories have not been developed except for Rubinstein's analysis [13] of the special case of alternating-offer sequential bargaining with complete information.

The pioneer of modern bargaining theory in economics was Nash [10, 11]. According to him [10], "A two-person bargaining situation involves two individuals who have the opportunity to collaborate for mutual benefit in more than one way." More explicitly, in Nash's definition bargaining has two characteristics:

1. There is a status quo or "threat point" (cf. Nash [11]).
2. In addition to the status quo, there are alternative agreements or outcomes, all Pareto-superior to the status quo.

This definition seems to include all voluntary agreements and is broader than the dictionary definition.

Of course, "bargaining" in the present symposium means something a good deal narrower than what is defined by characteristics (1) and (2) above. In almost all the papers:

3. Outcomes are reallocations of goods and money.
4. Some or all of the traders have incomplete information about each other's values and costs (i.e., about each other's preferences or the nature of what is being traded).
5. There is transferable utility.
6. Behavior is noncooperative within a given set of rules governing the process of negotiation.

Characteristic 6 implies that all papers within this symposium fall within what has become known as the "Nash program" (after Nash's paper [11]). This program suggests that the proper way to understand bargaining outcomes is to model various rules for bargaining as noncooperative games and solve for the corresponding equilibria. Several groups of questions then arise:

1. Does an equilibrium exist? Is it unique, or are there at least only
finitely many? Do the equilibria change continuously with the parameters of the model, at least for most values?

2. Can these equilibria be interpreted, at least in some approximate or qualitative sense, as describing observed bargaining behavior, either in laboratory experiments or in the field?

3. What are the welfare (efficiency) properties of these equilibrium outcomes, and how do these welfare properties depend on the rules under which the bargaining is carried out and on the distribution of information?

The papers included here cannot claim, individually and collectively, to solve all the theoretical problems that bargaining (even in the narrow sense) poses, but they do illuminate aspects of the problem that heretofore have been poorly understood and that further theoretical progress will have to acknowledge and deal with. In this Introduction we seek to give a short description of each paper. In addition to this we attempt informally to cross-reference the papers with respect to (i) the underlying model used, (ii) the specific questions asked, and (iii) the strength of the results obtained. Our hope is that this will help the reader to integrate the individual papers into his or her theoretical understanding of bargaining in simple economic situations.

A BASIC MODEL, INDIVIDUAL RATIONALITY, INCENTIVE CONSTRAINTS, AND THREE MECHANISMS

The model that underlies the majority of the papers is a particular bilateral bargaining situation. The seller has a single object that he may sell to the buyer if an acceptable price, \( p \), is agreed upon. The seller's reservation value for the object is \( C \) and the buyer's reservation value is \( V \). If trade occurs without delay, then the gains from trade are \( p - C \) and \( V - p \) for the seller and buyer, respectively. If negotiations fail, then neither party incurs any cost and the gain for each is zero. More generally, if the traders incur a delay of \( t \) units of time before agreeing to trade, then their gains from trade are respectively \( e^{-\delta t}(p - C) \) and \( e^{-\delta t}(V - p) \), where \( \delta \) is the traders' discount rate, the same for both.

Incomplete information may exist in this situation. It is modeled as follows. Each trader's reservation value is a random variable whose value is in some interval, e.g., \([0, 1]\). \( C \) and \( V \) are independently drawn from the distributions \( F \) and \( G \), respectively. The distributions \( F \) and \( G \) are common knowledge to the traders. If the incompleteness of information is two-sided, then the realization of \( C \) is private to the seller and the realization of \( V \) is private to the buyer. For one-sided incomplete information, the value \( C \) or
$V$ (but not both) is common knowledge to both seller and buyer. For the certainty case, both $C$ and $V$ are common knowledge. The majority of the Symposium’s papers concern two-sided uncertainty.

