Bankruptcy Codes and Innovation: A Model

Viral V. Acharya
London Business School, NYU-Stern & CEPR
vacharya@london.edu

Krishnamurthy V. Subramanian
Emory University
ksubramanian@bus.emory.edu

February 15, 2009

1This is the theoretical addendum to the paper “Bankruptcy Codes and Innovation,” forthcoming in the Review of Financial Studies.
Abstract

Bankruptcy Codes and Innovation: A Model

We consider the investment and financing choices of a firm as a function of the bankruptcy code under which it operates. We model investment as a choice between innovative exploration and pursuit of tried-and-tested strategy, and financing as the choice of leverage given the tradeoff between its tax benefits and deadweight costs in bankruptcy. Deadweight costs in bankruptcy arise due to creditors’ preference for liquidating assets or that of debtors in excessively continuing with assets, and their severity depends upon the nature of investment as well as on the relative creditor-friendly or debtor-friendly nature of the bankruptcy code. We show that under mild parametric restrictions, stronger creditor rights result in lower value and lower leverage-based financing of innovative investments relative to tried-and-tested strategies.

JEL: G3, K2, O3, O4, O5.

Keywords: Creditor rights, R&D, Technological change, Law and finance, Entrepreneurship, Growth, Financial development.
1 Introduction

The importance of innovation to country-level economic growth has been emphasized by the endogenous growth theory (Romer, 1990, Grossman and Helpman, 1991, and Aghion and Howitt, 1992). Figure 1 here shows this positive correlation between countries’ GNP per capita and their overall innovation intensity as measured by the total number of patents, citations to these patents, the number of patenting firms, and the average number of patents.¹

In the paper “Bankruptcy Codes and Innovation,” forthcoming in the Review of Financial Studies, we established a causal, empirical linkage between creditor rights in a country and the innovation intensity of its firms, suggesting that the design of bankruptcy codes may be an important part of the toolkit available to countries to foster innovation and economic growth. In this Addendum to that paper, we develop a theoretical model that formalizes the link between creditor rights and innovation. We illustrate in a simple setting the trade-offs that a levered firm faces under a creditor- or debtor-friendly bankruptcy code when deciding between investing in an innovative project and a conservative project.

We model a firm’s choice between innovative and conservative technologies. For example, suppose a pharmaceutical company could invest in either one of the following two projects: (1) inventing and launching a new drug for a hitherto incurable disease; or (2) manufacturing and launching a generic substitute for an existing drug. Launching a generic substitute involves uncertainties due to customer demand as well as competition from other manufacturers. In contrast, inventing a new drug entails additional uncertainties associated with the process of exploration and discovery, whether such a drug could be administered to humans, and whether it would receive FDA approval.

We formalize the essence of the above example in a two-period model. We model the firm’s choice between innovative and conservative technologies as a two-armed bandit problem.² The “explore” arm has higher value ex ante than the “exploit” arm, but exploration is risky. Exploration reveals at an interim date the quality of the innovation. Given the information revealed at the interim date, switching to the exploit strategy is optimal if (and only if) the news is bad. The firm finances its investment in either technology using debt and equity. As in the static trade-off theory, debt provides the benefit of tax shields. However, at the interim date, the firm may default on its debt payments. This entails deadweight costs of bankruptcy due to inefficient continuation or liquidation at the interim date. We model these deadweight costs as a function of the investment strategy followed by the firm (innovative or conservative), its capital structure (the debt-equity mix) and the bankruptcy code in place (creditor-friendly or debtor-friendly). The firm chooses simultaneously the nature of its real activity and its financing mix.

To fix ideas, consider two polar opposites of the bankruptcy code: First, the debtor-friendly code where equityholders retain all the control rights in bankruptcy; and, second, the

¹Note that GNP per capita of around USD 9000-10000 represents the widely accepted cutoff point for developed versus under-developed economies.
²See Sundaram (2003) for a theoretical survey of bandit problems with applications to economics.
creditor-friendly code where all control rights are transferred to a firm’s creditors. The non-linearity of creditors’ (and equityholders’) cash flow claims gives rise to the following effect: under a creditor-friendly code, the innovative technology may be inefficiently liquidated, whereas under a debtor-friendly code, it may be inefficiently continued. While this trade-off arises for the conservative strategy too, greater risk inherent in the innovative strategy accentuates the deadweight costs arising from liquidation under the creditor-friendly code but mitigates the deadweight costs from continuation under the debtor-friendly code. Thus, for a given financing mix, a creditor-friendly code discourages risk-taking and innovation relative to a debtor-friendly code. In fact, the inefficient continuation ex post under the debtor-friendly code can be efficient from the standpoint of ex-ante risk-taking.

Interestingly, since firms in our model also choose their financing mix, the optimal financing of an innovative strategy involves lower debt under creditor-friendly code than under debtor-friendly code. That is, in order to pursue the efficient real activity under the creditor-friendly code (which is to innovate in our model), the firm lowers its debt. This, however, comes at a cost to the firm in form of lower tax-shields. Thus, when the code becomes more creditor-friendly, the value of an optimally financed innovative firm reduces while that of an optimally financed conservative firm increases.

The empirical implication of this result can be understood using an example. Consider two industries: Biotechnology and Textiles & Apparel. Firms in the Biotechnology sector have a higher propensity to innovate than firms in the Textiles industry. Given this difference, the above result implies that the difference in innovation between Biotechnology and Textiles would be greater in the United States than in Germany since the rights provided to creditors in bankruptcy are weaker in the United States than in Germany. Figures 3, 4, 5, 6, and 7 illustrate this result for Biotechnology as well as other innovation-intensive industries such as Computer Pheripherals, Information Storage, Drugs, and Surgical and Medical Instruments.

To summarize, our theory argues that the nature of bankruptcy code alters not only the financing mix of the firm but also its ex-ante choice of real activity. The theory thus suggests an altogether different approach to thinking about the design of bankruptcy codes in a normative sense, in particular, an approach that focuses on the ex-ante real investments undertaken by firms in response to bankruptcy codes rather than on the ex-post efficiency of continuation outcomes when firms are in distress.

2 Model

We model a firm’s investment and financing decisions simultaneously to examine how the investment decision is affected by the way it is financed and by the bankruptcy regime under which it operates.

Figure 2 summarizes the time line and events in the model. There are three dates,

---

3 Starting with Gertner and Scharfstein (1991), such a tradeoff has been at the center of a large body of finance literature that focuses on the efficiency of bankruptcy mechanisms and their optimal design.
$t = 0, 1, 2$, an investment decision at date 0 as well as date 1, and a financing decision at date 0. We first explain the setup with regard to the investment decisions, taking financing as given, and explain the financing choice thereafter.

At date 0, the firm’s investment strategy consists of choosing whether it should innovate or it should continue to follow its “tried-and-tested” strategies. Each of these strategies requires one unit of investment and generates risky cash flows. The financing is raised in the form of debt and equity whose optimal mix is also determined by the firm at date 0. The debt matures at date 1. The investment strategies produce two streams of cash flows, one in the short term (which coincides with date 1) and one in the far horizon (date 2).

The firm’s short-term (date 1) cash flow serves two purposes. First, it generates the cash that can be used to service its debt. If the cash flow generated is insufficient to meet debt obligations, the firm is in financial distress, and the bankruptcy code in place determines whether control rights are bestowed upon equityholders or creditors. In particular, the bankruptcy code regulates whether it is the equityholders or the creditors who have the right to decide whether to liquidate the firm or to continue it. If the decision is made to continue the firm, then the decision maker also has to decide whether to switch the investment strategy or to continue with the earlier one. This investment choice at date 1 does not require any additional capital outlays. If the firm is continued at date 1, then the long-term cash flows from the investment strategy chosen at date 1 are realized at date 2. The investment strategy may however also be liquidated at date 1 as we explain in detail below. The inefficiencies in our model primarily arise from a conflict of interest in the investment decision at date 1, but this affects the investment and financing decisions at date 0 as well. It is this latter effect which is of primary interest to us.

Second, the firm’s short-term cash flow provides important information about future cash flows and liquidation values that may be expected from the project. In particular, the information about future cash flows will be especially relevant when the firm adopts the innovative strategy. Finally, all agents in the model are risk-neutral and the risk-free rate of interest is zero.

