A Model of Liquidity Hoarding and Term Premia in Inter-Bank Markets

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Abstract

Financial crises are associated with reduced volumes and extreme levels of rates for term inter-bank transactions, such as in one-month and three-month LIBOR markets. We provide an explanation of such stress in term lending by modeling leveraged banks’ precautionary demand for liquidity. When adverse asset shocks materialize, a bank’s ability to roll over debt is impaired because of agency problems associated with high leverage. Hence, a bank’s propensity to hoard liquidity is increasing, or conversely its willingness to provide term lending is decreasing, in its rollover risk over the term of the loan. High levels of short-term leverage and risk of assets can thus lead to low volumes and high rates for term borrowing, even for banks with profitable lending opportunities. In extremis, there can be a complete freeze in inter-bank markets.
1 Introduction

Extreme levels of inter-bank lending rates, particularly at longer maturities, were seen as a principal problem of the financial crisis of 2007-09 that caused intense financial distress among banks and resulted in large drops in lending to the real economy. Figure ?? shows that the spreads between London Interbank Offer Rate (LIBOR) and Overnight Indexed Swap (OIS) rate for 1-month, 3-month, and 6-month terms, seen as primary measures of this banking stress, increased to over 300 bps at the peak of the crisis, in comparison to spreads of less than 10 bps before the crisis.\(^2\) Rising inter-bank rates have been widely interpreted as a manifestation of rising counterparty risk of borrowing banks. However, during the crisis, banks with even the best credit quality borrowed in term markets at extremely high spreads to the risk-free rate, as shown by Kuo, Skeie and Vickery (2010). This is suggestive of lenders demanding heightened compensation for lending in term inter-bank markets even from relatively safe borrowers. Further, Figure ?? shows the weighted-average maturity of inter-bank term lending estimated by Kuo, Skeie and Vickery (2010). Lending maturities fell from a peak average term of over 40 days before the start of the crisis in August 2007 to less than 20 days after the bankruptcy of Lehman Brothers in September 2008. Consistent with this, Ashcraft, McAndrews and Skeie (2010) and Afonso, Kovner and Schoar (2010) document that volumes in overnight inter-bank markets did not fall much during the crisis in contrast to the collapse in term lending volumes.

We provide an explanation of such stress in term inter-bank markets – a rise in term lending spreads and a significant collapse in term lending volumes – by building a model of lending banks’ precautionary demand for liquidity. Our key insight is that each bank’s willingness to provide term lending (for a given counterparty risk

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\(^2\)The LIBOR-OIS spread is a measure of the credit and liquidity term spread to the risk-free rate for inter-bank loans. LIBOR is a measure of banks’ unsecured term wholesale borrowing rates. OIS is a measure of banks’ expected unsecured overnight wholesale borrowing rates for the period of the fixed-for-floating interest rate swap settled at maturity, where the floating rate is the effective (average) fed funds rate for the term of the swap.
of its borrower) is determined by its own rollover risk, i.e., the risk that it will be unable to roll over its debt maturing before the term of the loan. If adverse asset shocks materialize in the interim, debt overhang can prevent highly leveraged banks from being able to raise financing required to pay off creditors. Thus, during times of heightened rollover risk, such banks anticipate a high cost of borrowing (or even credit rationing) to meet future liquidity shocks and “hoard” liquidity by lending less and more expensively at longer term maturities. Elevated rates for term borrowing, in turn, aggravate the debt overhang and rollover risk problems of other banks. Even strong banks are thus forced to cut back on borrowing term in inter-bank markets and potentially bypass profitable investments such as real-sector lending for long-term and illiquid projects.

We develop these ideas in a model that builds upon the asset-substitution or risk-shifting model of Stiglitz and Weiss (1981), Diamond (1989, 1991), and more recently, Acharya and Viswanathan (2008). In essence, these papers provide a microeconomic foundation for the funding constraints of a leveraged financial firm: the firm can switch to a riskier, negative net present value investment (“loan”) after borrowing from financiers. In anticipation, the financiers are willing to lend to the firm only up to a threshold level of funding so as to ensure there is enough equity to keep the firm’s risk-shifting incentives in check.3 If there is an adverse asset shock, the funding level can fall low enough that the firm is unable to roll over its existing debt. We use this building block of rollover risk to consider inter-bank transactions between two banks: a bank that has access to profitable investment but not enough arm’s length financing to fund it (at least in the short run), and another bank that has surplus funds to potentially lend in the inter-bank market.

Absent rollover risk and risk-shifting (or alternative debt overhang) problems, the inter-bank market achieves the complete redistribution of liquidity that entails the surplus bank lending fully to the profitable one. We show, however, that the risk-shifting problem and attendant funding constraints can produce a fundamental

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3The idea that equityholders may prefer negative NPV risky projects to transfer wealth away from creditors was first noted and modeled by Jensen and Meckling (1976).
deviation in the equilibrium outcome from this frictionless benchmark. For the bank with profitable investment, the willingness to borrow declines as the inter-bank rate rises in order to avoid triggering the risk-shifting problem. For the surplus bank, the willingness to lend long-term against illiquid assets declines unless it is compensated by a suitably higher inter-bank rate. The equilibrium rate is determined by the clearing of this demand and supply of liquidity.

Our main result is that in the extremis there can be a complete freeze in the inter-bank market, in the sense that there is no interest rate at which inter-bank lending will occur. In particular, even when the bank with profitable investment opportunities does not have solvency or liquidity risk, it may be unable to access liquidity on the inter-bank market if the lending banks have high enough short-term leverage and moral hazard costs. In these cases, the lending banks face significant rollover risk so that paying their opportunity cost of liquidity renders borrowers’ investments unprofitable. More generally, when the banking sector is weak, for example, in a crisis (fewer profitable investments, high uncertainty about asset quality and high short-term leverage), lenders’ precautionary demand for liquidity manifests as low volumes and high rates in term inter-bank lending.

It is interesting to contrast our model and analysis to the traditional view of banking panics and runs. For instance, in the canonical model of Diamond and Dybvig (1983), banks fail because of liquidity demand by retail depositors. These depositors receive exogenous liquidity shocks and demand liquidity precipitating bank runs. Our model can be viewed in contrast as one in which liquidity needs of wholesale financiers play a crucial role, as they did in the financial crisis of 2007-09. These providers, such as banks and money market funds, are funded with short-term debt themselves. When adverse asset shocks materialize, for instance, house prices decline, they face the risk of being unable to roll over their debt. Thus, our model provides the microeconomics of what drives liquidity needs of wholesale financiers, prompting them to withdraw liquidity in term inter-bank markets. In Diamond and Dybvig style runs, liquidity demand by depositors leads to liquidations of banking
assets. In our model, liquidity demand by wholesale financiers leads to bypassing of profitable investment opportunities, such as lending to the real sector.\(^4\)

The most important empirical implication of our model concerns the determinants of inter-bank lending rates and volumes. First, controlling for the credit risk of the counterparties that borrow, a bank’s lending rate in the inter-bank market and to the real sector increases with its own credit risk (e.g., balance-sheet leverage) and liquidity risk (e.g., nature of leverage – wholesale deposits relative to retail deposits), and more so at longer maturities. Second, and more uniquely to our model, a bank’s borrowing rate for a particular maturity in the inter-bank market increases with the credit risk and liquidity risk of its lender, controlling for the borrower’s own credit risk. In the same vein, bilateral inter-bank borrowing and lending is more likely to freeze when banks are more leveraged, especially short-term, and are holding riskier and more illiquid assets. These implications suggest important borrower and lender fixed effects that are worthy of detailed empirical investigation of rates and volumes in bilateral inter-bank transactions. At a minimum, our model suggests caution in interpreting all rises in term inter-bank rates as being attributable to counterparty risk concerns.