A trading mechanism is a set of rules that govern the process of trade. Several mechanisms are considered in the papers. The sealed-bid double auction is the simplest of the mechanisms considered. Its rules are that simultaneously the seller submits an offer $c$ and the buyer submits a bid $v$. Trade occurs with no delay at price $(c + v)/2$ if $v \geq c$; if $v < c$, then negotiations are broken off. Given this mechanism, a pure strategy for the seller is a function $S(\cdot)$ that specifies an offer $c = S(C)$ for each of his possible reservation values. Likewise $B(\cdot)$ is a strategy for the buyer. A pair of strategies $(S, B)$ is an equilibrium if $S$ is a best response to $B$ and $B$ is a best response to $S$. $S$ is a best response to $B$ if, conditional on the seller’s knowledge of the distribution $G$, the strategy $B$, and the realization of $C$, $S(C)$ maximizes his expected gain from trade for each value of $C$.

The best known example of an equilibrium for the double auction with two-sided incomplete information is the Chatterjee–Samuelson (C-S) linear equilibrium [3]. In that example $F$ and $G$ are each the uniform distribution on $[0, 1]$ and, on the region where trade takes place, $S(C) = \frac{2}{3}C + \frac{1}{3}$ and $B(V) = \frac{2}{3}V + \frac{1}{3}$. Direct calculation shows that each trader’s ex ante expected gain from trade in this equilibrium is 0.0703, i.e., before his reservation value is drawn from $[0, 1]$, 0.0703 is his expected gain, given that both he and his bargaining partner play the linear C-S strategies. It is easy to verify that trade occurs $(c \leq v)$ if and only if $C + \frac{1}{3} \leq V$.

Ex post efficiency requires that the bargaining process result in trade if and only if $V \geq C$, i.e., it requires that bargaining exhaust all available gains from trade. Clearly the C-S equilibrium is not ex post efficient because some available gains from trade will be lost whenever $C < V < C + \frac{1}{4}$, an event with probability $\frac{7}{32}$. Myerson and Satterthwaite [9] showed that this inefficiency is necessary. Specifically, for two-sided incomplete information, they proved that if a particular equilibrium of a given mechanism satisfies interim individual rationality (interim IR hereafter), then its equilibrium allocations cannot always be ex post efficient. An equilibrium of a

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1 More generally, in the sealed-bid double auction price is set at $kc + (1 - k)v$, where $k$, a number between zero and one, is a parameter of the mechanism.

2 Our nomenclature for different forms of efficiency does not quite match that of Holmstrom and Myerson [7]. Our ex post efficiency is their ex post classical efficiency, our ex ante efficiency (see below) is their ex ante incentive efficiency, and our interim efficiency (again see below) is their interim incentive efficiency.

3 This result depends critically on the nature of the underlying uncertainty. The hypothesis Myerson and Satterthwaite employ is that $F$ has a positive probability density over the interval $[a_1, b_1]$, $G$ has a positive probability density over $[a_2, b_2]$, and the interiors of the two intervals have a nonempty intersection.
mechanism is interim IR for a trader if, after he learns the realization of his reservation value and before he begins bargaining, his expected gain from trade is nonnegative for each of his possible reservation values. Interim IR captures the idea that bargaining is a voluntary activity; if it holds, then each trader wants to participate no matter what the realization of his reservation value. Note that each trader in the double auction can guarantee interim IR for himself by choosing his strategy such that \( v \leq V \) if he is a buyer and \( C \leq c \) if he is a seller. In fact, this guarantees a strong form of IR, \textit{ex post} IR. That is, the trader's realized gain from trade is guaranteed to be either zero or positive. Interim IR only guarantees that in equilibrium his expected gain from trade, conditional on his reservation value, will be nonnegative.\(^4\)

Myerson and Satterthwaite's result that \textit{ex post} efficiency cannot be guaranteed by any mechanism in situations of two-sided uncertainty follows from the fact that each trader's reservation value is private. Conditional on his reservation value and the other trader's strategy, a trader can (and presumably will) select whatever action maximizes his expected utility. If that choice is different from the choice that some notion of efficiency in the presence of full information about reservation values would dictate, then so be it; the privacy of the trader's reservation value means that the efficient action cannot be imposed. This inability to force traders to behave in any other way than a maximizing way is called the incentive constraint (IC). By definition, all equilibrium outcomes necessarily satisfy it.

