In Defense of Leveraged and Inverse Funds *

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Abstract

This paper corrects several critical misunderstandings about leveraged and inverse ETFs. The key misconception, promoted by many academics, practitioners, regulators, and financial journalists, is that returns on these funds suffer from "volatility decay" – that the daily rebalancing inherent in these funds' strategies causes an erosion of value over long holding periods, especially when volatility is high. We show empirically that this is false. Daily rebalancing produces convex payoffs that differ from, but do not underperform, linear benchmarks. Regulators must correct their misconceptions so that ETF issuers can appropriately market these funds to investors who can then use them optimally.

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1 Introduction

Leveraged and inverse funds, i.e., funds that promise to deliver a positive or negative multiple of the daily return on an underlying asset, are a popular structured product in the ETF space. At the beginning of 2026, there were more than 260 of these funds, with total assets exceeding $150 billion.[1] However, in spite of their popularity with investors, they have come under significant criticism from academics, practitioners, regulators, and financial journalists.[2] This criticism takes various forms, but centers around the idea that these ETFs will underperform the promised multiple of the underlying return for holding periods of longer than one day, due to the effects of volatility, and thus they are not suitable as long-term holdings. This idea that the daily rebalance inherent in these funds causes an erosion of their value over time has come to be known as “volatility decay" or “volatility drag." Influential early papers that introduce these ideas include Cheng and Madhavan (2009) and Avellaneda and Zhang (2010). The Financial Industry Regulatory Authority (2009) issued a regulatory notice emphasizing the need for ETF issuers to flag these issues in their marketing materials and the Securities and Exchange Commission (2023) also raises these concerns. Given this regulatory scrutiny, it is hardly surprising that ETF issuers echo these same concerns (see, for example, ProShares (2026)).

Unfortunately, this criticism of leveraged and inverse ETFs is misguided and appears to stem from a severe misunderstanding about the returns that they generate. Specifically, the fundamental misconception is that these funds suffer from an erosion in value due to volatility. The existence of this erosion in value is a myth. The daily rebalance does change the distribution of the returns on these funds relative to a strategy that does not rebalance, and the level of the volatility of the underlying asset does affect the magnitude of this rebalance effect, but there is no inherent loss in value. The rebalancing is simply a pre-specified, dynamic trading strategy, which can never destroy value in an efficient market.[3]

In fact, the rebalancing strategy of leveraged and inverse ETFs is similar to the rebalancing strategy that dynamically replicates options. For example, for a leveraged fund, when the price of the underlying asset goes up during the day, the value of the fund goes up, and the amount of inherent leverage falls below the level promised by the fund. Thus, to restore the fund’s leverage to the promised daily multiple for the following day, the fund must borrow more money and buy more of the underlying asset. This is exactly the same direction of rebalancing as in the dynamic replication of a call option. When the underlying asset goes up, the call-replicating strategy borrows more money and acquires more of the underlying asset. Similarly, the dynamic strategy of an inverse fund resembles the dynamic replication of a put option. Consequently, it turns out that these daily rebalanced funds have cumulative returns over

[1] See https://etfdb.com/ for a current listing of these funds and their characteristics.

[2] Representative newspaper articles include Harris, Benjamin (2019) and Zweig (2020). From a practitioner perspective, a simple Google search of relevant terms yields too many webpages and articles to cite here, but Investopedia (Gratton (2026)) and ETF.com (ETF.com Editorial Staff (2026)) are two prominent examples.

[3] Two recent working papers, Bessembinder (2025) and Wang (2025) make this same point, although the focus of their analyses is different from that of this paper.

horizons longer than a day that are convex in the multi-day returns on the underlying asset, just like those of an option. However, no one would reasonably argue that this convexity is a source of value destruction in either the option or the associated dynamic replication of this option.

A simple, 2-day example illustrates the direction and magnitude of the effect of the daily rebalancing and may explain why the "volatility decay" myth came into existence in the first place. Before working through the example, note that leveraged funds should, absent fees and expenses, generate returns in excess of the risk-free rate that are the promised multiple of the excess returns on the underlying asset each day, under the assumption that leverage is funded through borrowing at the risk-free rate. Similarly, inverse funds produce a daily excess return that is the promised multiple of the underlying excess return under the assumption that they invest the proceeds from shorting the underlying asset at this same risk-free rate.[4] To achieve the promised leverage multiple of the underlying excess return each day, these ETFs must rebalance their positions at the end of each day, because the return on the underlying over the day alters the inherent leverage multiple of the leveraged or inverse position, which then needs to be reset.

To build intuition for the effect of this daily rebalancing, Table 1 compares the 2-day returns of levered and inverse funds under two alternative rebalancing strategies. The "no-rebalance" strategy starts with the promised leverage multiple at the beginning of day 1, but then never rebalances even though the leverage multiple changes after the first day. The "daily-rebalance" strategy starts with the promised leverage multiple at the beginning of day 1 and then trades to reset the leverage multiple back to the promised level at the end of day 1. The underlying index has an equal probability each day of generating an excess return of 10% or -10%. This example assumes a very high volatility in order to highlight the rebalancing effect. Over two days, there are four equally likely outcomes, corresponding to the four rows of the table. The first two columns show the underlying index returns on each of the two days. The remaining five columns show the cumulative 2-day excess returns on three different funds: a fund that replicates the underlying index, a leveraged fund that generates a 3x return, and an inverse fund that generates a -3x return.[5] The 3x fund leverages initially to a long position in the underlying equal to 3 times the initial capital, while the -3x fund takes a short position of 3 times the initial capital. The 3x fund and the -3x fund are assumed to borrow and lend, respectively, at a daily risk-free rate of zero. For the 3x and -3x funds, the table lists the 2-day returns under the two different strategies: the no-rebalance strategy and the daily-rebalance strategy that rebalances to the promised leverage multiple at the end of day 1. For the underlying index fund, the 2-day return is just the cumulative return on the index because its leverage multiple stays at 1 and does not require rebalancing. The 3x fund with no rebalance generates 3 times the cumulative excess return on the index. The 3x fund with a daily rebalance generates 3 times the daily excess return each day, either 30% or -30%, and then produces the cumulative 2-day return accordingly. Similarly, the -3x fund with no rebalance generates -3 times the cumulative excess return on the index, and the -3x fund with a daily rebalance generates -3 times the daily excess return each day, either -30% or 30%, and produces the

[4] In practice, leveraged and inverse ETFs generally use total return swaps to generate the promised return, but these swaps are functionally equivalent to borrowing to generate leverage or shorting to generate inverse payoffs.

[5] In practice, 3x and -3x are the most extreme multiples available in the ETF market.

associated cumulative 2-day return.

The effect of rebalancing is clear. If the underlying returns reverse, i.e., the sequence is either 10%, -10% or -10%, 10%, then the daily-rebalance fund underperforms the no-rebalance fund. It is easy to see that the same relative performance applies regardless of the ordering of returns. If the returns continue, i.e., the sequence is either 10%, 10% or -10%, -10%, then the daily-rebalance fund outperforms the no-rebalance fund, and again, that is true for both the 3x fund and the -3x fund. The point is that the daily-rebalance funds do not uniformly underperform the no-rebalance funds, rather, the return structures are just different.

Why do the daily-rebalance funds underperform the no-rebalance funds under reversals, and outperform under continuations? The underlying mechanism is simple. For example, for the 3x fund, if the first day's excess return is positive, the fund levers up. This rebalance is ex post good if the following day's return is positive and bad if it is negative. This outperformance of the daily-rebalance funds after return continuations, and their underperformance after reversals, causes the daily-rebalanced fund's excess return to be a convex function of the 2-day excess return on the underlying index. To illustrate this, Figure 1 plots the cumulative 2-day returns of the leveraged and inverse funds under the no-rebalance and daily-rebalance strategies as functions of the cumulative 2-day return of the underlying index. The figure shows that the 2-day returns of the funds with no rebalance are linear functions of the 2-day return of the underlying index, while the 2-day returns of the funds with daily rebalance are convex functions of the 2-day return of the underlying index.

Those criticizing leveraged and inverse funds tend to focus on the case of reversals, which by itself leads to a biased view of the rebalancing effect. However, even taking all four cases into consideration, it is possible to misinterpret the evidence and draw the wrong conclusion about volatility effects. In the example, the volatility of underlying returns each day is identical, and all four cases are drawn from the same underlying distribution with the same ex ante volatility. Moreover, the realized volatility of the underlying is also identical across all four cases, if the volatility is appropriately calculated using the true expected daily excess return of 0%. The mistake would be to compute the realized volatility over the 2-day sample using the realized 2-day average return. Using this latter, sample-specific volatility estimator, which uses the sample-specific mean, the return reversal scenarios look like high volatility and the continuations look like low volatility, even though they are not when we take into account the full distribution across all four possible underlying return sequences.

While the simple example above provides much of the intuition for the key insights of this paper, the paper provides both a rigorous theoretical analysis of the issue and a detailed empirical analysis of the most popular leveraged and inverse funds in the market, i.e., those whose underlying assets are the two most popular equity indexes: the S&P 500 and the NASDAQ-100. To establish the theoretical effects of rebalancing, we derive closed-form expressions for the cumulative fund returns produced by two alternative benchmark strategies, one with continuous rebalancing and one with no rebalancing. We prove two key results for the case in which the risk-free rate and the index volatility are constant. First, for both leveraged and inverse funds, the returns of the continuously-rebalanced strategy are convex functions of the returns of the underlying asset while those of the no-rebalance strategy are linear. The returns on the continuously-rebalanced strategy exceed those of the no-rebalance strategy when cumulative returns on the underlying asset over the holding period are large in magnitude, and they fall short when the cumulative returns on the underlying index are small in magnitude. Second, this intermediate range of index returns in which the continuously-rebalalanced strategy underperforms is wider the greater the index volatility, but this effect is offset by the fact that when the index volatility is greater, the probability of a large index return of either sign becomes higher. Finally, we derive an expression for fund returns with daily rebalancing to illustrate how multi-day returns on these funds inherit the properties of the continuously-rebalanced benchmark.

We then move to an empirical analysis of the returns on the full set of leveraged and inverse funds, from 3x to -3x, with the S&P 500 and the NASDAQ-100 as the underlying indexes. Together, these ETFs make up a substantial fraction of the leveraged and inverse ETF market, comprising close to 50% of the total assets at the beginning of 2026. Moreover, these funds have a long history, and we have data on their daily returns for a full 15-year sample from 2011 to 2025. The empirical analysis provides several key results. First, on a daily basis, these funds achieve an excess return fully in line with their stated strategies. Accounting for fees and expenses, they generate the multiple of daily excess returns on the underlying asset that they promise. Second, the multi-day fund returns exhibit exactly the convexity that the theory predicts. The ETFs outperform their linear benchmarks when cumulative returns on the underlying index are large in magnitude and underperform when they are small. Third, sorting on realized volatility, the convexity and regions of outperformance and underperformance again vary precisely as the theory would suggest. Finally, strategies that combine the leveraged and inverse funds to synthesize the return on the underlying index show no evidence of the value erosion that “volatility decay" would suggest.

This fundamental misunderstanding of the properties of the returns on leveraged and inverse funds has led to the conclusion that these funds are suitable only for short holding periods. The reality is that they are potentially useful as a permanent component of investors’ portfolios, albeit with periodic rebalancing, which is a feature of many investment strategies. For example, leveraged positions in equities might well be appropriate for some investors. Standard portfolio theory makes no economically meaningful distinction between portfolios that have positive weights in both the risky tangency portfolio and the risk-free asset and those that are leveraged, i.e., that have negative weights in the risk-free asset in order to leverage the risky tangency portfolio.[6] In fact, recent work by Choi et al. (2025) suggests that leverage will likely be optimal for younger investors whose financial wealth is small relative to the value of their future labor income.[7] A leveraged ETF would be a natural holding for these investors. Leveraged ETFs might also make sense for investors with more financial wealth if they have sufficiently high risk tolerance, or for investors who are constrained to hold some of their wealth in assets other than equities.

[6] See Merton (1969) for the original seminal work on portfolio choice and Bodie et al. (2024) for a current representative textbook treatment of the same issue.

