A Theory of Income Smoothing When Insiders Know More Than Outsiders*

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J.E.L.: G32, G35, M41, M42, O43, D82, D92

Keywords: payout policy, asymmetric information, under-investment, accounting quality, finance and growth.

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Abstract

We develop a theory of income smoothing by firms when insiders know more about income than outside shareholders, but property rights ensure that outside shareholders can enforce a fair payout. Insiders report income consistent with outsiders’ expectations and underproduce in order not to unduly raise expectations about future income. The observed income and payout process are smooth and adjust partially and over time towards a target. The underproduction problem is more severe the smaller is the inside ownership and results in an “outside equity Laffer curve”, but the problem is mitigated by the quality of independent auditing information and by introducing stock-based compensation.

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In this paper, we consider a setting in which insiders of a firm have information about income that outside shareholders do not, but property rights ensure that outside shareholders can enforce a fair payout based on available information. Under this setting, which is aimed to capture parsimoniously the relation between a firm’s insiders and outsiders, we ask the following questions: How is income of the firm reported? How is payout policy of the firm determined? Is there an effect from asymmetric information on insiders’ production decision, and how does inside ownership, quality of independent auditing and stock-based compensation affect this decision? Our model provides theoretical answers to these questions, which lie at the heart of firm and capital market interactions.

In a seminal paper concerning the firm and capital market interaction, Stein (1989) considers an environment where insiders can pump up current earnings by secretly borrowing at the expense of next period’s earnings. When the implicit borrowing rate is unfavorable, such earnings manipulation is value destroying. Stein (1989) shows that insiders do not engage in manipulation if they only care about current and future earnings. Incentives to manipulate arise, however, if insiders also care about the firm’s stock price. Since current earnings are linked to future earnings, pumping up current earnings raises outsiders’ expectations about future earnings, which in turn feed into the stock price. The market anticipates, however, that insiders engage in this form of “signal jamming” and is not fooled. Despite the fact that stock prices instantaneously reveal all information, insiders are “trapped” into behaving myopically. Thus, stock market pressures can have a dark side, even if markets are fully efficient.

Our paper’s central insight is that myopic behavior by insiders can arise even if the stock price does not explicitly enter into managers’ objective function. It is sufficient that similar “market pressures” apply with respect to earnings. We show therefore that myopic managerial behavior need not necessarily be attributed to stock price considerations. More importantly, we show that if outsiders cannot directly observe earnings and insiders face capital market
pressures then the Stein (1989) result can in fact get reversed in that insiders have an incentive to manage *downwards* outsiders’ beliefs about earnings. Unlike Stein (1989) we introduce the friction that insiders know more than outsiders regarding the firm’s marginal costs and that insiders face a capital market constraint. We then examine how this affects insiders’ incentives to engage in myopic behavior.

Asymmetric information leads to potential discrepancies between actual income and outsiders’ income estimate. This creates incentives for expropriation as insiders may try to fool outsiders, especially if outsiders’ ownership share is high. If outsiders cannot observe net income directly, but have to infer it indirectly from a noisy output measure (such as sales) then insiders try to “manage” downwards outsiders’ expectations of current and future income by underproducing and by making production less sensitive to economic shocks. Thus, in our asymmetric information setting, reported income and payout are smoothed even when insiders are not directly concerned about the stock price. Insiders do so because the capital market constraint implies that if they inflate earnings they increase outsiders’ current as well as future payout demands.

Formally, the model works as follows. For the firm to be able to attract outside equity-holders in the first place, we need investor protection and a credible mechanism that makes insiders disgorge cash to outside investors. To this end, we call upon the investor protection framework described in Fluck (1998, 1999), Myers (2000), Jin and Myers (2006), Lambrecht and Myers (2007, 2008, 2012), Acharya, Myers and Rajan (2011), among others. With the exception of Jin and Myers (2006) these papers assume symmetric information between insiders and outsiders. While under symmetric information outsiders know exactly what they are due, under asymmetric information outsiders refrain from intervention for as long as the

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1If outsiders and insiders own, say, 90% and 10% of the firm, respectively, then under symmetric information they get 90 and 10, respectively, if actual income is 100 (assuming property rights are strictly enforced). If, under asymmetric information, insiders could make outsiders believe income is, say, only 90 rather than 100, then insiders would get 19 instead of 10.