IC's bite can be seen in the double auction. \textit{Ex post} efficiency would be achieved if traders would truthfully announce their reservation values. In equilibrium, of course, a trader does shade his offer/bid upward if he is a seller (downward if a buyer) because the favorable effect on \( p \) more than offsets the possibility that a profitable trade will be foregone. This shading of offers/bids so that \( C < c \) and \( v < V \) is what produces \textit{ex post} inefficient outcomes.

Since IC and interim IR together may make \textit{ex post} efficiency unachievable, two alternative standards considered in some of the papers are \textit{ex ante} efficiency and interim efficiency. A mechanism, together with a pair of equilibrium strategies, is \textit{ex ante} efficient if no other mechanism has an equilibrium that gives both traders at least as great \textit{ex ante} gains from trade and one trader a greater \textit{ex ante} expected gain. For example, Myerson and Satterthwaite [9] showed that if \( F \) and \( G \) are each the uniform distribution on \([0, 1]\), then the C-S equilibrium for the double auction is \textit{ex ante} IR can also be defined. It requires that each trader's expected utility, before his reservation value is revealed to him, be nonnegative. D'Aspremont and Gerard-Varet [5] showed that if a mechanism is required only to satisfy \textit{ex ante} IR instead of interim IR, then achievement of \textit{ex post} efficiency can be guaranteed.

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Fig. 1. $U$ is the set of allocations of *ex ante* utility that are attainable in equilibrium. The upper right frontier (drawn as a heavy line) is the set of *ex ante* efficient allocations.  

*ex ante* efficient. Specifically, no mechanism and equilibrium exist that give one trader an *ex ante* expected gain greater than 0.0703 while giving the other an *ex ante* expected gain of at least 0.0703. If *ex post* efficiency were achievable, then both traders' *ex ante* expected gains would be 0.0833; the difference of 0.0130 represents the cost that IC and interim IR impose on the traders.

Figure 1 illustrates the concept of *ex ante* efficiency. Let $u_s(m, e)$ and $u_b(m, e)$ represent the *ex ante* expected gains from trade (utilities) that equilibrium $e$ of mechanism $m$ produces for the seller and buyer, respectively. For a given preference and information structure, the set $U$ in the figure is the set of utility pairs that satisfy both interim IR and IC. In other words, as IC requires, for each point $(u_s, u_b)$ in $U$ a mechanism $m$ and an equilibrium $e$ exist such that $(u_s(m, e), u_b(m, e)) = (u_s, u_b)$ and, as interim IR implies, $u_s \geq 0$ and $u_b \geq 0$. The upper right frontier of $U$ is the set of *ex ante* efficient allocations. The C-S equilibrium generates a point on that frontier.

Interim efficiency is a somewhat weaker form of efficiency. Define a trader's type to be his reservation value. Given a mechanism $m$ and pair of equilibrium strategies $e$, let $u_s(m, e|C)$ be the interim expected utility of a type $C$ seller and let $u_b(m, e|V)$ be the interim expected utility of a type $V$

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5 Holmstrom and Myerson [7] observe that *ex ante* efficiency implies interim efficiency, but not the reverse. Neither *ex ante* efficiency nor interim efficiency imply *ex post* efficiency.
buyer. The pair \((m, e)\) is interim efficient if no other pair \((m', e')\) exists such that,

1. for all \(C\), \(u_S(m', e' | C) \geq u_S(m, e | C)\),
2. for all \(V\), \(u_B(m', e' | V) \geq u_B(m, e | V)\); and
3. for some \(C\) or \(V\), either \(u_S(m', e' | C) > u_S(m, e | C)\) or \(u_B(m', e' | V) > u_B(m, e | V)\).

In essence, interim efficiency applies the Pareto criterion to each type of buyer and seller, while \textit{ex ante} efficiency applies it averaged across all types of buyers and all types of sellers. This distinction will arise below.