[7] Choi et al. (2025) imposes a no-leverage constraint in their paper, but it is clear from their results that, in many reasonable situations, an investor would optimally invest more than 100% of their financial wealth in the risky asset. In other words, they would want to access leverage to invest in the stock market if leverage were available.

Long-term demand for inverse ETFs might come from investors who wish to hedge a long position that they cannot or do not wish to sell. For example, unrealized capital gains make selling assets costly from a tax standpoint, and it might make more economic sense to hedge the exposure using an inverse fund rather than reducing it by liquidating the position. In addition, certain investors and managers are restricted from selling their equity or option positions, and again hedging them with inverse funds could achieve a similar economic goal.[8]

One objection to leveraged and inverse ETFs is that they charge fees that are substantially higher than those of ETFs that hold the underlying index, almost 1% higher in the case of the ETFs that we examine (see, for example, Bednarek and Patel (2022)). However, investors should reasonably expect to pay for access to leverage or short positions through a dynamically rebalanced portfolio. The fact that these ETFs have billions in assets when investors also have access to margin accounts in which they could leverage index ETFs themselves or sell index ETFs short shows, through revealed preference, that these fees are lower than the pecuniary and non-pecuniary costs associated with alternative ways of achieving a dynamically rebalanced leveraged or inverse position.

The bottom line is that the spread and promotion of these misconceptions has likely led to the current situation in which leveraged and inverse funds are primarily the province of short-term speculators, rather than tools for long-term investors to better achieve their financial goals.

The remainder of the paper is organized as follows. In Section 2 we develop our theoretical results. Specifically, we derive and compare the leveraged and inverse fund returns on the two natural benchmarks: one with continuous rebalancing and one with no rebalancing. We also compare these returns to those under daily rebalancing. Our empirical analysis follows in Section 3, where we examine daily and multi-day returns on the full spectrum of leveraged and inverse funds on the S&P 500 and the NASDAQ-100 indexes. Section 4 concludes.

2 Theoretical leveraged and inverse fund returns

First, we construct and compare two benchmark fund returns: one with continuous rebalancing to keep the degree of fund leverage constant and one with no rebalancing where fund leverage is set initially but then varies over time depending on the return on the underlying index. The intuition is that the daily rebalanced ETFs fall between these two extremes. At a daily horizon, the daily rebalanced ETFs have no rebalance, but as the horizon increases the rebalance frequency grows shorter relative to the length of the holding period and the fund returns more closely resemble those of a continuously rebalanced fund. We then examine returns on a daily rebalanced fund, for which the leverage is set before the beginning of the trading day, it evolves over the day with index returns, and is reset to the original level before the beginning of the subsequent trading day. These theoretical results allow us to understand what the properties of the returns on daily rebalanced ETFs should be and why. Throughout this analysis, we ignore fees and expenses for two reasons. First, the effect of fees paid to the fund and the expenses incurred by the fund in order to

[8] Of course, the hedging activities of investors and managers who face these restrictions might be restricted.

achieve the strategy returns are clear. Any fees or expenses will reduce the return received by the fund investor by exactly the amount of these fees and expenses. Second, and related, incorporating these fees and expenses complicates the notation and derivation with little or no payoff in terms of increased economic intuition. We will return to a consideration of these effects in the empirical analysis in the subsequent section.

2.1 Benchmark returns

First we state a key and well known result. Assume the value of an index S_t is a positive process that follows

$$\frac{dS_t}{S_t} = \mu_t\, dt + \sigma_t\, dB_t\,, \tag{1}$$

where B_t is a standard d-dimensional Brownian motion and the index appreciation rate μ_t and the d-dimensional volatility row-vector σ_t are suitably integrable processes that are measurable with respect to the information generated by the Brownian motion. Then

$$\left(\frac{S_T}{S_0}\right)^p = e^{p\int_0^T \mu_t\, dt + p\int_0^T \sigma_t\, dB_t - \frac{1}{2}p\int_0^T |\sigma_t|^2\, dt}\,. \tag{2}$$

In addition, assume there is a market for instantaneous risk-free borrowing and lending, where the risk-free interest rate process r_t is suitably integrable and measurable with respect to the information generated by the Brownian motion. If there are no arbitrage opportunities, then whenever $\sigma_t = 0$, i.e., the index return has no volatility, it must be that $\mu_t = r_t$, i.e., its return must equal the risk-free rate.

2.1.1 Benchmark 1: Fund returns with continuous rebalancing

As our first benchmark, consider a fund that borrows or lends at the instantaneous risk-free rate in order to maintain a continuously rebalanced proportion of the portfolio p in the index. There are three different cases. The case of $p > 1$ is that of positive leverage, i.e., the fund must borrow in order to maintain a position in the index that is larger than the value of the fund. The case of $p < 0$ corresponds to an inverse fund that takes a short position in the index and invests the proceeds of this short sale plus the capital in the fund at the risk-free rate. For $0 < p < 1$, what we call deleverage, the fund allocates a positive fraction of the capital to both the index and risk-free lending.

For all three cases, the value of wealth X_t under this dynamic self-financing trading strategy follows

$$\frac{dX_t}{X_t} = r_t\, dt + p(\mu_t - r_t)\, dt + p\sigma_t\, dB_t\,. \tag{3}$$

Therefore, the gross fund return is

$$\frac{X_T}{X_0} = e^{\int_0^T r_t\,dt + p\int_0^T (\mu_t - r_t)\,dt + p\int_0^t \sigma_t\,dB_t - \frac{1}{2}p^2\int_0^T |\sigma_t|^2\,dt} = e^{(1-p)\int_0^T r_t\,dt - \frac{1}{2}p(p-1)\int_0^T |\sigma_t|^2\,dt}\left(\frac{S_T}{S_0}\right)^p . \tag{4}$$

This result can be found in, for example, Jarrow (2010), and it is a generalization of the original result in Cheng and Madhavan (2009), with the correction of an error therein as noted in Jarrow (2010).

Under no arbitrage, this fund return is completely fair in the sense that it neither creates nor destroys value relative to the return on the underlying index. These dynamic trading strategies obviously affect returns, and thus they may generate returns that are more or less desirable than the return on the underlying index, in a utility sense, for any particular investor, but they neither dominate nor are dominated by this index return. This is true for any holding period. Why then has much of the literature concluded that these returns are undesirable? One possible explanation is that the coefficient on the volatility term is negative for leveraged and inverse funds. This may be the genesis of the term "volatility decay." In other words, there is the suggestion that this formula implies that returns are somehow lower for leveraged and inverse funds on volatile indexes. This is not correct, because the expected value of $\left(\frac{S_T}{S_0}\right)^p$ is increasing in volatility.

It is worthwhile to note that if r_t and σ_t are deterministic functions of time, then the return on the continuously rebalanced fund is a deterministic function of the index return that is increasing and convex for $p > 1$, decreasing and convex for $p < 0$, and increasing and concave for $0 < p < 1$. This convexity for leveraged and inverse funds seems to be the source of the negative reaction to these funds in at least some of the literature (see, for example, Avellaneda and Zhang (2010) and Cheng and Madhavan (2009)). However, if convexity is seen as a source of value destruction then surely concavity should be viewed as a source of value creation, and the literature should be advocating for deleveraged funds. Of course, there is no true value destruction or value creation, and neither leveraged or deleveraged funds should be inherently preferred on an ex ante basis.

2.1.2 Benchmark 2: Fund returns with no rebalancing

As our second benchmark, suppose the fund borrows or lends at the risk-free rate to establish an initial position q in the index but does not rebalance. Starting at time 0 with initial value Y_0, the fund value at time T is

$$Y_T = qS_T - (qS_0 - Y_0)e^{\int_0^T r_t\,dt} , \tag{5}$$

where $qS_0 - Y_0$ is the borrowing or lending at time 0. In order for q to correspond to the degree of leverage in the initial position, i.e., the analogue of p above, we need $Y_0 = S_0$. Then, the gross fund return is

$$\frac{Y_T}{Y_0} = q\frac{S_T}{S_0} - (q-1)e^{\int_0^T r_t\,dt} . \tag{6}$$

An equivalent result can be found in Jarrow (2010).

Note that the excess return on the fund is q times the excess return on the underlying index:

$$\frac{Y_T}{Y_0} - e^{\int_0^T r_t\,dt} = q\left(\frac{S_T}{S_0} - e^{\int_0^T r_t\,dt}\right) . \tag{7}$$

The no-rebalance strategy simply scales the excess return on the underlying index linearly. For the purpose of this analysis, we allow for the possibility that this excess return can be negative, i.e., there is no solvency constraint on the strategy. Interestingly, this issue does not arise for the continuously rebalanced strategy because the value of the fund can never go to zero under this strategy, regardless of the magnitude of p or the return on the underlying index, as long as the price process is continuous, i.e., does not exhibit jumps.

2.1.3 A comparison of the two benchmark fund returns

Assume $p = q$, i.e., both strategies start out with the same degree of leverage. Then the continuously rebalanced fund outperforms the fund with no rebalancing when

$$e^{(1-p)\int_0^T r_t\,dt - \frac{1}{2}p(p-1)\int_0^T |\sigma_t|^2\,dt}\left(\frac{S_T}{S_0}\right)^p > p\left(\frac{S_T}{S_0}\right) - (p-1)e^{\int_0^T r_t\,dt} . \tag{8}$$

For simplicity, suppose r_t and σ_t are constant. Panels A, B, and C of Figure 2 plot each of these two fund returns as functions of the underlying index return for the cases with $p = 3$ (leveraged), $p = 0.5$ (deleveraged), and $p = -2$ (inverse), respectively. The solid line in each plot represents the fund return with continuous rebalancing and the dashed line represents the fund return with no rebalancing. The underlying index has annualized volatility $|\sigma| = 15\%$, and the annualized risk-free rate is $r = 4\%$, which correspond to reasonable values for funds with the S&P 500 as the underlying index. The return horizon is $T = 1$ year.

As Figure 2 shows, neither fund uniformly outperforms the other, consistent with the absence of arbitrage. In Panels A and C, where the return on the continuously rebalanced fund is convex, i.e., in the cases of leveraged or inverse funds, then the continuously rebalanced fund outperforms the fund with no rebalancing for extreme values of the index return and underperforms on an interval of intermediate index returns. When the return on the continuously rebalanced fund is concave, i.e., in the case with deleverage, the reverse is true, as depicted in Panel B. That is, the continuously rebalanced fund underperforms the fund with no rebalancing for extreme values of the index return and outperforms for intermediate index returns. By definition, these results are due to the effects of rebalancing, and the intuition is relatively straightforward. Consider a continuously rebalanced leveraged fund ($p > 1$). As the underlying index goes up, maintaining constant leverage requires increasing leverage. If the index continues to rise, this increased leverage magnifies these positive returns even further, and the rebalanced fund outperforms. Similarly, a consistently declining index requires reductions in leverage, so subsequent negative returns have a smaller effect on fund value.

Similar logic applies to inverse funds. For deleveraged funds, the reverse is true. Index appreciation leads to decreases in leverage, while depreciation leads to increases in leverage. Thus, for deleveraged funds, it is reversals that benefit the rebalanced fund relative to its no-rebalance counterpart.

The propositions below state these results more precisely. Proofs are provided in Appendix A. The first proposition addresses the special case that $|\sigma| = 0$.

Proposition 2.1 *Suppose $|\sigma| = 0$. Then $\mu_t = r$ for all t,*

$$\frac{S_T}{S_0} = e^{rT} , \tag{9}$$

and both benchmark returns are

$$\frac{X_T}{X_0} = \frac{Y_T}{Y_0} = e^{rT} . \tag{10}$$

In other words, when the index volatility is zero, the index return equals the risk-free rate with certainty, so, in this special zero-volatility case, the return with continuous rebalancing and the return with no rebalancing are identically equal.

