Centralized versus Over-The-Counter Markets

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March 16, 2010

*We are grateful to Rob Engle for insightful discussions. We also thank comments from Eric Ghysels, Martin Oehmke, Marco Pagano (discussant) and seminar participants at the Federal Reserve Bank of New York and Econometric Society Meetings in Atlanta (2010).
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Abstract

The opacity of over-the-counter (OTC) markets – in which a large number of financial products including credit derivatives trade – appears to have played a central role in the ongoing financial crisis. We model such OTC markets for risk-sharing in a general equilibrium setup where agents have incentives to default and their financial positions are not mutually observable. We show that in this setting, there is excess “leverage” in that parties in OTC contracts take on short positions that lead to levels of default risk that are higher than Pareto-efficient ones. In particular, OTC markets feature a counterparty risk externality that we show can lead to ex-ante productive inefficiency. This externality is absent when trading is organized via a centralized clearing mechanism that provides transparency of trade positions, or centralized counterparty such as an exchange that observes all trades and sets prices.

J.E.L.: G14, G2, G33, D52, D53, D62

Keywords: OTC markets, leverage, counterparty risk, externality, transparency, centralized clearing, exchange
1 Introduction and motivation

An important risk that needs to be evaluated at the time of financial contracting is the risk that the counterparty will not fulfill its future obligations. This counterparty risk is difficult to evaluate because the exposure of the counterparty to various risks is generally not public information. Contractual terms such as prices, interest rates and collateral that affect the terms of trade can be tailored to mitigate counterparty risk, but the extent to which this can be achieved, and how efficiently so, depends in general on how contracts are traded.

One possible trading infrastructure is an over-the-counter (OTC) market in which each party trades with another, subject to a bankruptcy code that determines how counterparty defaults will be resolved.\footnote{The bankruptcy code may be specified in the contract or adhere to a uniformly applicable corporate bankruptcy code.} A key feature of OTC markets is their opacity. In particular, even within a set of specific contracts, for example, credit default swaps (CDS), no trading party has full knowledge of positions of others. We show theoretically in this paper that such opacity of exposures in OTC markets leads to an important risk spillover – a counterparty risk externality\footnote{The term “counterparty risk externality” is as employed by Acharya and Engle (2009). A part of the discussion below, especially related to A.I.G. is also based on that article.} – that leads to excessive “leverage” in the form of short positions that collect premium upfront but default ex post and result in inefficient levels of risk-sharing and/or deadweight costs of bankruptcy.

Counterparty risk externality is the effect that the default risk on one contract will be increased if the counterparty agrees to the same contract with another agent because the second contract increases the probability that the counterparty will be unable to perform on the first one. Put simply, the default risk on one deal depends on what else is done. The intuition for our result concerning the inefficiency of OTC markets is that in OTC markets it is not at all transparent what else is being done. Hence, counterparties cannot charge price schedules that effectively penalize the creation of counterparty risk. This makes it likely that excessively large short positions will be built by some institutions without the full knowledge of other market participants. In general, when such institutions were to default, their counterparties would also incur significant losses, creating systemic risk in the economy or more formally, inefficient ex-ante risk-sharing.
For example, in September 2008, it became known that A.I.G.’s liquidity position was inadequate given that it had written credit default swaps (bespoke CDS) for many investors guaranteeing protection against default on mortgage-backed products. Each investor realized that the value of A.I.G.’s protection was dramatically reduced on its individual guarantee. Investors demanded increased collateral – essentially posting of extra cash – which A.I.G. was unable to provide and the Treasury had to take over A.I.G. The counterparty risks were so widespread globally that a default would probably have spurred many other defaults generating a downward spiral.

The A.I.G. example illustrates well the cost that large OTC exposures can impose on the system when a large institution operating in OTC markets defaults on its obligations. But, more importantly, it also raises the question of whether A.I.G.’s true risk as a counterparty was reflected by investors in prices and risk controls for protections they purchased from A.I.G. We argue that the opacity of the OTC markets in which these credit derivatives trade was primarily responsible for allowing the build-up of such large exposures in the first place.

While a number of financial innovations in fixed income and credit markets have traded until now in OTC markets, many products linked to commodity and equities have traded successfully on centralized exchanges. A distinguishing feature of centralized exchange relative to OTC trading is that even though individual agents still do not see each others’ trades, there is a centralized counterparty – the exchange – that sees all trades (at least on all products traded on that particular exchange). Crucially, this enables the exchange to offer individual parties pricing schedules (e.g., collateral arrangements) for trades that are contingent not just on observable or public characteristics (e.g., credit ratings) but also on its own knowledge of other trades (e.g., net positions in CDS contracts). However, exchanges are often viewed as detrimental to ease of search facilitated by bilateral markets, especially for customized or non-standardized financial products.

Indeed, even if trades are not intermediated by a centralized exchange, a centralized clearing mechanism can be arranged to provide the transparency of trades precluded in OTC markets. We show formally that when trading is organized in the form of a centralized clearing mechanism, resulting transparency enables market participants to condition contract terms for each counterparty based on its overall positions. In turn, such conditioning is sufficient to get that party to internalize the counterparty risk externality of its trades. In other words, the moral hazard that a party wants to take
on excessively short positions – collect cash today and default tomorrow – is counteracted by the fact that they face a steeper price schedule by so doing. Effectively, centralized clearing mechanisms is sufficient to achieve the efficient risk-sharing outcome. A competitive centralized exchange also would induce efficient risk-sharing, but in practice, this would be at the cost of restricting all trades, including those involving of non-standardized financial assets, through a single intermediary.

1.1 Model and results

We derive these results in a competitive general equilibrium (GE) model with two periods but allowing for the possibility of default (Geanakoplos, 1997, Geanakoplos and Zame, 1998, Dubey, Geanakoplos and Shubik, 2005). There is a single financial asset, which can be interpreted as a contingent claim on future states of the world, and agents can take long or short positions in the asset. Trades are collateralized by agents’ endowments. When an agent has short positions that cannot be met by the pledgeable fraction of endowment, there is default. The possibility of default (the option to exercise limited liability, to be precise) implies that long and short positions do not yield the same payoff and indeed that there is counterparty risk in trading. We assume a natural bankruptcy rule that illustrates why counterparty risk potentially arises in such a setting. In particular, in any given state of the world, the payoff to long positions is determined pro-rata across positions based on delivery from short positions. This rationing of payments implies that each trade imposes a payoff externality on other trades. We call this spillover a counterparty risk externality.

In this setup, we consider various trading structures and ask whether a counterparty risk externality leads to inefficient risk-sharing. A centralized clearing mechanism guarantees that all trades are observable and agents can set pricing schedules that are conditional on this knowledge. A centralized exchange is a centralized counterparty that observes all trades and can set pricing schedules based on this knowledge. We show that competitive equilibria in economies with a centralized clearing mechanism or a centralized exchange are constrained Pareto efficient.

\[\text{In particular, in case of a centralized bankruptcy mechanism, delivery from short positions is the sum total of full payments from non-defaulting parties and partial payments from defaulting parties; whereas in case of a bilateral bankruptcy mechanism, delivery on short positions is just the partial payment from the defaulting counterparty.}\]
In economies with OTC markets, on the contrary, trades are not mutually observed and thus pricing schedules faced by agents are not conditional on their other trades (even though they might be conditioned on public information about their type, e.g., their endowment level or equivalently their credit rating). We show that competitive equilibria in economies with OTC markets are robustly constrained inefficient. We study two different cases, one in which OTC markets operate with a bilateral netting mechanism and one without bilateral netting. We show that OTC markets are robustly constrained inefficient in both cases. This makes it precise that it is the opacity or lack of transparency of positions in the OTC markets (rather than differences with centralized trading in how bankruptcy is resolved) that leads to ex-ante inefficiency.

The inefficiency in the OTC setting manifests as excessively large short positions as agents do not internalize the default risk these positions impose on other trades in the economy. Intuitively, as long as there is a “risk premium” on the insurance contract (e.g., because the risk being insured is aggregate in nature) and/or the costs of defaulting are not excessively large, the insurer perceives a benefit from building up short positions and defaulting ex post. We interpret this outcome as characterizing excessive “leverage” from an ex-ante standpoint. Interestingly, this implies a lower cost of insurance per unit of promised insurance payoff since the realized insurance payoff is smaller when insurer is more likely to default. In our model, we capture the resulting inefficiency in the form of deadweight costs of bankruptcy, but more generally, it could also manifest as excessive systemic risk from an ex-post standpoint.

Put together, these results imply that a centralized clearing mechanism (or a competitive exchange) are an efficient regulatory response to the moral hazard that in the absence of perfect observability of trades, agents have incentives to take on short positions that allow them to consume today and default tomorrow.

As an extension, we allow agents to alter their production schedules. In this case, the moral hazard of excessive leverage in the OTC case translates into an additional inefficiency in terms of excessive production. This result clarifies that the inefficiency of OTC markets extends beyond just inefficient

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4In other words, the counterparty risk externality is orthogonal to a netting externality – that default decision of one party depends on the default decision of its counterparties. The netting externality also arises in OTC markets operating without a bilateral netting mechanism.
risk-sharing. An example of this inefficiency could be ex-ante systemic risk. Suppose that there is insurance being provided on economy-wide mortgage default rates. This would carry a significant risk premium, giving rise to perverse insurer incentives to default. Thus, in equilibrium, the insurer would take on large and inadequately-collateralized short-selling (of protection) on pools of mortgages and the insured would feed the excessive creation of the housing stock backing such mortgages.5

The remainder of the paper is structured as follows. Section 2 provides a simple example of the counterparty risk externality and the insurance provision with default risk. Section 3 presents our basic equilibrium model, the various trading settings we study, and the welfare analysis. Section 4 discusses several possible extensions of the model. Section 5.3 considers the policy implications of our model for OTC versus centralized clearing. Section 6 relates our work to existing literature. Section 7 concludes.

