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A strategic analysis of global warming: Theory and some numbers☆

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A R T I C L E   I N F O

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A B S T R A C T

We model the global warming process as a dynamic commons game in which the players are countries, their actions at each date produce emissions of greenhouse gases, and the state variable is the current stock of greenhouse gases. The theoretical analysis is complemented by a calibration exercise. The first set of results establishes theoretically, and then with illustrative numbers, the over-emissions due to a "tragedy of the commons." The power of simple sanctions to lower emissions and increase welfare is then examined as is the effect of cost asymmetry. Finally, a complete theoretical charactrization is provided for the best equilibrium, and it is shown that it has a very simple structure; it involves a constant emission rate through time.

1. Introduction

The dramatic rise of the world's population in the last three centuries, coupled with an even more dramatic acceleration of economic development in many parts of the world, has led to a transformation of the natural environment by humans that is unprecedented in scale. In particular, on account of the greenhouse effect, *global warming* has emerged as a central problem, unrivalled in its potential for harm to life as we know it on planet Earth. Seemingly the consequences are everywhere: melting and break-up of the world's ice-belts in the Arctic and the Antarctic; heat-waves that set all-time temperature highs in Western Europe and sub-Saharan Africa; storms increased in frequency and ferocity including Hurricane Katrina, typhoons in Japan, and flooding in Mumbai; and immediately graspable economic consequences, such as the shut-down of part of the New York subway due to a heat wave, and the startling images from Al Gore's movie "An Inconvenient Truth" showing the possible disappearance of major parts of coast-lines in China, India, the United States, and throughtout the globe.[1] Here are three additional facts:

1. Eleven of the last 12 years (1995–2006) have been amongst the 12 warmest years in the instrumental record of global surface temperatures (since 1850). The 100 year linear trend (1906–2005) of 0.74 °C is larger than the corresponding trend of 0.6 °C given in the Third Assessment Report of the IPCC for 1901–2000.[2]

☆ The first version of this paper was written in 2000.

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[1] Global Warming and its consequences have been, of course, widely written about. Al Gore's recent book and movie "An Inconvenient Truth" is an excellent, easy-to-read examination of the issues. More detailed analysis can be found in Hansen (2006). Two authoritative recent treatments are the Stern Review on the Economics of Climate Change, October 2006 and the IPCC Synthesis Report, November 2007.

[2] From "Climate Change", IPCC (2007).

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2. The Intergovernmental Panel on Climate Change (IPCC) predicts that if we go on with "Business as Usual", by 2100 global sea levels will probably have risen by 9–88 cm and average temperatures by between 1.5 and 5.5 °C.

   Various factors contribute to global warming, but the major one is an increase in greenhouse gases (GHGs), primarily carbon dioxide, so-called because they are transparent to incoming shortwave solar radiation but trap outgoing longwave infrared radiation. Increased carbon emissions due to the burning of fossil fuel is commonly cited as the principal immediate cause of global warming. The "natural levels" of GHGs (e.g., pre-1800 levels) raised the earth's average temperature by some 33 °C, from −18 to +15. (Cline 1992, p. 4.).[3]
3. Before the Industrial Revolution, atmospheric CO_2 concentrations were about 270–280 parts per million (ppm). They now stand at almost 380 ppm and have been rising at about 1.5 ppm annually.

The IPCC Synthesis Report (2007, p. 257) says "Warming of the climate system is unequivocal, as is now evident from observations of increases in global average air and ocean temperatures, widespread melting of snow and ice, and rising global average sea level."

It is clear that addressing the global warming problem will require the coordinated efforts of the world's nations. In the absence of an international government, that coordination will have to be achieved by way of an international environmental treaty. For a treaty to be implemented, it will have to align the incentives of the signatories by way of rewards for cutting greenhouse emissions and punishments for not doing so. For an adequate analysis of this problem one needs a dynamic and fully strategic approach. A natural methodology for this is the theory of Nash equilibria of dynamic games, which we adopt in this paper. In other words, our goal is to characterize the equilibria of a well-defined dynamic global warming game, as well as the globally (Pareto) optimal outcomes (which are typically not sutstainable as equilibria of the game). We then supplement the theoretical analysis with a calibration exercise that yields numerical estimates of equibrium and optimal trajectories of the system.

The present paper is part of an ongoing research project in which we have addressed various aspects of the global warming problem. We describe some of the other results of this project, as well as goals for further research, in Section 8. While our approach is not entirely new, we believe that we have carried the analysis farther than what is currently available in the published literature; for some relevant bibliographic notes, see Section 9.

The rest of the introduction provides a brief discussion of background issues regarding global warming as well as the model and results of this paper.

1.1. Background and principal issues

Although there is considerable uncertainty about the exact costs of global warming, the two principal sources will be a rise in the sea-level and climate changes. The former may wash away low-lying coastal areas such as Bangladesh and the Netherlands. Climate changes are more difficult to predict: tropical countries will become more arid and less productive agriculturally; there will be an increased likelihood of hurricanes, fires and forest loss; and there will be the unpredictable consequences of damage to the natural habitat of many living organisms. On the other hand, emission abatement imposes its own costs. Higher emissions are typically associated with greater GDP and consumer amenities (via increased energy usage). Reducing emissions will require many or all of the following costly activities: cutbacks in energy production, switches to alternative modes of production, investment in more energy-efficient equipment, investment in R&D to generate alternative sources of energy, and so on.

The principal features of the global warming problem are

- *The Global Common*: although the sources of carbon buildup are localized, it is the total stock of GHGs in the global environment that will determine the amount of warming.
- *Near-irreversibility*: since the stock of greenhouse gases depletes slowly, the effect of current emissions can be felt into the distant future.
- *Asymmetry*: some regions will suffer more than others.
- *Non-linearity*: the costs can be very non-linear; a rise in one degree may have little effect, but a rise in several degrees may be catastrophic.

The theoretical framework that accommodates all of these features is an *asymmetric dynamic commons* model with the global stock of greenhouse gases as the (common) state variable. We study just such a model with all the above characteristics *except* non-linearity and, additionally, with two other features:

- *A Strategic Model*: although the players (countries) are relatively numerous, there are some very large players and some blocks of like-minded countries, such as the U.S., Western Europe, China, and Japan. That warrants a strategic analysis. Furthermore a lesson of history is that the only good international treaties are those that are incentive-compatible and

[3] The IPCC (2007) report declared unequivocally "most of the observed warming over the last 50 years is likely to have been due to the increase in greenhouse gas concentrations."

self-enforcing for the signatories. (Indeed an objection to the Kyoto Protocol, although not the one commonly cited, is that it was not incentive-compatible; more on this in Section 8.)

- *A Simple Linear Model*: the model we study embeds a simple linear description of how global climate change affects the physical and economic world; it is assumed that emissions add linearly to the stock of GHGs and the stock of GHGs affect country costs linearly. (Technically, we use a "dynamic affine model"; see Sobel, 1990.) In particular that implies that, say, a 20 percent increase in the stock of GHGs will cause a 20 percent increase in the costs of climate change.

The linearity of the model is undoubtedly restrictive in several ways. It implies that our model is unable to analyze catastrophes or certain kinds of feedback effects running back from climate change to economic costs. For instance, some scientists have argued that a doubling of pre-industrial GHG levels will lead to a 3–5 °C temperature increase, causing an annual loss of 5 percent of GDP, but any increase beyond that could cause up to a 20 percent loss in GDP (Stern, 2006 and less explicitly, IPCC Synthesis Report 2007). Similarly, many scientists have pointed out that there could be several potential non-linear feedback effects: the shutting down of the Atlantic Ocean thermohaline circulation, the release of methane from the melting of permafrost, the melting of polar ice-caps, to name a few.

We make the linearity assumption for three reasons. First, for a theoretical exercise to have any chance of informing policy-makers in even one country, let alone 200, its conclusions have to be simple, and the linear model delivers that. (Indeed all the proposals at Kyoto or follow-up conferences have involved simple across-the-board cuts in emissions.) Second, it allows us to derive closed-form solutions for global Pareto optimal outcomes and some equilibria and to calibrate our model and derive some numerical guideposts.[4] Third, there is little consensus on what is the correct form of non-linearity in costs. Partly the problem stems from the fact that some costs are not going to be felt for another 50–100 years, and forecasting the nature of costs on that horizon length is at best a hazardous exercise.[5] Hence, instead of postulating one of many possible non-linear cost functions, all of which may turn out to be incorrect for the long-run, we opt instead to work with a cost function that may be thought of as a linear approximation to any number of actual non-linear specifications. To the extent that the world might remain in the linear segment for the next 50 or 100 years, and since discounting makes this the more relevant time-frame, our results are approximations of what a fully non-linear model would deliver. One can also think of our exercise as suggesting a lower bound on the economic consequences of global warming since catastrophic consequences are not analyzed.[6]

Although a full discussion of the relevant literature may be found in Section 9, it is worth remarking here on two influential recent publications: the Stern Review (Stern, 2006) and the IPCC Synthesis Report (IPCC 2007). The Stern Review empirically examines two contrasting scenarios. The first looks at the economic costs of "Business as Usual", that is, the costs due to climate change that the world's economies would face if GHG emissions continued to increase at current projections.[7] Even without examining the non-linear feedback effects discussed above, it comes up with an estimate that BAU will cause a 5 percent drop in world GDP per year starting around 2050, and it argues that this cost could be significantly higher, upto 20 percent, once non-linearities are accounted for. The second scenario examines the costs associated with cutting emissions in such a fashion that the stock of GHG stabilizes around 550 parts per million (ppm).[8] It concludes that the costs to mitigation and adaptation will be around 1 percent of GDP per year. Hence, it delivers a call for international coordination on mitigation and adaptation (Executive Summary, especially pp. x–xiii.).

The IPCC Report is similar in its conclusions about mitigation costs although it focuses more on the range of possibilities. It considers stabilization by year 2050 in a band ranging from 435 to 710 parts per million (ppm) and concludes that these will cost in range between 5.5 percent of world GDP through −1 percent (Summary for Policymakers, pp. 23–24.) Again, when coupled with the 5 percent, and possibly higher, GDP reduction from inaction, this too is a call for urgent international coordination on reducing GHG emission levels.

Many scholars have pointed out that the focus on 2× pre-industrial CO_2 levels of 550 parts per million (ppm), and the pathways to that steady-state, are essentially arbitrarily chosen and that the right approach is to examine what is the globally optimal GHG dynamic. One way to view the contribution of this project, modest as it is, that we do just that: we examine the GPO and then the best equilibrium emission strategies. Furthermore, we take seriously the idea that these emission patterns will be followed only if the nations of the world wish to do so, something, again, that the Stern and IPCC Reports ignore.

[4] The calibration and extensive numerical calculations for a model with approximately 180 countries have been done by Dr. Sangwon Park. Space limitations prevent us from presenting the details here, but they can be found in (found in Park, 2004).

[5] This point has been made forcefully by, amongst others, Thomas Schelling. See, for example, Schelling (2002).

[6] One other restriction implied by linearity is that our model is unable to deliver complex dynamics including chaos. For an example of such behavior, please see Chen (1997) as well as the broader discussion in Rosser (2001).

[7] These costs include health costs, declining crop yields, flood and storm damage, loss of ecosystems, and so on. The Review uses an integrated assessment model to impute economic costs and includes a sensitivity analysis.

[8] Emission cuts are projected to arise from a combination of greater efficiency in energy usage, a switch to cleaner technologies such as nuclear power and hydro-electricity, and the generation of new technologies through R&D.