Proposition 2.2 *Suppose $|\sigma| > 0$. Suppose $p > 1$ or $p < 0$. Then there are exactly two values of the index return $\frac{S_T}{S_0}$, $L(r, |\sigma|)$ and $R(r, |\sigma|)$ where $L(r, |\sigma|) < R(r, |\sigma|)$, at which $\frac{X_T}{X_0} = \frac{Y_T}{Y_0}$. Furthermore, $\frac{X_T}{X_0} < \frac{Y_T}{Y_0}$ for $L(r, |\sigma|) < \frac{S_T}{S_0} < R(r, |\sigma|)$ and $\frac{X_T}{X_0} > \frac{Y_T}{Y_0}$ for values of the $\frac{S_T}{S_0}$ strictly outside the interval from $L(r, |\sigma|)$ to $R(r, |\sigma|)$. Finally, $L(r, |\sigma|) < e^{rT} < R(r, |\sigma|)$.*

Proposition 2.3 *Suppose $|\sigma| > 0$. Suppose $0 < p < 1$. Then there are exactly two values of the index return $\frac{S_T}{S_0}$, $L(r, |\sigma|)$ and $R(r, |\sigma|)$ where $L(r, |\sigma|) < R(r, |\sigma|)$, at which $\frac{X_T}{X_0} = \frac{Y_T}{Y_0}$. Furthermore, $\frac{X_T}{X_0} > \frac{Y_T}{Y_0}$ for $L(r, |\sigma|) < \frac{S_T}{S_0} < R(r, |\sigma|)$ and $\frac{X_T}{X_0} < \frac{Y_T}{Y_0}$ for values of the $\frac{S_T}{S_0}$ strictly outside the interval from $L(r, |\sigma|)$ to $R(r, |\sigma|)$. Finally, $L(r, |\sigma|) < e^{rT} < R(r, |\sigma|)$.*

Proposition 2.4 (Volatility Effects) *$L(r, |\sigma|)$ is decreasing in $|\sigma|$ and $R(r, |\sigma|)$ is increasing in $|\sigma|$.*

Figure 3 Panels A and B illustrate the volatility effects for the case of $p = 3$ and $p = -3$, with volatility varying from 0% to 30%. In both cases, the intuition is that as the index volatility increases, extreme index returns become more likely, so the region of index values where the continuously rebalanced fund underperforms the fund with no rebalancing must get larger in order for the return to maintain the same present value. In other words, the fact that for higher index volatility the leveraged fund return lies below the fund return with low index volatility cannot and should not be interpreted as suggesting that volatility reduces returns and decreases value. With higher volatility, larger magnitude index returns, which lead to more favorable fund returns, become more likely.

2.2 Returns with daily rebalancing

Section 2.1 above provides theoretical results for funds with continuous rebalancing and no rebalancing. Actual ETFs rebalance daily, so one might expect their return properties to lie in between those of funds with continuously rebalancing and no rebalancing. We now explore the return properties of funds with this intermediate, daily rebalancing frequency to understand the extent to which this intuition holds. Along the way, we also use our framework to correct a widely-held misconception that the effect of frequent rebalancing on cumulative px fund returns is driven by the sign of the realized first-order serial correlation in the path of the returns of the underlying index.

For simplicity, we assume a risk-free rate of zero, because in practice, daily risk-free rates are tiny compared with daily excess returns on typical underlying indexes. Let $r_1, r_2, \ldots, r_n, \ldots, r_N$ denote the daily returns on the underlying index over an N-day period. For a daily-rebalanced px fund, the return on any given day is pr_n, so the cumulative return over the N-day period is

$$r_d = (1 + pr_1)(1 + pr_2) \cdots (1 + pr_N) - 1 \, . \tag{11}$$

In contrast, the cumulative return on the no-rebalance strategy is p times the cumulative return on the underlying index, as we saw in continuous time in equation (7), under the assumption of a risk-free rate of zero:

$$r_{nb} = p[(1 + r_1)(1 + r_2) \cdots (1 + r_N) - 1] \, . \tag{12}$$

The first key insight comes from a simple inspection of Equations (11) and (12): For a given set of daily returns on the underlying index, the cumulative returns on both the daily-rebalanced px fund and the no-rebalance px fund are invariant to the *ordering* of the underlying daily returns. For example, for a 100-day holding period, alternating daily returns of 1% and -1% on the underlying index would generate the same cumulative returns on the daily-rebalanced px fund, and on the no-rebalance px fund, as would a sequence of 50 daily 1% underlying returns followed by 50 daily -1% underlying returns. Therefore, the rebalancing effect, i.e., the difference between the cumulative return on the daily-rebalanced px fund and the cumulative return on the no-rebalance px fund, is invariant to the ordering of the daily returns on the underlying index.

We also saw this invariance of the cumulative px fund return to the ordering of the returns on the underlying index for the both the daily-rebalanced and no-rebalanced funds in the 2-day example in Table 1. If one's intuition for the rebalancing effect rests on the idea that serial correlation in the underlying index returns causes the daily-rebalanced px fund to underperform the no-rebalance px fund, then this order-invariance result might seem counter-intuitive. For instance, in the 2-day example in Table 1, the rebalanced px funds underperformed the no-rebalance px funds when the realized path of the underlying index exhibited reversals, and outperformed when the underlying index exhibited continuations. In the 2-day example, it happens that paths with reversals correspond to negative first-order serial correlation in the underlying returns, and paths with continuations correspond to positive first-order serial correlation. However, when we consider N-day cumulative returns, with N large, this correspondence between reversals and negative daily first-order serial correlation breaks down. For example, in the 100-day scenarios described above, the path of underlying index returns with alternating days of 1% and -1% returns is characterized by strong negative first-order serial correlation, whereas the path with 50 daily 1% returns followed by 50 daily -1% returns is characterized by strong positive first-order serial correlation.

Thus, we see that what drives the performance of the daily-rebalanced px funds relative to the no-rebalance px funds is not the sign of the realized first-order serial correlation in the underlying daily returns, but rather, a more general notion: reversals vs. continuations in the realized path of the underlying returns. For example, if the cumulative multi-day return on the underlying index is close to zero, i.e., the price of the underlying at the end of the multi-day period is close to the price at the start, then, by definition, most of the daily returns during the period have been reversed, and in that case, the daily rebalanced funds underperform the no-rebalance funds. But it does not matter precisely when they are reversed in the sense that alternating positive and negative returns on daily underlying returns produces the same cumulative returns for all strategies as a prolonged run-up in the underlying price followed by a prolonged decline in price. In other words, if we try to understand the performance of daily-rebalanced funds relative to no-rebalance funds in terms of the serial correlation exhibited by the path of the underlying index, it is important to realize that serial correlations at all lags matter, not just the serial correlation at a lag of one day.

The second key insight is that while, for a given set of daily returns, the ordering does not matter, the cumulative return on the underlying index over the multi-day holding period is not a sufficient statistic for the cumulative return on the px funds with daily rebalancing. For example, Equation (4) shows that even if we assume a zero risk-free rate, the return on the continuously rebalanced strategy depends not only on the cumulative return of the underlying index, $\frac{S_T}{S_0}$, but also on the realized path of the volatility process, $\int_0^T |\sigma_t|\, dt$. On the other hand, Equation (7) shows that the cumulative excess return on the strategy with no rebalancing does depend only on the cumulative return of the underlying index, $\frac{S_T}{S_0}$, if we assume the risk-free rate is zero. Therefore, the rebalancing effect in cumulative fund excess returns, i.e., the difference between the return on the continuously rebalanced px fund and the return on the no-rebalance px fund, depends not only on the cumulative return of the underlying index, $\frac{S_T}{S_0}$ but also on the realized path of the volatility process, $\int_0^T |\sigma_t|\, dt$.

Now consider the rebalancing effect in the cases with daily rebalancing and no rebalancing in Equations (11) and (12) by examining their difference:

$$r_d - r_{nb} = (p^2 - p)\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} r_i r_j + (p^3 - p)\sum_{i=1}^{N-2}\sum_{j=2}^{N-1}\sum_{k=j+1}^{N} r_i r_j r_k + \cdots + (p^N - p) r_1 \cdots r_N \, . \tag{13}$$

It is clear from inspection that the magnitudes, or volatility, of the underlying daily returns matter. The products of returns are larger in magnitude if the individual returns are larger in magnitude, which is more likely in a more volatile environment. Moreover, this equation also highlights the fact that it is the products of returns across all lags within the holding period that matter, not just the lower order terms.

Returning to the question of whether the cumulative return properties of the daily-rebalanced px fund are similar to the those of the continuously-rebalanced px fund, we note that while daily and continuous rebalancing are different, the cumulative return on the px fund with daily rebalancing and the cumulative return on the px fund with continuous rebalancing both depend on the volatility structure of the underlying index. This is consistent with the idea that the properties of daily rebalance strategies will converge to those on continuously rebalanced strategies as the holding period grows large. In the next section, we provide empirical evidence on this convergence using data on the cumulative returns of existing leveraged and inverse ETFs.

3 Empirical analysis

3.1 Data

The dataset for the empirical analysis is the full range of levered and inverse funds, 3x to -3x, on the two indexes that are most heavily traded in the ETF market, the S&P 500 and the NASDAQ-100. We also include the two ETFs, SPY and QQQ, that replicate the underlying indexes. Table 2 lists the ETFs in this sample, their inception dates, and their current expense ratios.[9] For consistency, we use the ProShares leveraged and inverse ETFs, because ProShares offers the full range of funds, but it should be noted that Direxion also offers 3x and -3x funds on the NASDAQ-100.[10] Some of the ETFs started trading as early as June 2006, with the youngest products introduced in February 2010. We employ a common 15-year sample for all products: January 2011 to December 2025. During this period all the ETFs were well-established and sufficiently liquid that microstructure noise is of limited concern, even on a daily basis.

Table 2 also lists the total assets of the funds at the beginning and end of the sample. There is some evidence that assets have moved towards the more extreme funds, i.e., 3x and -3x, over time, especially for the funds based on the NASDAQ-100. Of some interest, while SPY is significantly larger than QQQ, the reverse is generally true for the leveraged and inverse funds, with those on the NASDAQ-100 generally being larger than their S&P 500 counterparts. The leveraged funds are significantly larger than the inverse funds, which is perhaps not surprising given the large positive returns on both indexes over the sample. Our analysis uses daily returns on the ETFs from CRSP and the daily risk-free rate from the Ken French data library (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html), which is based on Treasury bill rates. Multi-day returns are compounded from these daily returns.

[9] The expense ratios have varied somewhat over the sample, but generally have been slightly less than 1% per year. Currently, the leveraged and inverse ETFs on the NASDAQ-100 are subject to a small amount of fee forgiveness, i.e., they have a net expense ratio slightly lower than the gross expense ratio. This fee forgiveness is temporary in the sense it expires at a fixed point in the future, but it can be renewed indefinitely at the option of the ETF sponsor.

[10] Results are quantitatively similar using the returns on these funds.

3.2 Daily returns

Table 3, Panel A provides descriptive statistics for daily excess returns on the two indexes and their associated ETFs. Daily excess returns are the natural object of interest, since, absent fees and expenses, the leveraged and inverse funds should provide the promised multiple of excess returns not total returns, as shown in Equation (7). The first seven rows are for the S&P 500 funds, while the remaining seven rows are for the NASDAQ-100 funds. The second column shows statistics for the index excess returns, with dividends reinvested. The remaining six columns contain statistics for the leveraged and inverse fund excess returns, for p = 3, 2, 1, -1, -2, and -3, respectively. The fifth column, labeled 1x, reports statistics for the index ETFs.

The daily excess returns are largely in line with what one might expect. The mean index returns are both positive, with the NASDAQ-100 exhibiting a somewhat higher average return and volatility. Median returns are slightly larger than mean returns, suggesting slight positive skewness, which is also evident in the 25th and 75th percentiles of the return distributions, but not in the more extreme 10th and 90th percentiles. The index ETFs exhibit daily return distributions very close to those of the indexes themselves, and the leveraged and inverse funds have average returns, volatilities and percentiles approximately in line with their promised multiples. That is, the px fund has daily excess returns of approximately p times those of the index.