### 2.1 Investment Strategy and Work Methods

We now explain the two possible strategies that the firm can follow at date 0 in greater detail. The conservative work method (called the ‘Exploit’ strategy) is tried and tested and therefore involves minimal risk of bad cash flows at date 1. Instead, the firm could experiment with a new work method (called the ‘Explore’ strategy). The advantage of the Explore strategy is the following. If it is tried at date 0, and is successful, then the long-term cash flows of the firm are higher than under the Exploit strategy. However, the Explore strategy has a greater likelihood of producing a low cash flow at date 1. We denote the Exploit and Explore strategies by 1 and 2, respectively, in some of our notation to follow. The set-up resembles closely a two-armed bandit problem. See Sundaram (2003) for a theoretical survey of bandit problems with applications to economics and Manso (2005)
for a related model but without leverage.

2.2 Cash Flows

We denote the date 1 cash flow by $\tilde{x}$ and the date 2 cash flow by $\tilde{y}$.

The distribution of the first-period cash flow $\tilde{x}_i$ depends upon the strategy $i$ implemented at date 0. If the Explore strategy is followed ($i = 2$), then $\tilde{x}_2$ is distributed uniformly $U(0, 1)$ while if the Exploit strategy is followed ($i = 1$), then $\tilde{x}_1$ follows the distribution $U(\alpha, 1 + \alpha)$ where $0 < \alpha < 1$. This captures the fact that the Explore strategy has lower expected cash flow at date 1 and is in this sense riskier.

The second-period cash flow $\tilde{y}_j$ depends upon the strategy $j$ that is implemented at date 1 and also on the date 1 cash flow.

If the Exploit strategy is implemented at date 1, then the date 2 cash flow is equal to the realized date 1 cash flow with probability $p$ and is zero with probability $(1 - p)$. Thus, in this case, the date 1 cash flow does not provide any additional information about the likelihood of the high cash flow at date 2:

$$\tilde{y}_1 = \begin{cases} x_1 & \text{with probability } p \\ 0 & \text{with probability } 1 - p \end{cases} \quad (1)$$

The date 2 cash flow from the Explore strategy depends upon the signal provided by the date 1 cash flow. In particular, if the Exploit strategy ($i = 1$) was implemented at date 0, then the date 1 cash flow is completely uninformative about the likelihood of the high cash flow using the Explore strategy. In contrast, if the Explore strategy was implemented at date 0, then the date 1 cash flow provides important information about the probability of high cash flow. If the date 1 cash flow is greater than expected, then this indicates that the Explore strategy is likely to be successful. In this case, the likelihood of the high cash flow at date 2 increases substantially. If, in contrast, the date 1 cash flow is lower than expected, then this indicates that the Explore strategy is likely to fail. In this case, the likelihood of the high cash flow at date 2 decreases substantially:

$$\tilde{y}_2| (\tilde{x}_2 \geq 0.5) = \begin{cases} \gamma x_2 & \text{with probability } q_G \\ 0 & \text{with probability } (1 - q_G) \end{cases} \quad (2)$$

$$\tilde{y}_2| (\tilde{x}_2 < 0.5) = \begin{cases} \gamma x_2 & \text{with probability } q_B \\ 0 & \text{with probability } (1 - q_B) \end{cases}$$

where

$$\gamma > 1 \text{ and } \gamma q_B < p < q_G \quad (3)$$

Thus, the probability of a high cash flow using the Explore strategy increases significantly when the signal is good while it decreases considerably after a bad signal ($q_B < p < q_G$). Further, after a good signal, the expected date 2 cash flow from the Explore strategy is
greater than the expected cash flow from the Exploit strategy \( (p < \gamma q_G) \). In contrast, after a bad signal, the expected date 2 cash flow from the Explore strategy is lower than the expected cash flow from the Exploit strategy \( (\gamma q_B < p) \).

Thus, the cash flow streams capture two important differences between the two investment strategies. First, the Explore strategy has lower expected cash flows in the short-term. If successful, the Explore strategy provides high expected cash flows in the long-term. If unsuccessful, the expected cash flows in the long-term are lower for the Explore strategy than the Exploit strategy, whereby in the first-base case, it is optimal to switch from Explore to Exploit strategy. Second, the conditional distribution of the long-term cash flow in the case of the Explore strategy captures that innovation involves experimentation and learning: Either the pilot phase of exploration reveals success and there are substantial gains to be made by building upon it, or it fails and there is not much value to pursuing it any further.

2.3 Liquidation Values

The liquidation value that can be realized at date 1 is a function of the strategy that was implemented at date 0 and also the realized interim cash flow. If strategy \( i \) was implemented at date 0, then the uncertain liquidation value is assumed to be distributed as follows:

\[
\bar{L} = \begin{cases} 
\bar{I} x_i & \text{with probability } 0.5 \\
\bar{L} x_i & \text{with probability } 0.5 
\end{cases} \tag{4}
\]

where

\[
\bar{L} < p < \bar{L} < q_G \tag{5}
\]

Thus, when the firm chooses the Exploit strategy, liquidating the firm is better than continuing when \( \bar{L} = \bar{I} x_1 \) while continuing the firm is better than liquidating when \( \bar{L} = \bar{L} x_1 \). In contrast, when the firm implements the Explore strategy, continuing after a good signal is superior to liquidating irrespective of the liquidation value being low or high.

2.4 Leverage, Bankruptcy Code and Control Rights

The firm is financed at date 0 with debt of face value \( F \) (to be endogenously chosen) which matures at date 1. We assume that creditors are risk-neutral and individually rational, that is, they provide financing against face value of \( F \) equal to the market value of debt which is simply expected cash payout to creditors. The remaining investment of one minus the market value of debt is funded through equity. For simplicity, we do not model the provision of equity and it is perhaps cleanest to interpret it as inside equity in the form of past retained earnings. All payments to creditors are made from the firm’s cash-flows. To be precise, there is no additional equity financing at date 1 or date 2.

Consider the different states of the world at date 1. If \( x_i \geq F \) on date 1, then debt is paid off and the firm becomes an all equity firm. The remaining cash flow \( (x_i - F) \), net of
taxes (see below), goes to equityholders. For \( x_i < F \), the firm cannot meet its contractual payment fully and is in “default.” It pays the available amount \( x_i \) to creditors and is in arrears for the remaining amount \((F - x_i)\). Up to this remaining amount, creditors get the first claim on any date future (date 2) cash flows until they have been fully paid off. Future cash flows depend on whether the firm is continued or liquidated at this point, and on the strategy that is implemented at date 1 if the firm is continued.

The bankruptcy code determines who gets to make these decisions — the decision to liquidate/continue; and contingent on continuation, the decision to implement the strategy (Explore or Exploit). With probability \( \pi \in [0, 1] \), the control gets transferred to creditors while with probability \((1 - \pi)\), control remains with equityholders. The parameter \( \pi \) is exogenous and parametrizes the relative creditor-friendliness of the bankruptcy code. A higher \( \pi \) indicates that the bankruptcy code favors creditors. For example, \( \pi = 1 \) corresponds to creditors having control with perfect certainty in financial distress. In reality, most country’s creditor codes would be between the two polar cases of \( \pi = 0 \) and \( \pi = 1 \).

Finally, taxes are paid on the cash flows at date 1 at tax rate \( \tau \) after repaying the debt holders. The maximum tax shield available to the firm at date 1 is thus the face value of its debt \( F \). The firm pays no taxes in default, and for ease of analytical expressions, also in future states that follow default.

The nonlinear payoffs of the claimholders determine (a) deviations from the first-best decision to liquidate or to continue the firm at date 1 in states of default; and (b) contingent on continuation, the decision about which strategy is implemented at date 1.4

3 Analysis

3.1 First best

The first best corresponds to a world wherein the assumptions underlying the Modigliani and Miller (1958) theorem hold and the investment decision is independent of the financing strategy. In particular, there are no taxes or conflicts of interest between firmowners and financiers. Since financing mix is irrelevant, it suffices to focus on the investment strategy. In this world, let us first examine in which states at date 1 would the firm would be liquidated and when would it be continued.

\[4\] Note that renegotiation between claimholders in bankruptcy may be able to eliminate some of these inefficiencies. We follow Acharya, Sundaram and John (2004) in not modeling renegotiation between the claimholders. However, our results remain valid as long as there are frictions or costs that result in some remaining inefficiency. Dewatripont and Tirole (1994) provide a theoretical model of such frictions; see also Gertner and Scharfstein (1991) wherein coordination problems among public creditors result in inefficiencies in the workout process. The existence of some ex post inefficiencies in financial distress is also consistent with the empirical findings in Andrade and Kaplan (1998).
3.1.1 Exploit

When the Exploit strategy is implemented at date 0, it is optimal to continue at date 1 when $\bar{L} = \bar{l}x_1$ since the expected payoff from continuation $px_1$ is greater than that from liquidation. In contrast, it is optimal to liquidate at date 1 when $\bar{L} = \bar{l}x_1$ since the expected payoff from continuation $px_1$ is lower than that from liquidation.