1.2. The results

The present paper has five main parts: Benchmark Model, Baseline Global Pareto optima and "Business As Usual" Equilibria, Better Equilibria (including second and third-best), Asymmetry, and the Greenhouse Trap.

Model (Section 2): in the model, in each period each country adds to the global stock of greenhouse gases (GHGs) by its emission of those gases. The existing stock depreciates at a constant rate over time. Emissions arise from the production and consumption of energy, and other productive activities, and hence higher emissions are associated with, at least up to a point, higher benefits. The incremental cost of global warming is directly proportional to the amount of GHGs in the earth's atmosphere. Both the benefits as well as the costs vary across countries. The model has a relatively small set of parameters that guide it, and drawing on estimates for those parameters that are available in the literature, we are also able to calibrate the model.[9]

Baseline results (Section 3): we first show that *Global Pareto optimal* (GPO) emission profiles involve a constant emission of GHGs by each country (i.e., constant across GHG levels, and hence through time).[10] Next we characterize a Markov Perfect equilibrium (MPE) that is qualitatively similar to the GPO solutions; it too involves a vector a^* of constant emission rates by each country. Not surprisingly, this equilibrium, which we call *Business as Usual (BAU)*, exhibits the tragedy of the commons; a^* is strictly higher than the GPO vectors. The numerical estimates for this over-emission are illustrative. For base-line parameter values, the GPO is between 8 and 10 percent lower than the BAU level for developed economies and between 28 and 34 percent lower for developing economies and China. (Contrast this with the Kyoto-sanctioned 5 percent cut for developed economies.) Corresponding welfare loss in per period terms ranges from 0.1 percent for base-line costs to 2 percent for higher cost estimates.

Better equilibria/treaties (Sections 4 and 6): we then turn to the main question(s) of this paper: can countries sign a Kyoto-like agreement that slows down the pace of global warming *and* is honored by all signatories?

In Section 4 we start with the *third-best*: Nash or BAU reversion *trigger strategy equilibria* since these are the simplest to implement. The two main theoretical results here show that for all discount factors and all countries, one can do strictly better; the highest welfare level is strictly higher than in the BAU, and the minimum incentive-compatible emission levels are strictly lower than the BAU levels. Second, if discount factors are high enough, then, in fact, the GPO emission levels are themselves sustainable. For the base-line parameter values, the minimum sustainable emissions are lower than the GPO levels, for all regions except Eastern Europe; the greatest discrepancies arise for Russia and Ukraine where the GPO emission levels are, respectively, 39 percent and 40 percent lower than the BAU emission levels but the maximum sustainable cuts from BAU levels are only 20 percent and 19 percent, respectively.

Second-best: in Section 6 we characterize the full set of subgame perfect equilibria (SPE). We show that the *best equilibrium* has a remarkably simple structure; it too has a constant emission vector along the equilibrium path (at a rate strictly lower than the BAU rate a^*). The sanctions that support such a low emission level utilize the *worst equilibrium* for each player. That equilibrium is asymmetric (even in a symmetric game) and has an interesting two-stage structure. In the first stage, and for exactly one period, all countries other than the deviant country emit a "very high" amount, in excess of the BAU level. This behavior punishes all countries. For the sanctioning countries, the compensation is that in the second stage play proceeds to an equilibrium they prefer (and one that the sanctioned country does not). This is the equilibrium that maximizes a weighted sum of (equilibrium) payoffs, with zero weight on the sanctioned country. In such an equilibrium, the country with zero weight typically has to settle for very low emissions. This then is the long-term cost that a country suffers by breaking an agreement; its emission "quota" is permanently reduced *in an incentive-compatible fashion*.

The effect of *asymmetry* is studied in Section 5 by analyzing trigger strategy equilibria. Since there is no obvious measure of asymmetry with many countries, we restrict attention to two countries, or regions, and measure asymmetry by the difference in (marginal) costs of GHG. We show that increasing asymmetry leads to strict welfare loss (i.e., a reduction in the total payoffs of the two countries). Again we provide numerical analogs of these results by focussing on China and the United States.

Greenhouse traps: in Section 7 we consider emission strategies that are Markovian but not necessarily constant, and we demonstrate the existence of MPE that have simultaneously "basins of tragic attraction" , levels of GHGs that are inoptimally high so that once the system reaches them there are no incentives to escape, as well as "basins of low GHG buildup" (regions of carbon buildup that are no more than what a global planner would recommend). In other words, we show that there are MPE that simultaneously have characteristics of "bad" and "good" equilibria; where the system ends up depends then on where it started.

Section 8 discusses extensions of the model, including the incorporation of technological change, while Section 9 includes a literature review. Proofs are gathered in Section 10.

[9] The model has been generalized in Dutta and Radner (2004, 2006) and Dutta and Radner (in preparation) to account, respectively, for technical change, population growth, and endogenous capital stock dynamics. See Section 8 for a further discussion.

[10] The GPO emission is constant on account of the linearity in the damage-cost function; that, coupled with the constant depreciation rate, implies that the marginal valuation of a unit of emission is independent of the existing stock of GHG.

2. A simple model

In this section we present a simplified model to illustrate the basic strategic ideas. In this model there is no population growth and no possibility of changing the emissions producing technologies in each country.[11] However, the countries may differ in their "sizes," their emissions technologies, and their preferences.

There are I countries. The emission of (a scalar index of) greenhouse gases during period t by country i is denoted by $a_i(t)$. (Time is discrete, with $t = 0, 1, 2, \ldots,$ ad inf.) Let $A(t)$ denote the global (total) emission during period t;

$$A(t) = \sum_{i=1}^{I} a_i(t). \tag{1}$$

The total (global) stock of greenhouse gases (GHGs) at the beginning of period t is denoted by $g(t)$. The law of motion for the total GHG is

$$g(t+1) = A(t) + \sigma g(t), \tag{2}$$

where σ is a given parameter $(0 < \sigma < 1)$. We may interpret $(1-\sigma)$ as the fraction of the beginning-of-period stock of GHG that is dissipated from the atmosphere during the period. The "surviving" stock, $\sigma g(t)$, is augmented by the quantity of global emissions, $A(t)$, during the same period.

Suppose that the utility of country i in period t is

$$v_i(t) = h_i[a_i(t)] - c_i g(t). \tag{3}$$

The function h_i represents, for example what country i's gross national product would be at different levels of its own emissions, holding the global level of GHG constant. This function reflects the costs and benefits of producing and using energy as well as the costs and benefits of other activities that have an impact on the emissions of GHGs (e.g., the extent of forestation). For a given population there will be an optimal level of energy use, forestation, and so on, and hence an optimal level of emissions. It therefore seems natural to assume that h_i is a strictly concave C^2 function that reaches a maximum and then decreases thereafter.[12]

The parameter $c_i > 0$ represents the marginal cost to the country of increasing the global stock of GHG. Of course, it is not the stock of GHG itself that is costly, but the associated climatic conditions. As discussed above, in a more general model the cost would be non-linear. Similarly, the separability of the payoff function is a simplification; in a more general model, a country's GNP level might itself depend on the GHG stock level.[13]

The total payoff (utility) for country i is

$$v_i = \sum_{t=0}^{\infty} \delta^t v_i(t). \tag{4}$$

For the sake of simplicity, we have taken the discount factor, δ, to be the same for all countries.

At time t, a history of play is an enumeration of past GHG levels up to and including the current one as well as past emissions for all countries. A strategy for country i at time t, call it π_{it}, is a measurable map from the set of histories to the set of emissions. A *strategy* for the whole game, denoted by π_i, is a sequence of such strategies: $\pi_i \equiv [\pi_{i0}, \pi_{i1}, \ldots, \pi_{it}, \ldots]$.[14] Let π denote a profile of strategies, one for each country: $\pi = (\pi_1, \pi_2, \ldots, \pi_I)$. A particularly simple strategy for country i is to condition her action at period t only on the state at that period; such a strategy is called a *Markovian strategy.*

Each profile π, and the initial level of GHG, g_0, defines in an obvious manner country i's payoff in period t; denote this payoff by $v_i(t; \pi, g_0)$. Thus associated with each profile is a total discounted payoff for each player:

$$v_i(\pi, g_0) \equiv \sum_{t=0}^{\infty} \delta^t v_i(t; \pi, g_0).$$

As usual, a profile of strategies π^* forms a *Nash Equilibrium* (or just an *equilibrium*) from the initial GHG level g_0 if no player can increase her total discounted expected payoff by *unilaterally* changing her strategy. A profile π^* forms a *subgame perfect Equilibrium* (SPE) if after every history the continuation strategies of π^* form a Nash equilibrium. If a profile of Markovian strategies forms a Nash equilibrium from every initial state then it is called a *Markov Perfect Equilibrium* (MPE); note that an MPE is also an SPE.

Note that if there is imperfect monitoring then player i's history will contain only her own past actions. In this paper we shall restrict attention to the perfect monitoring case, although our results on Markovian strategies will apply to the case of

[11] Population growth is studied in Dutta and Radner (2006) while certain kinds of technological changes are allowed in Dutta and Radner (2004).
[12] However, none of the results depend on h having a finite argmax.
[13] We thank a referee for this suggestion.
[14] Without loss of generality we restrict attention to pure strategies.

imperfect monitoring, as well. This is because, by definition, Markovian strategies only condition on the observable part of history (i.e., the total stock of GHGs).

2.1. An illustrative example

The theoretical results of this paper will be illustrated throughout by use of a calibrated model using parameter values that have been considered in the literature. The aim is thereby to provide some numerical orders of magnitude for the results stated here. In this subsection we will briefly describe the numerical analysis, and in subsequent sections we will present the numerical analogs immediately following the appropriate theoretical results.

The numerical example is taken from a much more extensive analysis by Sangwon Park. In his model, 184 countries are grouped for calibration purposes into eight regions: the United States, Western Europe, Other High Income, Eastern Europe, Middle Income, Lower Middle Income, China, and Lower Income. (This grouping enables Park to utilize the data and estimates provided by Nordhaus and Boyer (2000).) The base year is 1998 ($t = 0$). Referring to Eq. (3), the utility of country i net of warming costs, its gross national product after subtracting the amount of capital needed to maintain the capital stock at its 1998 level, h_i, is given by

$$h_i[a_i(t)] = \phi_i K_i^{\gamma_i} L_i^{1-\gamma_i-\beta_i} \left[\frac{a_i(t)}{f_i}\right]^{\beta_i} - p_i \left[\frac{a_i(t)}{f_i}\right]$$

where K_i and L_i are the capital and labor inputs, respectively; f_i is the emission factor of country i and p_i is the price of "energy" (and, recall, a_i is the emission of country i). To explain the last two terms in greater detail, imagine that energy is a proxy for a scalar index of emissions-producing inputs, measured in coal–equivalent metric tons. Then for every unit of energy there are associated f_i units of emissions. Countries differ in their emission factors: countries with cleaner technologies have lower f_i than those with "dirtier" technologies. The Greek letters are parameters. These parameters as well as the price of energy, p_i, were the same for all countries in the same region (that is what defines a region) while the damage coefficient c_i is based on a combination of country and group-specific data and is drawn from Fankhauser (1995). All three production inputs – capital, labor, energy – are country-specific.

All countries have the same discount factor, δ, which for sensitivity analysis has been varied between 0.97 and 0.995. These would seem to bracket the values commonly discussed in the literature, motivated by different interpretations of δ as reflecting "social values" or the returns on investment. In what follows utility is measured in 1990 U.S. dollars and emissions are measured in gigatons of carbon.