2 Example

Consider a two-period economy with three types of agents. There are two states of the world at \( t = 1 \), denoted by Good (G) and Bad (B). The probabilities of these states are \( p \) and \( (1 - p) \), respectively. Agents' endowments in the two states are denoted as \( w^i(s) \), \( i = 1, 2, 3 \), and \( s = G, B \). Their initial endowments are denoted \( w^i_0 \). We assume throughout that initial endowments are large enough that there are no default considerations at \( t = 0 \). For simplicity, we also assume that

\[
 w^1(G) > w^2(G) > w^3(G) = 0, 
\]

and

\[
 w^1(B) = w^2(B) = 0 < w^3(B). 
\]

In other words, agents of type 1 and 2 have endowment in good state of the economy, but none in the bad state, whereas agents of type 3 are endowed in the bad state but not in the good state.

The utility of agents of each type satisfies the mean-variance representation:

\[
 E[u(x_0, x(s))] = x_0 + E(x(s)) - \frac{\gamma}{2} \text{var}(x(s)), 
\]

\footnote{This may be a partial explanation of the role played by credit default swap insurances and A.I.G. in fueling the credit boom preceding the crisis of 2007-09.}
where \( x_0 \) is the residual endowment at \( t = 0 \), and \( x(s) \) is the realized endowment at \( t = 1 \), taking account of trades structured at \( t = 0 \) and materialized at \( t = 1 \).

We assume that the only traded contract is an “insurance” (or a credit default protection) which resembles a put option on the bad state of the economy. The contractual payoff of the contract is \( R(G) = 0 \) and \( R(B) > 0 \). For simplicity, we will refer to \( R(B) \) simply as \( R \). Importantly, the economy will allow for default so that the actual payoff on the contract need not coincide with \( R \). The insurance contract must be paid for at \( t = 0 \) and we denote its price as \( q \).

To highlight our main point, we consider agents 1 and 2 purchasing insurance contract from agents 3. We denote the long positions of agents 1 and 2 as \( z_i \geq 0 \), \( i = 1, 2 \), and the short position of agents 3 as \( z_3 \geq 0 \). Note that the only agents that can default given our assumptions are agents 3. We assume that in case they default, they suffer a linear non-pecuniary penalty as a function of the positions defaulted upon, whose pecuniary equivalent in the bad state is given by \( \epsilon z^3 \). Broadly speaking, this penalty can be interpreted as loss of continuation of franchise value in a multi-period setting.

### 2.1 OTC markets

Consider the case of over-the-counter (OTC) trading: agents do not observe the size of the trades put on by other agents and hence prices cannot be conditioned on these. In other words, all agents take the price per unit of insurance as a given constant (and not a schedule depending on total insurance sold by agents 3 in the economy). Agents are fully rational, however, and anticipate correctly the likelihood of default, and its consequent effect on the realized payoff on the insurance contract \( (R^+) \) relative to the promised payoff \( (R) \). Suppose the realized positions on the long positions in state \( B \) is \( R^+ \leq R \). Then, the \( t = 0 \) payoffs to the three agents are

\[
(x_0^1, x_0^2, x_0^3) = (w_0^1 - z^1q, w_0^2 - z^2q, w_0^3 + z^3q),
\]

and \( t = 1 \) payoffs in good and bad states are given respectively as

\[
[x^1(G), x^2(G), x^3(G)] = [w^1(G), w^2(G), w^3(G)],
\]

and

\[
[x^1(B), x^2(B), x^3(B)] = [R^+ z^1, R^+ z^2, w^3(B) - R^+ z^3 - \epsilon z^3 1_D],
\]
where $1_D$ is an indicator variable which takes on value of one if there is default ($R^+ < R$) and zero otherwise.

Then, equilibrium in the economy is characterized by the trading positions, the payoff on the insurance contract (involving the possibility of default) and the cost of insurance: $(z^1, z^2, z^3, R^+, q)$, such that

1. Each agent maximizes its expected utility by choosing its trade positions (as we describe below);
2. Market for insurance clears: $z^3 = z^1 + z^2$; and,
3. In case of default, (we assume that) there is pro-rata sharing of agents' total endowment between the long positions of agents 1 and 2:

$$R^+ = \begin{cases} \frac{w^3(B)}{z^1+z^2} & \text{if } 1_D = 1 \\ R^+ & \text{else} \end{cases}$$

Now, consider agent 1’s maximization problem:

$$\max_{z^1} w^1_0 - z^1 q + pw^1(G) + (1 - p)R^+ z^1 - \frac{\gamma}{2} var(x^1(s)),$$

where

$$var(x^1(s)) = p(1 - p)[w^1(G) - R^+ z^1]^2.$$ 

Then, the first-order condition for agent 1 implies that

$$z^1(R^+, q) = \frac{1}{R^+} \left[ w^1(G) + \frac{(1 - p)R^+ - q}{\gamma p(1 - p)R^+} \right]. \quad (1)$$

Similarly, we obtain for agent 2’s long position that:

$$z^2(R^+, q) = \frac{1}{R^+} \left[ w^2(G) + \frac{(1 - p)R^+ - q}{\gamma p(1 - p)R^+} \right]. \quad (2)$$

In other words, all else equal, agents 1 and 2 purchase more insurance if they have greater endowment in the good state and less so if the cost of insurance rises.

On the other hand, the more insurance agent 3 sells the higher are its incentives to default in state $B$. To clarify agent 3’s choice with regards to
default, consider first the case in which it cannot default. In this case its problem is
\[
\max_{z^3} w_3^3 + z^3 q + (1 - p)[w_3^3(B) - Rz^3] - \frac{\gamma}{2} p(1 - p)[w_3^3(B) - Rz^3]^2,
\]
which yields
\[
z^3_{ND} = \frac{1}{R} \left[ w_3^3(B) - \frac{(1 - p)R - q}{\gamma p(1 - p)R} \right].
\] (3)
In the limit when there are no default costs, that is, \(\epsilon = 0\), agent 3 with position \(z^3_{ND}\) will not default in equilibrium only if
\[
w_3^3(B) \geq Rz^3_{ND},
\]
which turns out to be equivalent to requiring that \(q \leq (1 - p)R\). This condition has the intuitive interpretation that the insurer has incentives not to default ex post only if the price of insurance is smaller than or equal to the expected payoff on the insurance, or in other words, that there is no “risk premium” in the insurance price. This will, however, not hold in equilibrium in general, whenever the insurance is against a risk that cannot be fully diversified away.\(^6\)

More generally, consider then the problem of agent 3, the insurer, when we explicitly allow for default at proportional cost of default \(\epsilon > 0\):
\[
\max_{z^3} \ w_3^3 + z^3 q - (1 - p)\epsilon z^3 - \frac{\gamma}{2} p(1 - p)(\epsilon z^3)^2.
\]
Clearly, the insurer pledges the entire endowment in the bad state at \(t = 1\) in order to collect as much insurance premium as possible at \(t = 0\).\(^7\) Thus,

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\(^6\)This is an important point. Since default is inherently a macroeconomic phenomenon, it explains why there is the moral hazard of default on part of insurers selling credit default swaps (CDS): CDS inherently contain at least some portion of aggregate risk (or in the case of A.I.G., almost all portion). In contrast, there is less risk of such a moral hazard for traditional insurance businesses: Traditional insurances are on risks such as death, accidents, etc., which are easily diversified away across agents in the economy, so that insurers simply earn the actuarially fair premium and do not earn a significant risk premium.

\(^7\)Note that the no-default condition now takes the form:
\[
w_3^3(B) \geq (R + \epsilon)z^3.
\]
from the first-order condition, we obtain that
\[ z^3 = \frac{q - (1 - p)\epsilon}{\gamma p(1 - p)\epsilon^2}. \] (4)

Thus, the lower the cost of default \( \epsilon \) and greater the price of insurance \( q \), the greater is the quantity of insurance supplied by the insurers.

Substituting for \((z^1, z^2, z^3)\) in the market-clearing and bankruptcy conditions of the equilibrium yields two equations in the realized insurance payoff \( R^+ \) and insurance price \( q \) which can be solved to characterize the equilibrium:
\[ R^+(q) = \frac{w^3(B)\gamma p(1 - p)\epsilon^2}{q - (1 - p)\epsilon}, \] (5)
\[ w^3(B) = w^1(G) + w^2(G) + 2\frac{2q}{\gamma p} - \frac{2q}{\gamma p(1 - p)R^+}. \] (6)

Solving this system, we obtain a quadratic equation in the cost of insurance \( q \), which we can solve numerically.