3.2.1 Delevered excess return deviations

We use two approaches to evaluate the daily performance of the ETFs. The first is to examine daily delevered excess return deviations, defined as

$$e_{p,t} = \frac{r_{ETF,p,t}}{p} - r_{Idx,t} \, , \tag{14}$$

where $r_{ETF,p,t}$ is the excess return on the px ETF and $r_{Idx,t}$ is the daily excess return on its underlying index on day t. We choose to divide the ETF return by its degree of leverage rather than multiplying its underlying index excess return by this quantity so that the adjusted ETF returns will have approximately the same volatility for all funds linked to that index and will thus be easier to compare. Table 3, Panel B shows the means and volatilities of these daily deviations for all the ETFs. One important caveat is that the index returns are purely theoretical returns; they are not tradeable returns. The results must be interpreted in this context.

There are a number of interesting features of these data. First, the 1x ETFs, SPY and QQQ, have average daily return deviations of -0.05 and -0.1 basis points, respectively. These shortfalls are slightly larger than the associated expenses, suggesting that there are additional costs, perhaps trading costs, born by the investors in these funds. That said, these costs, and the expenses, are small, so, not surprisingly, the index ETFs exhibit average returns very close to those of their underlying indexes. The daily volatilities of these deviations are approximately ten times larger than the mean deviations, at 6 and 7 basis points for SPY and QQQ, respectively. These volatilities partially reflect microstructure effects, such as bid-ask bounce in the ETF prices, but they also reflect occasional deviations in the returns of both the index and the associated ETF from what one might call the “true" return. We will return to this issue in Section 3.2.2 below.

Second, the mean deviations for the leveraged funds are negative, while those for the inverse funds are positive, but smaller in magnitude. These positive deviations for the inverse funds may seem puzzling, but they are actually consistent with the presence of costs. It is just that the division by p in the definition of the delevered ETF excess returns in Equation (14) changes the sign of the effect of costs from negative to positive for inverse funds, which have $p < 0$.

To see this, let $r^*_{ETF,p,t}$ represent the gross excess return on the px ETF on day t, i.e., before fund expenses and any trading costs. Let $c_{p,t}$ equal the total fund costs on day t, i.e., $r_{ETF,p,t} = r^*_{ETF,p,t} - c_{p,t}$. Then the delevered excess return deviation is

$$e_{p,t} = \frac{r^*_{ETF,p,t} - c_{p,t}}{p} - r_{Idx,t} = \frac{r^*_{ETF,p,t}}{p} - r_{Idx,t} - \frac{c_{p,t}}{p} \, . \tag{15}$$

Thus, the presence of costs for leveraged funds, i.e., funds with $p > 0$, reduces the measured deviation, while the costs for inverse funds, i.e., those with $p < 0$, increases the measured deviation.

We do not have complete data on fund expense ratios over the full sample period, and there may also be other unobservable trading costs as well, so we cannot incorporate the $c_{p,t}$ directly into our performance measures. However, we can take time-series averages in Equation (15) and back out an implied average cost for each of the six px funds, under the assumption that the gross fund returns track their benchmarks perfectly, to see whether or not these implied average costs seem reasonable. Let

$$\bar{e}_p = \frac{\bar{r^*}_{ETF,p}}{p} - \bar{r}_{Idx} - \frac{\bar{c}_p}{p} \, , \tag{16}$$

where the bar notation indicates the average of the daily variable across the full sample period January 2011 to December 2025. Solving for the implied average cost $\bar{c}_p$, we get

$$\bar{c}_p = \bar{r^*}_{ETF,p} - p\bar{r}_{Idx} - p\bar{e}_p \, . \tag{17}$$

Under the assumption that $\bar{r^*}_{ETF,p} - p\bar{r}_{Idx} = 0$, i.e., that gross of costs, the ETFs are tracking their benchmarks perfectly on average, we get

$$\bar{c}_p = -p\bar{e}_p \, . \tag{18}$$

Table 3, Panel B reports the implied annualized ETF costs, i.e., $252 \times \bar{c}_p$, for each value of p and for each index, the S&P 500 and the NASDAQ-100. On the whole, these implied costs look reasonable. For example, for the S&P 500 1x fund, the implied annualized cost is 12 basis points per year, which is negligible on a per day basis, and aligns with the matching mean excess returns of the index and the 1x fund reported in Panel A. For the NASDAQ-100 1x fund, the implied annualized cost is 25 basis points per year, or about 0.1 basis point per day, which again matches the difference between the mean daily index return and the mean daily 1x fund return in Panel A.

For the leveraged and inverse funds, implied average costs are larger than for the 1x funds, which is consistent with the current expense ratios on these funds reported in Table 2. However, the implied average costs are higher for the leveraged funds, i.e., those with $p > 1$, than for the inverse funds, i.e., those with $p < 0$. This may be related to the fact that the way these funds achieve their promised leverage multiple is through the use of total return swaps on the underlying index with financial intermediaries as counterparties. In particular, if, in the broader market for total return swaps, financial intermediaries are naturally net short the underlying indexes, because the assets of the leveraged funds are greater than those of the inverse funds, then these intermediaries may try to resolve the imbalance by charging leveraged funds implicit borrowing rates that are higher than Treasury bill rates. This would show up as higher implicit costs for leveraged funds than for inverse funds.

3.2.2 Regressions of ETF excess returns on index excess returns

The alternative method for examining the daily performance of the ETFs is to run a simple linear regression of the excess return on the ETF on the excess return on the index:

$$r_{ETF,p,t} = a_p + b_p r_{Idx,t} + \varepsilon_{p,t} \; . \tag{19}$$

If replication is perfect, the intercept should be zero and the slope should be equal to the leverage or inverse ETF multiple. Table 3, Panel C provides the results from these regressions, with standard errors in parentheses.

Again, we can make a number of interesting observations. First, the slope coefficients are economically very close to the fund multiples, albeit statistically smaller in magnitude in all cases. At first glance, these deviations from the multiple are a slight puzzle, since noise in the ETF returns should show up in the error term and should not attenuate the slope coefficient. In fact, these discrepancies in the coefficient are evidence of noise in the index return. More generally, all of the returns are slightly noisy. Clear evidence of this noise can be seen by looking at days on which the delevered excess returns show abnormally large deviations. For each index we have seven returns, including the return on the index itself. On any given abnormal return day, one or two of the returns are out of line with the remaining returns. For example, on the very first day in the sample, January 3, 2011, the excess return on the NASDAQ-100 is 1.65%, while the delevered excess returns on all of the ETFs, including QQQ, are in the narrow range between 1.51% and 1.56%. These data suggest that it is the index return that is wrong, perhaps due to nonsynchronous trading. A similar, but even larger discrepancy occurs on January 19, 2011. The delevered excess return on UPRO, the 3x S&P 500 ETF, is -0.04%, while the other six delevered excess returns lie between -0.97% and -1.00%. Clearly, the 3x ETF return is somehow wrong, but the explanation for unusual ETF returns is different. Specifically, it appears that the creation and redemption process, which keeps ETF prices in line with the net asset values of the fund, i.e., the prices of the underlying securities, can break down if there are large moves in prices close to or at the close of trade.[11] However,

[11] See ETF Trends Editorial Staff (2020) for a discussion of this issue.

these price discrepancies appear to be very short-lived, and are corrected at the open on the following day. As a result, while the return on UPRO is almost 1% too high on January 19, it is approximately 1% too low, relative to the other six returns, on the following day. In other words, average returns are fine, but there are a handful of days in our 15-year sample on which the closing prices of one or more of the ETFs are clearly incorrect. These discrepancies are not tradable, since, by definition, the market is closed after these prices are visible, and the price corrects at the open the following day. However, they do create noise in the close-to-close returns, and they are the primary driver of the volatilities in the delevered excess return deviations reported in Panel B. This noise is also evident in the regression R-squareds, which all exceed 99.7% but are not 100%. Again, this is a somewhat interesting empirical phenomenon, but it is mostly a distraction from the main issues in this paper.

Second, the intercepts are close to, but statistically less than zero. It would be natural to attribute these negative intercepts to fund costs. However, the precise magnitudes of these intercepts are difficult to interpret because of the attenuation in the slope coefficients. For example, while the -3x S&P 500 fund may be generating the correct multiple of the index return on average, the slope coefficient is only -2.95 due to the effect discussed above. Thus, the intercept must adjust for the mean effect associated with this attenuated slope.

The data underlying Table 3 are plotted in Figure 4. The daily excess returns for each ETF are plotted against the excess returns on the index. There are 3,772 daily observations for each series, but the vast majority of the points are not visible since they effectively plot on top of each other. As a benchmark, the theoretical return multiple is also plotted as a solid line. In general, as the results in Table 3 illustrate, the ETF returns plot on this line. However, it is also possible to see some of the occasional return discrepancies that we discuss above. Specifically, if all the ETFs plot off their respective lines, then it is a day on which the index return is incorrect. Alternatively, if one or two ETF returns plot off their lines, then it is these returns that are incorrect.

The overall conclusion from these results is clear. The ETFs work as advertised, on a daily basis, i.e., they provide a multiple of the daily excess return on the index, with small deviations associated with fees and expenses and larger but much more infrequent and temporary deviations associated with an apparent breakdown of the creation and redemption process. This conclusion is broadly consistent with the results in the existing literature. We turn to the more controversial question, the properties of multi-day returns, in the next section.

3.3 Multi-day returns

We examine cumulative returns for holding periods of 5 days (1 week), 21 days (1 month), 63 days (1 quarter), and 252 days (1 year), as well as for the full 15-year sample. To introduce these data, Table 4 provides descriptive statistics for these returns. The sample consists of daily overlapping, multi-day returns, with the number of observations given in the final column. For example, there are 3,521 daily overlapping, 252-day returns, but there are obviously only 15 independent, non-overlapping annual returns in our 15-year sample.

Due to the daily rebalancing inherent in the strategies of leveraged and inverse funds, the return distributions exhibited by these funds over longer horizons should not be simple multiples of the returns on the corresponding index. For example, Equation (4) shows that continuous rebalancing generates a convex relation between the cumulative return on the leveraged or inverse fund and the cumulative return on the underlying index. Similarly, Figure 1 shows that the cumulative 2-day returns on daily-rebalanced leveraged or inverse funds are convex functions of the 2-day return on the underlying index. In addition, the analysis of N-day cumulative returns in Section 2.2 shows that the difference between the N-day cumulative return on the daily-rebalanced fund and that of the no-rebalance fund for a given sequence of the daily underlying index returns will depend on whether the sequence of those daily index returns exhibits continuations or reversals. Nevertheless, the descriptive statistics do provide preliminary evidence on multi-day leveraged and inverse fund returns.

Several observations are in order. First, in spite of the fact that these multi-day returns are not simple multiples of the index return, for horizons up to 1 year, these leveraged and inverse ETFs do exhibit mean returns that are broadly consistent with these multiples. However, this relation breaks down for cumulative returns over the full sample, for which the leveraged and inverse funds dramatically outperform their respective multiples. For inverse funds, cumulative returns converge to -100% over sufficiently long horizons, because the average index returns are large and positive over our sample period. However, these returns are much better than what would have been earned by simply shorting the various multiples of the index and holding the position for 15 years.

Second, the standard deviations of these multi-day fund returns exhibit a somewhat interesting pattern. Specifically, as the horizon increases, the volatility of the inverse fund returns declines relative to its corresponding multiple, while that of the leveraged fund returns increases, although not to the same extent. This result is consistent with the theoretical convexity effect documented earlier. For high return assets, the lower bound on the cumulative return of -100% dampens volatility for inverse funds at longer horizons. In contrast, the convexity on the upside increases volatility. Of course, these results do have to be interpreted with some caution, since while we have a relatively large number of daily returns, the sample is a single 15-year return history with the associated sampling variation.