Two mechanisms besides the double auction receive substantial attention within the symposium. The first is sequential bargaining, as studied by Rubinstein [13] for the certainty case. In sequential bargaining traders follow a predetermined order in making offers back and forth until one or the other accepts an offer and bargaining terminates. To be more specific, suppose uncertainty is two-sided. In seller’s-offer sequential bargaining, the seller makes an offer that the buyer can accept or reject. If the buyer accepts, trade occurs. If the buyer rejects, the seller makes another offer after a given delay. This process continues until either the traders reach agreement or they become convinced that no gains from trade exist and negotiations cease. (It is also possible, in principle, for bargaining to continue without end, at least in the abstract model.) If trade occurs after a delay of \(t\) units of time, then each trader’s gain is discounted by \(e^{-\delta t}\).

Strategies for this game are quite complicated. The buyer’s strategy is an infinite sequence of functions \(\{B_1(h_1, V), B_2(h_2, V), B_3(h_3, V), \ldots\}\) whose ranges are the set \{accept, reject, break off\} and whose arguments are \(h_j\), the history of the seller’s first \(j\) offers, and \(V\), the buyer’s reservation value. The seller’s strategy is also an infinite sequence of functions \(\{S_1(C), S_2(C, h_1), S_3(C, h_2), \ldots\}\) that specifies the series of price offers he will make to the buyer as a function of his reservation value and his previous offers. The equilibrium concept for sequential bargaining is a modification of Kreps and Wilson’s sequential equilibrium [6] that accommodates the infinite nature of sequential bargaining games. It requires traders to exhibit sequential rationality in their choices of strategies and to update their beliefs concerning their trading partner’s reservation value using Bayes’s rule whenever possible.

Sequential bargaining is attractive to study for two reasons. First, casual observation suggests that it is commonly used in practice. Second, sequential bargaining exhibits sequential rationality in a way that a one-shot mechanism such as the double auction does not. In particular, consider again the linear C-S equilibrium. If \(C = \frac{3}{8}\) and \(V = \frac{1}{2}\) (trade should occur), then \(c = \frac{1}{2}\) and \(v = \frac{5}{12}\) (trade does not occur). Because the C-S strategies are
invertible, it is common knowledge between the buyer and seller after they fail to trade that some unrealized gains from trade exist (provided each learns the other’s bid). They therefore have an incentive, contrary to the rules of the double auction, to engage in another round of bargaining. Consequently, if bargaining is regarded as a voluntary activity, the double auction rule that bargaining is restricted to a single offer/bid pair lacks credibility.

If additional rounds of bargaining were permitted in the double auction, making it into another variety of sequential bargaining, then \textit{ex post} efficiency would not be achieved in the second round. The reason is that the traders would anticipate that additional rounds are available and would no longer play the C-S linear strategies in the first round. In fact, generally they would play strategies in the first round that are not fully invertible. If they did play invertible strategies, then their reservation values would be revealed and an equilibrium in round two would exist that realizes all available gains from trade. If the time period between rounds were sufficiently short, then the delay would be trivial and this outcome would be essentially \textit{ex post} efficient. In particular, each trader’s \textit{ex ante} expected gain from trade would be almost 0.0833. This, however, is impossible because such a result would violate Myerson and Satterthwaite’s result \cite{9} that the C-S equilibrium, which gives each trader an \textit{ex ante} expected gain of 0.0703, is \textit{ex ante} efficient. (Their result applies to sequential bargaining as well as to the double auction.) In sequential bargaining the cost associated with interim IR and IC shows up as delay in trading, not as failure to trade.

The third mechanism that appears in several of the papers is the optimal mechanism, as introduced into bargaining theory by Myerson and Satterthwaite \cite{1}. Given distributions \( F \) and \( G \), an optimal mechanism is one that satisfies IC and interim IR and is designed to be \textit{ex ante} efficient. In an optimal mechanism the traders each submit an offer/bid. As a function of the submissions the mechanism specifies whether trade occurs and what payment the buyer should make to the seller. If no trade occurs negotiations are terminated. It follows from the Revelation Principle that, without loss of generality, optimal mechanisms may be designed so that truthful revelation of his reservation value by each trader constitutes an equilibrium strategy.\textsuperscript{6} Optimal mechanisms (sometimes called “optimal revelation mechanisms”) are analytically useful because they delineate the limits of achievable performance within a particular setting. As a set of rules for conducting actual bargaining they are not useful because an optimal mechanism’s rules change as the underlying distributions \( F \) and \( G \) vary.