2.2 Numerical example

We parametrize the above economy with \( w^1(G) = 10 \), \( w^2(G) = 5 \), and \( w^3(B) = 10 \). We set \( p = 0.9 \) and vary \( \epsilon \) in the range \([0.1, 1.0]\). Figures 1, 2 and 3 plot respectively the equilibrium quantity of insurance sold \( (z^3) \), its realized payoff \( (R^+) \), and its price \( (q) \), all as a function of \( \epsilon \), the proportional deadweight cost of default.

There is a critical value of \( \epsilon \) below which defaults take place and this value is around 0.548. Above this value, there is no default. Interestingly, for all \( \epsilon \) smaller than this threshold value, the equilibrium is effectively the same as far as risk-sharing is concerned. In particular, agents 3 transfer all their endowment in the bad state at \( t = 1 \) to agents 1 and 2.

To be precise, the equilibrium utilities (relative to \( t = 0 \) endowments) are \((U^1, U^2, U^3) = (-1.97, -0.84, 1.35)\) regardless of \( \epsilon \) in the default range. However, this is not true of equilibrium quantity of insurance and its price.

For example, when \( \epsilon = 0.5 \), the quantities traded are \((z^1, z^2) = (8.22, 2.74)\) with \( z^3 = z^1 + z^2 \); there is 9\% default on the contract \((R^+ = 0.91)\); and, insurance price is \( q = 0.30 \).

In contrast, with \( \epsilon = 0.01 \), the quantities traded become much larger: \((z^1, z^2) = (410.95, 136.98)\); there is 98\% default on the contract \((R^+ = 0.02)\); and, insurance price is \( q = 0.01 \).
In other words, as the default incentives for agent 3 become stronger, there is greater quantity of insurance sold. Thus, there is greater default and greater deadweight costs suffered by agents 3. In turn, the equilibrium insurance price is smaller too. Default by the insurer lowers the price of insurance since the payoff on the contract is rationally anticipated by those purchasing insurance to be smaller. Put simply, the quality of insurance has gone down given default risk of the insurer.

2.3 Inefficiency of OTC markets

The inefficiency of the equilibrium in example above stems from excessive deadweight costs of agent 3’s bankruptcy. This can be seen in Figure 4 which plots the sum of utilities of all three agents and also separately of agents 3. In effect, regardless of the value of $\epsilon$, we obtain full transfer of endowment in the bad state from agents 3 to the other two agents. However, when $\epsilon$ is small, this occurs in equilibrium with agents 3 selling quantities of insurance that lead to their default. Hence, agents 1 and 2 enjoy the same equilibrium utility as $\epsilon$ varies; in contrast, for $\epsilon < 0.548$, default leads to deadweight costs borne by agents 3 and their equilibrium utility is substantially lower compared to $\epsilon \geq 0.548$.

It is clear in this case that the planner can improve upon the OTC case when $\epsilon$ is smaller than 0.548. Essentially, the planner needs to enforce a “position limit” that restricts agents 3 from selling a quantity of insurance $z^3$ that is beyond their endowment in bad state $w^3(B)$. One way in which this position limit can be implemented is through a non-linear pricing schedule: $q(z^3) = 0$ if $z^3 > w^3(B)$, and $q(z^3)$ determined by the markets otherwise. While in our specific example, it is efficient for insurance to be fully collateralized so that any default is ruled out in equilibrium, this is in general not true. What is however true, and we will show below, is that the OTC markets always feature greater likelihood of default in equilibrium compared to its (Pareto) efficient level.

Also beyond this example, the high quantity of insurance sold in OTC markets may also be welfare-reducing if insurance has a moral hazard effect on part of hedgers, in terms of their changing productive investments towards aggregate risky assets. We extend the example to this case in Section 4.2. But first, we derive the result on inefficiency of OTC markets in a setup that generalizes the above example.
3 The economy

We now build on the above example to construct a general model of an OTC market with default risk and compare its properties with those of financial markets regulated by a centralized clearing mechanism. Effectively, we extend a two-period General Equilibrium (GE) exchange economy with default (Geanakoplos, 1997, Geanakoplos and Zame, 1998, Dubey, Geanakoplos and Shubik, 2005) to allow for different mechanisms for financial market trading.

Agents and endowments The economy is populated by $i = 1, ..., I$ types of agents. Let $x_i^0$ be consumption of agent $i$ at time 0. Let $s = 1, ..., S$ denote the states of uncertainty in the economy, which are realized at time 1. State $s$ occurs with probability $p_s$, and $\sum_s p_s = 1$. Let $x_i^1$ be agent $i$’s consumption at time 1, a random variable over the state space $S$: $x_i^1(s)$, for $s \in S$. Let $w_{i0}$ be the endowment of agent $i$ at time 0; and $w_i^1(s)$ her endowment at time 1 in state $s$. The utility of agent $i$ over consumption in state $s$ is denoted as $u_i(x_i^0, x_i^1(s))$ and belongs to the von-Neumann Morgenstern class of expected utility functions.

Financial markets and default We assume, for simplicity, that only one financial asset is traded in this economy, an asset whose payoff is an exogenous non-negative $S$-dimensional vector $R$. We can imagine it representing a derivative contract, e.g., a credit default swap.

Agents selling the asset might default on their required payments. In particular, agent $i$’s short positions are collateralized by the pledgeable fraction $\alpha$ of her endowment at time 1. In other words, in the event of default, creditors (counterparties holding long positions on the asset with the defaulting party) have recourse only to a fraction $\alpha \in [0, 1]$ of the debtor’s endowment $w_i^1(s)$. Only a fraction of the debtor’s endowment can be pledged as collateral, for instance, because part of the endowment is agent-specific. Other than the defaulting agent simply losing her collateral to counterparties, default is assumed to have a direct deadweight cost $\varepsilon z_i^1$ that is proportional on the position defaulted upon. Deadweight costs of default will serve the convenient purpose of providing a bound to short positions on the asset.

Agents trade bilaterally in financial markets. Even though one single asset is traded ex ante, the asset pay-off ex post depends on the type of the agent shorting it, as default decisions in turn do. Let $z_{i+}^0$ be long positions of
agents of type $i$ sold by agents of type $j$. Let $z^+_i = (z^+_i)^j_{j \in I}$ denote the long portfolio vector of agents of type $i$ (with $z^+_i = 0$, by construction). Let $z^-_i$ be the short position of agents of type $i$. As we will explain shortly, all short positions are symmetric for the agents shorting the asset, independently of the counterparty, so that there is no need to index short positions of an agent by the counterparty.

### 3.1 OTC markets

Consider first the case in which trading is intermediated in over-the-counter (OTC) markets. We model OTC markets as standard competitive markets with no centralized clearing or centralized counterparty (such as an exchange). We assume that no creditor has privileged recourse to a debtor’s collateral in case of default. Nonetheless, a bankruptcy mechanism operates to distribute the cash flow delivered on the short positions (full cash flow or endowment recovered in case of default) pro-rata amongst the long positions. To be precise, consider an agent of type $i$ shorting the asset. At equilibrium, the sum total of the repayment cash flow is distributed pro-rata among the holders of long positions with counterparty $i$.\(^8\)

#### The default condition

An agent of type $i$ with (long, short) portfolio position $(z^+_i, z^-_i)$ will default in period 1 in state $s$ iff her income after assets pay off is smaller than the non-pledgeable fraction of her endowment. Let $R^j_i(s)$ denote the payoff in state $s$ of her long asset portfolio with counterparty $j \in I \setminus \{i\}$. The payoff $R^j_i(s)$ is taken as given by each agent, though it is endogenously determined, depending on the equilibrium default rate of agents of type $j$ in the economy (as shown later).

Consider an agent $i$ with a net short position $z^-_i > 0$. She will default on her short position in state $s$ iff:

$$w^i_1(s) + \sum_j R^j_i(s)z^+_i - R(s)z^-_i < (1 - \alpha)w^i_1(s) - \varepsilon z^-_i. \quad (7)$$

Note that we allow for an agent to maintain at the same time both short and long positions on the asset: $z^+_i$ and $z^+_i > 0$, for some $j$. In other words,

\(^8\)In fact, in the model the bankruptcy mechanism will pool all repayments of agents of type $i$ and redistribute them pro-rata to the counterparties. This is without loss of generality, as we concentrate on symmetric equilibria.
we assume that the clearing mechanism provided by OTC markets does not include bilateral netting. We shall study netting later on in the section. Let $I^d(z_+, z_-; i, s)$ be an indicator variable taking on value 1 if agent $i$ with position $(z_+, z_-)$ will default at equilibrium in state $s$, and zero otherwise. Finally, let $I^{ud}(z_+, z_-; i, s) = 1 - I^d(z_+, z_-; i, s)$. Without loss of generality, $I^d(z_+, 0; i, s) = 0$.

Equilibrium payoffs on long and short positions: Since all long positions share pro-rata the payments from defaulting and non-defaulting short positions, the equilibrium payoff of the asset shorted by agent $j$, denoted $R^j(z^j_+, z^j_-; s)$, is given by

$$R^j(z^j_+, z^j_-; s) = \begin{cases} \alpha w^j_1(s) \frac{z^j_-}{z^j_+} \text{ if } I^d(z^j_+, z^j_-; j, s) = 1 \\ R(s) \text{ otherwise} \end{cases}$$

(8)

where $(z^j_+, z^j_-)$ is the portfolio of agents of type $j$ at equilibrium.

