Online Appendix for

Imperfect Competition in the Interbank Market for Liquidity as a Rationale for Central Banking

(not for publication)
1 Ex-ante liquidity insurance

We have considered liquidity transfers following a liquidity shock, ignoring the possibility for banks to insure against such shocks (e.g., Bhattacharya and Gale (1987), Allen and Gale (2000), Leitner (2005)). To consider this possibility, we modify the model as follows.

At $t = 0$, Bank A can seek liquidity insurance from Bank B. We assume Bank A cannot get liquidity insurance from outsiders. This seems consistent with banks being special in provision of lines of credit, not just to other borrowers, but also to each other.

At $t = 1$, Bank A’s assets need funding of $\rho$ with probability $x$, and no funding otherwise. Whether Bank A incurs a liquidity shock is verifiable. If Bank A incurs a liquidity shock, Bank B’s opportunity cost of capital is non-verifiable and equal to $\mu > 1$ with probability $y$ and 1 otherwise. We assume $p_H R < \mu \rho$, i.e., if Bank B’s cost is high, transfers from Bank B to Bank A are inefficient. At that point, the banks can renegotiate.

We also make simplifying assumptions. First, we assume that all of Bank A’s assets have the same characteristic $\theta$. Second, we assume that Bank A makes Bank B a take-it-or-leave it offer at $t = 0$. Third, we assume that Bank B cannot pledge any of its assets to Bank A. This ensures that if Bank B makes a transfer to Bank A but turns out to have a high cost of capital, Bank A will not transfer back the appropriate amount of liquidity to Bank B. Fourth, we assume that Bank B has full bargaining power in renegotiation. Finally, we assume that outside markets are so weak that only Bank B can re-finance Bank A’s assets, i.e., any loan not refinanced by Bank B must be terminated.

In principle, for each of the three states $\omega \in \{(\rho, 1), (\rho, \mu), (0, 1)\}$, Bank A’s offer specifies a transfer $T(\omega)$ from Bank B to Bank A, a set of assets of measure $\alpha(\omega)$ transferred to Bank B, and a claim $r(\omega)$ by Bank B on Bank A’s remaining assets. However, that Bank B’s cost of capital is unobservable constrains the set of feasible contracts at $t = 0$. It can be shown that the contract’s terms cannot differ across states $(\rho, 1)$ and $(\rho, \mu)$.

**Proposition 1** An optimal contract at $t = 0$ is as follows. Define

$$T^* \equiv \frac{(1 - x) p_H (R - R_b) \rho}{x ((1 - y) (p_B R - p_H (R - R_b)) + y R_H \mu \rho)}.$$

- If $T^* > \rho$, Bank A gets full liquidity insurance from Bank B so that no assets are sold to Bank B in the event of a liquidity shock, i.e., $T(\rho) = \rho$.

- If $T^* < \rho$, Bank A gets only partial liquidity insurance from Bank B, i.e.,

$$T(\rho) = T^*.$$

If Bank A incurs a liquidity shock, Bank B acquires a fraction $\alpha^*$ of Bank A’s assets with

$$\alpha^* = 1 - T^*/\rho.$$

1With heterogeneous values of $\theta$, the optimal contract would generally involve some asset sale to Bank B even when Bank A does not incur a liquidity shock. Indeed, the sale of more liquid loans absent a shock could avoid the sale of less liquid loans in the event of a shock.

2This contract is not uniquely optimal. Indeed, a contract with some asset sales when there is no shock and less sales when there is a shock can also be optimal.
Corollary 1 The fraction of assets Bank A sells after a liquidity shock, and the associated inefficiency increase with the probability $x$ of a liquidity shock, the probability $y$ that Bank B’s cost of capital is high, and with the value $\mu$ of the high cost of capital.

In other words, if a shock is more likely, there is less scope for liquidity insurance. As $y$ or $\mu$ increases, Bank B is less keen to commit to a transfer to Bank A. As an implication, when an aggregate liquidity shortage is more likely, there is less insurance. In turn, even absent an aggregate liquidity shortage, Bank B can exploit its market power against Bank A. Finally, as long as liquidity insurance is only partial, the central bank can improve efficiency by acting as a lender of last resort.

Proof of Proposition 1: Consider $\omega = (\rho, 1)$. Following the transfer $T(\omega)$, Bank A can fund a fraction $T(\omega)/\rho$ of its assets and hence, its expected payoff is $\pi_A(\omega) = p_H (R - r(\omega)) T(\omega)/\rho$. Hence Bank B’s best renegotiation offer ensures Bank A that same payoff, minimizing the fraction $\alpha'$ of assets sold to Bank B, i.e., $1 - \alpha' p_H R_b = \pi_A(\omega)$. (Note that since $r(\omega) \leq R - R_b$, we have $(1 - \alpha') \geq T(\omega)/\rho$, i.e., Bank B does not decrease its transfer to Bank A.) Hence Bank B’s expected payoff is

$$\begin{align*}
\pi_B(\omega) &= \alpha' p_B R + (1 - \alpha') p_H (R - R_b) - \rho \\
&= \left(1 - \frac{(R - r(\omega)) T(\omega)}{R_b}\right) p_B R + \frac{(R - r(\omega)) T(\omega)}{R_b R} p_H (R - R_b) - \rho
\end{align*}$$

Consider $\omega = (\rho, \mu)$. Bank A’s expected payoff is $\pi_A(\omega) = p_H (R - r(\omega)) T(\omega)/\rho$. Bank B’s best renegotiation offer ensures Bank A that same payoff, minimizing transfer $T'$, which amounts to minimizing the fraction $(1 - \alpha')$ of assets retained by Bank A, i.e., $(1 - \alpha') p_H R = \pi_A(\omega)$. (Note that since $r(\omega) \geq 0$, we have $(1 - \alpha') \leq T(\omega)/\rho$, i.e., Bank B does not increase its transfer to Bank A.) Hence Bank B’s expected payoff is

$$\pi_B(\omega) = -(1 - \alpha') \rho \mu = -\frac{(R - r(\omega)) T(\omega)}{R_b} \mu.$$

Consider $\omega = (0, 1)$. It is easily seen that the maximum expected payoff the contract can ensure without asset sales is $\pi_B(\omega) = p_H (R - R_b)$.

If there is no contract at $t = 0$, Bank B’s payoff is zero in all states except $\omega = (\rho, 1)$ in which it can acquire all of Bank A’s assets for no transfer and refinance them, so that its expected payoff is $\pi_B = x(1 - y) (p_B R - \rho)$.

The optimal contract chosen by Bank A at $t = 0$ maximizes $T(\rho)$ subject to

$$(1 - x) \pi_B(0, 1) + x (1 - y) \pi_B(\rho, 1) + xy \pi_B(\rho, \mu) \geq \pi_B.$$

This can be rewritten as

$$T(\rho) \leq \frac{(1 - x) p_H (R - R_b) \rho}{x (R - r(\rho)) \left(1 - y \frac{p_B R - p_H (R - R_b)}{R_b} + y \frac{\mu}{R} \rho\right)}.$$

The constraint is relaxed when $r(\rho)$ is maximized. Hence it is optimal to set $r(\rho) = (R - R_b)$. Given this, the constraint can be rewritten as $T(\rho) \leq T^*$ with $T^*$ as in (1).