\textsuperscript{6} See Myerson \cite{8} for a statement and discussion of the Revelation Principle.
The rules that are actually used in a broadly defined bargaining situation generally do not vary with the priors of the participants.

This discussion should make the Table understandable. It outlines the most salient aspects of the models used in the papers that follow. Needless to say its brevity does a certain amount of violence to the richness of the papers.

**RESULTS**

The several papers can be classified without too much arbitrariness into three main groups. The first set of papers investigates basic questions of equilibrium for the bilateral bargaining problem described above. Issues of existence, multiplicity, and efficiency of equilibria are considered. The papers of Ausubel and Deneckere; Broman; Leininger, Linhart, and Radner; Linhart and Radner; Radner and Schotter; Satterthwaite and Williams; and Vincent constitute this group. The second set of papers focuses on the effects of preplay communication. Preplay communication is nonbinding conversation between the traders that occurs before any of them takes any payoff-relevant actions. It is inconsequential in the real sense that it has no direct costs or benefits to the traders. Yet, as the papers show, it can increase the set of equilibria that exists for a particular bargaining situation and, paradoxically, it can play a role in selecting among a multiplicity of possible equilibria. The papers of Farrell and Gibbons, Matthews and Postlewaite, and Myerson are in this group. The third set of papers consists of the pair by Gresik and Satterthwaite and by Rob [12]. These two papers each explore the effect of increasing the number of agents from two, as is the case in all the other papers, to many.

**Existence, Multiplicity, and Efficiency**

Ausubel and Deneckere characterize the equilibrium outcomes that sequential bargaining can produce in the bilateral bargaining problem with one-sided uncertainty. The seller's reservation value is zero and the buyer's value has distribution $G$ on $[0, 1]$, so it is common knowledge that gains from trade exist and should be realized. All sequential bargaining equilibria must satisfy *ex post* IR and IC. Let the interim expected utility of the seller be $u_s(m, e \mid C)$, which is identical to $u_s(m, e)$ because $C = 0$ with certainty, and let the interim expected utility of the buyer be $u_b(m, e \mid V)$. Let $W = \{(u_s(m, e), u_b(m, e \mid \cdot)) : m$ and $e$ satisfy IC and *ex post* IR}. Clearly $W$ contains all outcomes that sequential bargaining can generate.

Given this setup, Ausubel and Deneckere prove the following. If the sequential bargaining is structured as seller-offer sequential bargaining in

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7 Rob's paper [12] was published in the April 1989 issue of the journal because of space limitations in the present issue.
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<td>Ausubel &amp; Deneckere</td>
<td>Seller-offer, alternating offer, and buyer-offer sequential bargaining. Time interval between bids/offers approaches zero.</td>
<td>Seller has reservation value 0. Buyer value drawn from distribution $G$ on $[0, 1]$. Delay reduces gains from trade exponentially.</td>
<td>Seller's value, $G$, and discount rate are common knowledge. Buyer's value is private.</td>
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<td>Vincent</td>
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<td>Leininger, Linhart, &amp; Radner</td>
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<td>Satterthwaite &amp; Williams</td>
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<td>Linhart &amp; Radner</td>
<td>Double auction with multiple units and dimensions.</td>
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<td>Farrell &amp; Gibbons</td>
<td>Double auction with restricted pre-play communication.</td>
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<td>Matthews &amp; Postlewaite</td>
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<td>Myerson</td>
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<td>Buyer and seller's reservation values are private. $F$ and $G$ are common knowledge.</td>
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<tr>
<td>Rob [12]</td>
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<td>The firm's profits and the distribution $F$ are common knowledge. Each neighbor's pollution costs are private.</td>
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which only the seller is permitted to make offers and if the time interval between offers is shrunk to zero, then \( W_{SO} \) is identical to \( W \) where \( W_{SO} = \{(u_s(m, e), u_g(m, e) \mid m = \text{seller-offer sequential bargaining and } e \text{ is an equilibrium}\}. \) Thus any outcome that can be achieved by an \textit{ex post} IR equilibrium of some arbitrary mechanism can also be achieved by seller-offer sequential bargaining. If the bargaining is structured as alternating offers, then \( W_{AO} \) is strictly contained in \( W \) where \( W_{AO} \), analogous to \( W_{SO} \), is the set of outcomes alternating-offer sequential bargaining generates. Finally, for buyer-offer sequential bargaining, \( W_{BO} \) is the set in \( W \) that awards all expected gains from trade to the buyer and no gains to the seller. These results are important because they demonstrate that the rules of bargaining shape the outcome of a negotiation and, more specifically, a trader's power to make offers is important in obtaining a favorable split of the gains from trade.