Opacity In OTC markets, there is no centralized clearing and disclosure, nor any centralized counterparty that sees all trades. Thus, the trades of each agent $i$, $(z^i_+, z^i_-)$, are not observed in OTC markets by other agents.

Prices and budget constraints Long and short bilateral positions will in general be traded at a price $q^j$, where the apex $j$ denotes the type of the agent in the short position. Importantly though, the price does not depend on her portfolio, since it is not observed.

The budget constraints of agent $i$ in the OTC market are thus given by:

$$x^i_0 + \sum_j q^j z^j_+ - q^j z^j_- = w^i_0,$$

$$x^i_1(s) = \max \left\{ w^i_1(s) + \sum_j R^j(s) z^j_- - R(s) z^j_+ (1 - \alpha) w^i_1(s) - \varepsilon z^i_- \right\}$$

(9)

where $z^j_+, z^j_- \geq 0$, for any $j$.

The unitary price of a bilateral asset trade $z^j_+$ depends on the short agent’s type $j$, as the type determines the agent’s endowment which is public knowledge and affects her probability of default. The fact that the ask price for agent $j$ is instead not conditioned on her trades, as we noted, is the primary
distinction between OTC and markets with a centralized clearing mechanism: contract terms (prices, interest rates, collateral requirements, etc.) are not conditioned on agents’ trades in the case of OTC markets whereas they will be in case of a centralized clearing mechanism (Section 3.3).

**Competitive equilibrium** At the competitive equilibrium, financial markets clear:

\[ \sum_i z^j_i + z^j_+ = 0, \text{ for any } j. \] (10)

Furthermore, at equilibrium, the payoff \( R^j(s) \) must satisfy the consistency condition:

\[
R^j(s) = R^j(z^j_+, z^j_-; s) = \begin{cases} 
\alpha w^j(s) \frac{\partial u^j}{\partial x^i} (x^0_i, x^1_i(s)) & \text{if } I^j(z^j_+, z^j_-; j, s) = 1 \\
R(s) & \text{otherwise}
\end{cases}
\] (11)

Let

\[
m^i(s) = \frac{\partial u^i}{\partial x^i} (x^0_i, x^1_i(s))
\] (12)

denote the marginal rate of substitution between date 0 and state \( s \) at date 1 for agents of type \( i \) at equilibrium. Equilibrium prices are then such that

\[
q^j = \max_i E \left( m^i R^j \right), \text{ for any } j.
\] (13)

### 3.2 OTC with netting

In the OTC markets modeled in the previous section an agent \( i \) is allowed to go both short and long on the asset, and in equilibrium it might be that \( z^+_i > 0 \) and, at the same time, \( z'^+_i > 0 \) with some counterparty \( j \). In this context, a mechanism for bilateral (ex-post, that is, state-by-state) netting might have welfare consequences. While bilateral netting might be difficult to implement in OTC markets, we nonetheless define here an economy with OTC markets and netting so as to be able to better distinguish the welfare effects of various distinct components of OTC and centralized market clearing mechanisms.
We model bilateral netting by requiring that agents are without loss of
generality on one side only of the market, that is, for an agent of type $i$:
\[ z_{ij} + z_i - = 0, \text{ for any } j. \] (14)

As a consequence, an agent of type $i$ with a short position $z_i - > 0$ will default
in state $s$ iff:
\[ w_i^1(s) - R(s)z_i - < (1 - \alpha) w_i^1(s) - \varepsilon z_i - . \] (15)

Let $I^d(z_+; i, s)$ be an indicator variable taking on value 1 if agent $i$ with short
position $z_+$ will default at equilibrium in state $s$, and zero otherwise. Short
positions payoffs are now written
\[ R^j(z_+; s) = \begin{cases} \alpha w_j^1(s) \frac{z_j^+}{z_+} & \text{if } I^d(z_+; j, s) = 1 \\ R(s) & \text{otherwise} \end{cases} \]

and budget constraints of agents $i$ are restricted by $z_{ij} z_i - = 0$, for any $j$.
Finally, at a competitive equilibrium in an economy with OTC markets and
netting, financial markets clear, the consistency condition $R^j(s) = R^j(z_+; s)$
is satisfied, and equilibrium prices are
\[ q_j^i = \max_i E (m_i R^j), \text{ for any } j. \]

### 3.3 Centralized clearing

In the previous section we formalized the competitive equilibrium of an econ-
yomy in which financial market trades are intermediated by an OTC market.
In this section we model instead the operation of a centralized clearing mech-
anism. We model centralized clearing mechanisms as composed of two fund-
amental functions: bilateral netting and transparency. Transparency is
obtained because a centralized clearing mechanism aggregates centrally all
the information about trades and disseminates it to market participants.\(^9\)

Regarding bankruptcy resolution, we continue to assume that no creditor
has direct privileged recourse to a debtor’s collateral, in case of default; and
that, at equilibrium, the sum total of cash flows is distributed pro-rata among

\(^9\)We stress that a centralized clearing mechanism need not centrally intermediate the
trades as, for instance, a centralized exchange would do. We study centralized exchanges
in Section 4.1.
the holders of long positions. Because of bilateral netting, as already noted, an agent \( i \) with a short position \( z^i_+ > 0 \) will default in state \( s \) iff

\[
w^i_1(s) - R(s)z^i_- < (1 - \alpha) w^i_1(s) - \varepsilon z^i_-;
\]

and the equilibrium payoff of the asset shorted by agent \( j \) is thus given by

\[
R_j^j(z^j_-; s) = \begin{cases} \frac{aw^j_1(s)}{z^j_-} & \text{if } I^d(z^j_-; j, s) = 1 \\ R(s) & \text{otherwise} \end{cases}
\]

Because of transparency, each agent in the economy has access to detailed information about all trades and can condition contract terms on this information. We assume that prices are set in a competitive manner. Specifically, agents are price-takers. However, the payoff on the short position of agent \( j \) depends on the position itself, \( z^j_- \), and prices will in general reflect such dependence. Different agents will face different prices, reflecting the probability of default implied by their characteristics: their type as well as their trading positions. This requires us to modify the price-taking assumption for short positions in an important manner (that is similar in spirit to modifications in Acharya and Bisin, 2008, and Bisin, Gottardi and Ruta, 2009).

Specifically, an agent of type \( j \) with short position \( z^j_- \) will face an ask price map

\[
q^j_-(z^j_-) = \max_i E \left( m^i R^j_i(z^j_-) \right).
\]

In other words, an agent of type \( j \) understands that the price he will face for a short position depends on the total short positions he sells, \( z^j_- \). Furthermore, an agent of type \( j \) understands that the price he will face for a short position is equal to the highest marginal valuation across all agents of the payoff associated to his/her position, \( R^j_-(z^j_-) \). Price taking is then represented by the fact that agents take the vector of pricing kernels \( (m^1, \ldots, m^i, \ldots, m^I) \) as given. On the other hand, regarding long positions, the payoff \( R^i(s) \) is taken as given by each agent, and so is the price \( q^i \). The budget constraints of agent \( i \) are thus given by:

\[
\begin{align*}
x^i_0 + \sum_j q^j_+ z^j_+ - q^j_-(z^i_-)z^i_+ &= w^i_0, \\
x^i_1(s) &= \max \left\{ w^i_1(s) + \sum_j R^j(s) z^j_+ - R(s)z^i_-, (1 - \alpha) w^i_1(s) - \varepsilon z^i_- \right\}
\end{align*}
\]

where \( z^j_+, z^j_- \geq 0, z^j_+ z^j_- = 0 \), for any \( j \).
At competitive equilibrium, the portfolios demanded by all agents clear:

\[ \sum z_{i+}^{ij} - z_{i-}^j = 0, \text{ for any } j, \]  

(18)

and the price maps and returns are rationally anticipated by agents:

\[ q^j = q_{i+}^j(z_{i+}^j) = \max_i E (m^i R^j (z_{i-}^j)), \]

(19)

\[ R^j (s) = \begin{cases} \frac{\omega w_i(s)}{z_{i-}^j} i f I_d(z_{i+}^j, z_{i-}^j, s) = 1 \\ R(s) \text{ otherwise} \end{cases} \]

(20)

3.4 Welfare

How does the competitive equilibrium under OTC markets compare in terms of efficiency properties to the competitive equilibrium under centralized exchange? To answer this question, we write down the constrained Pareto efficient outcome as the solution to the following problem:

\[ \max \left( x^i_0, x^i_1 \right) \sum \lambda^i E \left( u^i \left( x^i_0, x^i_1 \right) \right) \]

s.t.

\[ \sum_i x^i_0 - w^i_0 = 0, \sum_i x^i_1(s) - w^i_1(s) = 0, \text{ for any } s \]

\[ x^i_1(s) = \max \left\{ w^i_1(s) + \sum_j R^j(s) z^j_{i+} - R(s) z^j_{i-}, (1 - \alpha) w^i_1(s) - \varepsilon z^i_{i-} \right\}, \]

(21)

\[ R^j(z_{i+}^j; s) = \begin{cases} \frac{\omega w_i(s)}{z_{i-}^j} i f I_d(z_{i+}^j, z_{i-}^j, s) = 1 \\ R(s) \text{ otherwise} \end{cases} \]

(22)

where \( z^j_{i+}, z^i_{i-} \geq 0, z^j_{i+} z^i_{i-} = 0, \text{ for any } j. \)

(23)

(24)

where \( \lambda^i \) is the Pareto weight associated to agents of type \( i. \)

This is the standard constrained efficiency problem for a GE economy once it is assumed that default is not controlled by the planner. The constraint (21) serves two purposes:
(i) it restricts the planner’s allocations to those that can be achieved with the limited financial instruments available in the economy; and

(ii) it accounts for the fact that each agent can choose to default or not, in each state $s$; consumption in a default state $s$ is $(1 - \alpha) w^1_1(s) - \varepsilon z^1_-$, the non-pleadgable fraction of endowment net of the deadweight costs.