3.1.2 Explore

When the Explore strategy is implemented at date 0, and the cash flow at date 1 signals ‘success’, then it is always optimal to continue since the expected payoff from continuation $q_G\gamma x_2$ is greater than the maximum payoff from liquidation which is $\bar{l}x_2$. In contrast, when the interim cash flows signal ‘failure’ at date 1 after the Explore strategy is implemented at date 0, then switching to the Exploit strategy is better than continuing with the Explore strategy since $p > q_B\gamma$. Combining this with the optimal decision under Exploit strategy discussed above, we can infer that it is optimal to switch to the Exploit strategy when $\bar{L} = \bar{l}x_2$ while it is optimal to liquidate when $\bar{L} = \bar{l}x_2$.

3.1.3 Un-levered firm values

Since financing is irrelevant, for sake of future reference, we refer to the firm under first-best investments at date 1 as an un-levered or all-equity firm. We denote the date 0 values of an all-equity firm following the implementation of Exploit strategy and the Explore strategy at date 0 and efficient implementations thereafter, as $V_T$ and $V_R$, respectively. The value of the all-equity firm is simply the expected value of its cash flows.

When an all-equity firm follows the Exploit strategy at date 0, its value at date 0 is given by

$$V_T = E\left[\bar{x}_1\right] + 0.5E\left[\bar{l} \cdot \bar{x}_1\right] + 0.5E\left[p \cdot \bar{x}_1\right]$$

Note that since this value is being computed at date 0, it is correct to take all expectations over realized states of the world at date 1 as unconditional expectations from date 0 standpoint. The first term in the above expression is the expected value of the date 1 cash flow, the second term captures the expected liquidation value from states in which the firm is efficiently liquidated, while the third term is the expected value of the date 2 cash flow in states wherein the firm is efficiently continued at date 1. Note that we argued above that the firm is liquidated with probability $\frac{1}{2}$ when the liquidation value is $\bar{l}x_1$ and firm is continued with probability $\frac{1}{2}$ when the liquidation value is $\bar{l}x_1$.

When an all-equity firm follows the Explore strategy at date 0, its value at date 0 is given
by
\[
V_R = E[\bar{x}_2] + 0.5E[q_G \cdot \gamma \bar{x}_2] + 0.5(0.5E[\bar{l} \cdot \bar{x}_2] + 0.5E[p \cdot \bar{x}_2])
\]
(7)
\[
= (1 + 0.5\gamma q_G + 0.25\bar{l} + 0.25p)0.5.
\]

The first term in the above expression is the expected value of the date 1 cash flow, the second term captures the expected value of the date 2 cash flow when the signal about the Explore strategy at date 1 is good (i.e. \(x_2 \geq 0.5\)). The third term captures the expected value from liquidating the firm which is optimal in the first-best world when the date 1 signal about the Explore strategy is bad (\(x_2 < 0.5\)) and when the liquidation value is \(lx_2\). The fourth term is the expected value of the date 2 cash flow in states at date 1 wherein the firm is efficiently continued, but the investment strategy reverts back to the Exploit strategy since the expected payoff from either liquidating (\(lx_2\)) or continuing the Explore strategy (\(\gamma q_B x_2\)) is lower than the payoff from continuing with the Exploit strategy (\(pr_2\)).

Thus, the difference in un-levered firm values at date 0, under the Explore and Exploit strategy implementations at date 0 and efficient investments thereafter, is given by
\[
V_R - V_T = 0.25\gamma q_G - \alpha - 0.5(\bar{l} + p)(\alpha + 0.25).
\]
(8)

Throughout, we assume that at date 0 in the first-best world, the Explore strategy provides a higher value to the firm than the Exploit strategy:
\[
V_R > V_T.
\]
(9)

This requires a mild parametric restriction (see Assumption A1 in Section 5).

3.2 Second best

Our model set-up deviates from the first-best world of Modigliani and Miller (1958) by relaxing two of their assumptions. First, firms pay taxes on their cash flows but can shield taxes by financing in the form of debt. Second, firms incur deadweight costs when they go bankrupt. We derive these bankruptcy costs as a function of (a) the firm’s leverage, (b) the investment strategy that it chose at date 0, Explore or Exploit, and (c) the bankruptcy code.

The analysis of this second-best case proceeds as follows. We first identify equityholders’ and creditors’ decisions at date 1 to (a) liquidate the date 0 investment, (b) continue by implementing the Exploit strategy, or (c) continue by implementing the Explore strategy. Having identified the decision policy of equityholders and creditors, we then calculate the deadweight costs of bankruptcy for the Explore and Exploit strategy implementations at date 0 under equityholders’ control and creditors’ control, respectively. Having measured these deadweight costs, we derive the expressions for the levered firm values at date 0 under the two strategies as a function of leverage and the bankruptcy code. Then, we derive the optimal leverage that the firm would chose under the two strategies and the equilibrium levered
firm values given this optimal leverage. We finally compare these equilibrium firm values to
determine the optimal investment strategy for the firm as a function of the bankruptcy code.

Note that the cash flow is enough to pay creditors when \( x_i > F \) while the firm defaults
when \( x_i < F \). When the firm defaults, \((F - x_i)\) is owed to the creditors. So while calculating
bankruptcy costs as viewed from date 0, we consider cases at date 1 where \( x_i < F \), and then
take unconditional expectation of costs in these states.

3.2.1 Outcomes at date 1 for Exploit strategy implementation at date 0

When the Exploit strategy is chosen at date 0, the cash flow at date 1 does not provide any
signal about the efficacy of the Explore strategy in continuation. Therefore, implementing
the Explore strategy at date 1 is an inferior strategy for both the equityholders and the
creditors. In turn, the decision to be made at date 1 is whether to liquidate the firm or to
continue with the Exploit strategy.

If the date 1 cash flow is \( x_1 < F \), then \((F - x_1)\) is still owed to the creditors. Therefore,
their payoff when the firm is liquidated equals \( \min\left(\tilde{L}, F - x_1\right) \). If the firm is continued and
the Exploit strategy is implemented again, then the date 2 cash flow is \( x_1 \) with probability
\( p \) and 0 with probability \((1 - p)\). Therefore, the creditors’ payoff from continuation is \( p \cdot \min\left(x_1, F - x_1\right) \).

Since equityholders get the residual payoff after the creditors are paid \((F - x_1)\), the equityholders’ payoff from liquidation is \( \max\left(\tilde{L} - F + x_1, 0\right) \) while their payoff from continuation
is \( p \cdot \max\left(x_1 - F + x_1, 0\right) \).

Lemma 1 in Section 5 characterizes the liquidation versus continuation decision if cred-
itors take control in bankruptcy. In particular, creditors liquidate efficiently when the liq-
uidation value is high. In this case, creditors’ remaining claims are sufficiently large that
all future cash flows will accumulate to them. Thus, it is as if they own the firm, and in
turn they liquidate efficiently. In contrast, when the liquidation value is low, creditors liqui-
date inefficiently when the date 1 cash flow is relatively high. In this case, the concavity of
creditors’ claims kicks in, and they liquidate excessively (compared to the first best).

Lemma 2 in Section 5 describes formally the decision when equityholders take control
in bankruptcy. Equityholders continue efficiently when the liquidation value is low. In
contrast, when the liquidation value is high, equityholders continue excessively over a range
of date 1 cash flows where the convexity of their “option”-like claim drives them to continue
inefficiently.

3.2.2 Outcomes at date 1 for Explore strategy implementation at date 0

When the Explore strategy is chosen at date 0, the cash flow at date 1 provides a signal
about the efficacy of the Explore strategy in continuation. Conditional on this signal, there
are three possible outcomes for the firm at date 1: (a) liquidate the Explore strategy, (b)
continue with the Explore strategy, or (c) continue but switch to the Exploit strategy.
Creditors’ payoff when the firm is liquidated equals \( \min(\hat{L}, F - x_2) \). Creditors’ payoff from continuation using the Exploit strategy is \( p \min(x_2, F - x_2) \). Similarly, their payoff from continuation using the Explore strategy is \( q_j \min(x_2, F - x_2) \) where \( j = G \) if the signal from the date 1 cash flow is good while \( j = B \) if the signal from the date 1 cash flow is bad.