Vincent, like Ausubel and Deneckere, studies the sequential bargaining mechanism, but within a different structure of preferences and information. The value the seller places on the object is a random variable and privately known to him. The buyer does not know his own reservation value; he only knows that the object is more valuable to him than to the seller (gains from trade are certain) and how his value is correlated with the seller's value. This, for instance, is the preference structure that a buyer of a used car may face, i.e., Vincent's setup includes Akerlof's famous "lemons" problem [1] as a particular case. If trade is to take place, as it should, the seller must convince the buyer how valuable the object is; otherwise the buyer may be unwilling to pay the seller's reservation value. Delay is the means available to the buyer to test what value the seller actually places on the object. Vincent shows that if the mechanism is buyer-offer sequential bargaining, then a unique equilibrium exists and leads to trade occurring in finite time. For many correlated preference structures the equilibrium involves some, but not infinite, delay.

The next five papers concern the double auction. The first three of them are negative in their implications: an extraordinary number of equilibria exist, almost none of which can be guaranteed to have attractive efficiency properties. Leininger, Linhart, and Radner explore the variety of equilibria that can occur in the double auction with two-sided incomplete information. They first consider equilibria in which the strategies \( S(\cdot) \) and \( B(\cdot) \) are differentiable and symmetric and show that a one parameter continuum of such equilibria exists whenever the \((F, G)\) pair belongs to a parametric class of distributions that includes the uniform. For the case of uniform \( F \) and \( G \), these equilibria range in performance from the \textit{ex ante} efficiency of the C-S linear equilibria to the complete inefficiency of no trade. They also show that if \( S(\cdot) \) and \( B(\cdot) \) are permitted to be step functions, then a similarly large family of step-function equilibria exist. In fact, equilibria
that have differentiable strategies can be approximated arbitrarily closely by step-function equilibria.

Satterthwaite and Williams also investigate the multiplicity of differentiable equilibria, though without any assumption of symmetry, and show that, for any \((F, G)\) pair from a broad class of distributions, a 2-parameter continuum of differentiable equilibria exists. Their geometric representation of this 2-parameter family illustrates the broad range of performance that is possible with smooth equilibria. Their existence result implies that the multiplicity of equilibria for the double auction is generic. They then investigate the efficiency of these equilibria and show that, for a generic \((F, G)\) pair, no equilibrium exists that is \textit{ex ante} efficient. Thus for uniform \((F, G)\) the \textit{ex ante} efficiency of the linear C-S equilibrium is a special, knife-edge case. The double auction generically is \textit{ex ante} inefficient. On the other hand, they also show that the interim efficiency of the C-S equilibrium when \((F, G)\) are uniform is not a knife-edge case, thus establishing that the double auction is not generically interim inefficient.

Broman is concerned with the behavior of the equilibrium set of the two-sided incomplete information double auction as the pair \((F, G)\) of distributions approach certainty. Specifically, suppose a sequence of distribution pairs \((F_k, G_k)\) converge to \((\overline{F}, \overline{G})\), the certainty case, in which \(\overline{F}\)'s mass is concentrated on 0 and \(\overline{G}\)'s mass is concentrated on 1. She first shows that, for \((F, G)\) and any finite set \(\{a_1, ..., a_n\}\) such that \(0 < a_1 < \cdots < a_n < 1\), a mixed strategy equilibrium exists whose support is \(\{a_1, ..., a_n\}\). She then shows that a sequence of equilibria corresponding to the sequence \((F_k, G_k)\) exists that converges to the certainty equilibrium with support \(\{a_1, ..., a_n\}\).