3. Two benchmarks

In this section we characterize two benchmarks: the global Pareto optima, and a simple Markov Perfect Equilibrium, called "Business As Usual".

3.1. Global pareto optima

Let $x = (x_i)$ be a vector of positive numbers, one for each country. A *Global Pareto Optimum (GPO)* corresponding to x is a profile of strategies that maximizes the weighted sum of country payoffs,

$$v = \sum_i x_i v_i, \tag{5}$$

which we shall call *global welfare*. Without loss of generality, we may take the weights, x_i, to sum to I.

Theorem 1. *Given a vector of weights, x, let $\hat{V}(g)$ be the maximum attainable global welfare starting with an initial GHG stock equal to g; then*

$$\hat{V}(g) = \widehat{u} - wg, \tag{6}$$

$$w = \frac{1}{1-\delta\sigma} \sum_i x_i c_i,$$

$$\widehat{u} = \frac{\sum_i x_i h_i(\hat{a}_i) - \delta w \hat{A}}{1-\delta}, \quad \hat{A} = \sum_i \hat{a}_i,$$

and $\hat{a}_i$ is determined by

$$x_i h_i'(\hat{a}_i) = \delta w. \tag{7}$$

(*It is assumed that this last equation has a solution.*) *Furthermore, in the GPO solution, country i would be required to use a constant emission equal to* $\hat{a}_i$ *in all periods and after all histories.*

Theorem 1 states that, independently of the level of GHG, g, each country should emit an amount $\hat{a}_i$. The fact that the optimal emission is constant follows from the linearity of the model in g. Notice that on account of the linearity in the gas buildup equation Eq. (2) a unit of emission in period t can be analyzed in isolation as a surviving unit of size σ in period $t+1$, σ^2 in period $t+2$, σ^3 in period $t+2$, and so on. On account of the linearity in cost, these surviving units add $(\sum_i x_i c_i)\times\delta\sigma$ in period $t+1$, $(\sum_i x_i c_i)\times(\delta\sigma)^2$ in period $t+2$, and so on; therefore, the marginal lifetime cost is

$$\frac{1}{1-\delta\sigma}\sum_i x_i c_i,$$

or w_i, and that marginal cost is independent of g.

3.2. A Markov-perfect equilibrium: "Business as Usual"

This MPE shares the feature that the equilibrium emission rate of each country is constant in time, and it is the unique MPE with this property. We shall call it the "Business as Usual" equilibrium. Note that in this equilibrium each country takes account of the incremental damage to itself caused by an incremental increase in its emission rate, but does not take account of the damage caused to other countries. [In some contexts the phrase "Business as Usual" is used to describe a situation in which each country does not even take account of the damage to itself.]

Theorem 2. ("Business as Usual" Equilibrium). *Let g be the initial stock of GHG. For each country I, let* $a_i{}^*$ *be determined by*

$$h_i'(a_i^*) = \delta w_i, \tag{8}$$

$$w_i = \frac{c_i}{1-\delta\sigma},$$

and let its strategy be to use a constant emission equal to $a_i{}^*$ *in each period*; *then this strategy profile is a MPE, and country i's corresponding payoff is*

$$V_i^*(g) = u_i^* - w_i g, \tag{9}$$

$$u_i^* = \frac{h_i(a_i^*) - \delta w_i A^*}{1-\delta}, \quad A^* = \sum_j a_j^*.$$

The intuition for the existence of an MPE with constant emissions is similar to the analogous result for the GPO solution. As long as other countries do not make their emissions contingent on the level of GHGs, country i has a constant marginal lifetime cost to emissions; this follows from the linearity of the model. Notice that on account of the linearity in the transition (gas buildup) equation – Eq. (2) – a unit of emission in period t can be analyzed in isolation as a surviving unit of size σ in period $t+1$, σ^2 in period $t+2$, σ^3 in period $t+2$, and so on. On account of the linearity in cost, these surviving units add $c_i \times \delta\sigma$ in period $t+1$, $c_i \times (\delta\sigma)^2$ in period $t+2$, and so on, i.e., the marginal lifetime cost to country i on account of its own emission is $(c_i/(1-\delta\sigma))$ or w_i. And that marginal cost is independent of g. There are, however, other MPEs in which countries do condition on g, and some of them will be described in Section 7.

3.3. Comparison of the GPO and business as usual

The preceding results enable us to compare the emissions in the GPO with those in the Business as Usual MPE:

$$\text{GPO}: h_i'(\hat{a}_i) = \frac{\delta\sum_j x_j c_j}{x_i(1-\delta\sigma)}, \tag{10}$$

$$\text{BAU}: h_i'(a_i^*) = \frac{\delta c_i}{1-\delta\sigma}.$$

Since

$$x_i c_i < \sum_j x_j c_j,$$

it follows that

$$\frac{\delta c_i}{1-\delta\sigma} < \frac{\delta \sum_j x_j c_j}{x_i(1-\delta\sigma)}.$$

Since h_i is concave, it follows that

$$a_i^* > \hat{a}_i. \tag{11}$$

Note that this inequality holds except in the trivial case in which all welfare weights are zero (except one). In other words, there is a tragedy of the commons whenever there is some externality to emissions. In turn, all this follows from Eq. (8), which says that in the BAU equilibrium each country only considers its own marginal cost and ignores the cost imposed on other countries on account of its emissions; in the GPO solution that additional cost is, of course, accounted for. It follows that the GPO is strictly Pareto superior to the MPE for an open set of welfare weights x_i (and leads to a strictly lower steady-state GHG level for all welfare weights).

3.4. Numerical results on the GPO and BAU

In this subsection we present numerical results on the emission levels and welfare values (especially in a comparative sense) of the GPO and BAU solutions. As indicated above, the results are derived by using numerical estimates for parameter values that have been used elsewhere by Fankhauser (1995) and Nordhaus and Boyer (2000). We start with the benchmark case in which $\delta = 0.97$ and cost coefficients c_i are as given in Fankhauser. This case has been calibrated so that the BAU matches available data and estimates for 1998. The benchmark GPO then uses the same parameter values and initial conditions. (Throughout, in computing GPO trajectories, we have taken the country-welfare weights x_i to be equal to 1. The consequences of using other welfare weights have been explored in Park.) (Table 1).

The most striking results are (a) BAU emission levels are most significantly higher (relative to GPO) for Eastern Europe, presumably on account of Soviet-era energy technologies; (b) the next significant departures are for China and Lower Middle Income countries (that includes India), and this is particularly worrisome if the major growth spurt of the next 50 years comes from those countries, which will then move them way past Eastern Europe in over-emission; and (c) that the United States and Western Europe account for over 40 percent of current total emissions and, hence, a cut-back to GPO levels in those regions alone can significantly narrow the overall GPO–BAU gap.

The differences between GPO and BAU are magnified if either of the two basic parameters are increased. Consider an increase in the discount factor to 0.995 (reflecting the idea that environmental discount factors ought to be closer to 1 for inter-generational equity reasons); this is Table 2. The main findings while qualitatively similar (Eastern Europe is the

Table 1
Benchmark case ($\delta = 0.97$, cost = Fankhauser c_i, Year = 1998).

Region/emissions	BAU (Gtc)	GPO (Gtc)	% Difference ((BAU − GPO)/BAU)
United States	1.50	1.36	9
Western Europe	0.86	0.79	8
Other high income	0.59	0.53	9
Eastern Europe	0.74	0.45	39
Middle income (MI)	0.41	0.36	12
Lower MI	0.58	0.41	30
China	0.85	0.56	34
Lower income	0.66	0.48	28
Total	6.18	4.93	20

Table 2
Sensitivity analysis I ($\delta = 0.995$, cost = Fankhauser c_i, Year = 1998).

Region/emissions	BAU (Gtc)	GPO (Gtc)	% Difference ((BAU − GPO)/BAU)
United States	1.50	1.15	23
Western Europe	0.86	0.69	20
Other high income	0.59	0.46	22
Eastern Europe	0.74	0.27	64
Middle income (MI)	0.41	0.29	28
Lower MI	0.58	0.27	54
China	0.85	0.32	62
Lower income	0.66	0.32	52
Total	6.18	3.76	39

Table 3
Value loss when $\delta = 0.97$.

Value loss/cost parameter	FC	2XFC	3XFC	4XFC	5XFC
(GPO – BAU)/BAU in 1998 dollars	0.11%	0.4%	0.82%	1.34%	1.95%

Table 4
Value loss when $\delta = 0.995$.

Value loss/cost parameter	FC	2XFC	3XFC	4XFC	5XFC
(GPO – BAU)/BAU in 1998 dollars	0.68%	2.14%	4.06%	6.37%	9.02%

greatest problem) show that China and Lower Income Countries have almost as much over-emissions. This could pose a huge problem down the line if these economies continue to grow strongly.[15]

We now turn to value comparisons; that is how much is the welfare loss due to over-emission and its consequent increase in global temperatures? The welfare analysis takes account of the infinite future. In particular, what is computed is the present discounted average value of GNP less carbon costs (i.e., $(1-\delta)V_i$)so that when δ is close to 1, the discounted average value is close to the long-term yearly average. Note that if 2 percent of a country's GNP were invested every year, the rate of growth of its economy would be dramatically increased.

Table 3 provides value loss estimates for $\delta = 0.97$ while Table 4 provides the parallel estimates for $\delta = 0.995$. The carbon costs parameter is drawn from Fankhauser (1995), denoted FC, and the sensitivity analysis is to vary them between his estimates (=FC) and five times his estimates (=5XFC):

4. Other equilibria I: BAU sanctions

In this section we analyze a "third-best" problem: maximize a weighted sum of country utilities when the emission policies are incentive-compatible under the threat of BAU emissions. Recall that an equilibrium is *second best* if its outcome is Pareto optimal among the outcomes that can be sustained by some(subgame-perfect) equilibrium. The set of second best equilibria will be characterized in Section 6.2. For some given subset $\mathcal{E}$ of equilibria, an equilibrium is third best relative to $\mathcal{E}$ if it is Pareto optimal among the outcomes that can be sustained by some equilibrium in $\mathcal{E}$. In the present section, $\mathcal{E}$ is the set of equilbria that are sustained by a threat of reverting to the BAU in case there is a defection.

There are three main results. First, for all discount factors, the third-best solution qualitatively mirrors the BAU and GPO solutions; there is a constant emission level a'_i that country i emits, independently of the stock of GHGs. Second, the set of sustainable emissions includes the BAU level but is strictly larger than that. In particular, the minimum incentive-compatible emission level is strictly lower than the BAU level, and that is true for every country. Hence, BAU sanctions are strictly welfare improving for all countries and all discount factors. Third, if discount factors are high enough, then, in fact, the GPO emission levels are themselves the third-best solution. Finally, we provide empirical illustration for these results.

4.1. BAU Sanctions: third-best and minimum emissions

Let $x = (x_i)$ be a vector of positive numbers, one for each country. A *Third-Best Optimum (TBO)* corresponding to x is a profile of "norm" strategies that maximizes the weighted sum of country payoffs,

$$v = \sum_i x_i v_i, \tag{12}$$

subject to BAU reversion (i.e., subject to the constraint, detailed below, that should any country i not follow the norm, all countries would switch to BAU emissions vector $a^* = (a_1^*, a_2^*, \ldots, a_I^*)$ starting in from the following period). As before, and without loss of generality, we may take the weights, x_i, to sum to I. The following result characterizes the TBO:

Proposition 1. *There exists a vector* ã *of constant emission levels* $\tilde{a}_i$ *such that on the equilibrium path country i's TBO strategy is to use a constant emission equal to* $\tilde{a}_i$ *in all periods, where* $\tilde{a}'_i$ *satisfies the incentive constraint*:

$$\text{for every } i, \quad h_i(\widetilde{a}_i) - \delta w_i\left(\widetilde{a}_i + \delta\sum_{j\neq i}\widetilde{a}_j\right) \geq h_i(a_i^*) - \delta w_i\left[a_i^* + \delta\sum_{j\neq i}a_j^*\right].$$

[15] A change in the second parameter, raising the cost estimates to 5× Fankhauser (reflecting possible catastrophic costs) increases the range of over-emissions to 56 percent (Western Europe) to 90 percent (Eastern Europe).

