Allowing for Real Effects while Assets are Sold

Model

Assume that there is asset-specificity even within the financial intermediaries, so that when assets are sold, the new buyer can only obtain a mean \( \delta \theta y^2 \), where \( \delta \in (0, 1) \). So the new buyer faces a choice between two projects, one with payoff \( \delta y^2 \) with probability \( \theta^2 \) and 0 with probability \( 1 - \theta^2 \), and the second with payoff \( \delta y_1 \) with probability \( \theta^1 \) and 0 with probability \( 1 - \theta^1 \). Hence the new buyer faces a scaled down version of the original owner’s risk shifting choice. We also assume that \( \delta \theta^2 > \rho^* > \theta^1 y^1 \). We denote \( \rho^*_b \equiv \delta \rho^* \).

Thus the secondary market can only support a price that is less than or equal to \( \delta \theta^2 y^2 \) because of the deadweight loss on transfer.

The market for assets at date 1 now operates slightly differently. The original buyer of the asset still sells only if \( p > \rho^* \) which yields the exact same formula as before: the fraction sold is \( \alpha = \min \left[ \frac{p - \rho^*}{p - \rho^*_b}, 1 \right] \). In contrast, a new buyer faces a slightly different problem. If \( p < \delta \theta^2 y^2 \), he is willing to buy a quantity

\[
\alpha = \frac{\rho^* - \rho}{p - \rho^*_b} = \frac{\rho^* - \rho}{p - \delta \rho^*}
\]  

(1)

In effect, the new buyer buys a lower amount than in absence of asset-specificity because he can finance less than the original buyer at the same price.

Equating demand and supply we obtain

\[
\int_{p_{\min}}^{p^*} \frac{\rho^* - \rho}{p - \rho^*_b} g(\rho) d\rho = \int_{p^*}^{p_{\max}} \frac{\rho - \rho^*}{p - \rho^*} g(\rho) d\rho + \int_{p}^{p_{\max}} g(\rho) d\rho
\]  

(2)

Rearranging yields

\[
p \leq \rho^* + \frac{p - \rho^*}{p - \delta \rho^*} \int_{p_{\min}}^{\rho^*} G(\rho) d\rho + \int_{\rho^*}^{p} G(\rho) d\rho
\]  

(3)

where a strict inequality implies that \( p = \delta \theta^2 y^2 \). Overall, this yields a lower price relative to the original case where \( \delta = 1 \).

Existence of solution to excess demand equation

The excess demand equation is now given by:

\[
E(p, \rho^*) = \int_{p_{\min}}^{\rho^*} \frac{\rho^* - \rho}{p - \rho^*_b} g(\rho) d\rho - \int_{\rho^*}^{p} \frac{\rho - \rho^*}{p - \rho^*} g(\rho) d\rho - \int_{p}^{p_{\max}} g(\rho) d\rho
\]
$\frac{1}{p - \rho_b} G(\rho) d\rho + \int_{\rho^*}^{p} \frac{1}{p - \rho^*} G(\rho) d\rho$

First, if the excess demand function is positive for all $p < \delta \theta y$, the price is $\delta \theta y^2$.

We wish to prove next that if the excess demand has an intersection before $\delta \theta y^2$, it is a unique intersection.

Rather than work with the excess demand function directly, we rescale it by a positive function $p - \delta \rho^*$ and use the fact that if this function intersects zero once, then the excess demand intersects zero only once.

So the equivalent function which must be non-negative when the excess demand is nonnegative is given by:

$-(p - \delta \rho^*) + \int_{\rho_{min}}^{\rho^*} G(\rho) d\rho + \frac{p - \rho_b^*}{p - \rho^*} \int_{\rho^*}^{p} G(\rho) d\rho$

Differentiating this re-scaled function with respect to $p$, we obtain the derivative (after some simplification) as

$-1 + \frac{p - \rho_b^*}{p - \rho^*} G(p) - \frac{p - \rho_b^*}{(p - \rho^*)^2} \int_{\rho^*}^{p} G(\rho) d\rho$

$\leq -1 + G(p)$

so the excess demand function has negative slope when it intersects zero and hence can only intersect once, i.e., the solution is unique.

Existence of solution to dynamic equilibrium

We now derive the ex-ante equilibrium. The three equilibrium conditions are:

(Price equation)

1. $p(\theta) \leq \rho^*(\theta) + \frac{p(\theta) - \rho^*(\theta)}{p(\theta) - \delta \rho^*(\theta)} \int_{\rho_{min}}^{\rho^*(\theta)} \hat{G}(u) du + \int_{\rho^*(\theta)}^{p(\theta)} \hat{G}(u) du$

In particular, $\hat{G}(u)$ is the truncated equilibrium distribution of liabilities given by $\hat{G}(u) = \frac{R(\rho^{-1}(u))}{R(s)}$.

Formally, $\hat{G}(u)$ is induced by the distribution of financing amounts, $R(s)$, via the function
Prob[\rho(s_i) \leq u | s_i \leq \hat{s}]. A strict (<) inequality in equation above leads to \( p^*(\theta_2) = \bar{p}(\theta_2) = \delta\theta_2 y_2 \).

(Breakeven for lender on each loan)

2. Given the price function \( p^*(\theta_2) \), for every shortfall \( s_i \in [0, \hat{s}] \), the promised face value \( \rho \) is determined by the requirement that lenders receive in expectation the amount being lent:

\[
\begin{align*}
  s_i &= \int_{\theta_{\min}}^{\theta_{\max}} p^*(\theta_2) h(\theta_2) d\theta_2 + \int_{p^*^{-1}(\rho)}^{\theta_{\max}} \rho h(\theta_2) d\theta_2. \\
  &\quad (8)
\end{align*}
\]

(Last marginal loan yields lender zero NPV)

3. The truncation point \( \hat{s} \) for maximal external financing is determined by the condition

\[
\hat{s} \leq \int_{\theta_{\min}}^{\theta_{\max}} p^*(\theta_2) h(\theta_2) d\theta_2,
\]

with a strict inequality implying that \( \hat{s} = s_{\max} \) (all borrowers are financed).

Existence of unique solution

For brevity, we do not detail the existence proof. Using differential versions of Equations (7) and (8), we follow the contraction approach to integro-differential equations that is in the Appendix of the paper. However, we now have a system of two equations, one of which is an ODE and one of which is an integro-differential equation. However, our approach to existence can be extended relatively easily.

Welfare

We conjecture that with asset-specificity within the financial intermediaries, there is “excessive” leverage undertaken at date 0, simply because the cost of fire sales imposed on other firms’ creditors is not fully internalized in each firm’s leverage decision. Hence, some form of capital requirement that restricts entry of the most leveraged intermediaries might be desirable in general. Formally, reducing \( \hat{s} \) slightly only takes out the marginal borrower at time 0 as far as investment is concerned (a second-order effect), but by increasing time 1 prices and reducing face values, it has a first-order effect on all other firms as it affects the volume of transfers in each future state of the world and thereby reduces allocation inefficiency. Thus, the second-best social welfare must be enhanced by reducing \( \hat{s} \) from the outcome under the dynamic competitive equilibrium.