The paper by Radner and Schotter reports on an experimental study that used students to explore the empirical properties of the double auction with two-sided incomplete information. The \((F, G)\) pair for each experiment was picked so that a linear equilibrium with good efficiency properties can be found among the continuum of equilibria that exist. The results they obtained stand in contrast to the theoretical results reported just above. Rather than observing a great variety of different equilibria with widely varying efficiency properties, the experimental results show a strong tendency for the students to play linear strategies that attain reasonably efficient performance. The strategies sellers tended to use were close to the linear equilibrium strategies. The buyers, however, tended not to shave their bids under their reservation values as much as the linear equilibrium predicts they should.

Linhart and Radner's paper on minimax regret and the double auction can be seen as seeking a way out of the difficulty posed by the multiplicity of equilibria in the double auction with incomplete information. Regret is defined to be the difference between (i) the gain from trade that a trader would have realized if he had known before the fact what the other trader
was going to bid and (ii) the gain he actually realized. Linhart and Radner show that if a trader minimizes his maximum regret, conditional on his reservation value, then a resulting strategy is linear in his reservation value. If both traders adopt the minimax regret criterion, the coordination problem in the Bayesian Nash equilibrium approach to analyzing double auctions is eliminated. Moreover, within a particular parametric class of distributions \((F, G)\), the linear minimax strategies show reasonably good performance in terms of the \(ex\ ante\) expected gains from trade they realize.

**Preplay Communication**

Farrell and Gibbons develop an example of a double auction with two-sided uncertainty in which the rules are modified to allow each trader to send a preliminary message to the other before submitting his bid/offer. Specifically each trader gets to state whether he is “keen” or “not keen” to trade. They show, for the case of uniform \((F, G)\), that these messages create a new equilibrium that does not exist in the double auction without preplay communication. Thus plausible preplay communication expands the equilibrium set.

Matthews and Postlewaite demonstrate the degree to which preplay communication expands the equilibrium set within the double auction. Specifically, consider a game in which the buyer and seller, first, simultaneously exchange messages chosen from a sufficiently rich space and, second, play the sealed-bid double auction. They show that any allocation that satisfies two properties, \(ex\ post\ IR\) and \(IC^*\), is an equilibrium outcome of the game. \(IC^*\) is a strengthening of \(IC\); it, together with the \(ex\ post\ IR\) condition, can be thought of as giving each trader at the conclusion of the double auction the ability to enact the no-trade option rather than the trade prescribed by the auction’s outcome. Matthews and Postlewaite show that this set of allocations satisfying \(ex\ post\ IR\) and \(IC^*\) is very large in the sense that it does not exclude any allocations that are equilibrium outcomes of those specific trading mechanisms that have been studied in the literature. Thus the presence of preplay communication exacerbates the multiplicity of double auction equilibria.

Myerson seeks to turn the effect of preplay communication around by investigating how it can promote cooperation among rational traders. The idea is this. Suppose one trader, called the negotiator, is exogenously given the right to communicate with the other traders through a mediator. The negotiator makes a statement that consists of three parts: an allegation that describes his private information (e.g., his reservation value), a promise that says how he intends to act, and a request that states how he wishes the other trader to act. Myerson analyzes conditions under which the negotiator’s statement is credible. Clearly a necessary condition for
credibility is that obeying the negotiator's request be in the other trader's best interests if the negotiator in fact carries out his promise.

But, in addition, the negotiator's allegations about his own private information should be consistent with his promises and requests: given that his allegations are truthful, another set of promise and requests should not exist that would make him better off. Thus for the negotiator's statement to be credible it must be maximal in an appropriate sense. While Myerson does not prove that this analysis leads to a unique prescription for the statement the negotiator should make, he does sketch an argument that generically only finitely many statements are fully credible.

Multiple Traders

Gresik and Satterthwaite examine the bargaining problem for a market composed of \( n \) buyers and \( n \) sellers. Each seller has an object that he is willing to sell if he receives a payment greater than his reservation value and each buyer wishes to purchase an object if he makes a payment less than his reservation value. Each trader's reservation value is private; each seller's value is drawn from \( F \), each buyer's value is drawn from \( G \), and all draws are independent. \( Ex \ post \) efficiency could be achieved if reservation values were common knowledge by setting a market clearing price. Since this cannot be done, the bids and offers of the traders, who act strategically, must be used to set payments and decide who trades.