Table 5
Minimum emissions under BAU sanctions; $\delta = 0.97$ and Fankhauser costs.

Region/emissions	BAU (Gtc)	GPO (Gtc)	MIN (Gtc)
United States	1.50	1.36	1.22
Western Europe	0.86	0.79	0.67
Other high income	0.59	0.53	0.45
Eastern Europe	0.74	0.45	0.54
Middle income (MI)	0.41	0.36	0.28
Lower MI	0.58	0.41	0.32
China	0.85	0.56	0.45
Lower income	0.66	0.48	0.43
Total	6.18	4.93	4.36

It might help to sketch the derivation of the simple incentive constraint above. Consider any constant emission level vector $\widetilde{a} = \widetilde{a}_1, \widetilde{a}_2, \ldots, \widetilde{a}_I$. It can be easily shown that this emission level is sustainable as an equilibrium norm provided

$$\text{for every } i \text{ and } a_i, \quad h_i(\widetilde{a}_i) - c_i g + \delta\left[\widetilde{u}_i - w_i\left(\sigma g + \sum_j \widetilde{a}_j\right)\right] \geq h_i(a_i) - c_i g + \delta\left[u_i^* - w_i\left(\sigma g + a_i + \sum_{j \neq i} \widetilde{a}_j\right)\right], \quad (13)$$

where $\widetilde{u}_i$ and u_i^* are the present discounted values to emission policies that emit at constant rate-vectors of, respectively, $\widetilde{a}$ and a^* into the infinite future; recall that in the event of a deviation, the threat is to revert to the BAU emissions vector a^* forever. In turn, the above equation can be simplified to

$$\text{for every } i \text{ and } a_i, \quad h_i(\widetilde{a}_i) - \delta w_i \widetilde{a}_i + \delta \widetilde{u}_i \geq h_i(a_i) - \delta w_i a_i + \delta u_i^*.$$

From Eq. (8), the characterization of the BAU emission level, it follows that the deviation profit (the RHS of the equation above) is maximized at a_i^*. Using that fact and the definitions of $\widetilde{u}_i$ and u_i^*,[16] the incentive constraint can be rewritten in the form above.

It is immediate that the BAU emission policy is sustainable by the threat of BAU reversion (of course!) since the inequality is trivially satisfied when $\widetilde{a}_i = a_i^*$. What is also not very difficult to show is that the GPO emission policy also becomes sustainable at a high enough δ. Formally, we have

Proposition 2. *(a) The welfare that is achievable under the threat of BAU emissions is at least as high as* u^*.

(b) Suppose that the GPO solution under equal country weighting, $(x_i = x_j \text{forall} i, j)$ *Pareto-dominates the BAU solution for all high* $\delta \geq \delta'$*; then there is a cut-off discount factor* $\widetilde{\delta} \in (\delta', 1)$ *such that, above it, the GPO emission policy is sustainable as an equibrium norm.*

Consider the minimum emission problem:

$$Min \sum_i \widetilde{a}_i$$

over all incentive-compatible emission levels (i.e., over all $\widetilde{a}$ that satisfy the incentive constraint above). The next proposition characterizes the minimum emissions.

Proposition 3. *(a) There is a unique emission level* $\underline{a}$ *that is the minimum emission level sustainable by the threat of BAU reversion.*

(b) For all δ*, the minimum emission level is strictly lower than the BAU level* a^* (i.e., $\underline{a} << a^*$).

4.2. BAU sanctions: numerical evidence on minimum emissions

In this subsection we present empirical evidence on the emission cuts that are sustainable under the threat of BAU sanctions. We show that this is, for most regions, even lower than the GPO levels. There is one significant exception to this: Eastern Europe. We also present some country-specific evidence. Again countries from the ex-Soviet Bloc are the ones where significant cuts appear most difficult to implement; for Russia and Ukraine the GPO emission levels are, respectively, 39 percent and 40 percent lower than the BAU emission levels, but the maximum sustainable cuts are only 20 percent and 19 percent, respectively.

In Tables 5 and 6 below, MIN refers to the minimum emission level $\underline{a}$.

[16] Recall (see Eq. (9)) $u_i^* = (h_i(a_i^*) - \delta w_i \sum a_j^*/1 - \delta)$ and $\widetilde{u}_i = (h_i(\widetilde{a}_i) - \delta w_i \sum \widetilde{a}_j/1 - \delta)$. By extension, u^* is the vector $(u_1^*, u_2^*, \ldots, u_I^*)$ and $\widetilde{u}$ is the vector $(\widetilde{u}_1, \widetilde{u}_2, \ldots, \widetilde{u}_I)$.

Table 6
A comparison of minimum emissions and GPO emissions for selected countries.

Country/emissions	(MIN – GPO)/BAU (%)
UnitedStates	−9
Brazil	−31
China	−21
India	−9
Korea DPR	+22
Poland	+5
Russia	+17
Ukraine	+21

5. Asymmetry

One critical feature of the climate change problem is that countries differ greatly in the consequences they are likely to suffer. Whereas low-lying places such as Bangladesh may see parts of their countries washed away by rising sea-waters or equatorial countries may become uninhabitably hot, northern regions of the world such as parts of North America and northern Europe will bear limited costs and may even benefit from longer growing seasons and lower heating costs in the winter. In this section we analyze: how asymmetry (in costs) limits the extent of sustainable cuts. Are they optimal cuts? What are the welfare consequences of asymmetry? As in the last section, we will limit attention to equilibria that are sustainable by the threat of reversion to the BAU equilibrium. For this section, we will focus on

- "*Two countries*, (i.e., $I = 2$)."

We study two countries (or two regions) because it is easier to define asymmetry in terms of a single-dimensional parameter (namely the difference in the costs c_1 and c_2) and it is less obvious what we mean by an "increase" in asymmetry when we have I costs.[17]

5.1. Incentives and welfare

No matter what emissions policy we consider, when country i's marginal damage cost of global warming, c_i, increases, its total and marginal lifetime cost increases. As we saw in Section 3, the emission in any period t can be treated in isolation, and costs $c_i/(1-\delta\sigma)$ per unit of emission. Hence when c_i increases, *everything else being equal*, country i should be willing to cut emissions since benefits are unchanged and the marginal cost is higher (and conversely for a decrease in costs).

The problem though is that everything else does not actually remain unchanged when costs change. In particular, the sanction (i.e., the BAU solution) itself changes with the cost. In principle then the comparative statics of incentives are unclear; whereas a higher cost makes cheating on a treaty less profitable, it also makes the sanction to be suffered less severe. In this subsection we show that nevertheless the first effect always triumphs; a higher-cost country will agree to every cut that a lower cost country agrees to, and possibly more.

Consider any constant emissions policy with emissions at rate $\widetilde{a}$. Let $IC(c)$ denote the emission vectors that are incentive-compatible for both countries under the threat of BAU reversion, that is,

$$IC(c) = \{\widetilde{a}_1, \widetilde{a}_2 \geq 0 : h_i(\widetilde{a}_i) - \delta w_i(\widetilde{a}_i + \delta\widetilde{a}_j) \geq h_i(a_i^*) - \delta w_i(a_i^* + \delta a_j^*), \quad i = 1, 2, \quad i \neq j\}.$$

By extension, let $IC_i(c)$ denote the projection of that set (i.e., denote all emissions $\widetilde{a}_i$ of country i that are incentive-compatible given some incentive-compatible emission $\widetilde{a}_j$). The next result shows that as country i's cost increases, and j's cost decreases, its set of incentive-compatible emissions, $IC_i(c)$, gets larger as well. Conversely, the other country's set of incentive-compatible emissions, $IC_j(c)$, gets smaller.

Theorem 3. *Consider an emission vector $\widetilde{a}$ less than the BAU vector a^*: if $(\widetilde{a}_1, \widetilde{a}_2)$ is in $IC_i(c)$, that is, is incentive compatible (under costs c_1 and c_2) then it is still incentive-compatible for country i at $c_i + e$ and $c_j - e$, for $e > 0$. Conversely, $IC_j(c_i + e, c_j - e)$ is a subset of $IC_j(c)$.*

Recall the symmetric third-best problem:

$$Max\, V_1(g; a) + V_2(g; a), \tag{14}$$

where $V_i(g; a)$ is country i's lifetime payoff from emission policy $a(.)$, and this emission policy is incentive compatible under the threat of BAU reversion. Denote cost asymmetry by a single-dimensional parameter $d = c_1 - c_2$. Without loss of generality, let us consider cost changes in which c_1 increases while c_2 decreases, so that d is positive and increasing and let us start from

[17] Of course, the same analysis would work if the entire set I of countries were divisible into two groups, provided within each group every country had the same benefits and costs.

Table 7
Third-best emission levels as a function of cost difference d.

d	US (GtC)	China (GtC)
0.2	1.46400	0.77206
0.3	1.46369	0.77201
0.4	1.46208	0.77586
0.5	1.46048	0.77971
0.6	1.45601	0.79646
0.7	1.45332	0.81295

$d = 0$. In order to focus on the effect of incentive changes alone, let us keep the total cost, $c_1 + c_2$ constant since the total is the only way costs enter the global welfare function.

Theorem 4. *Suppose that the GPO is sustainable as a third-best equilibrium when costs are equal, $d = 0$, and suppose that $c_1 + c_2$ remains constant. Overall welfare, as measured by the sum of country payoffs are non-increasing in cost asymmetry d. It remains unchanged for small asymmetry, until a cut-off asymmetry level $\widehat{d}$, and thereafter strictly decreases.*

5.2. Asymmetry: some numbers

In this subsection the two countries are the US (country 1) and China (country 2), whose actual costs are quite similar to each other. Table 7 illustrates the *third-best* as it changes with asymmetry d.

6. Other equilibria II: a complete characterization of SPE

In this section we will characterize all possible subgame perfect equilibria (SPE) of the model. We will do that in two steps. In Section 6.1 we will characterize the SPE *payoff* correspondence. This characterization will be then used in Section 6.2 to study the *extreme equilibria*, the best and the worst equilibria. Although the theoretical analysis is complete, numerical estimates of extreme equilibria, and especially the worst equilibrium, are more difficult to compute. That exercise is left for future research.

6.1. The equilibrium payoff correspondence

We will show that the SPE payoff correspondence has a surprising simplicity; the *set* of equilibrium payoffs at a level g is a simple linear translate of the set of equilibrium payoffs from some benchmark level, say, $g = 0$. Consequently, it will be seen that the *set* of emission levels that can arise in equilibrium from level g is identical to those that can arise from equilibrium play at a GHG level of 0. Note that the fact that the set of equilibrium possibilities is invariant to the level of g is perfectly consistent with the possibility that, in a particular equilibrium, emission levels vary with g. (Indeed we will see some of those equilibria in Section 7.) However, the invariance property will make for a particularly simple characterization of the best and worst equilibria.