The remainder of the paper is organized as follows. Section 2 sets up our benchmark model of the inter-bank lending supply. Section 3 examines inter-lending frictions. Section 4 presents equilibrium analysis by including the inter-bank borrowing demand. Section 5 relates the results to existing empirical evidence and derives new implications. Section 6 discusses the related theoretical literature on hoarding by banks and stress in inter-bank markets. Section 7 concludes. Details of proofs and analysis of the borrowing bank demand are contained in the online appendix.

\(^4\)Although we take the presence of short-term leverage of wholesale financiers as given, there is now a large literature explaining why such form of leverage enables them to create liquidity. See Acharya and Viswanathan (2009) for a related model showing that debt with liquidation rights is the optimal contract when borrowers can risk-shift and there is coarseness of verifiable information on asset payoffs. Calomiris and Kohn (1991) and Diamond and Rajan (2001) also explain optimality of demandable debt in models with moral hazard and hold-up problems.
2 Benchmark model

We build a model in which a bank with surplus liquidity lends in the inter-bank market to another bank that has additional capacity for investment in a long term, illiquid asset. We start with the benchmark model in this section, in which the lending bank has existing risky assets and short-term debt in place. We show as a benchmark that absent any frictions, the bank’s short-term debt does not affect inter-bank lending.

The time-line of the overall model is shown in Figure {Timeline}.

There are three periods, dates \( t = 0, 1, 2 \), and two types of banks \( i = B, L \). At date 0, each bank has in place investment in one unit of a long-term illiquid asset that pays \( y \) with probability \( \theta \) and zero otherwise at date 2, where \( \theta y \geq 1 \). Bank \( i = B \) is called the “borrowing bank” because it has an opportunity at date 0 for additional investment of up to one unit into the long-term asset but has no additional liquid goods, which we call “liquidity,” required for the investment. The bank also does not have any additional borrowing sources from outside depositors (at least not in the very short term).

To start with, the focus of the model is on bank \( i = L \), called the “lending bank.” At date 0, this bank has an additional unit of liquidity but has no opportunity for additional investment in the long-term asset. The bank lends \( l \) for two-periods to the borrowing bank at an interest rate of \( r \) and stores liquidity \( (1 - l) \) for a return of one for one period. The borrowing bank invests any borrowed amount into the long-term asset and repays the loan at date 2 with probability \( \theta \).

At date 1, the lending bank has to repay short-term debt \( \rho^L \in [1, 2] \) held by depositors.\(^5\) The lending bank can repay the debt with liquidity \((1 - l)\) and also

\(^5\)The amount \( \rho^L \) reflects the lending bank’s effective short-term leverage in place. This leverage can alternatively be thought of as a broader type of liquidity shock, such as the drawdown of the bank’s extended credit lines. At the minimum value of \( \rho^L \), the bank has sufficient liquidity to repay all short-term debt at date 1. At the maximum value of \( \rho^L \), the bank’s one unit of the long-term asset in place is entirely financed by short-term debt.
attempt to roll over the remaining debt $\rho^L - (1 - l)$ by issuing new debt to depositors with a face amount $f^L$ due at date 2. The bank defaults if it cannot repay or roll over its debt, in which case the proceeds of the bank’s asset-in-place and inter-bank loan have no salvage value to either the bank’s depositors nor itself. Depositors must break even on their expected return $\theta f^L$ to be willing to roll over debt $\rho^L - (1 - l)$ according to their individual rationality constraint

$$\rho^L - (1 - l) \leq \theta f^L.$$  \hfill (1)

The lending bank’s expected profit conditional on rolling over debt is

$$\pi^L = \theta(y + lr - f^L).$$  \hfill (2)

Setting $\pi^L \geq 0$, solving for $\theta f^L$, and substituting into the individual rationality constraint (2), we find the bank’s rollover constraint:

$$\rho^L - (1 - l) \leq \theta(y + lr).$$  \hfill (3)

For a given inter-bank market rate $r$, the bank chooses $l$ to maximize expected profits $\pi^L$ conditional on the rollover constraint holding. Then, the bank’s solution is to lend fully $l^* = 1$ at any rate $r \geq \frac{1}{\theta}$ so that the expected return is greater than storage and to lend nothing otherwise: $l^* = 0$ for $r < \frac{1}{\theta}$. This lending choice maximizes expected profits and best relaxes the rollover constraint. With no agency problems, the bank lends its liquidity fully at any positive expected return because the bank can fully pledge expected returns on the asset-in-place and the inter-bank loan. For $\theta < 1$, credit risk of the borrower is reflected in the term lending rate in the inter-bank market of $r = \frac{1}{\theta} > 1$ but does not affect inter-bank lending volume.

\footnote{For instance, these assets are rendered worthless by disintermediation of the bank or illiquid for a while due to its bankruptcy.}
3 Inter-bank lending frictions

We now examine how frictions affect the supply of inter-bank lending. In Section 3.1, we add an agency problem aimed at capturing opacity of banking assets and activities. In particular, the lending bank can costlessly and unverifiably increase the risk of assets-in-place and cannot pledge any returns of the inter-bank loan. These agency problems limit the amount of inter-bank lending as the lending bank may retain all of its liquidity to ensure roll over of its short-term debt. In Section 3.2, we enrich the model with ex-interim information revelation regarding the credit risk of assets-in-place. This creates rollover risk, or in other words, uncertainty about the ability of the bank to roll over its short-term debt. Under the richer model, precautionary liquidity demand of the lending bank helps determine its supply of inter-bank lending, \( l(r) \), an increasing function of the inter-bank rate.

3.1 Agency problems

We add two agency problems for the surplus bank that capture opacity of financial intermediary activities and show how lending in the inter-bank market tightens its rollover constraint.

One, we assume that the lending bank faces moral hazard. After the bank rolls over its short term debt at date 1, the bank can increase the risk, while decreasing the expected return, of the asset-in-place. Specifically, the bank can receive a bank-specific, higher payoff \( y^L_K > y \); higher risk \( \theta^L_K < \theta \) that is uncorrelated with \( \theta \); and a lower expected return \( \theta^L_K y^L_K \leq \theta y \).\(^7\) The common payoff \( y \) reflects systematic risk, whereas the bank-specific payoff from risk-shifting reflects idiosyncratic risk.

Two, we assume that depositors have limits on the information they can verify about the lending bank’s three types of assets, depending on their opacity. First, inter-bank loans are the most opaque assets to depositors. The lending bank itself

\(^7\)An interpretation of the risk-shifting problem is that the bank decreases its risk management and monitoring of the asset, which leads to the decrease in probability and increase in return of the payoff.
can verify returns on its inter-bank loan, reflecting its ability for peer monitoring. However, the depositors of the lending bank cannot verify any information about the returns of the inter-bank loan because they are the furthest removed from the borrowing bank’s assets that ultimately back the inter-bank loan. Second, the asset in place is held directly by the lending bank and is less opaque to the bank’s depositors than inter-bank loan is. The depositors can verify whether the return on the asset in place is positive or zero. But depositors cannot verify whether the bank increases the asset’s risk and cannot distinguish between whether a positive return is $y$ or $y^L_{R}$. Third, liquidity held by the bank is perfectly transparent and verifiable by depositors, and can be paid out to depositors at date 1.