3.5 Results

We can derive the following results on the constrained efficiency of the centralized exchange economy and the (generic) constrained inefficiency of the economy with OTC markets:

**Proposition 1.** Any competitive equilibrium of an economy with a centralized clearing mechanism is constrained Pareto optimal.

The intuition for efficiency of the centralized exchange economy is that each agent $i$ that is short on the asset faces a price $q^i_1 (z^+_i) = \max_i E \left( m^i R^i_1 (z^+_i) \right)$ that is conditioned on her positions. Consequently, she internalizes the effect of her default on the payoff of long positions on the asset $R_1^j (s)$. The observability of all trades allows for conditioning of prices based on this information and hence in turn it enables the economy with default risk to get agents to internalize any externality their default choices impose (in terms of inefficient risk-sharing) on other agents due to positions that lead to “excessive” defaults ex post.

It is straightforward to show, on the contrary, that equilibria in economies with OTC markets are not in general constrained Pareto efficient. First of all consider an economy with OTC markets without bilateral netting. As we noted, in this case, an agent of type $i$ with a short position $z^+_i > 0$ will default in state $s$ iff:

$$x^i_1(s) = \max \left\{ w^i_1(s) + \sum_j R_j^i(s) z^j_+ - R(s) z^-_i, (1 - \alpha) w^i_1(s) - \varepsilon z^-_i \right\}. \quad (26)$$

Formally, the constraint includes the incentive compatibility constraint for each agent’s choice of default:

$$u^i(x^i_0, x^i_1(s)) \geq u^i(x^i_0, (1 - \alpha) w^i_1(s) - \varepsilon z^-_i). \quad (25)$$
The default decision of an agent of type $i$, therefore, depends on $R_j^i(s)$, that is, on agents $j$’s default decisions, which in turn depend on $i$’s default decisions, introducing a netting externality at equilibrium.

It is the case then that,

*Competitive equilibria of economies with OTC markets without netting are robustly not Pareto efficient.*

The proof of this statement, however, requires some complex differential computations and is omitted. It is an adaptation of that in Bisin, Geanakoplos, Gottardi, Minelli, and Polemarchakis (2001).

With netting, on the other hand, an agent $i$ with a short position $z_i^i > 0$ will default in state $s$ iff:

$$w_i(s) - R(s)z_i^i < (1 - \alpha) w_i(s) - \varepsilon z_i^i.$$  

(27)

No netting externality arises then in this case. We shall show however that equilibria of an economy with OTC markets and netting are also typically constrained inefficient. In other words, the transparency provided by centralized clearing mechanism (and not by OTC market economies, with or without netting) is typically required for constrained efficiency.

**Proposition 2.** Competitive equilibria of economies with a centralized clearing mechanism cannot be robustly supported as equilibria in economies with OTC markets, with or without netting. More specifically, any competitive equilibrium of the economy with centralized clearing mechanism in which default occurs with positive probability cannot be supported in the economy with OTC markets, with or without netting.

The intuition is that in OTC markets, with or without netting, each agent $j$ that is short on the asset faces a price $q^j$ that is not conditioned on her position $z_j^j$. Consequently, she does not internalize the effect of her default on the payoff of long positions on the asset $R^j$. This is a counterparty risk externality in addition to the netting externality. More generally, it is also the case that

\[\text{Formally, by robustly we mean: for an open set of economies parametrized by agents’ endowments and preferences.}\]
Competitive equilibria of economies with OTC markets and netting are robustly not constrained Pareto efficient.\textsuperscript{12}

Finally, let the leverage of agent \( j \), \( L^j \), be defined as the value of her (promised) debt divided by the value of her endowment

\[
L^j = \frac{E(m^j Rz^j)}{E(m^j w_1^j)}.
\]

(28)

Then,

**Proposition 3.** For deadweight costs \( \varepsilon \) small enough, competitive equilibria of economies with OTC markets, with or without netting, are characterized by weakly greater (and robustly by strictly greater) leverage and default with respect to equilibria of the same economy with a centralized clearing mechanism.

Since ask prices in economies with OTC markets, with or without netting, do not penalize the short positions for their own incentives to default, agents have incentives to exceed the Pareto efficient short positions. Indeed, the proof of these main propositions in the Appendix shows that as long as price on the short position is positive, which is robustly the case in equilibrium, there is incentive to go excessively short, collect the premia up front, and default ex post. This increases the equilibrium default rate and leads to inefficient risk-sharing.\textsuperscript{13} For efficient risk-sharing, it is in general necessary to be able to commit to future payoffs on financial assets, but in OTC markets, such commitment is not enforced through prices and incentives to go excessively short and default dilute the claims of shorting agent’s counterparties.

**Opacity and counterparty risk externality** When combined together, Propositions 1, 2, and 3 imply that a centralized clearing mechanism is an efficient response to the moral hazard that in the absence of perfect observability of trades, agents have incentives to take on short positions that allow

\textsuperscript{12}We omit the proof of this statement, once again, to avoid some complex differential computations.

\textsuperscript{13}If \( \varepsilon = 0 \), \( z^j \) is unbounded and, strictly speaking, the economy has no equilibrium. This is just a extreme case, which is of interest to identify the “force” towards borrowing and default built into our model of OTC markets. Positive deadweight costs, \( \varepsilon > 0 \), guarantee existence.
them to consume today and default tomorrow. Our analysis, especially in Propositions 2 and 3, makes it precise that it is the opacity or lack of transparency of the OTC markets that leads to ex ante inefficiency in terms of excessively large short positions or leverage. We call this inefficiency counterparty risk externality since it stems from counterparty risk, the risk of default of the short party on long positions. Excessive level of such counterparty risk lowers the payoff to all long positions in the economy, constituting an important negative externality on economy-wide risk sharing. In equilibrium, agents anticipate the lowering of payoff on long positions and the price of insurance falls. However, this is not sufficient to preclude the insurers from selling large quantities of insurance and defaulting ex post, resulting in inefficiently large deadweight costs of bankruptcy. While the externality is not fully internalized in OTC markets, it is internalized if there is a centralized clearing mechanism that provides disclosure or transparency of agents’ positions.

4 Extensions

4.1 Centralized exchange economy

In this section we study an economy in which all asset trades are operated by a competitive centralized exchange. In essence, the exchange is a centralized counterparty that observes all trades and conditions contract terms for individual agents on these trades.

We shall continue to assume that no creditor has direct privileged recourse to a debtor’s collateral, in case of default. The centralized exchange, on the other hand, has full recourse to the debtors’ pledgeable collateral. Furthermore, the exchange operates as a bankruptcy mechanism, by distributing the cash flow of the short positions of the asset, pro-rata with respect to the long positions.

An agent $i$, taking a long position $z_{ij}^+ > 0$ with short counterparty $j$, takes the return $R^i$ as well as the price $q^i$ as given. The equilibrium payoff of the asset shorted by agent $j$, denoted $R^i(z_i^+; s)$, continues to be given by

$$R^i(z_i^+; s) = \begin{cases} \frac{\alpha w_i(s)}{z_{i}^+} & \text{if } I^d(z_i^+, z_i^-; i, s) = 1 \\ R(s) & \text{otherwise} \end{cases}$$

(29)

Consequently, an agent of type $j$ with short position $z_j^-$ will face an ask price
map
\[ q^i_\perp(z^j) = \max_i E \left( m^i R^i_j(z^j) \right). \]  
(30)

Price taking is represented by the fact that agents take the pricing kernels \((m^1, \ldots, m^i, \ldots, m^I)\) as given.

We turn next to the decision problem of the competitive centralized exchange, which controls the supply of the asset to agents. Let the supply offered by the exchange to agent \(i\) for long and short positions be denoted \((\theta^i_+, \theta^i_-)\) s.t. \(\theta^i_+ \theta^-_i = 0)_{i,j}^{14}\).

Then, given the supplies, the exchange can compute the cash flow of the short positions of agents:
\[ R^j(\theta^j_-; s) = \begin{cases} \frac{\alpha w_j^i(s)}{\theta^-_j} i f \ I^d(\theta^j_-; j, s) = 1 \\ R(s) \text{ otherwise} \end{cases}. \]  
(31)

The exchange prices a unitary short position as \(\max_i E \left( m^i R^i_j(\theta^j_-) \right)\), taking as given the stochastic discount factor of the agent \(i\) who values it the most at the margin, that is the agent who would acquire it if offered.

