Online Appendix:
Aggregate Risk and the Choice between Cash and Lines of Credit*

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Appendix A  Characterization of the equilibrium when \( L^s < L^s_1(\theta) \)

Suppose first that \( q_1 > q_2 \), such that the firm’s budget constraint never binds in equilibrium. In this case, if \( L^s < L^s_1 \) we will have that \( q^* = q_2 > 1 \). To see why, notice that if \( q < q_2 \) then systematic firms would choose \( x^\theta = 1 \), which is not compatible with equilibrium. If \( q > q_2 \), then \( x^\theta = 1 \), generating an excess supply of cash. Thus, we must have \( q^* = q_2 \). Since systematic firms are indifferent between any \( x^\theta \) between 0 and 1 when \( q = q_2 \), we can sustain an equilibrium such that:

\[
\theta[x^\theta(q_2)(\rho - \rho_0) - w^{\text{max}}] = L^s. \tag{1}
\]

This is the unique equilibrium of the model. To see why, notice that for \( x^\theta > x^\theta(q_2) \), cash demand would be larger than supply, and if \( x^\theta < x^\theta(q_2) \), cash supply would be greater than demand and thus the cost of cash would drop to \( q = 1 \).

If \( q_1 < q_2 \), then the firm’s budget constraint will bind in equilibrium, and we will have \( q_1 < q^* \leq q_2 \). The cost of cash \( q^* \) is such that the demand for cash exactly equals supply:

\[
\theta[x^\theta(q^*)(\rho - \rho_0) - w^{\text{max}}] = L^s. \tag{2}
\]

Since \( q_1 < q^* \), then \( x^\theta(q^*) < 1 \). Since \( q^* \leq q_2 \), then systematic firms would like to increase their demand for cash beyond \( x^\theta(q^*) \), but they cannot afford to do so. Thus, \( q^* \) is the equilibrium cost of cash in this case.

Finally, notice that since the cost of cash cannot be greater than \( q_2 \), there is a level of liquidity supply (denoted by \( L^{s}_{\min} \)) such that for all \( L^s < L^{s}_{\min} \), the equilibrium is \( q^* = q_2 \). \( L^{s}_{\min} \) is such that the maximum level of \( x^\theta \) that satisfies the budget constraint when \( q = q_2 \) yields a demand for cash exactly equal to \( L^{s}_{\min} \):

\[
\theta[x^\theta(q_2)(\rho - \rho_0) - w^{\text{max}}] = L^{s}_{\min}. \tag{3}
\]

Appendix B  Computing Beta KMV and Var KMV

To compute Beta KMV and Var KMV we make the following assumptions. First, suppose that the total value of a firm follows:

\[
\frac{dV}{V} = \mu dt + \sigma_V dW \tag{4}
\]

where \( V \) is the total value, \( \mu \) is the expected continuously compounded return on \( V \), \( \sigma_V \) is the volatility of firm value, and \( dW \) is a standard Wiener process. In addition, assume that the firm issued one discount bond maturing in \( T \) periods. Under these assumptions, the equity of the firm is a call option on the underlying value of the firm with a strike price equal to the face value of the firm’s debt and a time-to-maturity of \( T \). The value of the “call option” is:

\[
E = VN(d_1) - e^{-rT}FN(d_2) \tag{5}
\]

where \( E \) is the market value of a firm’s equity, \( F \) is the face value of the firm’s debt, \( r \) is the instantaneous risk-free rate, \( N(.) \) is the cumulative standard normal distribution function, \( d_1 \) is given by

\[
d_1 = \frac{\ln(V/F) + (r + \frac{1}{2} \sigma_V^2)T}{\sigma_V \sqrt{T}}, \tag{6}
\]

\[
d_2 = d_1 - \sigma_V \sqrt{T}.
\]
and $d_2$ is given by

$$d_2 = d_1 - \sigma_V \sqrt{T}$$

Given the value of equity, the underlying value of the firm, or market value of asset is:

$$V = \frac{E + e^{-rT} FN(d_2)}{N(d_1)}$$  \hspace{1cm} (7)

Since the value of equity is a function of the value of the firm and time, using Ito’s lemma we obtain:

$$\sigma_E = V \frac{\partial E}{\partial V} \sigma_V = \frac{V}{E N(d_1)} \sigma_V$$  \hspace{1cm} (8)

To implement the model, we need to simultaneously solve equations (7) and (8). We follow Bharath and Shumway (2008), and adopt an iterative procedure as follows. First, equity volatility $\sigma_E$ is estimated from historical stock returns. We use the last 12 months to do so (e.g., $T = 12$ months). We also set $r = 0.03$. To compute the face value of debt for each firm, we use the firm’s total book value of short-term debt plus one-half of the book value of long-term debt. This is a known rule-of-thumb used to fit a KMV-type model to an annual horizon. Then, we propose an initial value for asset volatility, $\sigma_V$, which is computed as:

$$\sigma_V = \sigma_E \frac{E}{E + F}$$  \hspace{1cm} (9)

We use this value of $\sigma_V$, and equation (7) to infer the market value of the firm’s assets for every month. We then calculate the implied log monthly return on assets, and use that return series to generate new estimates of $\sigma_V$ and $\mu$. Finally, we iterate on $\sigma_V$ until the procedure converges. Similarly to unlevering volatility using (8), asset beta is then unlevered using:

$$\beta_{Asset} = \beta_{Equity} \frac{E}{V} N(d_1)$$  \hspace{1cm} (10)

Finally, we let $Var \ KMV = \sigma_V$, and $Beta \ KMV = \beta_{Asset}$.

**Appendix C Computing Bank VIX**

Three distinct forecasts of daily bank return volatility are computed. The purpose is to construct a forecast of volatility on day $t + 1$ given all information up to and including day $t$.

First, the daily estimates of volatility are computed using the return series available for the financial sector index from Kenneth French’s website. The data span July 1st 1963 through October 29th 2010.

Next, we compute a volatility forecast based on a Gaussian GARCH(1,1) model. This procedure is a fully parametric one and uses a statistical model to forecast future volatilities. The parametric approach requires the estimation of model parameters for which all data up to time $t$ are used. In the case of value-weighted financial sector return series, at least 105 days of observations were required to obtain reliable estimates of the parameters. Hence, the first run of the model uses the sample window $[t_0, t_{105}]$ to estimate the model parameters and subsequently forecasts the volatility on day $t_{106}$. To obtain volatility forecasts for all dates, the procedure is repeated for each individual day on an expanding sample size basis.

Finally, we compute the average yearly value of the expected volatility series ($Bank \ VIX$) to match the frequency of the other data that we use.