Gresik and Satterthwaite investigate the speed with which an optimal mechanism approaches \( ex \ post \) efficiency as \( n \), the number of traders on each side of the market increases. Recall that in the bilateral double auction case with uniform \((F, G)\) the C-S equilibrium is optimal inasmuch it is \( ex \ ante \) efficient. It is not \( ex \ post \) efficient, however, because under it a necessary condition for trade to occur is that \( V - C > \frac{1}{4} \). The quantity \( \frac{1}{4} \) is the "wedge" between the buyer and seller reservation values that causes the inefficiency. Gresik and Satterthwaite show that this wedge shrinks rapidly as \( n \) increases; its magnitude is at most \( O((\ln n)^{1/2}/n) \). In other words as \( n \) becomes large interim IR and IC, which are the source of the wedge, become less and less binding.

Rob [12] addresses much the same question, but does it in the context of public rather than private goods. He considers a model in which a plant, if it is allowed to operate, will earn profits \( R \). It will, however, damage each of its \( n \) neighbors through pollution. The damage to neighbor \( i \) is \( C_i \), where each \( C_i \) is independently drawn from a distribution \( F \). Knowledge of the cost \( C_i \) is private to neighbor \( i \). \( Ex \ post \) efficiency requires that the plant be allowed to operate if and only if \( \sum_i C_i < R \). This, however, is not directly observable and the decision must be made on the basis of the neighbors' self-reports of their individual costs.
Rob constructs an optimal mechanism for eliciting this information. It maximizes the firm's \textit{ex ante} expected profit subject to the interim IR and IC constraints. He shows that as the number of neighbors increases the performance of this mechanism worsens, i.e., it deviates increasingly from realizing \textit{ex post} efficiency. This result indicates that Coase's famous argument [4] that bargaining can resolve problems of externalities is dependent both on the structure of available information and on the number of agents involved. When compared with Gresik and Satterthwaite's result, it confirms the old intuition that increasing the number of agents improves performance when private goods are involved and worsens performance when public goods are involved.

An interesting sidelight is that a modified form of Rob's public goods problem is exactly the same mathematically as the bilateral sealed-bid double auction. A bridge, of unit cost, is to be built if and only if two parties are willing to finance it. Party $i$ ($i = 1, 2$) submits a sealed offer to pay amount $a_i$ towards the construction. The bridge will be built if and only if $\sum_i a_i \geq 1$. If it is built, the excess, $\sum_i a_i - 1$, is split in two and returned to the contributors.

It is easy to see that this scheme is mathematically equivalent to the double auction; a mapping between them is:

$$v \sim a_1,$$

$$c \sim 1 - a_2.$$

This formal identity between the private good problem and the public goods problem breaks down if the number of parties is increased beyond two. The reason is that in the public goods case nonexcludability implies that unanimity among the parties remains necessary. In the private goods case all traders need not agree on all trades; unanimity is necessary only in the case of a single buyer and a single seller. With two buyers and two sellers unanimity is not necessary; one of the buyers and one of the sellers can trade and exclude the other two parties.

\textbf{Conclusion}

Substantial progress has been made technically in analyzing bargaining situations within the Bayesian Nash equilibrium framework. This work, however, has created somewhat of an impasse: the theory permits an extremely wide range of behavior,\footnote{The generalization that the theory permits a wide range of behavior breaks down when one party (e.g., the seller) has a passive role that only involves accepting or rejecting the other party's proposals. See, for example, Ausubel and Deneckere's analysis of sequential bargaining with only the buyer making offers or Satterthwaite and Williams' analysis of sealed bid double auctions where only the buyer has any influence on price.} while the empirical evidence is that...
regularities do exist. This symposium contains two papers that suggest possible directions in which to move. First, Linhart and Radner demonstrate that approaches other than Bayesian Nash equilibrium may be more fruitful in generating testable (i.e., refutable) theory. Second, Myerson suggests that a full understanding of preplay communication may allow successful selection among the plethora of equilibria that exist in bargaining problems.

REFERENCES