To summarize, a competitive exchange takes as given the stochastic discount factors \((m^1, \ldots, m^i, \ldots, m^I)\). Crucially, the exchange anticipates the compositional effects on default risk of portfolios of different agent types, that is, it recognizes how each agent \(i\)'s incentives to default are affected by its positions \((\theta^i_+, \theta^i_-)\). Thus, the exchange solves the following problem:
\[ \max_{(\theta^i_+, \theta^i_-)} \left[ \sum_j \max_i E \left( m^i R^i_j(\theta^j_-) \right) \left( \theta^i_+ - \theta^j_- \right) \right] \]  
(32)

s.t.
\[ \sum_i \theta^i_+ - \theta^j_- = 0, \text{ for any } j. \]  
(33)

At competitive equilibrium, the portfolios demanded by the agents are offered by the competitive exchange and markets clear:
\[ \theta^i_+ = z^i_+, \theta^i_- = z^i_-, \forall i, j, \]  
(34)

\(^{14}\text{By construction, } \theta^i_+ = 0.\)
and the price maps and returns anticipated by agents are consistent with those perceived by the exchange:

\[ q^*_j(z^*_j) = \max_i E\left(m^i R^i_j(z^*_j)\right). \]  
\[ q^*_j = q^*_j(z^*_j) = \max_i E\left(m^i R^i_j(z^*_j)\right). \]  
\[ R^i(s) = R^i_j(z^*_j; s). \]

It is straightforward to show that the competitive equilibrium allocations of economies with a centralized exchange coincide with those of economies with a centralized clearing mechanism. Therefore, by Proposition 1, competitive equilibrium allocations of economies with a centralized exchange are constrained efficient. Note however that a centralized exchange which intermediates all financial market trades is a much more invasive institution than a centralized clearing mechanism.

4.2 Production risk

In our whole analysis the aggregate endowment of the economy, \( \sum_{i \in I} w^i_0 \) at time 0 and \( \sum_{i \in I} w^i_s \) in each state \( s \in S \), has been kept constant. Consequently, the inefficiency of OTC markets has only distributional effects: insurers have incentives to take on excessive leverage, asset prices rationally reflect equilibrium leverage, and hence insurance markets endogenously fail to serve their purpose. Therefore, resources are mis-allocated in equilibrium. Nonetheless, no resources are lost or altered in the aggregate other than the deadweight costs of bankruptcy.

This ceases to be the case if we allow for production in the economy, as we proceed to show in this section.\(^{15}\) Suppose each agent is endowed with a production function \( f \) which transforms consumption goods at time 0 into consumption goods at time 1. More precisely, consider the following technology. Let \( K = \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_A \end{pmatrix} \) denote a capital allocation vector over \( A \) activities (e.g., projects), so that \( k_1 + k_2 + \ldots k_A = k \). The production function can then

\(^{15}\)While we restrict to an economy with “backyard production” on the part of agents, for simplicity, the analysis directly extends to a firm-level production economy.
be defined as the output in state $s$ given capital allocation $K$: $f(s, K), \forall s$.\textsuperscript{16}

Note that, by allowing for multiple technological activities ($A \geq 2$), this formulation allows for some control of the agents over the distribution of capital across activities and hence over the probability distribution of outcomes, that is, over production risk.

With this technology in place the equilibrium analysis of centralized clearing and OTC economies can be extended to production. For instance, the budget constraints in the economy with a centralized clearing mechanism become:

$$x^i_0 + \sum_j q^j \pi^j_+ - q^j_- (\pi^j_-) z_+^i = w^i_0 - k^i$$

$$x^i_1(s) = \max \left\{ w^i_1(s) + \sum_j R^j \pi^j_- - R(s) \pi^j_- + f(s, K^i), (1 - \alpha) w^i_1(s) - \varepsilon \pi^j_- \right\}$$

and agent $i$ chooses a non-negative portfolio $(\pi^j_+, \pi^j_-)$ as well as a non-negative capital allocation $K^i$. Budget constraints in the OTC economies are similarly formulated.

It is easy to show that, in the production economy,

1. A centralized clearing mechanism continues to decentralize constrained Pareto efficient allocations; and

2. The generic inefficiency of OTC markets manifests itself as over-production or in other words undertaking of excessive production risk.

An example of this inefficient risk-taking is the possible effect of credit default swaps sold by A.I.G. to a large number of financial firms in the United States and the Europe, effectively insuring tail risk of corporate bond and loan portfolios and mortgage-backed securities. This is tantamount to selling insurance on economy-wide default risk. Our model implies that in an OTC setting for selling such insurance, the insurer would take on large and inadequately-collateralized selling of protection on pools of default risk; in turn, the insured would feed the excessive creation of the housing stock and corporate assets backing such pools of aggregate default risk.

To see this in a transparent manner, we revisit our example economy of Section 2. Suppose that agents 1 have access to a production technology,

\textsuperscript{16}We assume $f$ is continuously differentiable, strongly increasing, and strictly quasi-concave.
e.g., making mortgages, that yields per unit of investment $f(G)$ in the good state and $f(B)$ in the bad state, where the investment contains aggregate risk in that $f(G) > f(B)$. Then denoting the level of investment as $k$, agent 1’s maximization problem is:

$$
\max_{z^1, k} w^1_0 - k - z^1 q + p \left[ w^1(G) + f(G)k \right] + (1-p) \left[ R^+ z^1 + f(B)k \right] - \frac{\gamma}{2} \text{var}(x^1(s)),
$$

where

$$
\text{var}(x^1(s)) = p(1-p)[w^1(G) + f(G)k - R^+ z^1 - f(B)k]^2.
$$

Then, for a given level of insurance $z^1$, the first-order condition for agent 1’s investment decision implies that

$$
k(z^1) = \frac{[pf(G) + (1-p)f(B) - 1]}{\gamma p(1-p)[f(G) - f(B)]^2} - \frac{[w^1(G) - R^+ z^1]}{[f(G) - f(B)]}.
$$

Intuitively, the investment is more attractive the greater is its net present value ($pf(G) + (1-p)f(B) - 1$), and greater is the extent of insurance and its quality ($R^+ z^1$) as insurance lowers the attendant risk of the investment.

Now, assuming the investment is indeed net present value, insurance is desirable up to an extent. It reduces risk of the producer (agent 1) and facilitates greater investment in the economy. However, as is clear from the above maximization, agent 1 does not take account of the fact that in case insurance is associated with default of the insurer (agent 3), there may be deadweight costs ($\epsilon z^3$ whenever $z^3 > w^3(B)$) to facilitating production in the economy. On the other side, the insurer is unable to commit not to default given the attraction of collecting risk premium upfront and defaulting on insurance ex post.

As explained in last remarks of Section 2, default could be restricted (and if it be Pareto efficient, even eliminated) by requiring that insurers suitably (possibly fully) collateralize insurance sold. Doing so would restrict leverage in the economy and ensure productive efficiency in that investments take account of deadweight costs of risk transfers involved in managing the risk of investments. We leave the detailed analysis of the full-blown production economy, noting that centralized clearing can in fact achieve the outcome that planner would produce even in decentralized markets.
5 Discussion

5.1 Interpreting default and bankruptcy costs

A central feature of our modeling technology was the strategic nature of 
default – sell insurance today and default tomorrow – and the ex-post costs 
associated with such default. It is useful to interpret this feature in view of 
practical settings in which financial firms trade. Again, the example with 
A.I.G. as the protagonist is useful.

A.I.G. had traders (specifically, at A.I.G. Financial Products) who were 
engaged in the business of selling insurance through synthetic credit default 
swaps on portfolios of mortgages and corporate loans. Their incentive to 
sell a large quantity of such swaps is as in the model to collect premiums 
upfront and get paid salaries and bonuses based on these premiums. The 
result was a highly levered bet of A.I.G. on the tail risk of the economy, that 
is, the likelihood of default of A.I.G. in case aggregate risk materialized was 
rather large. Indeed, A.I.G. as an enterprise itself suffered substantial costs 
of the resulting default on these swaps. These costs can be interpreted as the 
parameter $\epsilon$ in the model. However, due to limited liability, these costs were 
not borne by traders who sold the swaps but by rest of A.I.G.’s businesses 
(such as life and property insurance) whose franchise value is at least in part 
being deployed to pay off A.I.G.’s (non bailed-out) positions.

In contrast to this example, our modeling of default had the same agent 
selling large quantities of insurance, defaulting ex post and suffering the 
bankruptcy costs. However, as seen in our examples and formal analysis, 
when these costs ($\epsilon$) were small, default moral hazard became a relevant 
concern, and in turn, induced counterparty risk externality. This case is 
more in the spirit of the A.I.G. example where traders bore only a small cost 
in case default materialized.

It should be noted that while we have focused on centralized clearing and 
exchange as mechanisms to eliminate the counterparty risk externality, an-
other possible remedy is to directly address the issue of default moral hazard. 
This would require the regulator to increase the bankruptcy costs suffered 
by defaulting financial firms, e.g., by imposing state-contingent penalties. 
To the extent such penalties are restricted by limited liability, our analysis 
highlights that improving trading infrastructure through centralized clearing 
can serve as an important part of regulatory design to contain systemic risk 
and leverage.
5.2 Implementing centralized clearing

The crucial aspect of centralized clearing, in our economy, is that agents condition the terms of the contracts they trade the total financial position of the counterparty and not just on bilateral positions. How could this be implemented in actual financial markets, where positions are contracted sequentially?