Under these risk-shifting and verifiability assumptions, the lending bank cannot pledge any returns from the inter-bank loan. An increase in lending by the bank decreases the liquidity $(1 - l)$ that is available to pay depositors at date 1. However, since returns on inter-bank loans are fully internalized by the lending bank, it would not attempt to increase the risk of these returns. To examine the bank’s risk-shifting incentives for the asset in place, consider the four possible states for the lending bank under risk-shifting, conditional on the bank first rolling over its short-term debt. For each state, the date 2 payoffs of the asset in place and inter-bank loan, the state probability, and the lending bank’s profit are as follows:

<table>
<thead>
<tr>
<th>Asset in place</th>
<th>Inter-bank loan</th>
<th>State probability</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y^L_{R}$</td>
<td>$lr$</td>
<td>$\theta^L_{R}\theta$</td>
<td>$y^L_{R} - f^L + lr$</td>
</tr>
<tr>
<td>$y_{R}$</td>
<td>0</td>
<td>$\theta^L_{R}(1 - \theta)$</td>
<td>$y^L_{R} - f^L$</td>
</tr>
<tr>
<td>0</td>
<td>$lr$</td>
<td>$(1 - \theta^L_{R})\theta$</td>
<td>$lr$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$(1 - \theta^L_{R})(1 - \theta)$</td>
<td>0</td>
</tr>
</tbody>
</table>

The expected profit from risk shifting conditional on rolling over debt is thus

$$\pi^L_{R} = \theta^L_{R}(y^L_{R} - f^L) + \theta lr. \quad (4)$$

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8See Acharya and Viswanathan (2009) for a related model showing that debt with liquidation rights is the optimal contract with risk-shifting and coarseness of verifiable information on asset payoffs. Calomiris and Kahn (1991) and Diamond and Rajan (2001) also explain optimality of demandable debt in models with moral hazard and hold-up problems.
Note that the payoff to the lending bank on the inter-bank loan \( lr \) that occurs with probability \( \theta \) is unverifiable to depositors and is not used to repay depositors. The lender only repays \( f^L \) from the payoff \( y^L_R \) of its assets in place.

The lending bank’s incentive constraint requires that profits without risk-shifting given by equation (2) are greater than profits with risk-shifting given by equation (4). This incentive constraint can be written as:

\[
\theta(y - f^L) \geq \theta^L_R(y^L_R - f^L). \tag{5}
\]

The bank’s incentive constraint (5) holds if and only if the realization of \( \theta \) is large enough that \( f^L \leq \frac{\theta y - \theta^L_R y^L_R}{\theta - \theta^L_R} \). For tractability, we assume the risk-shifting payout \( y^i_R \) to bank \( i \) increases as the probability of success \( \theta^i_R \) decreases. In the limit, as \( \theta^i_R \to 0 \), we assume that \( y^i_R \to \infty \) and \( \theta^i_R y^i_R \to k^i \), where \( k^i \) equals the expected return of the risk-shifting assets. The value \( k^i \) is equivalent to an amount of expected profits at date 2 that cannot be pledged at date 1 and represents the severity of the moral hazard problem, or in other words, illiquidity of the asset in place. We consider this limiting case and can write the incentive constraint (5) as

\[
\theta f^L \leq \theta y - k^L. \tag{6}
\]

The bank’s rollover constraint requires both the incentive constraint (6) and individual rationality constraint (1) to hold, which can be written as a constraint on the amount of inter-bank lending:

\[
l \leq \theta y - k^L - \rho^L + 1. \tag{7}
\]

Lemma 1. Interbank lending is constrained to be less than one unit, \( l < 1 \), according to the lending bank’s rollover constraint, if leverage plus moral hazard costs are greater than the expected return on investment: \( \rho^L + k^L > \theta y \).
Now, even if \( r > \frac{1}{\theta} \), such that lending is profitable under the benchmark case, the rollover condition requires the bank to hold enough liquidity, in order to avoid interim liquidation and loss of payoffs at date 2. This is because (i) returns on inter-bank loans \( rl \) are not pledgeable and (ii) the moral hazard amount \( k^L \) cannot be pledged from returns on the asset-in-place. It is worth observing which of the two agency problems plays a critical role in the model. The minimum assumption necessary is that returns on inter-bank loans are not fully verifiable, which gives insight into the illiquidity of inter-bank lending. If the bank could fully borrow against their inter-bank loans when rolling over debt, the bank would always lend its full liquidity for any profitable return \( r > \frac{1}{\theta} \), regardless of the amount of moral hazard \( k^L \). The decrease in inter-bank lending due to opacity of inter-bank loan is, however, exacerbated when a bank’s assets in place are illiquid or subject to moral hazard, which corresponds to the opacity depositors face on bank’s asset value.

Next, to develop this trade-off in a smooth manner and determine a continuous lending supply function \( l(r) \), we add to the model uncertainty about the realization of \( \theta \). This creates rollover risk, or uncertainty regarding the lending bank’s ability to roll over its short-term debt.

### 3.2 Rollover risk

We assume that the asset payoff probability \( \theta \) is drawn at date 1, where \( \theta \) has a distribution \( G(\theta) \) and density \( g(\theta) > 0 \) over \([\theta, \bar{\theta}]\). We define \( \hat{\theta}^L (l) \) as the bankruptcy cutoff value for the lending bank such that for a large enough realization \( \theta \geq \hat{\theta}^L (l) \), the rollover constraint (7) holds. The cutoff \( \hat{\theta}^L (l) \) is thus given by the rollover constraint (7) binding:

\[
\hat{\theta}^L (l) = \frac{\rho^L - (1 - l) + k^L}{y},
\]

where \( k^L \leq \theta y \). The following lemma shows that the rollover risk for the bank increases in leverage \( \rho^L \) and the severity of moral hazard \( k^L \).\(^9\)

\(^9\)We could instead assume that creditors of a bank that defaults at date 1 could collect the return on assets and the inter-bank loan at date 2. Under this assumption, the creditors might
Lemma 2. The lending bank cannot roll over its debt at date 1 if its probability of default is less than its bankruptcy cutoff value: \( \theta < \hat{\theta}^L(l) \). The bank’s cutoff value \( \hat{\theta}^L \), and hence rollover risk, is increasing in leverage \( \rho^L \), the severity of moral hazard \( k^L \), and the inter-bank lending amount \( l \), and is decreasing in the payoff of the asset \( y \) and liquidity held \( 1 - l \).