2Importantly, since both insiders and outsiders are risk neutral, smoothing does not result from risk aversion.
reported income (and corresponding payout) meets their expectations. Therefore, in Jin and Myers (2006) insiders pay out according to outsiders’ expectations of cashflows and absorb the residual variation, as is also the case in our model.

We assume that while shocks to marginal costs (modeled by an AR(1) process) are persistent, there is a value-irrelevant “noise” due to measurement error in the output level observed by outsiders. This noise is transitory, normally distributed, and i.i.d. over time. When observing an increase in sales (i.e. the noisy proxy of output) outsiders cannot distinguish whether the increase is due to a reduction in marginal costs (and therefore represents a real increase in income), or whether the increase is due to value-irrelevant measurement error. Outsiders try to disentangle the two influences by solving a Kalman filtering problem. Unlike Stein (1989) (where inference by outsiders is instantaneous and perfect) and Jin and Myers (2006) (where there is no learning) in our setting outsiders learn gradually over time. Since measurement errors are transitory and shocks to costs persistent, the underlying source of change becomes clear only as time passes by. Therefore, outsiders calculate their best estimate of income on the basis of not only current sales but also past sales. Indeed, while the current sales figure could be unduly influenced by measurement error, an estimate based on the full sales history smooths out the effect of these errors.

Then, in a rational expectations equilibrium outsiders calculate their expectation of actual income on the basis of the complete history of sales and of what they believe insiders’ optimal output policy to be. Conversely, insiders determine each period their optimal output policy given outsiders’ beliefs. We obtain a fixed point (a signal-jamming equilibrium) in which insiders’ actions are consistent with outsiders’ beliefs and outsiders’ expectations are unbiased conditional on the information available. Each period outsiders receive a payout that equals

3Formally, outsiders’ income estimate is the solution to a filtering problem. We adopt the Kalman filter because for our linear model with Gaussian disturbances the Kalman filter is optimal among all possible estimators and gives an unbiased, minimum variance and consistent estimate of actual (i.e., realized) income. For an early forecasting application of the Kalman filter in the context of earnings numbers, see Lieber, Melnick, and Ronen (1983), who use the filter to deal with transitory noise in earnings.
their share of what they expect income to be. Insiders also get a payout but they have to soak up any under (over) payment to outsiders as some kind of discretionary remuneration (charge): if actual income is higher (lower) than outsiders’ estimate then insiders cash in (make up for) the difference in outsiders’ payout.

Consequently, reported income and payout are smooth compared to actual income not because insiders want to smooth income, but because insiders have to meet outsiders’ expectations to avoid intervention. Two types of income smoothing take place simultaneously: “financial” smoothing and “real” smoothing. The former is value-neutral and merely alters the time pattern of reported income without changing the firm’s underlying cash-flows as determined by insiders’ production decision. Insiders also engage in real smoothing by manipulating production in an attempt to manage outsiders’ expectations. In particular, insiders under-invest and make output less sensitive to changes in the latent variable affecting marginal costs. This type of smoothing is value destroying.

Importantly, smoothing has an inter-temporal dimension. The first-best output level is determined in our model by considerations regarding the contemporaneous level only of the latent marginal cost variable. But, the current output decision not only affects current sales levels but also outsiders’ expectations of current and all future income. This exacerbates the previously discussed underinvestment problem for insiders because bumping up sales now means the outsiders will expect higher income and payout not only now but also in future.