We can think of one mechanism that requires prices to adjust continuously with each agent’s total position, much like margin and collateral arrangements. Such arrangements typically require counterparties to post cash (or treasury) collateral based on mark-to-market valuation of their positions and also based on an overall assessment of counterparty risk (e.g., through a credit rating). While such arrangements are not exactly equivalent to continuously observing each agent’s total position and conditioning price on that information, they serve as a way of dynamically responding to knowledge about such information. Further, such arrangements may in fact preclude an agent from positions beyond a certain size due to natural collateral constraints, in effect, implementing sufficiently non-linear pricing schedules (or “position limits” as are explicitly employed on exchanges).

But what if an agent has positions that are not centrally cleared (and indeed agents would have incentives to create such OTC “clones” given the default moral hazard)? This can effectively be handled via a second mechanism that specified a seniority rule in bankruptcy for centrally cleared positions over OTC positions. Under such a rule, OTC positions would be effectively voided if in the extreme case, the bankruptcy code legislates that OTC counterparties cannot be paid upon in case of default. In that case, these positions do not impose any counterparty risk externality on centrally cleared positions. In practice, subordinating OTC positions relative to centrally cleared ones may suffice, as counterparties may shy away from juniority of OTC claims and prefer to take centrally cleared positions with other agents.

5.3 Proposals to reform the OTC markets

We now consider the implications of our model for the debates raised by the financial crisis of 2007-09 on the desirability of over-the-counter versus centralized trading. In particular, analysis of the role played by OTC markets
in the ongoing financial crisis has led to several reform proposals. Our theoretical analysis can help provide a normative comparison of these proposals.

For example, Acharya, Engle, Figlewski, Lynch and Subrahmanyam (2009) divide the proposals into requiring a (i) centralized registry with no disclosure to market participants; (ii) centralized clearing with disclosure of aggregate trade information to market participants; and (iii) exchange with full public disclosure of prices and volumes. Our theoretical analysis makes it clear that a centralized registry by itself is not sufficient as it only gives regulators ex-post access to trade-level information but does not counteract the ex-ante moral hazard of institutions wanting to take on excessive leverage. Both centralized clearing and exchange improve on this ground but it is the centralized clearing that is more crucial rather than an exchange; in other words, it is sufficient that centralized clearing disseminates trade positions to market participants and they themselves set price schedules and risk controls conditional on that information. In particular, requiring all trades to take place through a centralized exchange is not necessary though in that case there would be no need to disclose information on all trades to individual agents.

Regulatory reforms announced in March 2009, and since then under debate in the United States (and similarly in the U.K. and the Europe), involve significant changes to the trading infrastructure of OTC markets, with the objective of reducing systemic risk in the financial sector. Under the proposed reforms, mature and standardized credit derivatives such as the credit default swaps (CDS) and indices linked to the CDS will be traded through a centralized counterparty; there is no proposal yet to mandate that these be traded on an exchange. Regulators will gain unfettered access to information on prices, volumes and exposures from the centralized counterparties, but the proposals do not require that such information be made public. While some aggregate information will be disseminated to all market participants, such as the recent data published by the Depository Trust and Clearing Corporation (DTCC) on all live positions in credit derivatives, full transparency is being required only for regulatory usage.

Our results suggest that these proposed changes are unlikely to be fully adequate, though they take some steps in the right direction. Importantly, not all products are being required, or are in fact amenable, to trade on

\footnote{See Stulz (2009) for a summary of the dimensions along which OTC markets for credit derivatives likely contributed, and did not contribute, to the crisis.}
exchanges. In particular, many financial products such as the customized or “bespoke” collateralized debt and loan obligations (CDOs and CLOs) are being recommended to remain OTC. On these products, our results suggest that disclosure might be the required alternative: At a minimum, these products should have centralized clearing (just a registry) so as to aggregate trade-level information and provide transparency among market participants. This will be necessary to improve the assessment and pricing of counterparty risk associated with these products.

6 Related Literature

The bilateral nature of contracts in the OTC markets has been stressed in the recent literature on the subject. Duffie, Garleanu and Pedersen (2005, 2007) focus on search frictions, dynamic bargaining and valuation in OTC markets; Caballero and Simsek (2009) analyze the role of complexity introduced by bilateral connections and their role in causing financial panics and crises; and, Golosov, Lorenzoni, and Tsyvinski (2009) examine what kind of bilateral contracts will get formed when agents have private information about their endowment shocks.

Specifically in the context of insurance provision through financial contracts, the literature (e.g., Duffee and Zhou, 2001, Acharya and Johnson, 2007, Parlour and Winton, 2008) has largely focused on moral hazard on part of the insured due to presence of information frictions, rather than moral hazard on part of the insurer, the latter being the focus of our paper. On this focus, our paper is more closely related to Allen and Carletti (2006), Thompson (2009) and Zawadowski (2009).

Allen and Carletti (2006) consider contagion from insurance sector to the financial sector when there is credit risk transfer, but they do not consider agency-theoretic issues. In contrast, we allow for default incentives of the insurer and model credit risk transfer more generally as risk-sharing through financial contracts in a GE setting. Zawadowski (2009) analyzes counterparty risk in entangled financial systems. The system is “entangled” because banks hedge risks using bilateral OTC contracts but do not internalize the cost of their own failure on other banks (through counterparty risk exposures). As a result of this network externality, banks purchase less insurance against low probability events. Thus, there is less insurance in his model whereas due to moral hazard on part of the insurer, there is in fact excessive insurance in our
setup but it is of low quality and entails default by the insurer. Thompson
(2009) considers the moral hazard of default on part of the insurer when
there is credit risk transfer in the financial sector. His focus is on analyzing
how this moral hazard provides incentives to the insured parties to reveal
information about their type, so that the two agency problems interact and
reduce each others’ adversity.

More specifically on the benefits of OTC versus centralized markets, our
analysis did not consider practical issues relating to the extent of netting of
positions that is possible under different market structures. Duffie and Zhu
(2009) explain that for a centralized exchange for credit default swaps to
reduce counterparty risk more than in the OTC setting, it would require net-
ting not just across CDS but also across other products such as interest-rate
swaps. In our model, the primary role of the centralized clearing mechanism
or the exchange is not to reduce or eliminate counterparty risk but to improve
its price by aggregating information on trades.

We conjecture that if there were a centralized registry of positions (cen-
tralized or OTC) that is observed by different clearing platforms or exchanges
and disseminated to market participants, then the pricing (or collateral) ar-
rangements would be efficient ex ante, and so would be the levels of ex-post
default risk. This is related to Leitner (2009)’s result that a clearinghouse-
style mechanism, allowing each party to declare its trades and revealing pub-
licly those that hit pre-specified position limits, can prevent agents from
promising the same asset to multiple counterparties and then defaulting.

From a theoretical standpoint our paper studies competitive equilibria
of economies with moral hazard, where the moral hazard is induced by the
agents’ default decisions. In particular, in the terminology of this literature,
we compare competitive equilibria in exclusive contractual environments (in
the case of economies with a centralized clearing mechanism or with a cen-
tralized exchange) with competitive equilibria in non-exclusive contractual
environments (in the case of economies with OTC markets, with and with-
out netting). This distinction is central in competitive economies with moral
hazard; see e.g., Bisin and Gottardi (1999) and Bisin, Geanakoplos, Gottardi,
Minelli and Polemarchakis (2001). In finance, several papers have exploited
the distinction between exclusive and non-exclusive markets in different con-
texts: e.g., Bizer and DeMarzo (1992) in a sequential model of banking.

\[18\text{In the context of principal agent models, see the early work of Arnott and Stiglitz (1993) and Hellwig (1983).}\]

7 Conclusion

We formalized in this paper an important market failure arising due to opacity of over-the-counter (OTC) markets, in particular that the payoff on each position depends in default on other positions sold by the defaulting party, but there is no way for market participants to condition their trades or prices based on knowledge of these other positions. We showed that this counterparty risk externality can lead to excessive default and production of aggregate risk, and more generally, inefficient risk-sharing. Centralized clearing, by enabling transparency of trades, and exchanges, by creating a centralized counterparty to all trades, can help agents fully internalize the counterparty risk externality. Our model provides one explanation for the substantial buildup of OTC positions in credit default swaps in the period leading up to the crisis of 2007-09, their likely contribution to over-extension of credit in housing and corporate sectors, and possible remedies for avoiding this excess in future.

Our analysis focused on competitive markets. In future, we plan to consider bilateral OTC markets in the presence of a “large” individual agent that effectively observes the trades of all others but whose trades are not seen by others. Such an agent would enjoy monopoly rents in the OTC setting, which in turn would reduce private incentives in the economy to coordinate on a centralized trading platform and achieve Pareto improvement. Furthermore, our model suggests that excessive leverage and excessive production arising due to the OTC nature of trading can lead to a “bubble” in the market for goods (e.g., the housing stock), a subsequent crash upon realization of adverse shocks, and a breakdown of risk transfer (credit or insurance markets) in those states. Our analysis also suggests that the possibility of regulatory forbearance of “too big to fail” positions can result in ex-ante inefficiencies even with centralized exchange trading. These phenomena are worthy of detailed modeling and study in extensions of our basic setup.

Finally, we focused on symmetric information about states of the world in our analysis of centralized versus over the counter markets. However, the model can be extended to relax this assumption. In particular, it is possible to add adverse selection to the model, e.g., in the form of unobservable
probability distributions over $S$, the uncertainty states at time 1. The model would combine features of Rothschild and Stiglitz (1976) and Akerlof (1970), with separating equilibria in the economy with centralized exchanges, and excessive lemons trading (in the form of risky short positions) in the case of OTC markets.\footnote{Santos and Scheinkman (2001) have adverse selection as well in their model of competition of exchanges.} It remains an important exercise for future to confirm that the relative inefficiency of OTC markets relative to centralized markets persists in this setting. Indeed, we conjecture that the inefficiency of OTC markets will be exacerbated in a setting with adverse selection.