The bank’s optimization is to maximize expected profits, subject to the rollover constraint, which is given by\(^{10}\)

\[
\pi^L \equiv \int_{\hat{\theta}^L(l)}^{\theta^L} \left[ \theta (y + lr) - (\rho^L - (1 - l)) \right] g(\theta) d\theta.
\]

(9)

For an interior solution \( l^*(r) \in (0, 1) \), the first order condition is

\[
\int_{\hat{\theta}^L(l)}^{\theta^L} (\theta r - 1) g(\theta) d\theta = (k^L + \hat{\theta}^L lr) g(\hat{\theta}^L) \frac{\partial \hat{\theta}^L}{\partial l}.
\]

(10)

The left-hand side of the first order condition gives the benefit of a marginal increase in lending, which is the expected rate of return to the bank on the inter-bank loan whenever bank survives the asset shock at date 1. The right-hand side of the first order condition gives the cost of lending at the margin, which is the marginal increase in bankruptcy risk, \( g(\hat{\theta}^L) \frac{\partial \hat{\theta}^L}{\partial l} \), applied to the bank’s moral hazard cost on asset in place, \( k^L \), and the expected gross lending return \( lr \) at the bankruptcy cutoff \( \hat{\theta}^L \).

Assumption 1. We assume that the second order condition holds for an interior lending solution. We show in the appendix that a uniform distribution \( g(\cdot) \) and sizable enough frictions for leverage \( \rho^L \) and moral hazard \( k^L \) are sufficient.

Lemma 3. The lending bank’s supply on the inter-bank market \( l^*(r) \in [0, 1] \) is increasing in \( r \) and is decreasing in leverage \( \rho^L \) and the severity of moral hazard \( k^L \).

\(^{10}\)We can confine our analysis to considering \( \hat{\theta}^L \leq \bar{\theta} \). The lending bank would not choose \( l(r) > 0 \) such that \( \hat{\theta}^L(l > 0) > \bar{\theta} \). For the case of \( \hat{\theta}^L < \bar{\theta} \), we define \( g(\theta < \bar{\theta}) \equiv 0 \).
The lending bank holds precautionary liquidity to reduce the rollover risk that arises from its short-term leverage and agency costs when there is adverse information revelation in the ex-interim period about its asset quality. The precautionary demand for liquidity held by the lending bank \((1 - l)\) reduces the bank’s inter-bank lending \(l\). When there are no agency problems or rollover risk as in the benchmark model, inter-bank lending helps to overcome bank’s leverage as a part of borrowing bank’s investment opportunity is pledgeable to lending bank’s creditors. However, with agency problems and rollover risk, there is reduction in lending in the inter-bank market.

4 Equilibrium analysis

In this section, we consider loan demand by the borrowing bank to study the inter-bank market in equilibrium. Agency problems and rollover risk for the borrowing bank gives rise to demand for inter-bank borrowing, \(b(r)\), that is a decreasing function of the rate. In the online appendix, we derive the borrowing bank’s demand \(b^*(r)\) to increase investment in the asset-in-place when it faces a risk-shifting problem and has short-term debt, analogous to the lending bank’s optimization for \(l^*(r)\). For the analysis in this section, we simply assume the properties of \(b^*(r)\) that we derive in the appendix: the borrowing demand \(b^*(r) \in (0,1)\) is decreasing in the inter-bank rate \(r\), borrowing bank’s short-term leverage \(\rho^B\), and the severity of its moral hazard \(k^B\); and, the bank does not borrow at an interest rate greater than the return on the asset: \(b^*(r > y) = 0\).

Then, an equilibrium in the inter-bank market is a lending quantity and interest rate pair \((l^*, r^*)\) such that market clearing is satisfied \(l^*(r^*) = b^*(r^*)\), and \(l^*(r^*) \in [0,1]\) and \(b^*(r^*) \in [0,1]\) satisfy the lending and borrowing banks’ optimizations, respectively. We show below that in equilibrium, there is lower inter-bank lending compared to the benchmark, driven both by the demand side of the market as well as the supply side contraction.
4.1 Inter-bank market freeze

We first consider the case of no leverage or moral hazard costs for the borrowing bank, \( \rho^B = k^B = 0 \). In this case, the borrowing bank has a perfectly elastic demand to borrow at a rate that is not greater than the rate on return that investment pays, \( b^*(r) = 1 \) for \( r \leq y \), and not borrow otherwise, \( b^*(r) = 0 \) for \( r > y \). Consider for sake of illustration a uniform distribution of \( g(\theta) \) on the interval \([0, \bar{\theta}]\). Then, there is an explicit solution for the lending bank’s problem:

\[
r^*(l) = \frac{2[y(\bar{\theta} - \hat{\theta}^L) + k^L]}{y[\theta^2 - (\hat{\theta}^L)^2] - 2\hat{\theta}^L l}.
\]

(11)

If the lending bank has no leverage and moral hazard costs, \( \rho^L = k^L = 0 \); as in the benchmark case, then equilibrium lending equals the full available unit of the lending bank’s liquidity, \( l^* = 1 \); and the rate is inconsequential, \( r^* \leq y \).

With frictions, however, the equilibrium lending may fall below the full unit amount that is available. In particular, the lending bank’s leverage and moral hazard can cause a lending freeze in the inter-bank market. In a lending freeze, the lending bank does not supply any amount of the inter-bank loan at an interest below the return on the investment assets. Thus, for parameters such that \( r(0) > y \), a lending freeze occurs. Formally, equation (11) implies that \( r(0) > y \) iff

\[
(\bar{\theta}y - 1)^2 < (\rho^L + k^L - 2)^2 + k^L,
\]

(12)

which is the condition for a lending freeze, satisfied whenever lending bank’s leverage \( \rho^L \) and moral hazard cost \( k^L \) are sufficiently large.

Figure ?? illustrates such an inter-bank lending freeze. The lending bank does not supply lending at any interest below the return on assets: \( l^*(r \leq y) = 0 \).

Next, we consider the general case including rollover risk and moral hazard costs for the borrowing bank, that is, with positive \( \rho^B \) and \( k^B \). The condition for a freeze

\[\text{All numerical illustrations are for a uniform distribution } g(\cdot) \text{ of } \theta \text{ on the interval } [0.4, 1].\]
in the inter-bank market in this case is

\[
\max r(b = 0) \leq \min r(l = 0), \tag{13}
\]

under which there is no interest rate at which inter-bank lending will occur.

Figure ?? demonstrates that in general a freeze can arise at lower interest rates \( r < y \) as well. In this case, an inter-bank market freeze occurs because at rates that generate a positive borrowing demand, the lending bank prefers to hoard liquidity for precautionary reasons rather than lend. This is even though the expected return on lending at these rates is greater than its return on storage of one. The borrowing bank is unwilling to borrow at rates that generate a positive lending supply, even though the rates are less than the rate that additional investment pays. This is because the borrowing bank too is concerned about its own ability to rollover short-term debt.

### 4.2 Inter-bank market stress

More broadly, in an interior equilibrium the amount of inter-bank lending is less than a full unit. Figure ?? illustrates such a case where the lender supply curve and borrower demand curve intersect such that an interior equilibrium is obtained.

In this general case, our general result is that the inter-bank lending quantity is decreasing as the lending bank or the borrowing bank is more leveraged or their moral hazard costs are greater. As the moral hazard problem becomes more severe or as short-term leverage increases for the lending bank, its supply of term lending decreases, driving the equilibrium inter-bank loan amount down. As the moral hazard problem becomes more severe or leverage increases for the borrowing bank, its demand for term borrowing decreases, driving the equilibrium inter-bank loan quantity down as well. As a result, the equilibrium inter-bank lending quantity falls (ceteris paribus). Note that neither bank risk-shifts in equilibrium, but the possibility for risk-shifting in the future creates rollover risk and leads the lending
bank to hoard liquidity in advance and the borrowing bank to reduce borrowing at high rates.