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4 For example, DeFond and Park (1997) show that managers increase (decrease) current period discretionary accruals when current earnings are low (high) and in doing so are borrowing (saving) earnings from (for) the future. For an illustrative case example of financial smoothing, we refer to the highly publicized settlement that Microsoft reached with the SEC in 2002. The settlement marked the end to years of investigation by the SEC over allegations that Microsoft was employing “cookie jar” accounting practices in which it put aside income in certain quarters to pad future financial results when the company did not meet expectations. Under the settlement agreement Microsoft is admitting no explicit wrongdoing and is not obliged to pay a fine.

5 For empirical evidence regarding real smoothing, we refer to section 4.2.

6 We do not model how real and financial smoothing are implemented in practice. In Ronen and Sadan (1981), various smoothing mechanisms are discussed and illustrated in great detail.
Even though the spillover effect of a one-off increase in sales on outsiders’ future expectations wears off over time, it still causes insiders to underproduce even more.

Smoothing increases with the degree of information asymmetry between insiders and investors. Holding constant the degree of information asymmetry (as determined by the variance of the measurement error), smoothing and underproduction in particular also increase with outside shareholders’ ownership stake because it increases insiders’ incentives to manage outsiders’ expectations. Conversely, a higher level of inside ownership leads to less real smoothing. Indeed, the under-investment problem disappears as insiders move towards 100% ownership. We show that these effects lead to an “outside equity Laffer curve”: the value of the total outside equity is an inverted U-shaped function of outsiders’ ownership stake.7

This result suggests that low inside ownership could have detrimental consequences for the firm. If outside equity is crucial for the development and expansion of owner-managed firms given their financing constraints, then our results offer a rationale for imposing disclosure requirements on publicly listed companies and for improving their accounting and auditing quality. We show that, all else equal, introducing independent accounting information, such as an unbiased but imprecise income estimate, improves economic efficiency, increases the outside equity value, and acts as a substitute for a higher inside ownership stake. The implication is that accounting quality, investments, size of public stock markets, and economic growth are all positively correlated in our model, as found in the empirical literature on finance and growth (King and Levine (1993), Rajan and Zingales (1998)).

In a final extension, we embed in Section 3.4 the effect of stock-based compensation for insiders as in the Stein (1989) model. We show that this dampens but does not eliminate insiders’ incentives to engage in under-production in our model: there is now a tradeoff that insiders face between raising stock price (which benefits them through stock-based compensa-

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7 The analogy with the taxation literature is straightforward: outsiders’ ownership stake acts ex post like a proportional tax on distributable income and undermines insiders’ incentives to produce. Note that our under-investment result does not require the presence of costly effort by insiders.
tion) and paying out higher dividends in future (since higher stock prices arise due to greater outsider expectations).

The paper is organized as follows. Section 1 presents the benchmark case with symmetric information between outsiders and insiders. Section 2 analyzes the asymmetric information model. Section 3 discusses the robustness and extensions of the model, in particular, the insiders’ participation constraint, the value of audited disclosure, and the effect of stock-based insider compensation. Section 4 presents novel empirical implications that flow from our model for (1) the time-series and cross-sectional properties of corporate income, (2) real smoothing by firms, (3) corporate ownership structure, and (4) public versus private firms. Section 5 relates our paper to existing literature. Section 6 concludes. Proofs are in the appendix.

1 Symmetric information case

Consider a firm with access to a productive technology. The output from the technology is sold at a fixed unit price, but its scale can be varied. Marginal costs of production follow an AR(1) process and are revealed each period before the output scale is chosen. A part of the firm is owned by risk-neutral shareholders (outsiders) and the rest by risk-neutral insiders who also act as the technology operators. To start with, we focus on the first-best scenario in which there is congruence of objectives between outsiders and insiders, and information about marginal costs is known symmetrically to both outsiders and insiders.