Since equityholders get the residual payoff after the creditors are paid \((F - x_2)\), equityholders’ payoff from liquidation is \( \max(\hat{L} - F + x_2, 0) \) while their payoff from continuation using the Exploit strategy is \( p \max(x_2 - F + x_2, 0) \). Their payoff from continuation using the Explore strategy is \( q_j \max(\gamma x_2 - F + x_2, 0) \), where \( j = G \) if the signal from the date 1 cash flow is good while \( j = B \) if the signal from the date 1 cash flow is bad.

Lemma 3 of Section 5 characterizes the outcome when creditors hold control rights in bankruptcy. Intuitively, when the signal about the Explore strategy is good, the outcomes are similar to that in Lemma 1 and creditors liquidate inefficiently when the date 1 cash flow is relatively high. They make it efficient to continue with the Explore strategy when the cash flow is relatively low. Note that when the signal is good, the first-best action is to continue irrespective of the liquidation value. When the signal is bad, creditors always liquidate when the liquidation value is high. This is efficient because conditional on the signal being bad, continuing with the Explore strategy is dominated by continuing with the Exploit strategy, which is in turn dominated by the decision to liquidate when the liquidation value is high (again, as in Lemma 1). However, when liquidation value is low, creditors liquidate inefficiently for relatively high values of the date 1 cash flow and continue efficiently with the Exploit strategy for relatively low values of the date 1 cash flow. The intuition for this result is analogous to that in Lemma 1.

Lemma 4 provides the case when equityholders take control in bankruptcy. In this case, when the signal is good, equityholders always continue with the Explore strategy. This decision is efficient ex post. In contrast, when the signal is bad, equityholders continue with the Explore strategy for a certain range of date 1 cash flows. This is inefficient since in a first-best world, continuing with the Explore strategy is strictly inferior to either liquidating or continuing with the Exploit strategy. However, given the convexity of the equityholders’ claims, the small likelihood of a high cash flow gives them risk-shifting motives. Similarly, for a certain range of date 1 cash flows, equityholders continue inefficiently with the Exploit strategy. Importantly, this range of cash flows is higher than the range over which equityholders continue inefficiently with the Explore strategy. This is also to be expected given the convexity of equityholders’ claim and the induced preference for the greater risk in cash flows from the Explore strategy when the signal is bad.

These characterizations of the continuation and liquidation outcomes at date 1 under different decision-makers and under different strategy choices at date 0 lead to the following important result comparing the attendant inefficiencies.
3.2.3 Expected deadweight costs of bankruptcy

When the firm chooses the Exploit strategy at date 0, the expected deadweight costs from date 1 bankruptcy as viewed from date 0, denoted by $DW_T$, can be expressed in the following simple form when creditors and equityholders are in control in bankruptcy, respectively:

\[
DW_T (\pi = 1) = a_T F^2 , \quad (10)
\]
\[
DW_T (\pi = 0) = b_T F^2 .
\]

Similarly, when the firm follows the Explore strategy at date 0, the expected deadweight costs from bankruptcy, $DW_R$, take the form:

\[
DW_R (\pi = 1) = a_R F^2 , \quad (11)
\]
\[
DW_R (\pi = 0) = b_R F^2 .
\]

The exact expressions for $a_T, a_R, b_T, \text{and} b_R$ are provided in Section 5.

3.2.4 Levered firm’s values

We now consider the more general bankruptcy code where control rights are transferred to creditors with probability $\pi$ and to equityholders otherwise. Note that the levered firm’s value at date 0 is equal to the value of the firm under the first-best minus the taxes paid (taking account of the tax shields from debt) and minus the deadweight costs of bankruptcy. Therefore, the levered firm’s values at date 0 under the Exploit and Explore strategies, as a function of face value of debt $F$ and bankruptcy code $\pi$, are as follows:

\[
V_T (F, \pi) = (1 - \tau) V_T + \int_0^F \tau x_1 dx_1 + \int_{F}^{1+\alpha} \tau F dx_1 - \left[ \pi a_T F^2 + (1 - \pi) b_T F^2 \right] , \quad (12)
\]
\[
V_R (F, \pi) = (1 - \tau) V_R + \int_0^F \tau x_2 dx_2 + \int_{F}^{1} \tau F dx_2 - \left[ \pi a_R F^2 + (1 - \pi) b_R F^2 \right] . \quad (13)
\]

In particular, the second and third term in the expression for $V_T$ represent the tax benefits of debt: firm does not pay taxes in default and shields taxes up to the debt payment when it does pay taxes. The last term in the expression for $V_T$ inside the square parentheses reflects the expected deadweight costs of bankruptcy.

3.2.5 Optimal leverage and equilibrium levered firm values

We first examine the optimal leverage for a given investment strategy, compute the equilibrium firm value under this optimal leverage for that strategy, and then compare firm values across strategy to determine the equilibrium investment strategy.
The firmowners choose \( F \) at date 0 to maximize their equity value given the investment strategy. This is given by the expected value of the firm after payment to creditors minus the investment firmowners have to put up as equity. Since the latter is equal to one minus the market value of debt and debt is fairly priced, firmowners’ problem is effectively one of choosing \( F \) to maximize expected firm value at date 0.

Let the optimal leverage under the Explore and Exploit strategies be \( F^*_R \) and \( F^*_T \), respectively, and the corresponding equilibrium firm values be \( V^*_R \) and \( V^*_T \). We show in Section 5 that for \( i = R, T \)

\[
F^*_i = \frac{c_i \tau/2}{0.5 \tau + a_i \pi + b_i (1 - \pi)} \quad \text{and} \quad V^*_i = K_i + \frac{c_i^2 \tau^2/4}{0.5 \tau + a_i \pi + b_i (1 - \pi)} \tag{14}
\]

where \( c_R = 1 \) and \( c_T = 1 + \alpha \).

4 Results

**Proposition 1:** Under creditor-friendly bankruptcy system \((\pi = 1)\), the deadweight costs of bankruptcy are higher for firms following the Explore strategy compared to those for firms following the Exploit strategy. In contrast, under debtor-friendly code, the deadweight costs of bankruptcy are lower for firms following the Explore strategy compared to those firms following the Exploit strategy. Formally, under Assumption A2,

\[
\begin{align*}
(a) \quad a_R & > a_T, \quad \text{and} \\
(b) \quad b_R & < b_T. 
\end{align*} \tag{15}
\]

This result arises due to the fact that equityholders continue the firm too often while creditors liquidate the firm too often. Since the Explore strategy implementation at date 0 is less likely to succeed at date 1, the net effect is that deadweight costs are lower for firms following the Explore strategy than for firms following the Exploit strategy under debtor-friendly code, and the converse holds under the creditor-friendly code.

**Proposition 2:** As the bankruptcy code becomes more creditor friendly \((\pi \text{ increases})\), the difference in the optimal leverage between firms following the Exploit strategy and the Explore strategy increases. Formally, under Assumption A2,

\[
\frac{d (F^*_R - F^*_T)}{d \pi} < 0. \tag{17}
\]

Intuitively, as the bankruptcy regime becomes more creditor-friendly, firms that follow the Explore strategy reduce their leverage since the deadweight costs for such firms are comparatively higher under creditor control than under equityholder control. In contrast,
firms that follow the Exploit strategy increase their leverage since the deadweight costs for such firms are comparatively lower under creditor control than under equityholder control.

Given this, we obtain our main theoretical result:

Proposition 3: As the bankruptcy code becomes more creditor friendly, the firm value from following the Explore strategy decreases while the firm value from following the Exploit strategy increases. Formally, under Assumption A2,

\[ \frac{d(V_R^* - V_T^*)}{d\pi} < 0. \]  

(18)

In other words, as the bankruptcy regime becomes more creditor friendly, firms find it more valuable at the margin to follow the Exploit strategy than to follow the Explore strategy. From Proposition 1, we know that firms that follow the Explore strategy face greater bankruptcy costs under creditor control than under equityholder control. Therefore, when the bankruptcy code becomes more creditor friendly, these firms respond by lowering their leverage. This reduces the tax shields from debt (effectively raises the cost of capital for the firm) and reduces the value of such firms. In contrast, firms that follow the Exploit strategy face lower bankruptcy costs under creditor control than under equityholder control. Therefore, when the bankruptcy code becomes more creditor friendly, these firms respond by increasing their leverage. This increases the tax shields from debt and thus enhances the value of such firms. Therefore, when the bankruptcy code becomes more creditor friendly, the value of the firms following the Explore strategy decreases while the value of firms following the Exploit strategy increases.