**Proposition 1. Stress in inter-bank lending.** The equilibrium inter-bank lending quantity \( l^* \) is decreasing in the lending and borrowing banks’ leverage \( \rho^i \) and moral hazard \( k^i \): \( \frac{dl^*}{d\rho^i} \leq 0 \) and \( \frac{dl^*}{dk^i} \leq 0 \) for \( i = B, L \).

It is also the case that as moral hazard or leverage increases for the lending bank, the equilibrium inter-bank rate increases. Conversely, as the moral hazard problem becomes more severe or leverage increases for the borrowing bank, its demand for term borrowing decreases, and drives the inter-bank rate down.

**Proposition 2. Stress in inter-bank rates.** The equilibrium inter-bank rate \( r^* \) is increasing in the lending bank’s leverage \( \rho^L \) and moral hazard \( k^L \), \( \frac{dr^*}{d\rho^L} \geq 0 \) and \( \frac{dr^*}{dk^L} \geq 0 \), and decreasing in the the borrowing bank’s leverage \( \rho^B \) and moral hazard \( k^B \), \( \frac{dr^*}{d\rho^B} \leq 0 \) and \( \frac{dr^*}{dk^B} \leq 0 \).

For sake of illustration, Figure ?? shows the decrease in equilibrium lending with increasing moral hazard costs \( k^B \) and \( k^L \). Lending freezes entirely for large enough moral hazard, and similar illustrations are obtained for large enough leverage. Finally, Figure ?? shows that the equilibrium inter-bank rate \( r^* \) increases (sharply) in lending bank leverage \( \rho^L \) and increases (only very gradually) in the borrowing bank leverage \( \rho^B \).

## 5 Empirical predictions and relevance of results

These results on hoarding of liquidity by banks and its effect on inter-bank rates are corroborated by empirical findings in the extant literature. Further, the model also provides testable implications for teasing out borrower demand and lender supply effects in inter-bank markets. We discuss these in turn.
5.1 Extant empirical evidence

5.1.1 Liquidity hoarding and inter-bank markets

Acharya and Merrouche (2009) show empirically that during the first year of the crisis (August 2007 to June 2008), some settlement banks in the United Kingdom voluntarily revised upward their reserve balance targets with the Bank of England. Such revisions followed critical dates of the crisis, such as the asset-backed commercial paper (ABCP) market freeze of August 8, 2007, the collapse of Northern Rock in mid-September 2007 and that of Bear Stearns in mid-March of 2008. In the cross-section of banks, hoarding of liquidity – as measured by increases in reserve balance targets – was greater for banks that had suffered greater equity losses in the crisis (low $\theta$ realization in the model) and had greater reliance on overnight wholesale financing (greater $\rho$). They also document that this hoarding caused increases in inter-bank lending rates for all other banks, in line with our Proposition 2.

Ashcraft, McAndrews and Skeie (2010) provide similar evidence for the United States during the crisis: to insure themselves against intraday liquidity shocks, weaker banks facing heightened rollover risk held larger reserve balances. In particular, they show evidence that banks sponsoring ABCP conduits witnessed increased payments shocks, and that greater payments shocks led to an increase in bank’s reserves. In addition, banks appeared to have responded to higher uncertainty about payments during the crisis by becoming more reluctant to lend excess reserves to other banks when reserves were high. These results are also suggestive of a precautionary demand for liquidity and are entirely consistent with Lemma 3.

5.1.2 Overnight versus term inter-bank markets

Ashcraft, McAndrews and Skeie (2010) and Afonso, Kovner and Schoar (2010) show evidence that overnight inter-bank lending in the fed funds and Eurodollar market increased through much of the early crisis and held up well even after the Lehman bankruptcy. However, this has not been the case for maturities longer than
overnight, especially one-month onwards, for which inter-bank lending volumes generally fell during the heart of the crisis while term spreads increased. Kuo, Skeie and Vickery (2010) show evidence of the decline in the maturity-structure (as illustrated in Figure ??), in line with our Proposition 1. The decrease in term lending provides an explanation for the steady and even increasing levels of overnight inter-bank lending as borrowing banks, facing heightened costs of term borrowing, shift from term to overnight lending. Indeed, Ashcraft, McAndrews and Skeie (2010) show evidence that banks with borrowing constraints increased overnight fed funds lending relative to term lending. Focusing exclusively on the resilience overnight inter-bank volumes can thus paint too rosy a picture of inter-bank markets as it ignores the underlying stress felt in term inter-bank markets.

5.1.3 Counterparty risk or lender’s rollover risk?

Large one-month and three-month LIBOR-OIS spreads during the crisis, as shown in Figure ?? (Kuo, Skeie and Vickery, 2010), measure the cost of inter-bank borrowing for term maturities relative to the expected cost of rolling over overnight borrowing. In empirical work studying these spreads, McAndrews, Sarkar and Wang (2008), Michaud and Upper (2008) and Schwarz (2009) attribute most of the spread to liquidity risk. In contrast, Taylor and Williams (2008a, 2008b) attribute the spread primarily to counterparty credit risk, whereas Smith (2010) argues that time-varying risk premia explain half of the variation in spreads.

Our model clarifies that the term inter-bank spread consists of not just counterparty risk of borrowers but also the rollover risk of lenders (which varied over the course of the crisis and in fact ignited the crisis on August 8 2007 in the first place). Such rollover risk has been considered a key driver of the financial crisis of 2007-09. Acharya, Schnabl and Suarez (2009), for instance, show empirically that the onset of the crisis in August 2007 was due to commercial bank exposures to off-balance sheet vehicles (conduits and SIVs) that held asset-backed (primarily, sub-prime mortgage backed) securities that were funded with extremely short-dated
asset-backed commercial paper (ABCP). When hit by worsening house prices (low \( \theta \) in our model) and the BNP Paribas’s public announcement on 8 August 2007 that sub-prime assets had become practically illiquid (large \( k^L \) and \( k^B \) in our model), these vehicles – and, in turn, the sponsoring commercial banks – faced significant rollover risk.

5.2 Novel empirical predictions

Our model also offers several new testable implications:

(1) First, Lemma 3 shows that a bank’s lending rate for a particular maturity in the inter-bank market and to the real sector increases with its own credit risk (e.g., balance-sheet leverage), illiquidity of assets (e.g., holding of complex assets) and rollover risk (e.g., nature of leverage – wholesale deposits relative to retail deposits), controlling for the credit risk of the counterparties that borrow. More uniquely to our model, Proposition 2 shows that a bank’s borrowing rate for a particular maturity in the inter-bank market increases with credit risk, illiquidity and rollover risk of its lender, controlling for the borrower’s own credit risk.