The most direct evidence on the convex relation between multi-day fund returns and multi-day index returns is provided by simply plotting realized multi-day fund returns vs. multi-day index returns. We expect these convexity effects to become more pronounced as the holding period increases, because the time between rebalancing, i.e., one day, grows small relative to the length of holding period. In addition, the theoretical fund return under continuous rebalancing from Equation (4) shows that the convexity effects become more pronounced as the fund multiple increases in magnitude. Moreover, Proposition 3 and Figure 3 show that in cases in which the volatility of the underlying returns is greater, the returns on the leveraged and inverse funds, as functions of the returns of the underlying, form curves that lie below the curves for cases with lower volatility, with the portion of the curve underperforming the linear benchmark becoming wider. Again, this wider region of underperformance is not evidence of value destruction. Rather, it is offset by the fact that extreme returns on the index, where the curve outperforms the linear benchmark, become more likely with higher volatility. To see if the volatility effects are present in the data, we color-code fund returns based on the level of the realized index volatility over the same holding period. We compute the realized volatility of the underlying index as

$$v_{t,t+N} = \sqrt{\sum_{s=0}^{N} r_{t+s}^2} \, . \tag{20}$$

Note that this measure implicitly assumes a mean daily return of zero. It is also important to note that we are measuring realized volatility, whereas the quantity of theoretical relevance is the ex-ante conditional volatility. However, there is reason to believe that ex-post realized volatility is a reasonable proxy.

Figure 5 plots the realized N-day excess returns for the -3x and 3x S&P 500 funds against the corresponding N-day excess return on their index, along with a black line representing the return on the linear, no-rebalance benchmark. Panel A shows the plots for $N = 63$ and Panel B shows the plots for $N = 252$. Figure 6 plots the returns for the -3x and 3x NASDAQ-100 funds. Both figures are strikingly consistent with the theory illustrated in Figure 3. For each volatility level, the associated points trace out a convex curve. This convexity is more pronounced for $N = 252$ than for $N = 63$. In addition, consistent with the theory, curves associated with higher index-volatility levels lie below those associated with lower volatility levels. On its own, this phenomenon would appear to justify the "volatility drag" intuition, i.e., that volatility hurts the performance of leveraged and inverse ETFs. However, the figure also clearly illustrates the offsetting volatility effect: when the cumulative index return is large in magnitude, then the leveraged and inverse fund returns outperform the linear benchmark. Most important, one only observes these high returns in the high-volatility groups. As the theory indicates, it is the high returns in these extreme states that compensates for the underperformance when cumulative returns are small in magnitude. The only missing piece is that there are no extreme-negative 252-day index returns in the Panel B plots for either index, because realized 252-day returns on both indexes were so high during the 15-year sample period. Thus, we do not observe inverse-fund outperformance of the linear benchmark in the extreme-negative index-return regions.

As a final exercise to illustrate that leveraged and inverse funds do not destroy value, consider combining pairs of funds, one leveraged and one inverse fund, in order to create a synthetic 1x fund.[12] There are six combinations of levered and inverse funds that could replicate a 1x fund using positive weights in levered and inverse funds. For example, a position consisting of a 50% portfolio weight in a 3x fund and a 50% portfolio weight in a -1x fund should produce the same excess return as a 1x fund, ignoring expenses. Table 5 shows the six combinations of available funds, and their associated portfolio weights, that achieve this goal. The table reports the means and standard deviations of differences of the daily returns on these strategies from the associated index return over the 15-year sample.[13] These differences average less that 0.1 basis points per day, on the same order as the fees and expenses on the funds, and the standard deviations are only slightly higher than those reported in Table 3 for the individual funds, including

[12] Of course, it makes no sense to actually do this in practice since the available 1x fund is cheaper in terms of fees and expenses than the leveraged and inverse funds.

[13] Since we keep the weights constant over time there is an implicit daily rebalance across the leveraged and inverse funds each day. However, we are not interested in the implementation issues, and any associated costs, but rather in the implied returns.

the 1x fund. In other words, there is essentially no evidence that leveraged and inverse funds destroy economically meaningful value.

4 Conclusion

The demonization of leveraged and inverse equity index ETFs, particularly the false accusation that these funds erode value via “volatility decay," is a major disservice to investors. It has likely discouraged long-term investors from using these products in ways that would help them achieve their financial goals. These funds provide leverage and short exposure, plus a dynamic rebalancing scheme that ensures that the degree of this exposure resets to the target level at the beginning of each day. They accomplish these objectives at fees that are greater than those on the underlying index funds, but are reasonable, or even attractive, compared with the costs individual investors would face to match these exposures through other means, such as costs associated with borrowing on margin, short selling, and the attention it would require to implement daily rebalancing. While these funds are suitable for short-term speculation or short-term adjustment of portfolio exposures, they are also a potentially useful long-term holding. In order to achieve this potential, the SEC needs to revise its misguided position on these funds and allow the funds to educate investors and market themselves appropriately. Moreover, practitioners and financial journalists need to admit the mistakes they have made in the past and attempt to rectify them by changing the way they characterize these funds going forward.

A Proofs

Proposition A.1 *Suppose $|\sigma| = 0$. Then $\mu_t = r$ for all t,*

$$\frac{S_T}{S_0} = e^{rT} \; , \tag{21}$$

and both benchmark returns are

$$\frac{X_T}{X_0} = \frac{Y_T}{Y_0} = e^{rT} \; . \tag{22}$$

Proof This follows from the absence of arbitrage and substitution of Equation (21) into Equations (4) and (6).

Proposition A.2 *Suppose $|\sigma| > 0$. Suppose $p > 1$ or $p < 0$. Then there are exactly two values of the index return $\frac{S_T}{S_0}$, $L(r,|\sigma|)$ and $R(r,|\sigma|)$ where $L(r,|\sigma|) < R(r,|\sigma|)$, at which $\frac{X_T}{X_0} = \frac{Y_T}{Y_0}$. Furthermore, $\frac{X_T}{X_0} < \frac{Y_T}{Y_0}$ for $L(r,|\sigma|) < \frac{S_T}{S_0} < R(r,|\sigma|)$ and $\frac{X_T}{X_0} > \frac{Y_T}{Y_0}$ for values of the $\frac{S_T}{S_0}$ strictly outside the interval from $L(r,|\sigma|)$ to $R(r,|\sigma|)$. Finally, $L(r,|\sigma|) < e^{rT} < R(r,|\sigma|)$.*

Proof In the cases $p > 1$ or $p < 0$, $p(p-1) > 0$. It follows from inspection of Equation (8) that when $\frac{S_T}{S_0} = e^{rT}$, $\frac{X_T}{X_0} < \frac{Y_T}{Y_0}$. By continuity of the fund returns as functions of the index return, there is a unique maximal interval $(L(r,|\sigma|), R(r,|\sigma|))$ containing e^{rT} on which $\frac{X_T}{X_0} < \frac{Y_T}{Y_0}$.

To establish that both $L(r,|\sigma|)$ and $R(r,|\sigma|)$ are finite, suppose first that $p > 1$. As the value of $\frac{S_T}{S_0}$ goes to zero the right-hand side of Equation (8) falls below zero while the left-hand side remains positive, so $L(r,|\sigma|)$ is finite. As the value of $\frac{S_T}{S_0}$ goes to infinity, the difference between the left-hand and right-hand sides of Equation (8) goes to infinity, so $R(r,|\sigma|)$ is finite. Now suppose $p < 0$. Then as the value of $\frac{S_T}{S_0}$ goes to infinity, the right-hand side of Equation (8) goes to minus infinity while the left-hand side remains positive, so $R(r,|\sigma|)$ is finite. As the value of $\frac{S_T}{S_0}$ goes to zero, the difference between the left-hand and right-hand sides of Equation (8) goes to infinity, so $L(r,|\sigma|)$ is finite.

It remains to show that $\frac{X_T}{X_0} > \frac{Y_T}{Y_0}$ whenever $\frac{S_T}{S_0} < L(r,|\sigma|)$ or $\frac{S_T}{S_0} > R(r,|\sigma|)$. This follows from the strict convexity of $\frac{X_T}{X_0}$ and the linearity of $\frac{Y_t}{Y_0}$ as functions of $\frac{S_T}{S_0}$.

Proposition A.3 *Suppose $|\sigma| > 0$. Suppose $0 < p < 1$. Then there are exactly two values of the index return $\frac{S_T}{S_0}$, $L(r,|\sigma|)$ and $R(r,|\sigma|)$ where $L(r,|\sigma|) < R(r,|\sigma|)$, at which $\frac{X_T}{X_0} = \frac{Y_T}{Y_0}$. Furthermore, $\frac{X_T}{X_0} > \frac{Y_T}{Y_0}$ for $L(r,|\sigma|) < \frac{S_T}{S_0} < R(r,|\sigma|)$ and $\frac{X_T}{X_0} < \frac{Y_T}{Y_0}$ for values of the $\frac{S_T}{S_0}$ strictly outside the interval from $L(r,|\sigma|)$ to $R(r,|\sigma|)$. Finally, $L(r,|\sigma|) < e^{rT} < R(r,|\sigma|)$.*

The proof of Proposition A.3 is similar to that of Proposition A.2.

Proposition A.4 (Volatility Effects) *$L(r,|\sigma|)$ is decreasing in $|\sigma|$ and $R(r,|\sigma|)$ is increasing in $|\sigma|$.*

Proof. This follows from inspection of Equation (8). On the right-hand side, the fund return with no rebalancing is invariant to $|\sigma|$. On the left-hand side, the fund return with continuous rebalancing is uniformly decreasing in $|\sigma|$ for the case $p > 1$ or $p < 0$, so the interval on which it lies below the fund return with no rebalancing gets larger. For the case $0 < p < 1$, the fund return with continuous rebalancing is uniformly increasing in $|\sigma|$, so the interval on which it lies above the fund return with no rebalancing gets larger.

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Table 1: Two-Day Example: Cumulative Returns for No-Rebalance and Daily-Rebalance Strategies

		Cumulative 2-Day Returns				
Daily Index Returns		Index Fund	3x Fund		-3x Fund	
Day 1	Day 2	No-Reb	No-Reb	Daily-Reb	No-Reb	Daily-Reb
10%	-10%	-1%	-3%	-9%	3%	-9%
-10%	10%	-1%	-3%	-9%	3%	-9%
10%	10%	21%	63%	69%	-63%	-51%
-10%	-10%	-19%	-57%	-51%	57%	69%

This table shows cumulative fund returns for five different leverage strategies in a simple 2-day example. The daily excess returns on the underlying index are 10% or -10% with equal probability. The first two columns show the underlying index returns on each of the two days. The remaining five columns show the cumulative 2-day excess returns on 3 different funds: a fund that replicates the underlying index, a leveraged fund that generates a 3x return, and an inverse fund that generates a -3x return. The 3x fund leverages initially to a long position in the underlying index equal to 3 times the initial capital, while the -3x fund takes a short position of 3 times the initial capital. The 3x fund and the -3x fund are assumed to borrow and lend, respectively, at a daily risk-free rate of zero. For the 3x and -3x funds, the table lists the 2-day returns under two different strategies: a no-rebalance strategy (No-Reb) and a daily-rebalance strategy (Daily-Reb) that rebalances to the promised leverage multiple at the end of day 1. For the underlying index fund, the 2-day return is just the cumulative return on the index because its leverage multiple stays at 1 and does not require rebalancing. The 3x fund with no rebalance generates 3 times the cumulative excess return on the index. The 3x fund with a daily rebalance generates 3 times the daily excess return each day, either 30% or -30%, and then produces the cumulative 2-day return accordingly. Similarly, the -3x fund with no rebalance generates -3 times the cumulative excess return on the index, and the -3x fund with a daily rebalance generates -3 times the daily excess return each day, either -30% or 30%, and produces the associated cumulative 2-day return.