Formally, we consider a firm with the following income function:

\[ \pi_t = q_t - \frac{q_t^2}{2x_t} \]

where \( x_t = Ax_{t-1} + B + w_{t-1} \) with \( w_{t-1} \sim N(0, Q) \).

where \( q_t \) denotes the chosen output level. The (inverse) marginal production cost variable \( x_t \) follows an AR(1) process with auto-regressive coefficient \( A \in [0, 1) \), a drift \( B \), and an
i.i.d. noise term $w_{t-1}$ with zero mean and variance $Q$. The output level $q_t$ is implemented after the realization of $w_{t-1}$ is observed.

All shareholders are risk-neutral, can borrow and save at the risk-free rate, and have a discount factor $\beta \in (0, 1)$. Therefore -unlike Stein (1989)- changing the time pattern of cash flows (without changing their present value) through more borrowing or saving is costless. The value of the firm is given by the present value of discounted income:

$$V_t = \max_{q_{t+j}, j=0, \ldots, \infty} E_t \left[ \sum_{j=0}^{\infty} \beta^j \pi_{t+j} \right] = \max_{q_{t+j}, j=0, \ldots, \infty} E_t \left[ \sum_{j=0}^{\infty} \beta^j \left( q_{t+j} - \frac{q_{t+j}^2}{2x_{t+j}} \right) \right]$$

(3)

Then, the first-best production policy that maximizes firm value is as follows.

**Proposition 1** The first-best production policy is $q_t^* = x_t$. The firm’s realized income and total payout under the first-best policy are given by: $\pi_t^* = \frac{x_t}{2}$.

The first-best output level $q_t^*$ equals $x_t$. Recall that a higher value for $x_t$ implies lower marginal costs. Therefore, the output level rises with $x_t$. As $x_t$ goes to zero, marginal costs spiral out of control and the first-best output quantity goes to zero.

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8Our model generalizes to the case where $x_t$ follows a random walk with drift (i.e. $A = 1$). Mean reversion (i.e. $A < 1$) is, however, a more realistic assumption for production costs. For example, commodity prices (which constitute a large component of production costs in some industries) are often mean reverting due to the negative relation between interest rates and prices.

9Since the shocks that drive $x_t$ are normally distributed, marginal costs could theoretically become negative. The solution in proposition[1] no longer makes sense for negative $x_t$ because marginal costs can, of course, not be negative. The likelihood of negative values for $x_t$ arising is, however, negligibly small if the stationary unconditional mean of $x_t$ (given by $B - \frac{1}{A}$) is sufficiently large relative to the unconditional variance of $x_t$ (given by $Q - \frac{A}{1-A}$). We assume this condition to be satisfied so that we can safely ignore the occurrence of negative costs. To rule out negative values for $x_t$ altogether one could assume that $x_t$ is log-normally distributed. This would, however, make the Bayesian updating process deployed in next section completely intractable. The normality assumption is standard in the information economics literature (for example, Grossman (1976) and papers that originated from this seminal paper).
1.1 Inside and outside shareholders

We now introduce inside and outside shareholders who, respectively, own a fraction \((1 - \varphi)\) and \(\varphi\) of the shares, \(\varphi \in [0, 1]\). For example, insiders (managers and even board members involved in the firm’s operating decisions) typically own the majority of shares of private firms \((\varphi < 0.5)\), whereas for public firms it is more common that outsiders own the majority of shares \((\varphi > 0.5)\). Insiders set the production \((q_t)\) and payout \((d_t)\) policies. Analogous to Myers (2000), Jin and Myers (2006), Lambrecht and Myers (2007, 2008, 2012), and Acharya, Myers and Rajan (2011), we assume that insiders operate subject to a threat of collective action. Outsiders’ payoff from collective action is given by \(\varphi \alpha V_t\) where \(\alpha \in (0, 1]\) reflects the degree of investor protection (or specificity of the firm’s technology).\(^{10}\)