(2) Second, Figures ?? and ?? suggest tests that combine inter-bank rates and volumes. An increase in the lender or borrower bank’s leverage or illiquidity drives down the bank’s lending supply or borrowing demand for loans, respectively, which decreases the amount of inter-bank lending (Proposition 1). An increase in the lender leverage or illiquidity, however, increases the equilibrium inter-bank rate; in sharp contrast, an increase in the borrower leverage or illiquidity decreases the equilibrium inter-bank rate (Proposition 2). A joint analysis of inter-bank rates and volumes can thus help tease out the effects of lender supply versus borrower demand shifts. To the best of our knowledge, such analysis of inter-bank rates and volumes with borrower and lender fixed effects (or characteristics) has not yet been conducted.

(3) Third, our model suggests that an increase in the risk of asset-level shocks increases term inter-bank rates, and reduces term inter-bank volumes. This is not
just due to an increase in borrower’s credit risk but also due to an increase in the rollover risk of the lender were adverse asset shocks to materialize, as in Propositions 1 and 2. That is, term inter-bank rates and volumes should contain an interaction effect between risk (e.g., realized or implied volatility, as reflected in the VIX) and both borrower and lender leverage and rollover risk.\textsuperscript{12}

(4) Finally, our results indicate that measures of inter-bank market rates such as LIBOR do not necessarily indicate the full breakdown that may occur in the inter-bank market as they do not focus on volumes. When there is a complete breakdown of terms between some borrowers and lenders, the inter-bank rate between some parties is not even well-defined. Rates based on actual or quoted transactions may mask the breakdown in some parts of the market. Hence, measurement and reporting of \textit{volumes} in term inter-bank markets are crucial for understanding the stress and collapse (or lack thereof) in these markets.\textsuperscript{13}

\section{Related literature}

In addition to the empirical literature discussed in the preceding section, there is a growing body of theoretical literature on inter-bank markets. A part of this literature examines the micro-economics of these markets, particularly peer monitoring issues, and another part focuses on consequences of central bank policy on inter-bank markets. Our focus in this paper is on the positive implications for the terms (quantity and interest rates) of liquidity transfers in inter-bank markets when banks have leverage and face attendant agency problems. Hence, we restrict discussion of the related literature on this theme.

Goodfriend and King (1988) provide the benchmark result that with complete markets, inter-bank lending allows for the efficient provision of lending among banks.\textsuperscript{12} Furfune (2010) finds that the LIBOR-OIS spread is related to VIX over time, supporting our prediction, and results could be further tested by examining separate borrower and lender effects.\textsuperscript{13} Documenting transaction volumes that go with one, three and six month LIBOR rates is potentially also important as they are used to index over $360 trillion of notional financial contracts, as estimated by the British Bankers’ Association (BBA), ranging from interest rate swaps and other derivatives to floating-rate residential and commercial mortgages.
The literature has relaxed the assumption of complete markets to obtain deviations from this benchmark result. Flannery (1996), Freixas and Jorge (2007), Freixas and Holthausen (2005) and Heider, Hoerova and Holthausen (2009) consider asymmetric information among banks whereas Donaldson (1992) and Acharya, Gromb and Yorulmazer (2007) consider imperfect competition in inter-bank markets and strategic behavior by relationship-specific lenders. In contrast to these papers, we consider agency problems relating to short-term leverage of banks, illiquidity of assets in place, and the attendant rollover risk.14

Rollover risk in our model induces banks to hold liquidity and raise inter-bank rates or withdraw liquidity altogether from inter-bank markets. The literature has also explored other motives for banks’ desires to hold liquidity in crises. Acharya, Shin and Yorulmazer (2008) derive a strategic motive for holding cash. When banks’ ability to raise external financing is low, they anticipate fire sales of assets by troubled banks and as a result hoard liquidity and forego profitable but illiquid investments. Diamond and Rajan (2009) also study long-term credit contraction that operates through a channel of asset fire sales. During a crisis, banks delay asset sales as part of their efforts to stay alive (a version of the risk-shifting problem). In turn, high rates are required ex ante on term loans to the real sector. Finally, Caballero and Krishnamurthy (2008) derive a propensity for firms to hoard liquid assets and reduce risk-sharing when there is Knightian uncertainty about their risks.

While these papers focus on aggregate liquidity shortages and strategic or behavioral demand for liquidity by bank(er)s, we derive instead a precautionary demand for liquidity by (weak) banks as contributing to heightened borrowing costs for (even safe) banks. In a contemporary paper, Gale and Yorulmazer (2010) model both the precautionary and the strategic motive for holding cash and show that banks may hoard liquidity and lend less than the maximum possible amount, as in our model. In our paper as well as in these other papers, a common theme is that the increase

14Acharya, Gale and Yorulmazer (2008) show how rollover risk can arise upon adverse news even in absence of agency problems. In their model, small liquidation costs can get amplified if debt has to be rolled over frequently relative to the likelihood of arrival of better news. Such rollover risk would also suffice to generate the effects on term-lending we derive in our model.
in bank propensity to hold liquidity is in *anticipation* of crises, rather than (just) upon their incidence. Diamond and Rajan (2005) show how asset liquidations by some banks can ex post cause a decrease in the endogenous amount of aggregate liquid resources available to even fundamentally healthy banks. The contagion in their paper also operates through an increase in inter-bank market rates and results in a decrease lending to the real sector. This is, however, an *ex post* contagion rather than an *ex ante* one (as in our model), i.e., in anticipation of insolvency or rollover risk.

7 Concluding remarks

In this paper, we provided an explanation for stress, and potentially freezes, in term inter-bank lending due to rollover risk of highly leveraged lenders and illiquidity of assets underlying term loans. We showed that the term inter-bank lending rates and volumes are jointly determined, reflecting the precautionary demand for liquidity of lenders and aversion of borrowers to trade at high rates of interest, both induced by their respective rollover risks. The model’s implications are consistent with a range of phenomena observed in inter-bank markets during financial crises. The model also provides implications for future empirical work, especially through its main result that the borrowing rate for a bank is also tied to the lender’s rollover risk, measurable through the lender’s reliance on short-term leverage and illiquidity of its assets.

In future work, it would be fruitful to conduct a fuller welfare analysis of inter-bank market outcomes. We conjecture that if inter-bank loans were partially pledgeable to external financiers, then lending banks’ precautionary demand for liquidity can be excessive relative to its socially efficient level. In such a setting, it seems interesting and important to analyze possible interventions that can address the excessive hoarding of liquidity by highly leveraged banks. Is an unconditional (traditional) lender of last resort (LOLR) in which a central bank provides liquidity to
strong as well as weak banks desirable? Or would it be better to have a solvency-
contingent LOLR in which the central bank provides liquidity only to sufficiently
strong banks? And should there instead (or in addition) be a resolution authority
that forces weak banks to reduce their rollover risk? We conjecture that (i) a reso-
lution authority to address weak banks’ rollover risk, and (ii) a solvency-contingent
LOLR by a central bank that has lower credit and rollover risks than its banks, are
likely to be more effective interventions than the traditional, unconditional LOLR.