Table 2: Sample of ETFs

Ticker	Name	p	Inception	Current Expense Ratio	Assets ($Mill.) 1/3/2011	Assets ($Mill.) 12/31/2025
A. S&P 500 ETFs						
UPRO	ProShares UltraPro S&P 500	3	6/25/2009	0.91%	211	4,735
SSO	ProShares Ultra S&P 500	2	6/19/2006	0.89%	1,623	7,556
SPY	State Street SPDR S&P 500	1	1/22/1993	0.0945%	91,905	712,185
SH	ProShares Short S&P 500	-1	6/19/2006	0.89%	1,537	910
SDS	ProShares UltraShort S&P 500	-2	7/11/2006	0.89%	2,670	360
SPXU	ProShares UltraPro Short S&P 500	-3	6/25/2009	0.89%	281	417
B. NASDAQ-100 ETFs						
TQQQ	ProShares UltraPro QQQ	3	2/9/2010	0.84% (0.97%)	131	29,502
QLD	ProShares Ultra QQQ	2	6/19/2006	0.95% (0.97%)	869	10,745
QQQ	Invesco QQQ	1	3/10/1999	0.18%	21,994	407,779
PSQ	ProShares Short QQQ	-1	6/19/2006	0.95% (0.99%)	197	541
QID	ProShares UltraShort QQQ	-2	7/11/2006	0.95% (1.01%)	609	281
SQQQ	ProShares UltraPro Short QQQ	-3	2/9/2010	0.95% (0.98%)	51	2,305

This table provides descriptive information on the ETFs in the sample: ticker symbols, names, degrees of leverage, inception dates, net expense ratios (with gross expense ratios in parentheses if different), and total assets at the beginning and end of the sample period.

Table 3: Daily Returns

	Index	3x	2x	1x	-1x	-2x	-3x
A. Descriptive Statistics for Daily Excess Returns							
				S&P 500			
Mean	0.058%	0.153%	0.104%	0.058%	-0.050%	-0.102%	-0.155%
Std Dev	1.09%	3.24%	2.16%	1.08%	1.09%	2.16%	3.22%
10	-1.08%	-3.25%	-2.15%	-1.07%	-1.12%	-2.27%	-3.39%
25	-0.37%	-1.10%	-0.74%	-0.37%	-0.57%	-1.14%	-1.70%
Median	0.08%	0.21%	0.13%	0.07%	-0.07%	-0.13%	-0.22%
75	0.57%	1.70%	1.13%	0.58%	0.37%	0.75%	1.11%
90	1.15%	3.41%	2.28%	1.14%	1.08%	2.17%	3.22%
				NASDAQ-100			
Mean	0.077%	0.207%	0.141%	0.076%	-0.069%	-0.141%	-0.214%
Std Dev	1.32%	3.87%	2.61%	1.31%	1.30%	2.61%	3.88%
10	-1.43%	-4.22%	-2.83%	-1.42%	-1.43%	-2.88%	-4.28%
25	-0.48%	-1.42%	-0.96%	-0.47%	-0.74%	-1.45%	-2.20%
Median	0.12%	0.36%	0.23%	0.12%	-0.11%	-0.24%	-0.36%
75	0.74%	2.17%	1.46%	0.74%	0.50%	0.97%	1.42%
90	1.47%	4.28%	2.88%	1.45%	1.42%	2.82%	4.21%
B. Delevered Excess Return Deviations							
				S&P 500			
Mean		-0.0034%	-0.0033%	-0.0005%	0.0030%	0.0014%	0.0011%
Std Dev		0.058%	0.052%	0.057%	0.068%	0.055%	0.065%
Implied Ann. Costs		2.57%	1.66%	0.12%	0.76%	0.71%	0.83%
				NASDAQ-100			
Mean		-0.0043%	-0.0038%	-0.0010%	0.0037%	0.0021%	0.0019%
Std Dev		0.069%	0.053%	0.067%	0.073%	0.058%	0.073%
Implied Ann. Costs		3.25%	1.92%	0.25%	0.93%	1.06%	1.44%
C. Regressions							
				S&P 500			
Intercept		-0.009%	-0.006%	0.000%	-0.003%	-0.004%	-0.006%
		(0.003%)	(0.002%)	(0.001%)	(0.001%)	(0.002%)	(0.003%)
Slope		2.971	1.984	0.992	-0.995	-1.981	-2.955
		(0.003)	(0.002)	(0.001)	(0.001)	(0.002)	(0.003)
R^2		99.72%	99.77%	99.73%	99.61%	99.75%	99.66%
				NASDAQ-100			
Intercept		-0.008%	-0.006%	0.000%	-0.005%	-0.006%	-0.010%
		(0.003%)	(0.002%)	(0.001%)	(0.001%)	(0.002%)	(0.003%)
Slope		2.932	1.976	0.990	-0.987	-1.975	-2.938
		(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)
R^2		99.77%	99.85%	99.75%	99.70%	99.82%	99.72%

Panel A reports means, volatilities, and various percentiles for daily excess returns on leveraged and inverse ETFs and their corresponding underlying indexes. Panel B reports the means and volatilities of delevered daily ETF excess return deviations from their underlying indexes, as well as implied average annual fund costs. Panel C reports coefficients, standard errors, and R-squareds from simple regressions of the daily excess returns of the leveraged and inverse ETFs on the daily excess returns of the underlying index. The sample period is 2011-2025.

Table 4: Multi-Day Returns

Horizon		Index	3x	2x	1x	-1x	-2x	-3x	Obs.
					S&P 500				
5	Mean	0.26%	0.71%	0.48%	0.26%	-0.28%	-0.56%	-0.84%	3768
	Std Dev	2.24%	6.70%	4.47%	2.23%	2.23%	4.45%	6.66%	
21	Mean	1.07%	2.87%	1.96%	1.06%	-1.19%	-2.38%	-3.65%	3752
	Std Dev	4.24%	12.67%	8.47%	4.24%	4.21%	8.34%	12.37%	
63	Mean	3.16%	8.23%	5.68%	3.14%	-3.59%	-7.24%	-11.15%	3710
	Std Dev	6.51%	20.55%	13.40%	6.50%	5.95%	11.34%	16.18%	
252	Mean	13.28%	35.50%	24.20%	13.16%	-13.91%	-26.62%	-38.62%	3521
	Std Dev	12.41%	47.85%	28.09%	12.38%	9.24%	15.81%	19.82%	
Full		477.91%	3356.22%	1571.80%	469.54%	-90.11%	-99.29%	-99.97%	
					NASDAQ-100				
5	Mean	0.35%	0.96%	0.65%	0.35%	-0.38%	-0.76%	-1.15%	3768
	Std Dev	2.69%	8.05%	5.37%	2.68%	2.67%	5.34%	7.99%	
21	Mean	1.45%	3.95%	2.69%	1.44%	-1.61%	-3.25%	-4.98%	3752
	Std Dev	5.14%	15.67%	10.37%	5.14%	4.99%	9.83%	14.45%	
63	Mean	4.36%	11.60%	7.97%	4.30%	-4.87%	-9.80%	-14.96%	3710
	Std Dev	8.15%	26.85%	17.10%	8.15%	7.29%	13.76%	19.40%	
252	Mean	18.63%	51.87%	35.13%	18.37%	-18.20%	-33.94%	-47.81%	3521
	Std Dev	16.96%	60.08%	36.95%	16.92%	12.69%	20.88%	24.70%	
Full		967.02%	11425.64%	4436.29%	933.17%	-95.75%	-99.89%	-100.00%	

This table presents means and volatilities of multi-day excess returns on leveraged and inverse ETFs for holding periods of 5, 21, 63 and 252 days and the multi-day excess returns on the corresponding underlying index. The sample period is 2011-2025.

Table 5: Synthetic 1x Return Deviations

ETF	3x	3x	3x	2x	2x	2x
Weight	0.50	0.60	0.67	0.67	0.75	0.80
ETF	-1x	-2x	-3x	-1x	-2x	-3x
Weight	0.50	0.40	0.33	0.33	0.25	0.20
			S&P 500			
Mean	-0.0061%	-0.0067%	-0.0074%	-0.0049%	-0.0052%	-0.0055%
Std Dev	0.0719%	0.0829%x	0.0969%	0.0561%	0.0603%	0.0673%
			NASDAQ-100			
Mean	-0.0073%	-0.0084%	-0.0094%	-0.0053%	-0.0058%	-0.0062%
Std Dev	0.0744%	0.0876%	0.0845%	0.0550%	0.0564%	0.0644%

This table provides means and volatilities of daily excess return on synthetic 1x strategies less the return on the corresponding 1x fund. Each column is a different strategy. The first four rows indicate the leveraged and inverse funds used in the strategy and the daily weights in these funds. The underlying index is either the S&P 500 or the NASDAQ-100 and the strategies use the corresponding ETFs. The sample period is 2011-2025.

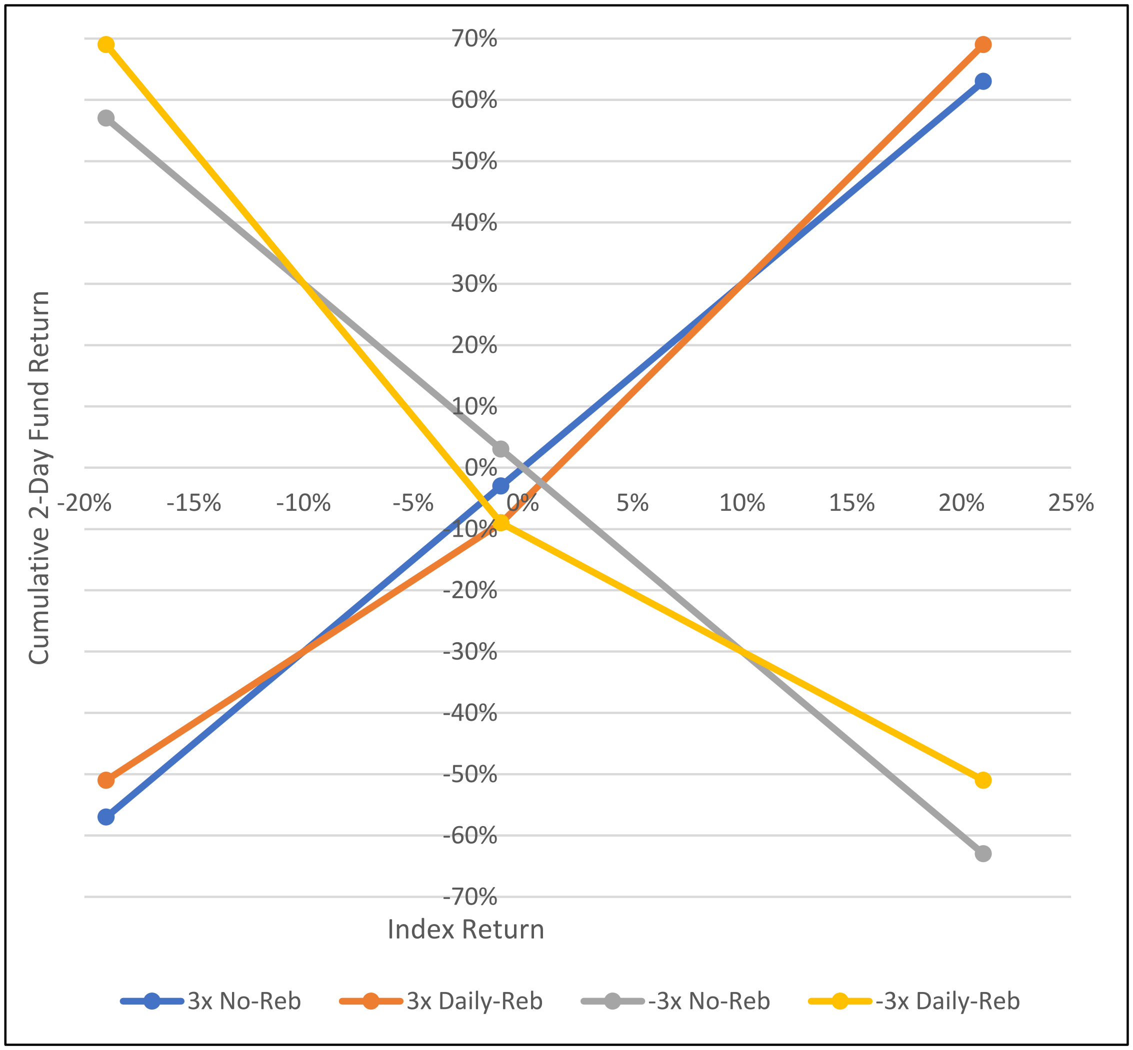
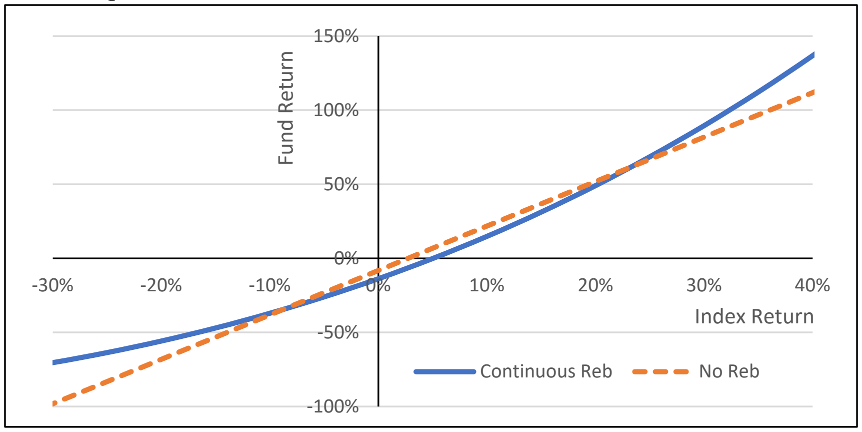