We also find, given some restrictions on our parameters, that the firm value under the Exploit strategy is higher than the firm value under the Explore strategy in countries with stronger creditor rights ($\pi > \overline{\pi}$) while the value under the Explore strategy is higher in countries with weaker creditor rights ($\pi \leq \overline{\pi}$). Therefore, firms in all industries would pursue more innovation in countries with weaker creditor rights. This is because in countries with stronger creditor rights, firms in all industries find it ex ante beneficial to switch to the Explore strategy to avoid the ex post higher likelihood of the creditor liquidating their investments.

Formally, as $\gamma$ increases, the value from pursuing the innovative project increases. However, the deadweight costs under the creditor-friendly code ($a_R$) increase too. However, it turns out that the increase in value from pursuing the innovative strategy is more than the decrease in value from having to give up some leverage. Thus, we obtain that when $\gamma$ is high, that is, ($\gamma > \overline{\gamma}$), for some threshold value $\overline{\gamma}$ (characterized in Section 5), then $V_R^*(1) > V_T^*(1)$, where the argument in the parentheses corresponds to the value of $\pi$. Combined with $V_R^*(0) > V_T^*(0)$, it follows that firms always choose the innovative strategy for this set of values of $\gamma$, irrespective of the bankruptcy code. However, if $\gamma$ is sufficiently small, that is, $\gamma < \overline{\gamma}$, for some threshold $\overline{\gamma}$ (again characterized in Section 5), then $V_R^*(1) < V_T^*(1)$. 

13
Combined with $V^*_R(0) > V^*_T(0)$ and Proposition 3, it follows that there is a single crossing at $\overline{\pi} \in (0, 1)$.

**Proposition 4** (i) If $\gamma > \overline{\pi}$, then $V^*_T(\pi) \leq V^*_R(\pi)$ $\forall \pi \in [0, 1]$. (ii) If $\gamma < \gamma < \overline{\pi}$, then there exists a $\overline{\pi} \in (0, 1)$ such that $V^*_T(\pi) \leq V^*_R(\pi)$ for $\pi \leq \overline{\pi}$ and $V^*_T(\pi) > V^*_R(\pi)$ for $\pi > \overline{\pi}$.

The overall effect can be summarized as follows. The firm attempts to pursue innovation by lowering its debt financing in case the bankruptcy code accords too strong control rights to creditors. If innovation is sufficiently valuable, the firm may in fact employ no debt at all. In general, however, reducing leverage is costly to the firm as it results in a higher cost of capital (e.g., foregoing of tax shields in our model and more generally some other benefits related to debt). If this cost becomes too high, then the firm may be forced to forego its first-best real investment and switch to tried and tested methods. This intuition is formalized in Proposition 3 which states that the difference in value from following innovative and conservative strategies decreases as the bankruptcy code becomes more creditor friendly. Proposition 4 shows that unless the innovative technology is far too dominant compared to the conservative technology, there is a single-crossing property: For weak creditor rights, the value of the conservative technology is lower than that of the innovative one, and beyond a critical level of creditor rights, the innovative technology is worth more. Our main empirical implication is based on Propositions 3 and 4.

## 5 Parametric assumptions and proofs

**Assumption 1.** The following assumption ensures that the value of the un-levered firm under the Explore strategy, $\overline{V}_R$, is greater than that under the Exploit strategy, $\overline{V}_T$:

$$\gamma q_G > 4\alpha (1 + 0.5\overline{T} + 0.5p) + 0.5 (\overline{T} + p) \tag{A1}$$

In other words, we assume that the expected cash flows from the Explore strategy when it succeeds are sufficiently higher than the total cash flows from the Exploit strategy. The assumption is a natural one to make given the considerable returns that firms realize from successful innovations.

**Assumption 2.** The following assumptions are required for Propositions 1, 2 and 3 to hold:

$$\gamma \geq 2 \text{ and } p \geq 0.5 \tag{A2}$$

In other words, we assume that the cashflows from the Explore strategy are at least twice the cash flows from the Exploit strategy ($\gamma \geq 2$), and the likelihood of success on the less risky Exploit strategy is at least a half.\(^5\)

\(^5\)In fact, the assumption that $p$ be greater than half is a rather weak sufficient condition. The condition
**Lemma 1:** If the Exploit strategy was implemented at date 0 and control rests with creditors in bankruptcy, then conditional upon default

(a) Creditors liquidate and this decision is efficient ex-post if $\bar{L} = \bar{L}x_1$.

(b) Creditors liquidate and this decision is *inefficient* ex-post if $\bar{L} = \bar{L}x_1$ and $\frac{pF}{p^* + l} < x_1 < F$.

(c) Creditors continue and this decision is efficient ex-post if $\bar{L} = \bar{L}x_1$ and $\alpha \leq x_1 \leq \frac{pF}{p^* + l}$.

**Proof of Lemma 1:** First consider $\bar{L} = \bar{L}x_1$.

Case 1: $F - x_1 \leq \bar{L}x_1 < x_1 : \min (\bar{L}, F - x_1) = F - x_1 > p \min (x_1, F - x_1) = p (F - x_1)$. So, creditors liquidate.

Case 2: $\bar{L}x_1 < F - x_1 \leq x_1 : \min (\bar{L}, F - x_1) = \bar{L}x_1$ while $p \min (x_1, F - x_1) = p (F - x_1) \leq px_1$. Since $p < l$, creditors liquidate.

Case 3: $\bar{L}x_1 < x_1 < F - x_1 : \min (\bar{L}, F - x_1) = \bar{L}x_1$ while $p \min (x_1, F - x_1) = px_1$. Since $p < \bar{L}$, creditors liquidate.

Now, consider $\bar{L} = \bar{L}x_1$.

Case 1: $F - x_1 \leq \bar{L}x_1 < x_1 : \min (\bar{L}, F - x_1) = F - x_1 > p \min (x_1, F - x_1) = p (F - x_1)$. So, creditors liquidate.

Case 2: $\bar{L}x_1 < F - x_1 \leq x_1 : \min (\bar{L}, F - x_1) = \bar{L}x_1$ while $p \min (x_1, F - x_1) = p (F - x_1)$. So, creditors liquidate if $x_1 > \frac{pF}{p^* + l}$ and continue if $x_1 \leq \frac{pF}{p^* + l}$.

Case 3: $\bar{L}x_1 < x_1 < F - x_1 : \min (\bar{L}, F - x_1) = \bar{L}x_1$ while $p \min (x_1, F - x_1) = px_1$. Since $p > \bar{L}$, creditors continue.

The result follows by comparing with the first best which is to liquidate when $\bar{L} = \bar{L}x_1$ and continue when $\bar{L} = \bar{L}x_1$. ◊

**Lemma 2:** If the Exploit strategy was implemented at date 0 and control rests with equityholders in bankruptcy, then conditional upon default

(a) Equityholders always continue and this decision is efficient ex-post if $\bar{L} = \bar{L}x_1$.

(b) Equityholders continue and this decision is *inefficient* ex-post if $\bar{L} = \bar{L}x_1$ and $\frac{F}{\alpha + l} \leq x_1 \leq \frac{(1 - p)F}{1 + \alpha - 2p}$.

(c) Equityholders liquidate and this decision is efficient ex-post if $\bar{L} = \bar{L}x_1$, and $\alpha < x_1 < \frac{F}{\alpha}$ or $\frac{(1 - p)F}{1 + \alpha - 2p} < x_1 < F$.

**Proof of Lemma 2:** First consider $\bar{L} = \bar{L}x_1$.

Case 1: $F - x_1 \leq \bar{L}x_1 < x_1 : \max (\bar{L} - F + x_1, 0) = \bar{L}x_1 - F + x_1$ while $p \max (x_1 - F + x_1, 0) = p (2x_1 - F)$. So, equityholders continue if $x_1 \leq \frac{(1 - p)F}{1 + \alpha - 2p} < F$ since $\bar{L} > p$. \[\bar{L} > 0.265\text{, meaning that liquidation, when the high liquidation values are realized, provides at least 26.5\% of the first period cash flow, suffices. Since the high liquidation value must be higher than the probability of success under the Exploit strategy (p) for liquidations to occur in equilibrium, this assumption holds for even very low probabilities of success with the Exploit strategy.}\]

15
Case 2: If \( Lx_1 < F - x_1 \leq x_1 : \max(\bar{L} - F + x_1, 0) = 0 < p \max(x_1 - F + x_1, 0) = p(2x_1 - F) \). So, equityholders continue.