Let $\Xi(g)$ denote the *set* of equilibrium payoff vectors with initial state g (i.e., each element of $\Xi(g)$ is the payoff to *some* SPE starting from g).

Theorem 5. *The equilibrium payoff correspondence Ξ is linear; there is a compact set $U \subset \Re^I$ such that for every initial state g*

$$\Xi(g) = U - \{w_1 g, w_2 g, \ldots, w_I g\}$$

where $w_i = c_i/(1 - \sigma\delta)$, $i = 1, \ldots, I$. In particular, consider any SPE, any period t and any history of play up until t. Then the payoff vector for the continuation strategies must necessarily be of the form

$$v - (w_1 g_t, w_2 g_t, \ldots, w_I g_t)$$

where $v \in U$ (and g_t is the state at period t).

The theorem is proved by way of a bootstrap argument. We presume that a (candidate) payoff set has this invariance and show that the linear structure of the model confirms the conjecture. Consequently, we generate another candidate payoff set that is also state-invariant. Then we look for a fixed point of that operator.[18]

[18] In other words, we employ a generalized version of the Abreu et al. (1990) operator to generate the SPE correspondence. We need to generalize the APS argument since that was formulated for repeated games alone.

6.2. Extreme equilibria

We will now use the result of the previous subsection to characterize the best (and the worst) equilibria in the global climate change game. Consider the *second-best problem* (from initial state g and for a given vector of welfare weights $x = (x_i; i = 1, \ldots, I)$), the problem of maximizing a weighted sum of *equilibrium payoffs*:

$$\max \sum_{i=1}^{I} x_i V_i(g),\; V(g) \in \Xi(g).$$

Note that we consider all possible equilibria, so, we consider equilibria that choose to condition on current and past GHG levels as well as equilibria that do not. The result states that the best equilibrium *need not* condition on GHG levels.

Theorem 6. *There exists a constant emission level $\bar{a} \equiv \bar{a}_1, \bar{a}_2, \ldots, \bar{a}_I$ such that no matter what the initial level of GHG, the second-best policy is to emit at the constant rate $\bar{a}$. In the event of a deviation from this constant emissions policy by country i, play proceeds to i′s worst equilibrium. Furthermore, the second-best emission rate is always strictly lower than the BAU rate* (i.e., $\bar{a} < a^*$). *Above a critical discount factor* (*less than* 1), *the second-best rate coincides with the GPO emission rate $\widehat{a}$.*

The theorem is attractive for three reasons. First, it says that the best possible equilibrium behavior is no more complicated than BAU behavior, so there is no argument for delaying a treaty (to cut emissions) merely because the status quo is simple. Second, the cut required to implement the second-best policy is an across-the-board cut; independently of anything else, country i should cut its emissions by the amount $a_i^* - \bar{a}_i$.[19] Third, the second-best is exactly realized at high discount factors rather than asymptotically approached as the discount factor tends to 1.

Sanctions will be required if countries break with the second-best policy,[20] and without loss of generality we can restrict attention to the worst such sanction. We turn now to a characterization of this worst equilibrium (for, say, country i). One definition will be useful for this purpose.

Definition 1. *An i-less second-best equilibrium is the solution to a second-best problem in which the welfare weight of i is set equal to zero* (i.e., $x_i = 0$).

By the previous theorem, every such problem has a solution in which on the equilibrium path, emissions are a constant. Denote that emission level $a(x_{-i})$.

Theorem 7. *There exists a "high" emission level $\bar{\bar{a}}(i)$(with$\sum_{j \neq i} \bar{\bar{a}}_j(i) > \sum_{j \neq i} a_j^*$) and an i-less second-best equilibrium $a(x_{-i})$ such that at the country i′s worst equilibrium*:

1. *Each country emits at rate $\bar{\bar{a}}_j(i)$ for one period* (*no matter what g is*), $j = 1, \ldots, I$.
2. *From the second period onwards, each country emits at the constant rate* $a_j(x_{-i})$, $j = 1, \ldots, I$.

If any country k deviates at either stages 1 *or* 2, *play switches to k′s worst equilibrium starting with the very next period after the deviation*.

Put another way, for every country i, a sanction is made up of two emission rates, $\bar{\bar{a}}(i)$ and $a(x_{-i})$. The former imposes immediate costs on country i. The way it does so is by increasing the emission levels of countries $j \neq i$. The effect of this is a temporary increase in incremental GHG, but due to the irreversibility of gas accumulation, a permanent increase in country *i′s* costs, enough of an increase to wipe out any immediate gains that the country might have obtained from the deviation. Of course this additional emission also increases country *j′s* costs. For the punishing countries, however, this increase is offset by the subsequent permanent change, the switch to the emission vector $a(x_{-i})$, which permanently increases their quota at the expense of country *i′s*.

The fact that there is a temporary loosening of environmental regulations as part of environmental sanctions is reminiscent of GATT rules where tariffs can be temporarily imposed by countries that seek to punish illegitimate trade practices on the part of others.

7. Greenhouse trap

In every equilibrium that we have studied so far (BAU, the third and second-best), each country emits at a constant rate regardless of GHG level. Hence, the dynamics of every such equilibrium is also simple; at a constant cumulative rate A, the stock of greenhouse gases (GHGs) grow to a steady-state of $(A/(1-\sigma))$. In particular, there is no such thing as starting "too

[19] Our model operates at the aggregative level alone and, in particular, we do not address the issue of how national governments will implement cuts that they agree to (in the national interest). However, it seems quite likely that an across-the-board cut will be easier to implement (and will be perceived to be fairer to all) than one which is sensitively tied to levels of GHG.

[20] A major criticism of the Kyoto accord is that it did not incorporate sanctions and hence would never be carried out. For details, see Barrett (2003, Chapter 15) and the further discussion in Section 8.

late" on emission control. As long as the current GHG level is less then the target steady-state, a successful effort to bring emissions down to A will guarantee that the world will not zoom past that steady-state. Put yet another way, the current GHG level has no long-term implication.

In this section we demonstrate the richness of the model by identifying some MPEs in which current GHG levels matter, MPEs in which there is a so-called "greenhouse trap". If the world starts below some critical level of GHG, say $\widetilde{g}$, then it grows no farther than that level. However if the system starts above $\widetilde{g}$ (or somehow crosses into the higher region) then greenhouse gases are trapped into growing and eventually grow to the BAU steady-state.

The key to these equilibria is a richer interaction between the (Markov) emission levels of country i and the rest of the world. In particular, we will consider emission policies that are Markovian but not constant, say an emission policy (vector) such as $a(g)$. Now country i has an incentive, everything else being equal, to emit in such a fashion that the global stock of GHG grows towards a region where $a_{-i}(.)$ are lower. Of course every country has such an incentive, so each country will wish to drive g_t towards a region where emission levels are low for the other countries. Hence, the conjecture for the group as a whole is that there will be MPEs with regions of "abnormally" low emissions, and every country will (a) have an incentive to stay in such regions once the system gets there and (b) have an incentive to participate in pushing the system towards such regions.

The conjecture is almost correct. We will show that there are indeed such equilibria (in fact there are many) each of which has such a "good" region of low emissions. Furthermore, from *most* (but not all) initial GHG levels outside this good region, countries will have an incentive to drive the system to the good region. The one additional complication is that in order for there to be a good region there must also be some "bad" region (of high emissions) from which countries do *not* find their way to the good region. The presence of a bad region, and the associated spectre of landing there, is what keeps the countries honest in the good region.[21]

To keep the exposition simple, we are going to present results only for the symmetric case.[22] Accordingly, when we speak of the Pareto optimal solution we will refer to the symmetric solution, and to avoid clutter we will simply denote that solution by $\widehat{a}$ (with associated steady state $\widehat{g}$). We shall present two results of increasing generality.

Consider the following symmetric Markovian strategy $a(.)$: if the GHG level is below the Pareto optimal steady state $\widehat{g}$, emissions take the game immediately to that state. On the other hand, if the GHG level exceeds $\widehat{g}$, then emissions are at the (high) BAU level of a^*. In other words,

$$\begin{aligned} a(g) &= \frac{\widehat{g} - \sigma g}{I}, \quad g \le \widehat{g} \\ &= a^*, \quad g > \widehat{g}. \end{aligned}$$

In the terminology of the immediately preceding discussion, the region below the Pareto optimal steady state $\widehat{g}$ is the "good" region of (relatively) low emission levels whereas the region above is the "bad" region of high emissions. Consider the following condition ("L" for "large"), which says that the BAU emission level is sufficiently larger than the GPO level.

Condition L:

$$\frac{a^*}{\widehat{a}} > \max\left(\frac{I}{I-1}, \frac{1}{1-\sigma}\right),$$

where (recall) I is the number of players and σ is the persistence of CO_2 in the earth's atmosphere.[23]

Theorem 8. *Suppose that Condition L holds. Then there is a cut-off value of the discount factor, say $\widehat{\delta}$, such that $a(.)$ is an MPE for all $\delta \ge \widehat{\delta}$. In such an equilibrium, the GHG level converges in one period to the Pareto optimal steady state $\widehat{g}$ if the initial level is below $\widehat{g}$ whereas it converges asymptotically to the BAU steady state g^* if the initial level is above $\widehat{g}$.*

The reader might wonder how useful this last theorem is if we suspect that the world is currently already past the Pareto optimal steady state. Are there MPE that have steady states less than the BAU steady state of g^* (but higher than the desired but unattainable steady state of $\widehat{g}$)?

We will now demonstrate the existence of an MPE that differs in three ways from the one above. First, in addition to the good and bad regions, there will be a "latent good" region, a region of low GHG levels from which the stock will gradually grow until it reaches the good region (whereupon it will stay there). Second, the good region (with emissions lower than the BAU level a^*) will extend beyond the Pareto optimal steady state $\widehat{g}$; in fact it will extend quite close to the BAU steady

[21] Readers familiar with repeated games will note the obvious connection with the idea of history-dependent punishments. The point to note though is that these are not history-dependent equilibria since they only depend on the GHG level and not on past emissions; hence they are more difficult to construct. They are also more sparing in terms of informational requirements since they do not require i to condition on (or even know) past emission levels of the other countries.

[22] The first of the two results is easily generalized to the asymmetric case.

[23] Since $(I/I-1)$ approaches 1 for I large, Condition L really boils down to $(a^*/\widehat{a}) \ge (1/1-\sigma)$ whenever we have a large number of players. This condition will always hold under standard asymptotic conditions on h. To see this, note that simple algebra shows that $(h'(\widehat{a})/h'(a^*)) = I$, and hence $(a^*/\widehat{a})$ is large whenever I is large.

state g^*. (So no matter where the world is today, however close to the worst possibility, we can still put the brakes on in an incentive-compatible way!) Third, we will considerably weaken Condition L.

Let $\widetilde{g}$ be any GHG level that is higher than $\widehat{g}$ but no more than $((I-1)/I)g^*$; $\widetilde{g}$ will be our candidate low steady state. We will consider Markov strategies in which stocks from a left neighborhood of $\widetilde{g}$ (to be defined shortly) come in one step to $\widetilde{g}$ and stocks above $\widetilde{g}$ exhibit BAU behavior:

$$\begin{aligned}\widetilde{a}(g) &= a^*, \quad g > \widetilde{g} \\ &= \frac{\widetilde{g} - \sigma g}{I}, \quad \widetilde{g}_1 \leq g \leq \widetilde{g}\end{aligned} \tag{15}$$

where

$$\widetilde{g}_1 \equiv \frac{I-1}{I}\widetilde{g}.$$

For stocks lower than $\widetilde{g}_1$, the "latent good" region, the emission levels will be such that GHG levels grow (gradually) until they get into the $[\widetilde{g}_1, \widetilde{g}]$ region. These emission levels cannot, however, be solved for in closed form. Instead we will employ a fixed point argument to show the following.