To avoid collective action, insiders pay out each period a dividend \(d_t\) that leaves outsiders indifferent between intervening and leaving insiders unchallenged for another period. If \(S_t\) denotes the value of the outside equity then \(d_t\) is defined by\(^{11}\)

\[
S_t = d_t + \beta \alpha \varphi E_t[V_{t+1}] = \alpha \varphi V_t \quad (4)
\]

\[
\iff d_t + \beta \alpha \varphi E_t[V_{t+1}] = \alpha \varphi \pi_t + \alpha \varphi \beta E_t[V_{t+1}] \iff d_t = \alpha \varphi \pi_t \quad (5)
\]

Equation \((4)\) can be interpreted as a capital market constraint that requires insiders to provide

\(^{10}\)When we have a public corporation with a large outside ownership stake, then collective action is as described in the Myers (2000) “corporation model”. Outsiders take over the firm and displace insiders. The cost of collective action reflects the loss in managerial human capital, deadweight costs of getting organized, etc. If we have a private company with a small outside ownership stake then outsiders are minority stakeholders and the inside majority rules. Minority shareholders are, however, not entirely impotent as company law or commercial code grants minority shareholders either a judicial venue to challenge the decisions of management or the right to step out of the company by requiring the company to purchase their shares. The payoff from collective action to outside minority shareholders under this “oppressed minorities mechanism” (see La Porta et al. (1998)) is therefore the fair value of their stake, net of any costs of intervention (such as a possible minority discount or legal costs).

\(^{11}\)It is not strictly necessary that all income is paid out each period. If reported income earns the risk-free rate of return within the firm (e.g. through a high yield cash account) and is protected from expropriation by insiders, then outsiders do not require income to be paid out (see Lambrecht and Myers, 2012).
an adequate return to outside investors.\textsuperscript{12}

ϕ denotes outsiders’ “nominal” ownership stake. Scaling the nominal ownership stake by the degree of investor protection \(\alpha\) gives outsiders’ effective or “real” ownership state \(\theta \equiv \varphi \alpha\). It follows that the payouts to outsiders \((d_t)\) and insiders \((r_t)\) are respectively given by \(\theta \pi_t\) and \((1 - \theta) \pi_t\). Income \((\pi_t)\) is shared between insiders and outsiders according to their real ownership stake. The following corollary results at once.

**Corollary 1** With symmetric information, insiders adopt the first-best production policy, and payout to outsiders (insiders) equals a fraction \(\theta\) \((1 - \theta)\) of realized income \(\pi_t\).

2 **Asymmetric information**

We now add two new ingredients to the model. First, we assume that the actual realizations of the stochastic marginal cost variable \(x_t\) are observed by insiders only. All model parameters remain common knowledge, however. Outsiders also have an unbiased estimate \(\hat{x}_0\) of the initial value \(x_0\).\textsuperscript{13} Second, outsiders cannot observe the output level \(q_t\), but have to rely on some noisy proxy. This introduces measurement error. Instead of observing \(q_t\), outsiders observe \(s_t \equiv q_t + \epsilon_t\) where \(\epsilon_t\) is an i.i.d. normally distributed noise term with zero mean and variance \(R\) (i.e., \(\epsilon_t \sim N(0, R)\)). The measurement error is uncorrelated with the marginal cost variable \(x_t\) (i.e., \(E(w_k \epsilon_l) = 0\) for all \(k\) and \(l\)). In what follows we refer to \(s_t\) as the firm’s “sales” as perceived by outsiders, i.e., outsiders perceive the firm’s revenues to be \(s_t\), whereas in reality they are \(q_t\). \(s_t\) can, for example, be interpreted as analysts’ estimate of output. We assume this estimate to be noisy, but unbiased (i.e. \(E(\epsilon_t) = 0\)).

\textsuperscript{12}Graham et al. (2005) provide convincing evidence of how capital market pressures induce managers to meet earnings targets at all costs. As one surveyed manager put it: “I miss the target, I’m out of a job.” Mergenthaler et al. (2012) find that CEOs are penalized via bonus cuts, fewer equity grants, and forced turnover when they just miss the latest consensus analyst forecast.