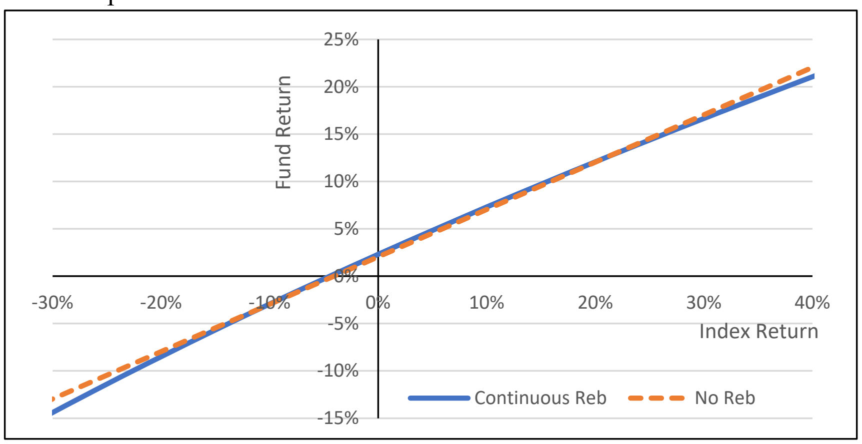
Figure 1: Two-Day Example: Cumulative Returns for No-Rebalance and Daily-Rebalance Strategies

This figure plots the cumulative 2-day returns on leveraged and inverse funds under two different strategies, one with no rebalancing and one with daily rebalancing, based on the example in Table 1.

Panel A: p=3



Panel B: p=0.5



Panel C: p=-2

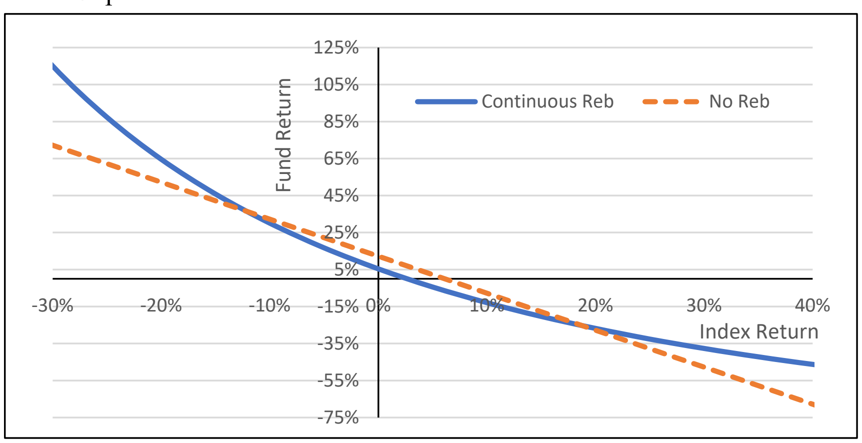
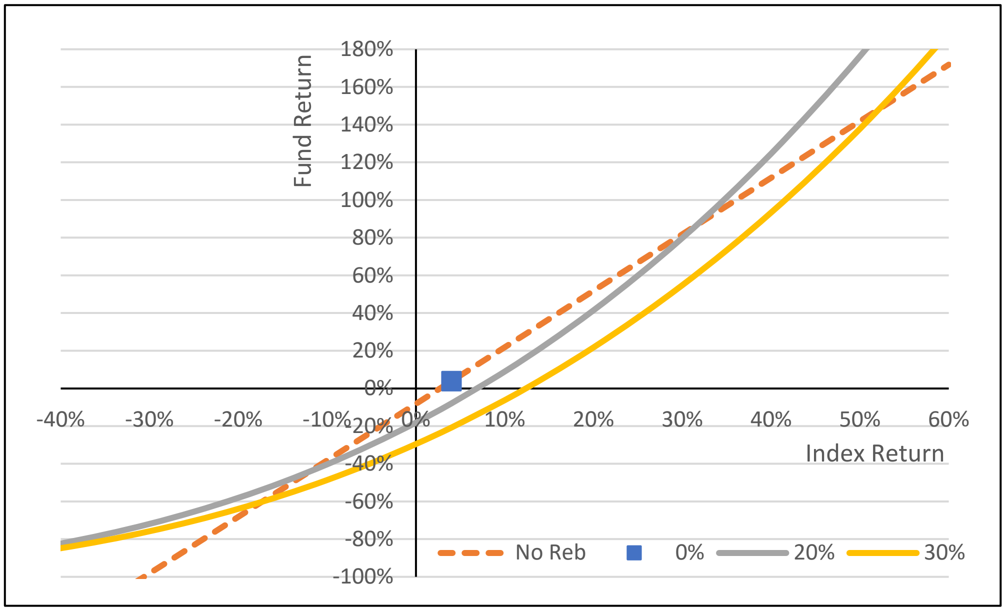


Figure 2: Fund Returns Under Alternative Rebalancing Strategies

This figure plots fund returns under continuous rebalancing (solid line) and no rebalancing (dashed line). The underlying index has a volatility of $|\sigma| = 15\%$, the risk-free rate is $r = 4\%$, and the return horizon is $T = 1$ year. Panels A, B, and C show the cases with multiples of $p = 3$ (leveraged), $p = 0.5$ (deleveraged), and $p = -2$ (inverse), respectively.

Panel A: p=3



Panel B: p=-3

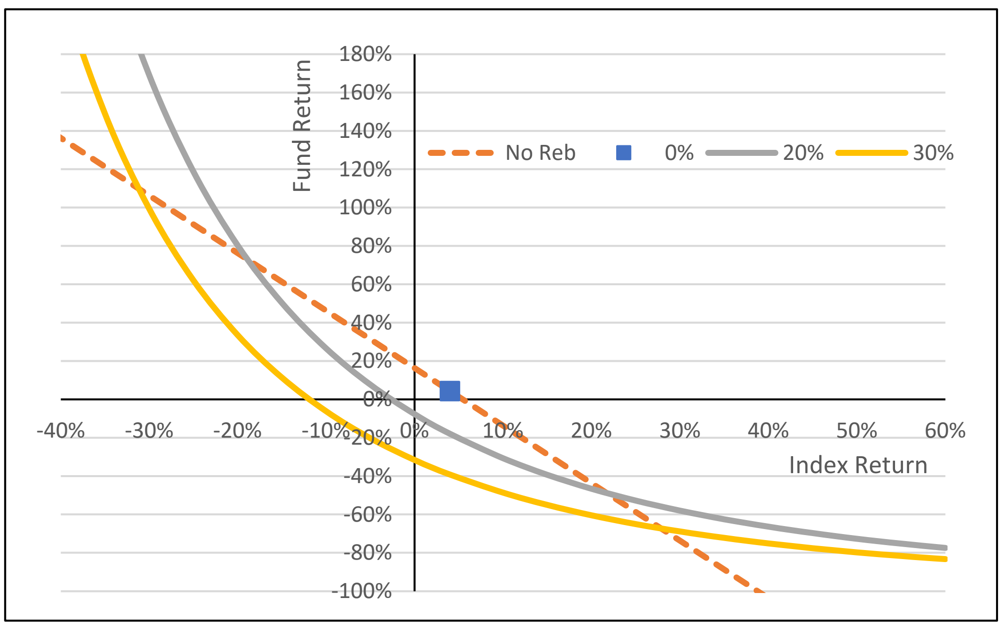
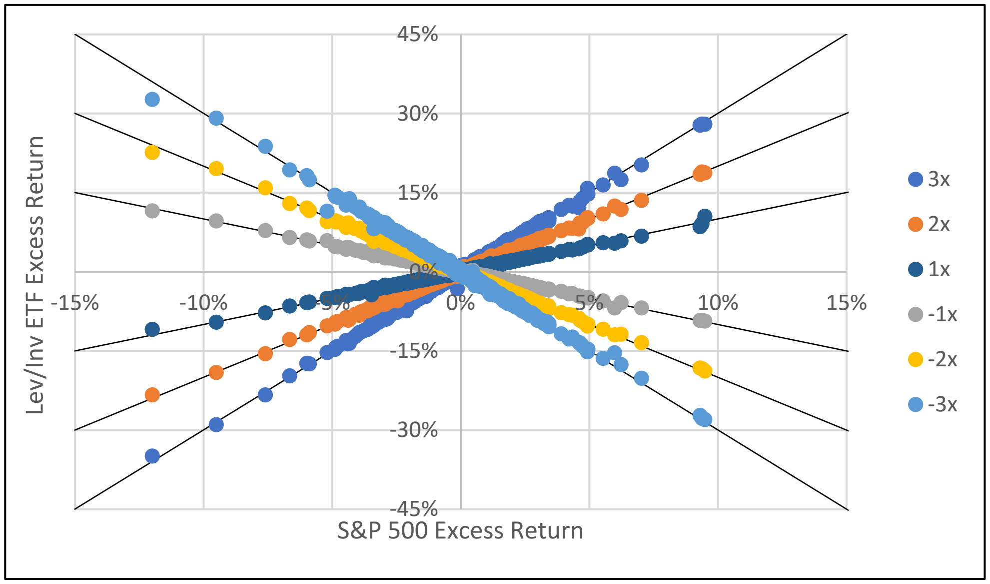


Figure 3: The Effect of Volatility on Fund Returns Under Alternative Rebalancing Strategies

This figure plots fund returns under strategies with continuous rebalancing (solid line) and no rebalancing (dashed line). The underlying index has a volatility of $|\sigma| = 0\%$ (blue), $|\sigma| = 20\%$ (grey), or $|\sigma| = 30\%$ (yellow); the fund has leverage of $p = 3$ (Panel A) or $p = -3$ (Panel B); the risk-free rate is $r = 4\%$; and the return horizon is $T = 1$ year.

Panel A: S&P 500



Panel B: NASDAQ-100

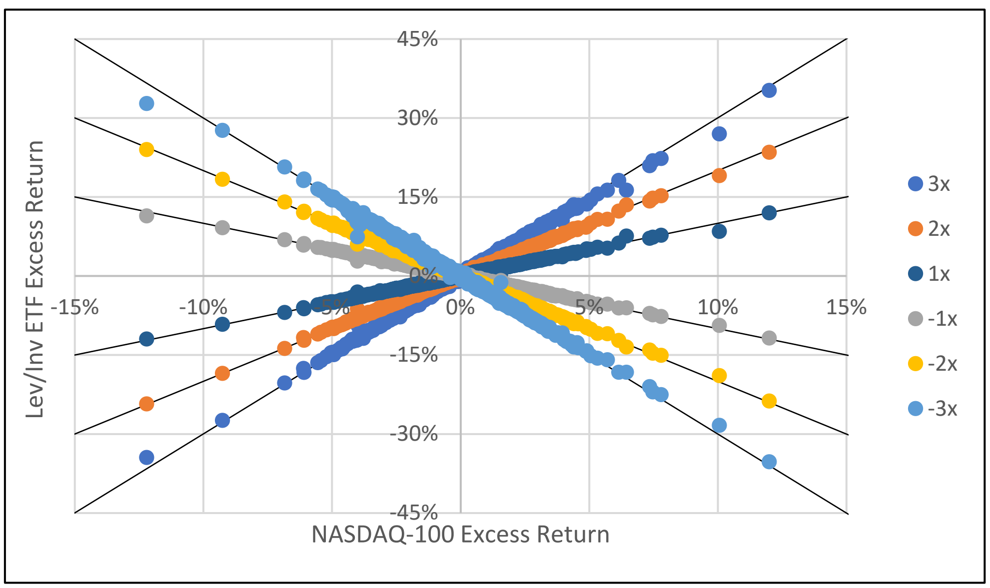


Figure 4: Daily Returns

This figure plots daily excess returns over the period 2011-2025 for leveraged and inverse ETFs ranging from 3x to -3x versus the return on the underlying index. The underlying index is the S&P 500 in Panel A and the NASDAQ-100 in Panel B.