Theorem 9. *Suppose that*

$$\frac{a^*}{\widehat{a}} > \frac{I}{I-1}.$$

Then there is a cut-off value of the discount factor, say $\widetilde{\delta}$, such that for all $\delta \geq \widetilde{\delta}$, there is a MPE $\widetilde{a}(.)$ whose behavior above $\widetilde{g}_1$ is as given above. Below $\widetilde{g}_1$, the stock grows although it remains below $\widetilde{g}$, that is,

$$\sigma g + I\widetilde{a}(g) \in (g, \widetilde{g}), \text{for all} g < \widetilde{g}.$$

In such an equilibrium, the GHG level converges to the steady state $\widetilde{g}$ if the initial level is below it whereas it converges asymptotically to the BAU steady state g^* if the initial level is above $\widetilde{g}$.

8. Discussion and extensions

In terms of conclusions, our findings are similar in terms of welfare loss to those of other authors, including those who focus on the first-best issue. Nordhaus and Boyer calculate a United States loss from global climate change of the order of 0.5 percent of GDP (for doubling of pre-industrial levels); other estimates have been in the 1–2 percent range. This agrees with our findings of a decrease in global welfare (respectively, US welfare) in the range of 0.6–2 percent (respectively, 0.3–1 percent) for different discount rates with our benchmark damage cost coefficients. Where we differ is in the size of the emission cuts that our model predicts. Whereas Kyoto had called for cuts in the 10 percent range relative to 1990 levels and some of the literature has proposed that even that is too much to be globally optimal (see Nordhaus and Boyer), our numerical computations ask for much deeper emission cuts. We conjecture that the dynamic element of our model (that gases can persist in the atmosphere for a 100 years or more, which is in line with the known evidence) make the deeper cuts optimal. Furthermore, our results for both potential welfare increases and emission cuts are quite sensitive to increases in the damage cost coefficients.

The Kyoto Protocol appears not to be the basis of a self-enforcing treaty, principally because it does not build in any effective sanctions that would be applied if countries fail to meet their targets. This contrasts with GATT and its attendant institutional structure, the World Trade Organization (WTO). All that is said in the Kyoto protocol (in Article 18) is that procedures and mechanisms for compliance should be determined by the parties at their first meeting and should include "an indicative list of consequences". Subsequently, at the Hague November 2000 meeting, the most popular proposal (which came from the Dutch Environment Minister Jan Pronk) was that countries would face an escalating series of target reductions in the future if they failed to comply in the current stage. A watered-down version of this proposal was adopted in Bonn in March, 2001, after the United States had pulled out of the treaty, yet even this version had several problems; in principle, countries could postpone retribution indefinitely, and the base from which the enhanced reduction would be required would be worked out in the future, and so on (Barrett, 2003, Chapter 15). One contribution of this paper is to show that fairly simple sanctions (BAU reversion) can be quite effective, and even the most severe sanctions (the worst equilibria) are fairly simple.

A key feature of our model is that (1) the one-period payoff for each country is additively separable in GDP and the damage cost, and (2) the (incremental) damage cost is linear in the current stock of greenhouse gas. This property of the model allows us to get closed-form solutions for the Business as Usual and Pareto optimal solutions, characterize the equilibrium payoff correspondence, and investigate other special classes of equilibria. It also facilitates the calibration of the model, the numerical calculation of various trajectories, and sensitivity analyses. The disadvantage is that it results in a number of cases in unrealistic "bang-bang" strategies, that is, strategies in which the emission rates are constant after the first period. This aspect of the results needs to be taken "with a grain of salt." In a more realistic model, one would expect that these strategies would display a more gradual adaptation to the current levels of greenhous gas. Our conjecture is that the analysis of the affine model yields reasonable approximations to equilibrium and optimal trajectories, except in the very short run. However, precise tests of this conjecture will have to await future research.

In Dutta and Radner (2006) we generalized our current model to allow for population change and demonstrate qualitatively similar theoretical results. In Dutta and Radner (2004) we allowed for simple technological change and presented some theoretical and numerical results on the GPO and BAU solutions. In Dutta and Radner (in preparation) we incorporate capital accumulation (and technical change). The main question that we hope to address in that model is under what conditions does the prevention of global warming slow down the rate of economic growth. A second question that we hope to look at is: (how) does asymmetry in the current level of economic development affect sustainability of agreements about emission cuts. We have some preliminary results on the first question but not a complete solution. Finally, we hope to develop and analyze a "complete" model that incorporates all of the above features.

9. Bibliographic notes

The literature on (symmetric) dynamic commons games is exceedingly rich and goes back over 25 years. The earliest model was that of Levhari and Mirman (1980) who studied a particular functional representation of the neo-classical growth model with the novel twist that the capital stock could be "expropriated" by multiple players. Subsequently several authors (Sundaram, 1989; Sobel, 1990; Benhabib and Radner, 1992; Rustichini, 1992; Dutta and Sundaram, 1992, 1993; Sorger, 1998) studied this model in great generality and established several interesting properties relating to existence of equilibria, welfare consequences, and dynamic paths.[24]

Another variant of that model has been studied by Tornell and Velasco (1992) and, subsequently, Long and Sorger (2006). In that model, the public capital stock grows linearly at a fixed rate of growth. The expropriated capital need not be consumed immediately but can instead be made to grow privately at a rate lower that the public growth rate. The effect of such expropriation (or capital outflows) is examined by the various authors.

More recently in a series of papers by Dockner and his co-authors, the growth model has been directly applied to environmental problems including the problem of global warming. The paper closest to the current one is Dockner et al. (1996). It studies a model of global warming that has some broad similarities to the one we have studied here. In particular, the transition equation is identical in the two models. What is different is that they impose linearity in the emissions payoff function h (whereas we have assumed it to be strictly concave) while their cost to g is strictly convex (as opposed to ours which is linear).

The consequence of linearity in the benefit function h is that the GPO and BAU solutions have a "most rapid approach" property; if $(1-\sigma)g$, the depreciated stock in the next period, is less than a most preferred g^*, it is optimal to jump the system to g^*. Else it is optimal to wait for depreciation to bring the stock down to g^*. In other words, linearity in benefits implies a "one-shot" move to a desired level of gas g^*, which is thereafter maintained, while linearity in cost (as in our model) implies a constant emission rate.[25] The authors then show that a trigger strategy can realize the GPO for high enough discount factors. The paper also shows that there might be other MPE, and it gives an example of an MPE with complex dynamics. That MPE in this environmental model is identical, as the authors acknowledge, to a complex dynamics example first provided by Dutta and Sundaram (1993) in the context of the growth model.

Where the two papers truly differ in terms of analysis is that the Dockner at al. paper has a) no discussion of the effects of asymmetry or b) a full characterization of all subgame perfect equilibria. Additionally, that paper does not attempt to calibrate its model to generate illustrative numbers.[26]

A large volume of literature exists that directly focuses on the economics of climate change. A central question there is to determine the level of emissions that is globally optimal. An excellent example of this is Nordhaus and Boyer. Several of those papers, including the Nordhaus and Boyer paper, analyze only the "competitive" model, not taking strategic considerations fully into account.[27] A smaller volume of literature emphasizes the need for treaties to be self-enforcing, presenting a strategic analysis of the problem (see Barrett, 2003 and Finus, 2001). Where we depart from that literature is in the dynamic modelling; we allow GHGs to accumulate and stay in the environment for a (possibly long) period of time. By contrast the Barrett and Finus studies

[24] Some of these papers allow asymmetry; however, none of them analyzes the effect of asymmetries. One significant exception is the recent paper of Long and Sorger that explicitly considers asymmetry in appropriation costs within the Tornell and Velasco model. They show that increasing asymmetry lowers equilibrium rate of growth, a result broadly consistent with the one shown in this paper that asymmetry is deletrious.

[25] What is unclear in their model is why the multiple players would have the same target steady-state g^*. It would appear natural that, with asymmetric payoffs, each player would have a different steady-state. The existence of a MRAP equilibrium would consequently appear problematical. The authors impose a condition that implies that there is not too much asymmetry.

[26] There are is one other interesting and relevant paper (Dockner and Nishimura, 1999) that focuses on complex dynamics example and shows that it emerges in a couple of related models as well (for example, a model in which every country's pollution affects one immediate neighbor).

[27] To be fair, Nordhaus and Boyer (2000) and Nordhaus and Yang (1996) do consider strategic models but restrict themselves to open-loop strategies.

restrict themselves to purely repeated games, which implies that the state variable, gas stock, remains constant over time.

10. Proofs

10.1. Global pareto optima and BAU equilibrium

Proof of Theorem 1. We shall show by dynamic programming arguments that the Pareto-optimal value function is of the form $\widehat{V} = \sum_{i=1}^{I} x_i[\widehat{u}_i - w_i g]$. We need to be able to find the constants $\widehat{u}_i$ to satisfy

$$\sum_{i=1}^{I} x_i[\widehat{u}_i - w_i g] = Max_{a_1,\ldots,a_I} \sum_{i=1}^{I} x_i \left[h_i(a_i) - c_i g + \delta \left(\widehat{u}_i - w_i \left(\sigma g + \sum_{j=1}^{I} a_j \right) \right) \right]. \tag{16}$$

Collecting terms that need maximization we can reduce the equation above to

$$\sum_{i=1}^{I} x_i \widehat{u}_i = Max_{a_1,\ldots,a_I} \sum_{i=1}^{I} x_i [h_i(a_i) - \delta w_i \sum_{j=1}^{I} a_j] + \delta \sum_{i=1}^{I} x_i \widehat{u}_i. \tag{17}$$

It is clear that the solution to this system is the same for all g; call this (first-best) solution $\widehat{a}_i$. Elementary algebra reveals that

$$\widehat{u}_i = \frac{h_i(\widehat{a}_i) - \delta w_i \sum_{j=1}^{I} \widehat{a}_j}{1-\delta} \quad \text{and} \quad w_i = \frac{c_i}{1-\delta\sigma}.$$

It is also obvious that $x_i h_i'(\hat{a}_i) = \delta w$, where $w = \sum_{i=1}^{I} x_i w_i$. □

Proof of Theorem 2. Consider any constant emission strategy for players other than i; call these constants $\bar{a}_j$. We shall now show by dynamic programming arguments that the best response value function is of the form $\bar{u}_i - w_i g$, where

$$\bar{u}_i = \frac{h_i(a_i^*) - \delta w_i (a_i^* + \sum_{j \neq i} \bar{a}_j)}{1-\delta}.$$

We therefore need to show that

$$\bar{u}_i - w_i g = Max_{a_i} \left[h_i(a_i) - c_i g + \delta(\bar{u}_i - w_i) \left(\sigma g + a_i + \sum_{j \neq i} \bar{a}_j \right) \right]. \tag{18}$$

Collecting the terms that need maximization we can reduce the above equation to

$$\bar{u}_i = Max_{a_i}[h(a_i) - \delta w_i a_i] + \delta \bar{u}_i. \tag{19}$$

It is clear that the solution to this system is the same for all g and equals a_i^* where $h_i'(a_i^*) = \delta w_i$. □

10.2. Equilibria sustained by BAU sanctions

Proof of Proposition 1. The proof follows the lines of that for the analogous result on the second-best along the lines of the proof of Theorems 6 and 7(see below). Evidently, equilibria that are sustained by the threat of BAU reversion are SPE, and hence their values belong to the SPE payoff correspondence that we identify in Theorem 6- $U - \{w_1 g, w_2 g, \ldots, w_I g\}$ where U is a compact subset of $\Re^I$. Define therefore the set of incentive-compatible emission vectors a (associated with the threat of BAU reversion) as

$$A = \{a \in \Re_+^I : h_i(a_i) + \delta(u_i' - w_i a_i) \geq h(\widetilde{a}_i) + \delta(u_i^* - w_i \widetilde{a}_i), \forall \widetilde{a}_i, i, \text{for some} u' \in U\} \tag{20}$$

where

$$u_i^* = \frac{h_i(a_i^*) - \delta w_i \sum_{j=1}^{I} a_j^*}{1-\delta}$$

is the BAU equilibrium payoff.