Case 3: If \( Lx_1 < x_1 < F - x_1 : \max(\bar{L} - F + x_1, 0) = 0 \) and \( p \max(x_1 - F + x_1, 0) = 0 \). Since equityholders are indifferent, they liquidate efficiently.

Now, consider \( \bar{L} = lx_1 \).

Case 1: If \( F - x_1 \leq lx_1 < x_1 : \max(\bar{L} - F + x_1, 0) = lx_1 - F + x_1 \) while \( p \max(x_1 - F + x_1, 0) = p(2x_1 - F) \). Since \( \bar{L} < p \) and \( x_1 < F \), \( p(2x_1 - F) > lx_1 - F + x_1 \). Therefore, equityholders continue.

Case 2: If \( lx_1 < F - x_1 \leq x_1 : \max(\bar{L} - F + x_1, 0) = 0 < p \max(x_1 - F + x_1, 0) = p(2x_1 - F) \). So, equityholders continue.

Case 3: If \( lx_1 < x_1 < F - x_1 : \max(\bar{L} - F + x_1, 0) = 0 \) and \( p \max(x_1 - F + x_1, 0) = 0 \). Since equityholders are indifferent, they continue efficiently.

The result follows by comparing with the first best which is to liquidate when \( \bar{L} = lx_1 \) and continue when \( \bar{L} = lx_1 \).

**Lemma 3:** If the Explore strategy was implemented at date 0 and control rests with creditors in bankruptcy, then conditional upon default

(a) Creditors liquidate and this decision is efficient ex-post if the signal is bad and \( \bar{L} = lx_1 \).

(b) Creditors liquidate and this decision is inefficient ex-post in the following three cases:

(i) signal is good, \( \bar{L} = lx_2 \) and \( \frac{qG}{qG + L} < x_2 < F \) (ii) signal is good, \( \bar{L} = lx_2 \), and \( \frac{qG}{qG + L} < x_2 < F \)

(iii) signal is bad, \( \bar{L} = lx_2 \), and \( \frac{pE}{pE + L} < x_2 < F \).

(c) Creditors continue with the Explore strategy and this decision is efficient ex-post in the following two cases: (i) signal is good, \( \bar{L} = lx_2 \) and \( 0 \leq x_2 < \frac{qG}{qG + L} \) (ii) signal is good, \( \bar{L} = lx_2 \) and \( 0 \leq x_2 < \frac{qG}{qG + L} \).

(d) Creditors continue with the Explore strategy and this decision is efficient ex-post if signal is bad, \( \bar{L} = lx_2 \) and \( 0 \leq x_2 < \frac{pE}{pE + L} \).

**Proof of Lemma 3:** We first consider the case when the signal is good. So, the payoff from continuing with the Explore strategy is \( qG \min(\gamma x_2, F - x_2) \geq p \min(x_2, F - x_2) \) since \( qG > p \) and \( \gamma > 1 \). So, creditors always prefer continuing with the Explore strategy to continuing with the Exploit strategy. Now we check the creditors’ decision to liquidate versus continuing with the Explore strategy.

Case 1: If \( F - x_2 \leq \bar{L} < \gamma x_2 : \min(\bar{L}, F - x_2) = F - x_2 > qG \min(\gamma x_2, F - x_2) = qG(F - x_2) \). So, creditors liquidate.

Case 2: If \( \bar{L} < F - x_2 \leq \gamma x_2 : \min(\bar{L}, F - x_2) = \bar{L} \) while \( qG \min(\gamma x_2, F - x_2) = qG(F - x_2) \). So, creditors liquidate if \( x_2 > \frac{qG}{qG + (L/x_2)} \) and continue if \( x_2 \leq \frac{qG}{qG + (L/x_2)} \).

Case 3: If \( \bar{L} < \gamma x_2 < F - x_2 : \min(\bar{L}, F - x_2) = \bar{L} \) while \( qG \min(\gamma x_2, F - x_2) = qG\gamma x_2 \). Since \( qG\gamma > \bar{L} \), creditors continue.

Next consider the case when the signal is bad.
Case 1: $F - x_2 > \gamma x_2 > \tilde{L}$: Payoff from the new work method is $q_B \min(\gamma x_2, F - x_2) = q_B \gamma x_2 > p \min(x_2, F - x_2) = px_2$ since $p > \gamma q_B$. So, creditors prefer continuing with the Explore strategy to liquidation.

Case 2: $x_2 < F - x_2 \leq \gamma x_2$: Payoff from the Explore strategy is $px_2$ while the payoff from the Explore strategy is $q_B (F - x_2) \leq \gamma q_B x_2 < px_2$ since $p > q_B \gamma$. So, creditors prefer the Exploit strategy.

Case 3: $F - x_2 \leq x_2 < \gamma x_2$: Payoff from the Explore strategy is $p (F - x_2)$ which is the payoff from the Explore strategy. This inequality follows from $p > q_B$. So, creditors prefer the Exploit strategy.

Therefore when the signal is bad, creditors always prefer continuing with the Exploit strategy to continuing with the Explore strategy. The decision to liquidate versus continue with the Explore strategy remains the same as in Lemma 1. \(\diamondsuit\)

**Lemma 4:** If the Explore strategy was implemented at date 0 and control rests with equityholders in bankruptcy, then conditional upon default

(I) If the signal is good, equityholders continue with the Explore strategy and this decision is ex post efficient.

(II) If the signal is bad, then

(a) Equityholders continue with the Explore strategy and this decision is ex post inefficient in the following cases: (i) $\tilde{L} = lx_2$ and $\frac{F}{2} \leq x_2 < \frac{(p-q_B)F}{2p-q_B-q_B}$ (ii) $\tilde{L} = lx_2$ and $\frac{F}{2} \leq x_2 < \frac{(p-q_B)F}{2p-q_B-q_B}$.

(b) Equityholders continue with the Exploit strategy and this decision is ex post efficient if (i) $\tilde{L} = lx_2$ and $x_2 < \frac{F}{1+\gamma}$ (ii) $\tilde{L} = lx_2$ and $\frac{(p-q_B)F}{2p-q_B-q_B} \leq x_2 < F$.

(c) Equityholders continue with the Explore strategy and this decision is ex post inefficient if $\tilde{L} = lx_2$ and $\frac{(p-q_B)F}{2p-q_B-q_B} \leq x_2 < \frac{F(1-p)}{1+1-2p}$.

(d) Equityholders liquidate and this decision is ex post efficient if (i) $\tilde{L} = lx_2$ and $x_2 < \frac{F}{1+\gamma}$ (ii) $\tilde{L} = lx_2$ and $\frac{F(1-p)}{1+1-2p} \leq x_2 < F$.

**Proof of Lemma 4:** We first consider the case where the signal is good. Payoff from the Explore strategy is $q_G \max(\gamma x_2 - F + x_2, 0) \geq p \max(x_2 - F + x_2, 0)$ since $q_G > p$ and $\gamma > 1$. Therefore, when the signal is good, equityholders always prefer the Explore strategy to the Exploit strategy. The maximum value from liquidation is $\max(\tilde{L}x_2 - F + x_2, 0)$. Now using $\tilde{L} < q_G$ we get $\tilde{L}x_2 - F + x_2 - q_G (\gamma x_2 - F + x_2) < (1-q_G)(\gamma x_2 - F + x_2) < 0$. Since $\gamma > 1$ and $x_2 < F$, it follows that $(1-q_G)(\gamma x_2 - F + x_2) < 0$. Therefore, $\tilde{L}x_2 - F + x_2 < q_G (\gamma x_2 - F + x_2)$. So, $\max(\tilde{L}x_2 - F + x_2, 0) \leq q_G \max(\gamma x_2 - F + x_2, 0)$. So, equityholders prefer continuing with the Explore strategy to liquidation. Therefore, when the signal is good, equityholders continue with the Explore strategy.

When the signal is bad, payoff from the Explore strategy is $q_B \max(\gamma x_2 - F + x_2, 0)$.

Case 1: $F - x_2 > \gamma x_2 > x_2 > \tilde{L}$: $q_B \max(\gamma x_2 - F + x_2, 0) = p \max(x_2 - F + x_2, 0) = \max(\tilde{L} - F + x_2, 0) = 0$. Therefore, equityholders implement the first best in this case which is to liquidate when $\tilde{L} = \tilde{L}x_2$ and continue with the Exploit strategy when $\tilde{L} = lx_2$. 17
Case 2: $\bar{L}x_2 < x_2 < F - x_2 \leq \gamma x_2 : p \max(x_2 - F + x_2, 0) = \max(\bar{L} - F + x_2, 0) = 0$. Therefore, equityholders continue with the Explore strategy.