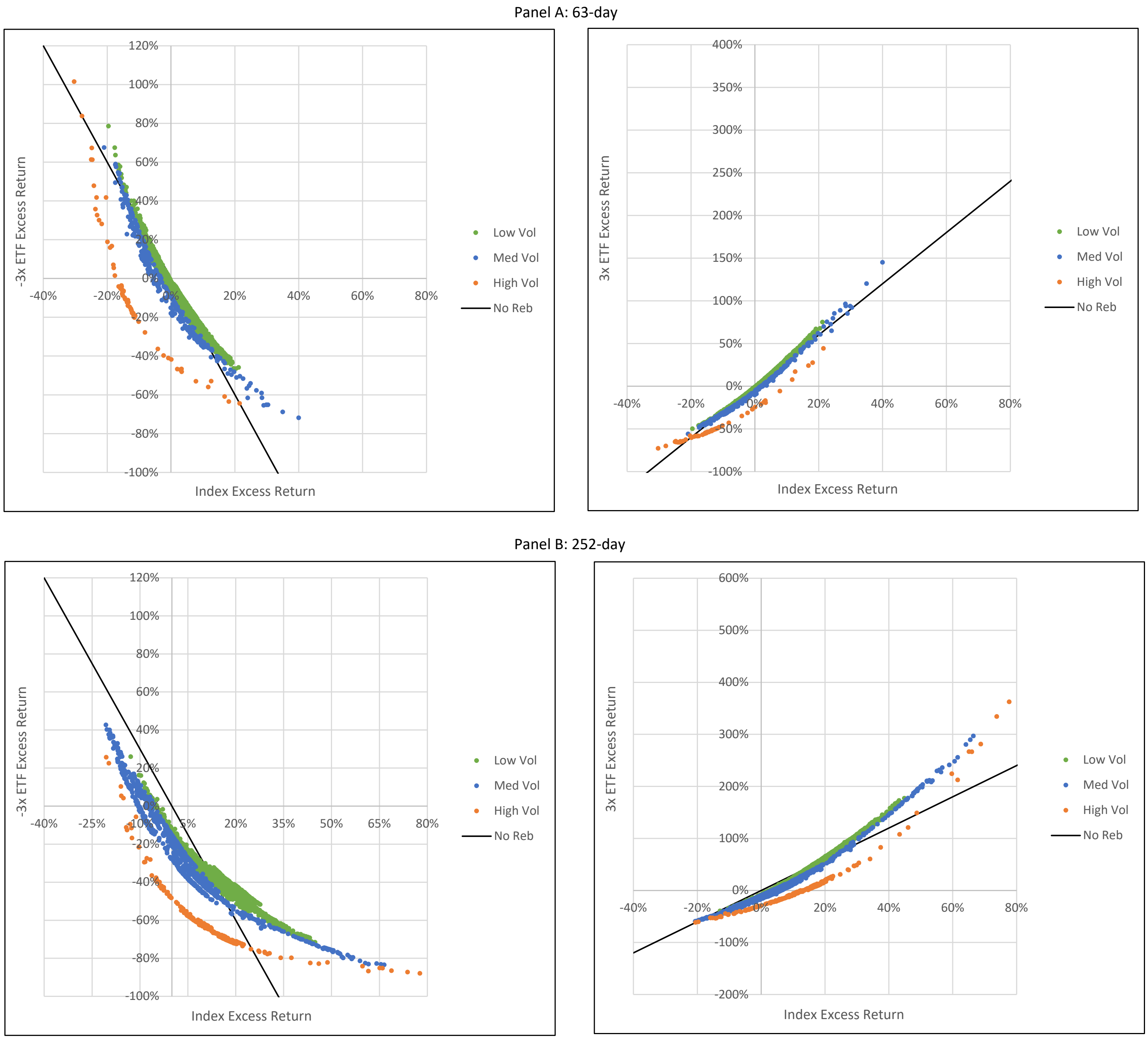
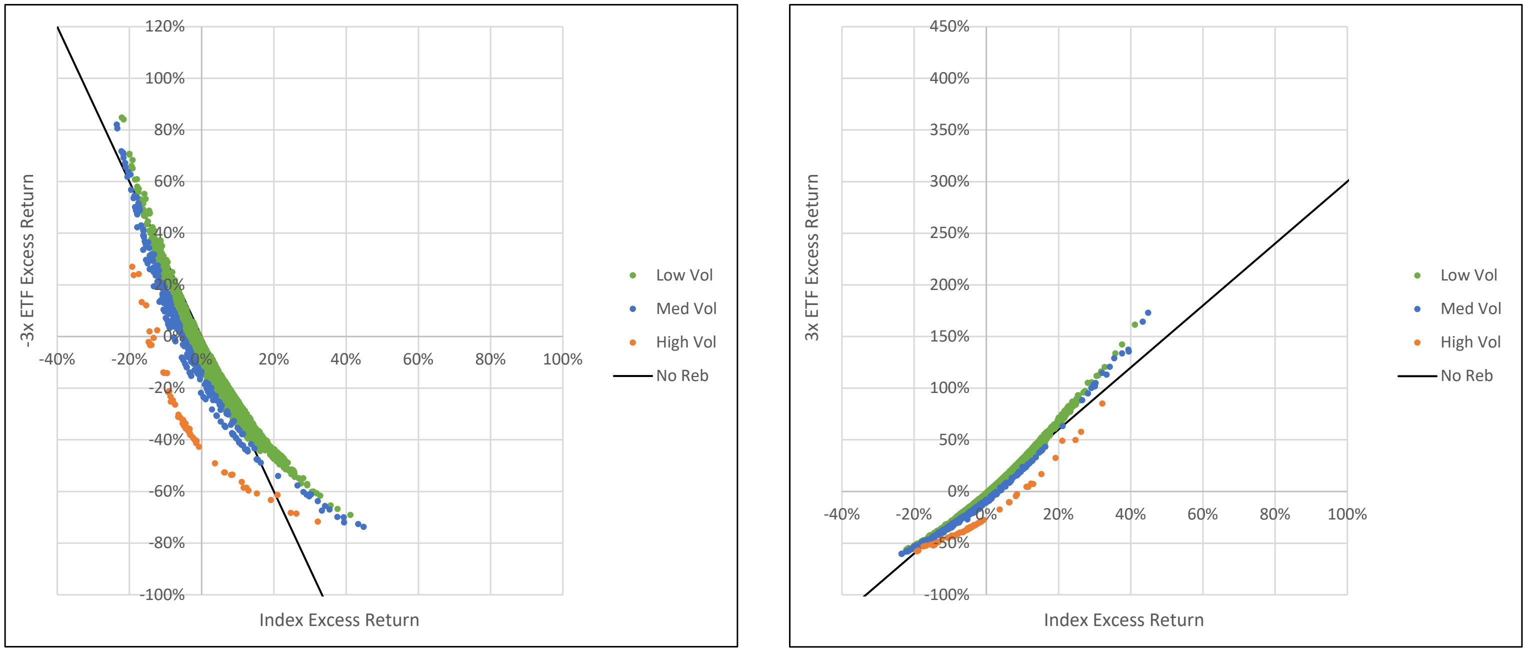


Figure 5: Convexity and Volatility Effects in Realized Leveraged and Inverse ETF Returns: S&P 500 Index
This figure plots 63-day (Panel A) and 252-day (Panel B) excess returns over the period 2011-2025 for -3x and 3x ETFs versus the excess return on their underlying index, the S&P 500, color-coded by the level of the contemporaneous realized volatility of the excess returns on the S&P 500 index.

Panel A: 63-day



Panel B: 252-day

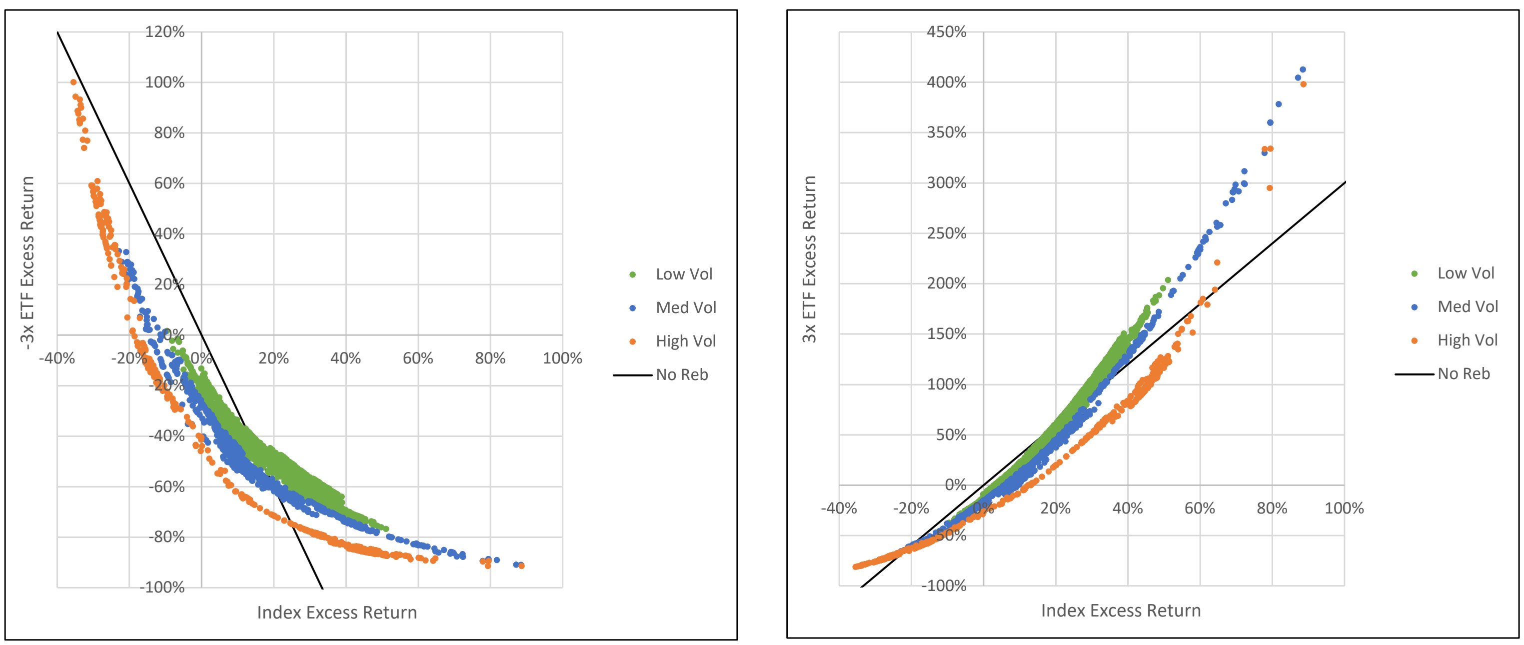


Figure 6: Convexity and Volatility Effects in Realized Leveraged and Inverse ETF Returns: NASDAQ-100 Index
This figure plots 63-day (Panel A) and 252-day (Panel B) excess returns over the period 2011-2025 for -3x and 3x ETFs versus the excess return on their underlying index, the S&P 500, color-coded by the level of the contemporaneous realized volatility of the excess returns on the NASDAQ-100 index.