The third-best equilibrium is therefore found by solving the following problem:

$$Max_{a_i,u_i}\sum_i x_i v_i = Max_{a_i,u_i}\sum_{i=1}^{I} x_i\left[h_i(a_i)+\delta\left[u_i - w_i\sum_{k=1}^{I}a_k\right]\right],$$

subject to the incentive compatibility constraint Eq. (20). Since the constraint is independent of g, and so is the objective function, it follows that the solution is as well. The proposition is proved. □

Proof of Proposition 2. The only part left to prove is part (b). Note that as $\delta \uparrow 1$, the incentive constraint, Eq. (20), becomes

$$h_i(\widetilde{a}_i) - w_i(1)\left(\widetilde{a}_i + \sum_{j\neq i}\widetilde{a}_j\right) \geq h_i(a_i^*) - w_i(1)\left(a_i^* + \sum_{j\neq i}a_j^*\right), \forall i, \tag{21}$$

where

$$w_i(1) = \lim_{\delta\uparrow 1} w_i(\delta) = \lim_{\delta\uparrow 1}\frac{c_i}{1-\sigma\delta} = \frac{c_i}{1-\sigma}.$$

The expression on the LHS, $h_i(\widetilde{a}_i) - w_i(1)\sum_j \widetilde{a}_j$, is precisely the long-run average (LRA) value from a policy that emits at the constant rate $\widetilde{a}$. In particular then, if we pick the candidate $\widetilde{a}$ to be the GPO emission under the LRA criterion with equal weights for all countries, the incentive constraint Eq. (21) is satisfied by the definition of GPO, at $\delta = 1$. Add to that (a) that the LRA GPO is the limit of discounted average GPO values (see Dutta, 1991) and (b) the assumption that the equal weight GPO solution Pareto-dominates the BAU value for all high discount factors. The proof is complete. □

Proof of Proposition 3. We shall first prove part (b). Start then at the BAU solution a^* and consider a reduction of every country's emission by an amount ε. We shall show that the emission vector $a^* - \varepsilon$ is sustainable by the BAU sanction for appropriately small but positive ε. Note that the LHS of the incentive constraint, Eq. (20), becomes

$$h_i(a_i^* - \varepsilon) - \delta w_i\left(a_i^* - \varepsilon + \delta\sum_{j\neq i}(a_j^* - \varepsilon)\right),$$

and that can be rewritten as

$$h_i(a_i^*) - \delta w_i\left(a_i^* + \delta\sum_{j\neq i}(a_j^* - \varepsilon)\right) - [h_i'(z_i^*) - \delta w_i]\varepsilon$$

for some $z_i^* \in (a_i^* - \varepsilon, a_i^*)$. That in turn equals

$$h_i(a_i^*) - \delta w_i\left(a_i^* + \delta\sum_{j\neq i}a_j^*\right) + \{\delta^2 w_i(I-1) - [h_i'(z_i^*) - \delta w_i]\}\varepsilon$$

As $\varepsilon \downarrow 0$, $z_i^* \to a_i^*$. Since $h_i'(a_i^*) - \delta w_i = 0$, it follows that $\delta^2 w_i(I-1) - [h_i'(z_i^*) - \delta w_i] > 0$ for small ε. In particular, the incentive constraint

$$h_i(a_i^* - \varepsilon) - \delta w_i\left(a_i^* - \varepsilon + \delta\sum_{j\neq i}(a_j^* - \varepsilon)\right) \geq h_i(a_i^*) - \delta w_i\left(a_i^* + \delta\sum_{j\neq i}a_j^*\right), \forall i. \tag{22}$$

holds. The proof of part (b) is complete.

At the minimum incentive-compatible emission level, every IC constraint must hold with equality. Consider instead the alternative case; suppose the constraint is a strict inequality for country *j*. In that case we can further reduce that country's emission without violating its constraint. However lowering *j's* emission only helps satisfy every other country's constraint. Hence, we will have lowered total emissions without violating incentive constraints, which is a contradiction.

Rewriting the incentive constraints as equalities yields

$$h_i(a_i) - \delta w_i\left(a_i + \delta\sum_{j\neq i}a_j\right) = \Psi_i, \forall i$$

where $\Psi_i = h_i(a_i^*) - \delta w_i(a_i^* + \delta\sum_{j\neq i} a_j^*)$. Let us suppose that $\underline{a}$ is a solution to the emission minimization problem and let $\underline{A} = \sum_i \underline{a}_i$. Rewriting the immediately preceding equation we have

$$h_i(\underline{a}_i) - (1-\delta)w_i\underline{a}_i = \Psi_i + \delta^2 w_i \underline{A}.$$

Given the strict concavity of h, it is evident that $\underline{a}_i$ is uniquely defined by the above equation. The proof is complete. □

10.3. Asymmetry

Proposition 1 above greatly simplifies the incentive analysis for the third-best problem since we can now restrict ourselves to constant emissions policies and the consequent incentive constraint:

$$h_i(a_i) - \delta w_i(a_i + \delta a_j) \geq h_i(a_i^*) - \delta w_i(a_i^* + \delta a_j^*),\ i, \quad \text{and for } j \neq i. \tag{23}$$

Proof of Theorem 3. Consider an increase in w_1 (to w_1') and a simultaneous decrease in w_2. Denote $S_i(a)$ as the incentive slack of country i; therefore,

$$S_i(a) = h_i(a_i) - \delta w_i(a_i + \delta a_j) - [h_i(a_i^*) - \delta w_i(a_i^* + \delta a_j^*)].$$

Similarly, denote the incentive slack under the new costs S_i'. Evidently, for country 1, the difference in the two slacks, $S_1'(a) - S_1(a)$, equals

$$\{h_1(a_1) - \delta w_1'(a_1 + \delta a_2) - [h_1(a_1) - \delta w_1(a_1 + \delta a_2)]\} - \{h_1(a_1'*) - \delta w_1'(a_1' * + \delta a_2' *) - [h_1(a_1^*) - \delta w_1(a_1^* + \delta a_2^*)]\}$$

where $a'*$ is the BAU emission level for costs w'. Call the term in the first (curly) bracket, $\{\}_1$ and the second one $\{\}_2$. Simple algebra yields

$$\{\}_1 = \delta(w_1 - w_1')[a_1 + \delta a_2]. \tag{24}$$

Note that

$$h_1(a_1'*) - \delta w_1(a_1'*) < h_1(a_1^*) - \delta w_1(a_1^*),$$

by the definition of a_1^*. Furthermore, $a_2'* > a_2^*$ since country 2's costs have gone down. Combining those two facts we have that

$$\begin{aligned}\{\}_2 &< h_1(a_1'*) - \delta w_1'(a_1' * + \delta a_2'*) - [h_1(a_1'*) - \delta w_1(a_1' * + \delta a_2'*)],\\ &= \delta(w_1 - w_1')[a_1' * + \delta a_2'*].\end{aligned} \tag{25}$$

Since $a_1' * + \delta a_2'* \geq a_1 + \delta a_2$, it further follows $\{\}_2 < \{\}_1$; therefore, $S_1'(a) - S_1(a) = \{\}_1 - \{\}_2 > 0$.

To prove the result for country 2, we mimic the above argument except in reverse. The theorem is proved. □

Proof of Theorem 4. Consider starting from the symmetric situation where $c_1 = c_2$ or, equivalently, the symmetric situation where the lifetime costs are equal:

$$w_1 = \frac{c_1}{1-\delta\sigma} = \frac{c_2}{1-\delta\sigma} = w_2.$$

Now consider varying the costs in a manner that the costs of country 1 "South" are increased while those of country 2 "North" are decreased. Let us measure the asymmetry in costs by the difference $d = c_1 - c_2$ or, equivalently, the difference in lifetime costs:

$$\frac{d}{1-\delta\sigma} = \theta = w_1 - w_2.$$

In other words, we shall consider an increase in asymmetry as exemplified by an increase in d (or θ). Furthermore, to ensure that the only effect is that of an increase in asymmetry, we shall hold constant the sum of the two costs $w_1 + w_2$.

By hypothesis, the GPO is third-best when $w_1 = w_2$.

Even when the cost difference becomes positive, the GPO continues to be incentive-compatible for both countries as long as d is small. However, beyond a cut-off value, it is no longer incentive-compatible for the low cost country 2 "North" (see Theorem 3). At that point, in the third-best solution only one of the incentive constraints is binding. Furthermore, and again from Theorem 3 above, that has to be the incentive constraint for country 2 (the lower-cost country).

Consider two different θ^1 and θ^2, say $\theta^1 > \theta^2$ and let $a^1 = (a_1^1, a_2^1)$ and $a^2 = (a_1^2, a_2^2)$ be the corresponding third-best emission vectors. Since the incentive constraint for country 1 "South" is slack, it follows that if θ^1 is chosen sufficiently close to θ^2, then a_1^1 is also incentive compatible at θ^2. By Theorem 3 above, country 2's incentive region shrinks as its costs decrease (i.e., its emission under θ^1, a_2^1, must also have been incentive compatible at θ^2). Putting the two together, the emission vector a^1 is incentive compatible at θ^2, but the choice made for that parameter is a^2. That implies that total welfare is higher at a^2 than at a^1 at the parameter θ^2, but since the total welfare function is unchanged as the cost difference changes (since $w_1 + w_2$ is constant), it follows that total welfare is lower at θ^1 compared to θ^2. □

10.4. All equilibria

Proof of Theorem 5. We shall employ the generalization of the Dynamic Programming Bellman equation that was introduced by Abreu et al. to study the set of SPE in repeated games. Let us start with the conjecture that the correspondence of SPE payoffs, V, has the structure claimed in the result (i.e., is of the form $U - \{w_1 g, w_2 g, \ldots, w_I g\}$ where U is a compact subset of $\Re^I$). Let $\underline{u}^i$ denote the vector in U that gives country i the worst payoff,

$$\underline{u}^i = \min\{u_i | u \in U\},$$

and denote i's payoff in that minimum as $\underline{u}^i_i$. Define the set of incentive-compatible emission vectors a (associated with this candidate equilibrium payoff set) as

$$A = \{a \in \Re^I_+ : h_i(a_i) + \delta(u'_i - w_i a_i) \geq h(\widetilde{a}_i) + \delta(\underline{u}^i_i - w_i \widetilde{a}_i), \forall \widetilde{a}_i, i, \text{for some} u' \in U\}. \tag{26}$$

Now define the APS operator BV (that takes the correspondence V into another correspondence) as follows:

$$BV = \Big\{v(.) : \exists \text{selections} a(.) \text{and} u(.,.) \text{satisfying} a(g) \in A, \text{and} u(g', g) \in U, \forall g', g.$$
$$v_i(g) = h_i(a_i) - c_i g + \delta\left(u_i\left(\sigma g + \sum_{j=1}^{I} a_j(g), g\right) - w_i\left(\sigma g + \sum_{j=1}^{I} a_j(g)\right)\right)\Big\}.$$

Note that $v(.)$ can be rewritten as

$$v_i(g) = \left[h_i(a_i) + \delta\left(u_i\left(\sigma g + \sum_{j=1}^{I} a_j(g), g\right) - w_i \sum_{j=1}^{I} a_j(g)\right)\right] - w_i g$$

Note from the incentive set A that exactly the same set of actions satisfy the incentive constraint at every value of g. By hypothesis the "continuation payoffs" $u(g', g) \in U$, a set that is independent of g as well. It follows that BV is a correspondence of the same form as V (i.e., $BV = U' - \{w_1 g, w_2 g, \ldots, w_I g\}$ for some U' that is a subset of $\Re^I$). Standard arguments using the Maximum theorem can be employed to show that U' must be a compact set as well.