Such welfare analysis can also help comparisons with the type of interventions
that were put in place during the crisis, including the Term Auction Facility (TAF)
by the Federal Reserve and several policy interventions by the European Central
Bank (ECB). For instance, our model suggests that the introduction of the Federal
Reserve’s Term Auction Facility (TAF) for 28-day and later 84-day loans should
have decreased rates and increased volumes of lending to the real sector, not only for
banks that used the facilities but also by other banks. In essence, we conjecture that
by acting as a relatively risk-free intermediary, the Federal Reserve intermediated
liquidity hoardings of riskier banks to safer banks that had profitable opportunities.
Appendix

Assumption 1. We make two assumptions that ensure that the second order condition is satisfied. First, we assume a uniform distribution for \( g(\cdot) \), which is always sufficient to satisfy the condition needed for \( g'(\hat{\theta}^L) \) to be not too small. This ensures that the lending bank has a minimal enough increase in its marginal bankruptcy risk for marginal increases in its bankruptcy cutoff value \( \hat{\theta}^L \). Second, we assume large enough parameters for \( k^L \) and \( \rho^L \) relative to \( y \) such that

\[
l > \frac{y}{2r} - \frac{1}{2}(\rho^L + k^L - 1).
\] (14)

Proof of Lemma 3. To study the second order condition of lender’s optimization problem, note that

\[
\frac{\partial^2 \pi^L}{\partial l^2} = -\frac{1}{y}(2\hat{\theta}^L r + \frac{lr}{y} - 1)g(\hat{\theta}^L) - \frac{1}{y^2}(\hat{\theta}^L lr + k^L)g'(\hat{\theta}^L)
\] (15)

\[
= -\frac{g(\hat{\theta}^L)}{y} \left[ 2\hat{\theta}^L r + \frac{lr}{y} - 1 + \frac{1}{y^2}(\hat{\theta}^L lr + k^L)\frac{g'(\hat{\theta}^L)}{g(\hat{\theta}^L)} \right].
\] (16)

For \( g'(\hat{\theta}^L) \geq 0 \), which is satisfied by a uniform distribution for \( g(\cdot) \), condition (14) is sufficient for \( \frac{\partial^2 \pi^L}{\partial l^2} < 0 \). For \( l \in [0, 1] \), we can see that lending is increasing in \( r \), since

\[
\frac{\partial^2 \pi^L}{\partial l \partial r} = \int_{\hat{\theta}^L (l)}^{\hat{\theta}} \theta g(\theta) d\theta - \hat{\theta}^L l \frac{1}{y} g'(\hat{\theta}^L)
\] (17)

\[
\geq \hat{\theta}^L g(\hat{\theta}^L)(1 - \frac{l}{y})
\] (18)

\[
\geq 0,
\] (19)

where the last inequality holds since \( l \leq 1 < y \). Lending is decreasing in \( \rho^L \), since

\[
\frac{\partial^2 \pi^L}{\partial l \partial \rho} = -(\hat{\theta}^L r - 1)g(\hat{\theta}^L) \frac{1}{y} - \frac{lr}{y} g(\hat{\theta}^L) \frac{1}{y} \leq 0,
\] (20)
which is satisfied by condition (14). Lending is also decreasing in $k^L$, since
\[
\frac{\partial^2 \pi^L}{\partial L \partial k^L} = -(\dot{\theta}^L r - 1)g(\dot{\theta}^L)\frac{1}{y} - (1 + \frac{lr}{y})g(\dot{\theta}^L)\frac{1}{y} \leq 0,
\] (21)
which is always satisfied for $l \leq 1$ and $r \leq y$; the borrowing bank demand is never positive for $r > y$, which can be excluded. ■

**Proof of Proposition 2 and 3.** In equilibrium, $l^*(r^*, x) = b^*(r^*, x)$ and hence $\frac{dr^*}{dx} = \frac{db^*}{dx}$ for $x \in \{k^L, k^L, \rho^L, \rho^L\}$. Thus,
\[
\frac{\partial l^*}{\partial x} + \frac{\partial l^*}{\partial r^*} \frac{dr^*}{dx} = \frac{\partial b^*}{\partial x} + \frac{\partial b^*}{\partial r^*} \frac{dr^*}{dx},
\] (22)
so we have
\[
\frac{dr^*}{dx} = \left[\frac{\frac{\partial b^*}{\partial x} - \frac{\partial b^*}{\partial r^*}}{\frac{\partial l^*}{\partial x} - \frac{\partial l^*}{\partial r^*}}\right].
\] (23)
Now, $\frac{\partial b^*}{\partial x} \leq 0$ and $\frac{\partial b^*}{\partial r^*} \geq 0$, therefore $\text{sign}(\frac{dr^*}{dx}) = \text{sign}(\frac{\partial b^*}{\partial x} - \frac{\partial b^*}{\partial r^*})$. For $x = k^L$, $\frac{\partial l^*}{\partial x} = 0$ and $\frac{\partial b^*}{\partial x} \leq 0$, thus $\frac{dr^*}{dx} \leq 0$. For $x = k^L$, $\frac{\partial l^*}{\partial x} \leq 0$ and $\frac{\partial b^*}{\partial x} \leq 0$, thus $\frac{dr^*}{dx} \geq 0$. For $x = \rho^L$, $\frac{\partial l^*}{\partial x} \leq 0$ and $\frac{\partial b^*}{\partial x} \leq 0$, thus $\frac{dr^*}{dx} \leq 0$. For $x = \rho^L$, $\frac{\partial l^*}{\partial x} \leq 0$ and $\frac{\partial b^*}{\partial x} \leq 0$, thus $\frac{dr^*}{dx} \geq 0$.

Consider
\[
\frac{dl^*}{dx} = \frac{\partial l^*}{\partial x} + \frac{\partial l^*}{\partial r^*} \frac{dr^*}{dx}.
\] (24)
For $x = k^L$, as shown above,
\[
\frac{dr^*}{dk^L} = \frac{\frac{\partial l^*}{\partial k^L}}{\frac{\partial l^*}{\partial x} - \frac{\partial l^*}{\partial r^*}},
\] (25)
therefore
\[
\frac{dl^*}{dk^L} = \frac{\partial l^*}{\partial k^L} \left[1 - \frac{\partial l^*}{\partial k^L} \frac{\partial l^*}{\partial r^*} - \frac{\partial l^*}{\partial r^*} \right].
\] (26)
Now
\[
\frac{\frac{\partial l^*}{\partial x}}{\frac{\partial l^*}{\partial r^*}} - \frac{\partial l^*}{\partial r^*} \leq 1
\] (27)
as $\frac{\partial l^*}{\partial r^*} \leq 0$; hence, $\frac{dl^*}{dk^L} \leq 0$. Similarly, as $\frac{\partial l^*}{\partial x} \leq 0$, $\frac{\partial l^*}{\partial r^*} \geq 0$, and $\frac{dr^*}{dx} \geq 0$, we have
\[ \frac{dt^B}{dp^B} \leq 0. \] As \( \frac{dt^B}{dp^B} = 0 \), \( \frac{dr^*}{\partial \tau} \geq 0 \), and \( \frac{dr^*}{\partial \tau} \leq 0 \), we have \( \frac{dt^*}{\partial \tau} \leq 0 \). Finally, as \( \frac{dt^*}{\partial \tau} \leq 0 \), \( \frac{dr^*}{\partial \tau} \geq 0 \), and \( \frac{dt^*}{\partial \tau} \leq 0 \), we have \( \frac{dt^*}{\partial \tau} \leq 0 \). ■

**Borrowing bank**

At date 1, the borrowing bank needs to roll over short-term debt \( \rho^B \) by issuing new debt to depositors with a face amount \( f^B \) due at date 2. Depositors of the borrowing bank can verify whether the quantity \( (1 + b) \) invested in the asset pays off a positive amount but not whether the bank risk-shifts. The bank’s incentive constraint not to risk shift is

\[
\theta[(1 + b)y - br - f^B] \geq \theta^B_R[(1 + b)y^B_R - br - f^B].
\] (28)

Greater amounts of inter-bank borrowing and additional investment into the asset increase the borrowing bank’s moral hazard problem and tighten the incentive constraint.