Case 3: $F - x_2 \leq x_2 \leq \gamma x_2 : \max(x_2 - F + x_2, 0) = p(x_2 - F + x_2)$ and $q_B \max(\gamma x_2 - F + x_2, 0) = q_B(\gamma x_2 - F + x_2)$. Therefore, equityholders continue with the Explore strategy when $F \left(\frac{p - q_B}{p - q_B + p - q_B} - x_2 \right) \leq \gamma x_2$: $p_{\max}(x_2 - F + x_2, 0) = \max(\bar{e}_L - F + x_2, 0) = 0$. Therefore, equityholders continue with the Explore strategy.

Sub-case A: $\frac{F}{2} \leq x_2 < F \left(\frac{p - q_B}{p - q_B + p - q_B} \right)$: If $x_2 \leq F \left(\frac{1 - q_B}{1 + l - q_B} \right)$, then equityholders continue with the Explore strategy while they liquidate if $x_2 > F \left(\frac{1 - q_B}{1 + l - q_B} \right)$. $\diamond$

Sub-case B: $F \left(\frac{p - q_B}{p - q_B + p - q_B} \right) \leq x_2 < F$ : If $x_2 \leq F \left(\frac{1 - p}{1 + l - p} \right)$, then equityholders continue with the Exploit strategy while they liquidate if $x_2 > F \left(\frac{1 - p}{1 + l - p} \right)$.

**Proof of Proposition 1**: From Lemma 1, the deadweight costs from bankruptcy when the firm uses the Exploit strategy and when creditors are in control, equal

$$0.5 \int_{\frac{F}{1 + l - 2p}}^F (px_1 - \bar{L}x_1) \, dx_1 = 0.25 (p - \bar{L}) \left[ 1 - \left( \frac{p}{p + \bar{L}} \right)^2 \right] \bar{F}^2 \equiv a_T \bar{F}^2,$$

where

$$a_T \equiv 0.25 (p - \bar{L}) \left[ 1 - \left( \frac{p}{p + \bar{L}} \right)^2 \right].$$

From Lemma 2, the deadweight costs from bankruptcy when the firm follows the Exploit strategy and equityholders are in control equals

$$0.5 \int_{\frac{F}{1 + l - 2p}}^{(1 - p)F} (\bar{L}x_1 - px_1) \, dx_1 = 0.25 (\bar{L} - p) \left[ \left( \frac{1 - p}{1 + l - 2p} \right)^2 - \frac{1}{4} \right] \bar{F}^2 \equiv b_T \bar{F}^2$$

where

$$b_T \equiv 0.25 (\bar{L} - p) \left[ \left( \frac{1 - p}{1 + l - 2p} \right)^2 - \frac{1}{4} \right].$$

Similarly, from Lemma 3, the deadweight costs from bankruptcy when the firm follows
the Explore strategy and creditors are in control equal

\[
0.25 \int_{\gamma qG + l}^{\gamma qG} (q_G \gamma x_2 - \bar{t} x_2) \, dx_2 + 0.25 \int_{\gamma qG + l}^{p} (q_G \gamma x_2 - \bar{t} x_2) \, dx_2 + 0.25 \int_{\gamma qG + l}^{p} (px_2 - \bar{t} x_2) \, dx_2
\]

\[
\equiv a_R F^2
\]

where

\[
a_R \equiv 0.125 \left\{ \left( \gamma q_G - \bar{t} \right) \left[ 1 - \left( \frac{q_G}{q_G + l} \right)^2 \right] + \left( \gamma q_G - \bar{p} \right) \left[ 1 - \left( \frac{q_G}{q_G + l} \right)^2 \right] \right\}.
\]

Finally, from Lemma 4, the deadweight costs from bankruptcy when the firm follows the Explore strategy and equityholders are in control equal

\[
0.25 \int_{\gamma qB}^{\gamma qB + l} (px_2 - q_B \gamma x_2) \, dx_2 + 0.25 \int_{\bar{t} - q_B}^{\bar{t}} (\bar{t} x_2 - q_B \gamma x_2) \, dx_2
\]

\[
+0.25 \int_{\gamma qB}^{\gamma qB + l} (\bar{t} x_2 - px_2) \, dx_2
\]

\[
\equiv b_R F^2,
\]

where

\[
b_R \equiv 0.125 \left\{ \left( p - \gamma q_B \right) \left[ \left( \frac{p - q_B}{2p - q_B - \gamma q_B} \right)^2 - \frac{1}{4} \right] + \left( \bar{t} - \gamma q_B \right) \left[ \left( \frac{p - q_B}{2p - q_B - \gamma q_B} \right)^2 - \frac{1}{4} \right] \right\}.
\]

We now proceed to prove the results.

(a) From the expressions above, we obtain that

\[
8 (a_R - a_T) = \left( \gamma q_G - \bar{t} \right) \left[ 1 - \left( \frac{q_G}{q_G + l} \right)^2 \right] + \left( \gamma q_G - \bar{p} \right) \left[ 1 - \left( \frac{q_G}{q_G + l} \right)^2 \right] - \left( \bar{t} - \bar{p} \right) \left[ 1 - \left( \frac{p}{p + l} \right)^2 \right].
\]

Because \( q_G > \bar{t} \) from (5), it follows that

\[
\gamma q_G - \bar{t} > \gamma \bar{t} - \bar{t} = (\gamma - 1) \bar{t}.
\]
Also, since \( p < \bar{t} \) from (5), we obtain that

\[
p - \underline{l} < \bar{t} - \underline{l}.
\]

In turn, \( \gamma_{qG} - \bar{t} > p - \underline{l} \) because \( \gamma > 2 \) (by Assumption A2) and \( \bar{t} > 0 \), and \( \underline{l} > 0 \).

Next, since \( \underline{l} < \bar{t} \), it follows that

\[
\gamma_{qG} - \underline{l} = \gamma_{qG} - \bar{t} + \bar{t} - \underline{l} > 2 (p - \underline{l})
\]

where we have used the facts \( \gamma_{qG} - \bar{t} > p - \underline{l} \) and \( \bar{t} > p \). Since \( \bar{t} > \sqrt{\frac{8}{5}} - 1 \) (by Assumption A2) and \( q_{G} < 1 \), it follows that

\[
\frac{q_{G}}{q_{G} + \bar{l}} < \sqrt{\frac{5}{8}}.
\]

Thus,

\[
8 (a_{R} - a_{T}) > (p - \underline{l}) \left[ 2 - 2 \left( \frac{q_{G}}{q_{G} + \bar{l}} \right)^{2} - \left( \frac{q_{G}}{q_{G} + \bar{l}} \right)^{2} + \left( \frac{p}{p + \bar{l}} \right)^{2} \right]
\]

\[
> (p - \underline{l}) \left[ 1 - \frac{5}{8} + \left( \frac{p}{p + \bar{l}} \right)^{2} + 1 - \left( \frac{q_{G}}{q_{G} + \bar{l}} \right)^{2} \right]
\]

\[
= (p - \underline{l}) \left[ \left( \frac{p}{p + \bar{l}} \right)^{2} - \frac{1}{4} + 1 - \left( \frac{q_{G}}{q_{G} + \bar{l}} \right)^{2} \right] > 0,
\]

where the last step follows from \( \underline{l} < p \). Therefore, \( a_{R} > a_{T} \). ♦

(b) Similarly, it can be shown that

\[
8 (b_{R} - b_{T}) = (p - \gamma_{qB}) \left[ \left( \frac{p - q_{B}}{2p - q_{B} - \gamma_{qB}} \right)^{2} - \frac{1}{4} \right] + (\bar{t} - \gamma_{qB}) \left[ \left( \frac{p - q_{B}}{2p - q_{B} - \gamma_{qB}} \right)^{2} - \frac{1}{4} \right]
\]

\[
+ (\bar{t} - p) \left[ \left( \frac{1 - p}{1 + \bar{t} - 2p} \right)^{2} - \left( \frac{p - q_{B}}{2p - q_{B} - \gamma_{qB}} \right)^{2} \right] - 2 (\bar{t} - p) \left[ \left( \frac{1 - p}{1 + \bar{t} - 2p} \right)^{2} - \frac{1}{4} \right]
\]

\[
= (\bar{t} - p) \left[ \frac{1}{2} - \left( \frac{1 - p}{1 + \bar{t} - 2p} \right)^{2} - \left( \frac{p - q_{B}}{2p - q_{B} - \gamma_{qB}} \right)^{2} \right]
\]

\[
+ (p + \bar{t} - \gamma_{qB}) \left[ \left( \frac{p - q_{B}}{2p - q_{B} - \gamma_{qB}} \right)^{2} - \frac{1}{4} \right].
\]