Start then with a set U_0 (or correspondence V_0) that is compact and "large" (one that contains all feasible payoffs in the game). Define U_1 via the APS operator as above; U_1 is compact as well. It must also be non-empty because it must contain the BAU payoff u^*. By definition, $U_1 \subset U_0$. Define U_n recursively in this fashion. It is easy to check that the operator is monotone, and hence U_n is a decreasing sequence of non-empty compact sets. Associated with each is the correspondence $V_n = U_n - \{w_1 g, w_2 g, \ldots, w_I g\}$. Let U be the intersection of these U_n sets. APS type arguments then show that $BV = V$ and that V contains all SPE payoffs. The theorem is proved. □

Proof of Theorem 6. Given the previous theorem, the second-best equilibrium is found by solving the following problem:

$$Max_{a, u(.,g) \in U} \sum_{i=1}^{I} x_i \left[h_i(a_i) + \delta\left[u_i\left(\sigma g + \sum_{k=1}^{I} a_k, g\right) - w_i \sum_{k=1}^{I} a_k\right]\right]$$

where, for every g, $u(\sigma g + \sum_{k=1}^{I} a_k, g) \in U$, and the emissions have to satisfy the incentive compatibility constraint

$$h_j(a_j) + \delta\left(u_j\left(\sigma g + \sum_{k=1}^{I} a_j, g\right) - w_j a_j\right) \geq h_j(\widetilde{a}_j) + \delta(\underline{u}^j_j - w_j \widetilde{a}_j), \forall \widetilde{a}_j, j.$$

Since the constraint is independent of g, as is the objective function, it follows that the solution is as well. □

Proof of Theorem 7. In light of our previous results, the worst equilibrium for player i, if the state is g, is a solution to the following problem:

$$Min_{a, u(.,g) \in U} h_i(a_i) + \delta\left[u_i\left(\sigma g + \sum_{k=1}^{I} a_k, g\right) - w_i \sum_{k=1}^{I} a_k\right] \tag{27}$$

$$s.t. h_j(a_j) + \delta\left(u_j\left(\sigma g + \sum_{k=1}^{I} a_j, g\right) - w_j a_j\right) \geq h_j(\widetilde{a}_j) + \delta(\underline{u}^j_j - w_j \widetilde{a}_j), \forall \widetilde{a}_j, j. \tag{28}$$

Since $u(.,g) \in U$ (a state independent set) it follows that the solution to the problem is going to be state independent as well. Furthermore, by arguments that we have seen before, it is clear that if player j is going to deviate (and expects the worst equilibrium for her to be the continuation regardless of how much she deviated by), then her optimal deviation is a_j^*. In other words, Eqs. (27) and (28) can be simplified to yield the following:

$$\underline{u}_i = Min_{a,\widetilde{u} \in U} h_i(a_i) + \delta\left[\widetilde{u}_i - w_i \sum_{k=1}^{I} a_k\right] \tag{29}$$

$$s.t. h_j(a_j) + \delta(\widetilde{u}_j - w_j a_j) \geq h_j(a_j^*) + \delta(\underline{u}_j^j - w_j a_j^*), \forall j. \tag{30}$$

Note that it must be the case that the incentive constraint for player i, whose worst equilibrium we are analyzing, is binding in the above problem. If not, we can perturb a_i in such a manner (without changing $a_j, j \neq i$) such that $h_i(a_i) - \delta w_i a_i$ is lowered *and* player *i's* incentive constraint continues to be satisfied. Since the other's actions do not change, their incentives are untouched and we lower the value of the minimand in Eq. (29). In light of that observation, we can rewrite the problem as

$$Max_{a,\widetilde{u} \in U} \sum_{k \neq i} a_k$$

$$s.t. h_j(a_j) + \delta(\widetilde{u}_j - w_j a_j) \geq h_j(a_j^*) + \delta(\underline{u}_j^j - w_j a_j^*), \forall j \tag{31}$$

It immediately follows that in the solution, $\bar{\bar{a}}(i)$, $\sum_{j \neq i} \bar{\bar{a}}_j(i) \geq \sum_{j \neq i} a_j^*$. We know however that there are equilibrium payoffs that are strictly higher than the BAU payoffs (for example, the third-best payoffs are strictly higher). Hence from the incentive constraints it follows that there are emission vectors for the other players that are incentive compatible and whose sum is strictly higher than the BAU emissions.

A little reflection also shows that the optimization problem of Eq. (31) must be equivalent to some $i - less$ second problem. After all the higher is the equilibrium payoff $\widetilde{u}_j$, the easier it is to get country j to emit a high amount this period (in order to punish i). From the result for second-best problems, it then follows that from the second period onwards, the emission must be at some constant level $a(x_{-i})$. The theorem is proved. □

10.5. Greenhouse trap

Proof of Theorem 8. Suppose that the initial GHG level g_0 is above the Pareto optimal steady state $\widehat{g}$. In that case, country i faces a best response problem that is identical to the one it would face if the rest of the world emitted a^* at *all* levels of GHG. Note that even if i drops her emission to zero, the GHG level would still remain above $\widehat{g}$; after all, $\sigma g + (I-1)a^* \geq \sigma g + I\widehat{a} > \widehat{g}$. (Recall Condition L: $(I-1)a^* \geq I\widehat{a}$ and that $(\sigma\widehat{g} + I\widehat{a} = \widehat{g})$. It follows therefore that, if $g > \widehat{g}$, the best response for i is to play the BAU level a^*.

Suppose instead that $g \leq \widehat{g}$. There are two possibilities for *i's* emission: either the emission level is chosen to be so high that the next period's GHG stock is above $\widehat{g}$, or it is not. Consider the two cases separately.

Case 1 Emissions such that $g_{t+1} > \widehat{g}$, that is, $a_i > a(g_t)$.

In this case, the lifetime payoffs are $h(a_i) - cg_t + \delta[u^* - wg_{t+1}]$, where u^* is the BAU payoff, $u^* = (h(a^*) - \delta wIa^*)/1 - \delta$. Substituting for the relevant expressions, the optimal deviation in this case is then determined by

$$Max_{a_i > a(g)} h(a_i) - cg + \delta\left[\frac{h(a^*) - \delta wIa^*}{1-\delta} - w(\sigma g + (I-1)a(g) + a_i)\right].$$

Collecting only the terms that involve a_i, the above expression immediately simplifies to

$$Max_{a_i > a(g)} h(a_i) - \delta w a_i,$$

and its solution is, in fact, the BAU emission level a^*.[28] Hence, the best deviation payoff in this case reduces to

$$u^* - wg + \delta w(I-1)(a^* - a(g)).$$

Case 2 Emissions such that $g_{t+1} \leq \widehat{g}$, that is, $a_i \leq a(g_t)$.

It will be more convenient to think of emission levels in this case as $a(g) - \epsilon$, $\epsilon > 0$. Since we only need to consider one-shot deviations, and because lowering emissions unilaterally will simply delay by one period the convergence to the steady state $\widehat{g}$, it follows that the optimal deviation can be derived from the following exercise:

$$Max_{\epsilon \geq 0} h(a(g) - \epsilon) + \delta\left[h\left(\widehat{a} + \frac{\sigma\epsilon}{I}\right) + c\epsilon\right]$$

[28] Another way of saying what we just did is that *if* country i is going to push the game into the BAU region, then she might as well take the unilaterally optimal action a^* to do so.

where we have used the following facts: (i) an ϵ cut-back by country i leaves the state of the system at $\widehat{g}-\epsilon$ in the next period, and (ii) that requires an additional $(\sigma\epsilon/I)$ emission by each country to bring the GHG level two periods hence up to the Pareto optimal steady state of $\widehat{g}$. It is straightforward to verify that the maximand above is concave. A sufficient condition for the maximizer to be $\epsilon = 0$ (which is what we want to show) is that the derivative at that point be non-positive, or,

$$\frac{\sigma\delta}{I}h'(\widehat{a}) \le h'(a(g)) - \delta c.$$

By the second part of Condition L, $a^* \ge a(g)$, and hence by the concavity of h, a sufficient condition for the last inequality is

$$\frac{\sigma\delta}{I}h'(\widehat{a}) \le h'(a^*) - \delta c.$$

Note that $h'(\widehat{a}) = (\delta Ic/1-\sigma\delta)$ and that $h'(a^*) = (\delta c/1-\sigma\delta)$. After making the appropriate substitutions, it is easy to see that the two sides of the inequality are actually equal to each other (and to $\delta c(\sigma\delta/1-\sigma\delta)$). Hence, the optimal deviation is not to deviate.

Combining the two cases we have the following: $a(g)$ is a best response (to $a(g)$) provided it yields higher lifetime payoffs than a^*. Note that the lifetime payoffs to $a(g)$ are

$$h[a(g)] - wg - \delta wIa(g) + \delta\widehat{u}.$$

Elementary algebra says that the lifetime payoffs to $a(g)$ are higher if and only if

$$h[a(g)] - \delta wIa(g) + \delta\widehat{u} \ge h(a^*) - \delta wIa^* + \delta u^* + \delta w(I-1)(a^* - a(g)).$$

Since $h(a) - \delta wIa$ is a concave function that is maximized at $\widehat{a}$ and since $a(g) \in [\widehat{a}, a^*]$, it follows that $h[a(g)] - \delta wIa(g) \ge h(a^*) - \delta wIa^*$. Hence a sufficient condition for the last inequality to hold is that

$$\widehat{u} \ge u^* + \delta w(I-1)(a^* - a(g)).$$

In turn the second term in the right-hand side is strictly smaller than $\delta wI(a^* - \widehat{a})$, so a sufficient condition for the inequality is

$$\widehat{u} \ge u^* + \delta wI(a^* - \widehat{a}).$$

From the definitions, it follows that the condition is equivalent to

$$h(\widehat{a}) - \delta wI\widehat{a} \ge h(a^*) - \delta wIa^* + (1-\delta)\delta wI(a^* - \widehat{a}).$$

As $\delta \uparrow 1$, the inequality converges to

$$h(\widehat{a}) - w(1)I\widehat{a} \ge h(a^*) - w(1)Ia^*,$$

where the present undiscounted cost is $w(1) = c/(1-\sigma)$, and $\widehat{a}$ and a^* are respectively the solutions to $Max h(a) - \delta Iw(1)a$ and Max $h(a) - \delta w(1)a$ (and hence $\widehat{a} \ne a^*$). By definition, the last inequality is actually a strict inequality. It follows therefore that the sufficient condition for $a(g)$ to be a (strictly) better response than a^* holds for all δ appropriately close to 1. □

Proof of Theorem 9. The proof may be found in Dutta and Radner (2007). □

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