Similar to the lending bank, the borrowing bank’s incentive constraint (28) holds if and only if the realization of \( \theta \) is large enough that

\[
f^B \leq \frac{(1 + b)(\theta y - \theta^B_R y^B_R)}{\theta - \theta^B_R} - br.
\] (29)

In the limit as \( \theta^B_R \to 0 \) and \( \theta^B_R y^B_R \to k^B \), the incentive constraint (28) is

\[
\theta f^B \leq (1 + b)(\theta y - k^B) - \theta br.
\] (30)

Subject to the incentive constraint holding, the depositors’ individual rationality constraint for rolling over the short-term debt amount \( \rho^B \) is

\[
\rho^B \leq \theta f^B.
\] (31)

The rollover constraint depends on both the incentive constraint (28) and individual
rationality constraint (31) holding:

$$\rho^B \leq \theta[(1 + b)y - br] - (1 + b)k^B. \tag{32}$$

We define $\hat{\theta}^B(b)$ as the bankruptcy cutoff value for the lending bank such that for a large enough realization $\theta \geq \hat{\theta}^B(\cdot)$, the rollover constraint (32) holds. The cutoff $\hat{\theta}^B(b)$ is thus given by the rollover constraint (32) binding:

$$\hat{\theta}^B(b) = \frac{\rho^B + k^B(1 + b)}{(1 + b)y - br}. \tag{33}$$

The rollover risk for the borrowing bank increases in leverage $\rho^B$ and the severity of moral hazard $k^B$.

**Assumption B-1.** We assume large enough moral hazard $k^B$ and not too large leverage $\rho^B$ such that the bankruptcy cutoff is increasing in borrowing, $\frac{\partial \hat{\theta}^B}{\partial b} \geq 0$. Writing

$$\frac{\partial \hat{\theta}^B}{\partial b} = \frac{(k^B + \rho^B)r - \rho^By}{(1 + b)y - br}, \tag{34}$$

$\frac{\partial \hat{\theta}^B}{\partial b} > 0$ iff $k^B$ is large enough and $\rho^B$ is not too large relative to $y$ such that

$$r > \frac{\rho^B}{\rho^B + k^B y}. \tag{35}$$

**Lemma B-1.** The borrowing bank cannot rollover its debt at date 1 if its probability of default is less than its bankruptcy cutoff value: $\theta < \hat{\theta}^B(b)$. The bank’s cutoff value $\hat{\theta}^B$, and hence rollover risk, is increasing in the severity of moral hazard $k^B$, the inter-bank borrowing amount $b$, and the interest rate $r$, and is decreasing in leverage $\rho^B$ and the payoff of the asset $y$.

The borrowing bank’s optimization is $\max_b \pi^B$, where $\pi^B$ expected profits:

$$\pi^B \equiv \int_{\hat{\theta}^B(b)}^{\theta} \{\theta[y + b(y - r)] - \rho^B\} g(\theta) d\theta. \tag{36}$$
For an interior solution \( b^*(r) \in (0, 1) \), the first order condition is

\[
\int_{\theta^B(b)}^{\hat{\theta}} \theta(y - r) g(\theta) d\theta = k^B(1 + b) g(\hat{\theta}^B) \frac{\partial \hat{\theta}^B}{\partial b}. \tag{37}
\]

The LHS of the FOC gives the benefit of a marginal increase in borrowing, which is equals the expected rate of return on investing in the asset minus the rate of return on borrowing, conditional on the borrowing bank meeting its liquidity rollover needs at date 1. The RHS of the FOC gives the cost, which is the increase in bankruptcy risk, \( g(\hat{\theta}^B) \frac{\partial \hat{\theta}^B}{\partial b} \), multiplied by the moral hazard cost \( k^B \) and the amount of all assets \((1 + b)\).

**Remark B-1.** The borrowing bank does not borrow at an interest rate greater than the return on the asset: \( b^*(r > y) = 0 \).

**Proof.** We will show that \( b^*(r > y) = 0 \). To prove by contradiction, suppose instead that \( b(r > y) > 0 \). Positive borrowing \( b^*(r) > 0 \) requires that \( \frac{\partial \pi^B}{\partial b} \geq 0 \). However, with \( r > y \), both terms in the RHS of equation (37) are negative, which implies \( \frac{\partial \pi^B}{\partial b} < 0 \), a contradiction. Thus, \( b^*(r > y) = 0 \). ■

**Lemma B-2.** The borrowing bank’s demand on the inter-bank market \( b^*(r) \in (0, 1) \) is decreasing in the inter-bank rate \( r \), leverage \( \rho^B \), and the severity of moral hazard \( k^B \).

**Proof.** To study the second order condition, for \( g'(\cdot) = 0 \),

\[
\frac{\partial^2 \pi^B}{\partial b^2} = -g(\hat{\theta}^B)[(k^B r)^2 - (\rho^B)^2(y - r)^2] \left[ (1 + b)y - br \right]^3.
\]  \tag{38}

For \( r \leq y \), \( \frac{\partial^2 \pi^B}{\partial b^2} < 0 \) iff \( r > \frac{\rho^B}{\rho^B + k^B} y \), which holds by Assumption 2. Continuing assuming \( g'(\cdot) = 0 \) and \( r > \frac{\rho^B}{\rho^B + k^B} y \), we can see that borrowing is decreasing in \( r \),
since

\[
\frac{\partial^2 \pi^B}{\partial b \partial r} = - \int_{\theta^B(b)}^{\hat{\theta}} \theta g(\theta) d\theta - \hat{\theta}^B g(\hat{\theta}^B)(y - r) \frac{\partial}{\partial r} \hat{\theta}^B - \frac{k^B(1 + b)g(\hat{\theta}^B)(k^B + \rho^B)}{[(1 + b)y - br]^2}, \tag{39}
\]

\[- \frac{2k^B(1 + b)g(\hat{\theta}^B)(k^B + \rho^B)r - \rho^B y) b}{[(1 + b)y - br]^3} \leq 0. \tag{40}\]

Borrowing is decreasing in $\rho^B$ since

\[
\frac{\partial^2 \pi^B}{\partial b \partial \rho^B} = - \frac{\rho^B(y - r)g(\hat{\theta}^B)}{[(1 + b)y - br]^2} \leq 0. \tag{41}\]

Borrowing is decreasing in $k^B$ since

\[
\frac{\partial^2 \pi^B}{\partial b \partial k^B} = - \frac{(1 + b)[k^B(1 + b)y + k^B(1 - b)r] g(\hat{\theta}^B)}{[(1 + b)y - br]^2} \leq 0. \tag{42}\]

Also, note that when Assumption B-1 does not hold, $r < \frac{\rho^B}{\rho^B + k^B y}$, then $\frac{\partial \pi^B}{\partial b} > 0$ and $b^*(r) = 1$. ■

References