Since \( \gamma_{qB} < p < 2p \), it follows that \( (\bar{t} - p) < (p + \bar{t} - \gamma_{qB}) \). Therefore,

\[
8 (b_{R} - b_{T}) < (p + \bar{t} - \gamma_{qB}) \left[ \frac{1}{4} - \left( \frac{1 - p}{1 + \bar{t} - 2p} \right)^{2} \right] < 0,
\]
since $\frac{1-p}{1+l-2p} > \frac{1}{2}$. Therefore, $b_R < b_T$. $\diamond$

**Proof of Proposition 2:** The value of the levered firm can be written in general as (for the Explore strategy with sub-script $R$ on $V$, $a$ and $b$, and for the Exploit strategy with sub-script $T$, and with $c = 1$ for the Explore strategy and $c = 1 + \alpha$ for the Exploit strategy):

\[
V(F) = K - a\pi F^2 - b(1 - \pi) F^2 - 0.5\tau F^2 + cF\tau, \text{ so that (20)}
\]

\[
\frac{dV}{dF} = c\tau - 2a\pi F - 2b(1 - \pi) F - \tau F, \text{ and}
\]

\[
\frac{d^2V}{dF^2} = -2a\pi - 2b(1 - \pi) - \tau < 0.
\]

Therefore, setting $\frac{dV}{dF} = 0$, we obtain that optimal leverage and the optimized firm value are given respectively as

\[
F^* = \frac{c\tau/2}{0.5\tau + a\pi + b(1 - \pi)} \text{ and } V^* = K + \frac{c^2\tau^2/4}{0.5\tau + a\pi + b(1 - \pi)} (21)
\]

Then,

\[
\frac{dF^*}{d\pi} = -\frac{(a - b)c(\tau/2)}{[0.5\tau + a\pi + b(1 - \pi)]^2} \text{ and } \frac{dV^*}{d\pi} = -\frac{(a - b)c^2(\tau^2/4)}{[0.5\tau + a\pi + b(1 - \pi)]^2}.
\]

Thus, for the Exploit strategy, since $a_T < b_T$ from Proposition 1, we obtain that

\[
\frac{dF^*_T}{d\pi} = -\frac{(a_T - b_T)(c_T)(\tau/2)}{[0.5\tau + a_T\pi + b_T(1 - \pi)]^2} > 0.
\]

Similarly, for the Explore strategy, since $a_R > b_R$,

\[
\frac{dF^*_R}{d\pi} = -\frac{(a_R - b_R)(c_R)(\tau/2)}{[0.5\tau + a_R\pi + b_R(1 - \pi)]^2} < 0. \diamond
\]

**Proof of Proposition 3:** This follows directly from the expressions for optimized firm value in the proof of Proposition 2 above:

\[
\frac{dV^*_R}{d\pi} = -\frac{(a_R - b_R)(c^2_R)(\tau^2/4)}{[0.5\tau + a_R\pi + b_R(1 - \pi)]^2} < 0, \text{ and}
\]

\[
\frac{dV^*_T}{d\pi} = -\frac{(a_T - b_T)(c^2_T)(\tau^2/4)}{[0.5\tau + a_T\pi + b_T(1 - \pi)]^2} > 0. \diamond
\]
PROOF OF PROPOSITION 4: Define \( \gamma \) as

\[
0.25 (p - \gamma q_B) \left[ \left( \frac{p - q_B}{2p - q_B - \gamma q_B} \right)^2 \right] + 0.125 \left( \frac{1}{2} - \frac{p + \gamma}{4} \right) +
\]

\[
= \frac{1}{(1 + \alpha)^2} b_T - 0.125 (\bar{t} - p) \left( \frac{1 - p}{1 + \bar{t} - 2p} \right)^2 + 0.5 \tau \frac{(2\alpha + \alpha^2)}{(1 + \alpha)^2}
\]

and define \( \overline{\gamma} \) as

\[
0.25 \overline{\gamma} q_G + \frac{\tau^2/4}{0.5 \tau + a_R (\overline{\gamma})} = \alpha + 0.5 (\bar{t} + p) (\alpha + 0.25) + \frac{(1 + \alpha)^2 \tau^2/4}{0.5 \tau + a_T}
\]

Now

\[
V_R^* (\pi) = \overline{V}_R + \frac{\tau^2/4}{0.5 \tau + a_R \bar{p}_T + b_R (1 - \pi)}
\]

\[
V_T^* (\pi) = \overline{V}_T + \frac{(1 + \alpha)^2 \tau^2/4}{0.5 \tau + a_T \bar{p}_T + b_T (1 - \pi)}
\]

using the expressions for \( V^* (\pi) \) from equation (21). Therefore,

\[
V_T^* (0) - V_R^* (0) = \overline{V}_T - \overline{V}_R + \tau^2/4 \left( \frac{(1 + \alpha)^2}{0.5 \tau + b_T} - \frac{1}{0.5 \tau + b_R} \right)
\]

Since \( \gamma > \gamma \), it follows that

\[
0.25 \gamma q_G + \frac{\tau^2/4}{0.5 \tau + a_R (\gamma)} = \alpha + 0.5 (\bar{t} + p) (\alpha + 0.25) + \frac{(1 + \alpha)^2 \tau^2/4}{0.5 \tau + a_T}
\]

which employing the expression for \( b_R \) and \( b_T \) simplifies to

\[
\frac{(1 + \alpha)^2}{0.5 \tau + b_T} < \frac{1}{0.5 \tau + b_R}
\]

Since \( \overline{V}_T < \overline{V}_R \), it follows that \( V_T^* (0) < V_R^* (0) \).

Now

\[
V_T^* (1) - V_R^* (1) = \overline{V}_T - \overline{V}_R + \tau^2/4 \left( \frac{(1 + \alpha)^2}{0.5 \tau + a_T} - \frac{1}{0.5 \tau + a_R} \right)
\]

\[
= 0.25 \gamma q_G - \alpha - 0.5 (\bar{t} + p) (\alpha + 0.25) + \tau^2/4 \left( \frac{(1 + \alpha)^2}{0.5 \tau + a_T} - \frac{1}{0.5 \tau + a_R} \right)
\]

using the expression for \( \overline{V}_R - \overline{V}_T \) from equation (8). Now differentiating the above expression
w.r.t. $\gamma$, we get
\[
4 \left(0.5\tau + a_R\right)^2 \frac{d}{d\gamma} \left(V_R^* (1) - V_T^* (1)\right) \gamma q_G d\gamma = \left[\left(\frac{q_G}{q_G + l}\right)^2 + \left(\frac{q_G}{q_G + L}\right)^2\right] \frac{\tau^2}{8} + \tau a_R + (a_R)^2 > 0
\]
\[
\Leftrightarrow \frac{d}{d\gamma} \left(V_R^* (1) - V_T^* (1)\right) > 0
\]

Thus $V_R^* (1) - V_T^* (1)$ is monotonically increasing in $\gamma$. From the definition of $\overline{\gamma}$, it follows that
\[
\gamma > \overline{\gamma} \Rightarrow V_R^* (1) - V_T^* (1) > 0
\]
\[
\gamma < \overline{\gamma} \Rightarrow V_R^* (1) - V_T^* (1) < 0
\]

Since $V_T^* - V_R^*$ is monotonously increasing in $\pi$ from Proposition 3, it follows then that there exists a $\pi \in (0, 1)$ such that $V_T^* (\pi) \leq V_R^* (\pi)$ for $\pi \leq \pi$ and $V_T^* (\pi) > V_R^* (\pi)$ for $\pi > \pi$.

\[\Box\]

References


Figure 1: Correlation between country level economic development and measures of innovation.

Figure 2: Timing and Sequence of Events.
Figure 3: Ratio of innovation in Biotechnology to that in Textiles for US and Germany

Figure 4: Ratio of innovation in Computer Pheripherals to that in Textiles for US and Germany
Figure 5: Ratio of innovation in Information Storage to that in Textiles for US and Germany

Figure 6: Ratio of innovation in Drugs to that in Textiles for US and Germany
Figure 7: Ratio of innovation in Surgery and Medical Instruments to that in Textiles for US and Germany