\section*{References}


Appendix

Proof of Proposition 1. The proof proceeds by contradiction. Let \((x_i^0, x_i^1, z_{ij}^+, z_{ij}^-)_{i,j}\) denote an equilibrium of the economy with centralized clearing mechanism and let \((\hat{x}_i^0, \hat{x}_i^1, \hat{z}_{ij}^+, \hat{z}_{ij}^-)_{i,j}\) denote a constrained Pareto optimal allocation which dominates the equilibrium allocation. Both allocations satisfy netting, for any \(i\) and \(j\):

\[
\begin{align*}
  z_{ij}^+ z_{ij}^- = 0, & \quad \hat{z}_{ij}^+ \hat{z}_{ij}^- = 0. \tag{20}
\end{align*}
\]

The allocation \((\hat{x}_i^0, \hat{x}_i^1, \hat{z}_{ij}^+, \hat{z}_{ij}^-)_{i,j}\) must not have been budget feasible at equilibrium prices. That is,

\[
\begin{align*}
  \hat{x}_i^0 + \sum_j q_j^+ \hat{z}_{ij}^+ - q_j^- (\hat{z}_{ij}^-) \hat{z}_{ij}^- - w_i^0 & \geq x_i^0 + \sum_j q_j^+ z_{ij}^+ - q_j^- (z_{ij}^-) z_{ij}^- - w_i^0
\end{align*}
\]

with \(>\) for at least one agent \(i\). Summing over \(i\):

\[
\begin{align*}
  \sum_i (\hat{x}_i^0 - x_i^0) + \sum_i \left( \sum_j q_j^+ \hat{z}_{ij}^+ - q_j^- (\hat{z}_{ij}^-) \hat{z}_{ij}^- \right) - \sum_i \left( \sum_j q_j^+ z_{ij}^+ - q_j^- (z_{ij}^-) z_{ij}^- \right) > 0
\end{align*}
\]

But, since market clearing must hold for both \((x_i^0, x_i^1, z_{ij}^+, z_{ij}^-)\) and \((\hat{x}_i^0, \hat{x}_i^1, \hat{z}_{ij}^+, \hat{z}_{ij}^-)\), \(\sum_i (\hat{x}_i^0 - x_i^0) = 0\), and we obtain:

\[
\begin{align*}
  \sum_i \left( \sum_j q_j^+ \hat{z}_{ij}^+ - q_j^- (\hat{z}_{ij}^-) \hat{z}_{ij}^- \right) - \sum_i \left( \sum_j q_j^+ z_{ij}^+ - q_j^- (z_{ij}^-) z_{ij}^- \right) > 0.
\end{align*}
\]

Furthermore, at equilibrium, \(q_i^+ = q_i^- (z_i^+)\). Hence,

\[
\begin{align*}
  \sum_j q_j^+ (\hat{z}_{ij}^-) \sum_i (\hat{z}_{ij}^+ - \hat{z}_{ij}^-) - \sum_j q_j^+ (z_{ij}^-) \sum_i (z_{ij}^+ - z_{ij}^-) > 0.
\end{align*}
\]

But market clearing at equilibrium implies \(\sum_i (z_{ij}^+ - z_{ij}^-) = 0\), for any \(j\), and hence \(\sum_j q_j^+ (z_{ij}^-) \sum_i (z_{ij}^+ - z_{ij}^-) = 0\) for non-negative prices \(q_j^+ (z_{ij}^-)\). Therefore,

\[
\sum_j q_j^+ (\hat{z}_{ij}^-) \sum_i (\hat{z}_{ij}^+ - \hat{z}_{ij}^-) > 0,
\]

\(\text{Recall that } z_{ii}^+ = 0, \text{ for any } i, \text{ by construction.}\)
Proof of Proposition 2. We only prove the statement regarding economies with OTC markets and netting. The proof in the case of OTC markets without netting only requires straightforward modifications. Furthermore, assume $\varepsilon = 0$ and the proof below extends by continuity to $\varepsilon$ sufficiently small. Let $(z_i^{ij}, z_i^i)$ be the equilibrium portfolio for agent $i$ in an economy with centralized clearing mechanism. Let $S(i) \subseteq S$ denote the subset of the states of uncertainty in which, at equilibrium, an agent $i$ will default. Then, $S(i)$ is robustly non-empty. Furthermore, if $S(i)$ is non-empty, then $z_i^i > 0$. For any economy such that $S(i)$ is non-empty (and $z_i^i > 0$) for some $i$, at equilibrium of the centralized clearing mechanism, we must have

$$q_i^i(z_i^i) = \sum_{s \in S(i)} p_s m_i^i(s) \frac{\alpha w_i(s)}{z_i^i} + \sum_{s \in S \setminus S(i)} p_s m_i^i(s) R(s).$$

Suppose, by contradiction, that such a competitive equilibrium of the centralized exchange economy can be supported in an economy with OTC markets and netting. Then it is necessarily supported by price $q_i^i = \sum_{s \in S(i)} p_s m_i^i(s) \frac{\alpha w_i(s)}{z_i^i} + \sum_{s \in S \setminus S(i)} p_s m_i^i(s) R(s)$, such that $q_i^i = q_i^i(z_i^i)$ at the equilibrium portfolio $(z_i^{ij}, z_i^i)$. It is straightforward to see that in this case, at price $q_i^i$ agent $i$ prefers a portfolio $(z_i^{ij}, z_i^i + dz)$, for some $dz > 0$. This is because the marginal valuation of the discounted repayment of a unitary extra short portfolio $dz$, $\sum_{s \in S(i)} p_s m_i^i(s) \frac{\alpha w_i(s)}{z_i^i} + \sum_{s \in S \setminus S(i)} p_s m_i^i(s) R(s)$, depends negatively on $z_i^i$; while the price obtained at time 0 from the same unitary extra short portfolio, $dz$, $q_i$, does not. Since the portfolio $(z_i^{ij}, z_i^i + dz)$ is budget feasible, a contradiction is reached. This is the case for any equilibrium of the centralized exchange economy such that $S(i)$ is non-empty, for some $i$, and hence the contradiction holds robustly. ■

Proof of Proposition 3. Once again, we only prove the statement regarding economies with OTC markets and netting. The proof in the case of OTC markets without netting only requires straightforward modifications. Once again, assume $\varepsilon = 0$ and the proof below extends by continuity to $\varepsilon$ sufficiently small. Finally, the “weakly greater” part of the statement is straightforward. We turn to prove the robustly “strictly greater” part.

Consider the robust subset of economies for which, with centralized clearing at equilibrium, $S(i)$ is non-empty. An argument analogous to the one in
the proof of Proposition 2 guarantees that, for these economies, when $\varepsilon$ is small enough, at an equilibrium of the economy with OTC markets and netting, $S(i) = S$. Agents $i$, in other words, default in all states $s \in S$. This proves that default is robustly strictly greater at equilibria of the economy with OTC markets and netting than with centralized clearing.

Consider such an equilibrium with OTC markets and netting, to study leverage. At equilibrium it must be that $q_i > 0$. Suppose on the contrary that $q_i \leq 0$. In this case, we claim agents $i$ would rather choose $z_i^- = 0$ and hence would trivially not default. In fact, if $S(i) = S$, and $q_i \leq 0$, agents $i$ would consume

$$
x_0^i = w_0^i + q^i z_i^-
$$
$$
x_1^i(s) = (1 - \alpha) w_1^i(s) - \varepsilon z_i^-
$$

(recall that, because of netting, $\sum_j q^j z_{ij} = 0$). But then

$$
x_0^i \leq w_0^i + q^i z_i^- \leq w_0^i
$$
$$
x_1^i(s) = (1 - \alpha) w_1^i(s) - \varepsilon z_i^-
$$

By resorting to autarchy, $z_i^- = z_{ij}^+ = 0$, instead agents $i$ would guarantee themselves

$$
x_0^i = w_0^i
$$
$$
x_1^i(s) = w_1^i(s)
$$

which they prefer. Prices such that $q_i \leq 0$ therefore imply no default. This is the case for all agents of all types $i$. But then $R_+(s) = R(s)$, for all $s \in S$ and $z_{ij}^+$ is robustly $> 0$, for some $j$, a contradiction with market clearing. At an equilibrium of the economy with OTC markets and netting, therefore, it must be that $q_i > 0$. In this case $z_i^-$ grows unbounded as $\varepsilon \to 0$. This proves that leverage is robustly strictly greater in the economy with OTC markets and netting than with centralized exchange for $\varepsilon$ small enough. $lacksquare$
Figure 1: The quantity of insurance sold ($z^3$) as a function of the deadweight cost of default ($\varepsilon$)
Figure 2: The realized payoff on the insurance ($R'$) as a function of the deadweight cost of default ($\epsilon$)
Figure 3: The equilibrium price of insurance ($q$) as a function of the deadweight cost of default ($\varepsilon$)
Figure 4: The equilibrium utilities as a function of the deadweight cost of default (ε)