\textsuperscript{13}\(\hat{x}_0\) is revealed to outside investors when the firm is set up at time zero (see section 3.2).
Outsiders are aware that sales are an imperfect proxy for economic output and they know the distribution from which $\epsilon_t$ is drawn. Importantly, insiders implement output ($q_t$) \textit{after} the realization of $x_t$ but \textit{before} the realization of $\epsilon_t$ is known. Since $\epsilon_t$ is value-irrelevant noise, the firm’s actual income is still given by $\pi(q_t) = q_t - \frac{q_t^2}{2x_t}$. However, as $q_t$ and $x_t$ are unobservable outsiders have to estimate income on the basis of noisy sales figures. Therefore measurement errors can lead to misvaluation in the firm’s stock price (unlike Stein (1989) where stock prices are strong-form efficient).

We know from previous section that there is a mapping from the latent variable $x_t$ to both $q_t$ and $\pi_t$. The presence of the noise term $\epsilon_t$ obscures, however, this link and makes it impossible for outsiders exactly to infer $x_t$ and $\pi_t$ from sales. Assuming that insiders cannot trade in the firm’s stock and that the information asymmetry cannot be mitigated through monitoring or some other mechanism, the best outsiders can do is to calculate a probability distribution of income, $\pi_t$, on the basis of all information available to them. This information set $I_t$ is given by the full history of current and past sales prices, i.e., $I_t \equiv \{s_t, s_{t-1}, s_{t-2} \ldots\}$.

We show that on the basis of the initial estimate $\hat{x}_0$ and the sales history, $I_t$, outsiders can infer a probability distribution for the latent marginal cost variable $x_t$, which in turn maps into a probability distribution for income $\pi_t$. Outsiders then use this distribution to calculate their estimate $\hat{\pi}_t$ of the firm’s income, i.e. $\hat{\pi}_t = E[\pi_t | I_t] \equiv E_{S,t}(\pi_t)$, where the subscript $S$ in $E_{S,t}[\pi_t]$ emphasizes (outside) shareholders’ expectation at time $t$ of $\pi_t$ based on the information set $I_t$.

The capital market constraint requires that $d_t$ satisfies the following constraint:

$$S_t = d_t + \beta \varphi \alpha E_{S,t}[V_{t+1}] = \varphi \alpha E_{S,t}[V_t]$$

$$\iff d_t + \beta \varphi \alpha E_{S,t}[V_{t+1}] = \varphi \alpha E_{S,t}[\pi_t] + \varphi \beta E_{S,t}[V_{t+1}] \iff d_t = \theta E_{S,t}[\pi_t]$$

Therefore, to avoid collective action insiders set the payout equal to $d_t = \theta E_{S,t}(\pi_t)$. In other words, outsiders want their share of the income they believe has been realized according to all information available to them. While insiders cannot manage outsiders’ expectations through words (which are not credible) they can do so through their actions. Managers can influence
observable sales \(s_t\) and therefore \(\pi_t\) by their chosen output level \(q_t\).

Insiders’ optimization problem can now be formulated as follows:

\[
M_t = \max_{q_{t+j}, j=0,\infty} E_t \left[ \sum_{j=0}^{\infty} \beta^j (\pi(q_{t+j}) - \theta E_{S,t+j} [\pi(q_{t+j})]) \right]
\]

(6)

with insiders’ optimal output policy \(q_t\) being an equilibrium (fixed point) once outsiders’ beliefs are fixed. The complete derivation of the solution is given in the appendix. We briefly present a heuristic derivation of the rational expectations equilibrium here. Outsiders conjecture that insiders’ production policy is given by \(q_t = Hx_t\), where \(H\) is some constant. Therefore, \(E_{S,t+j} [\pi(q_{t+j})] = \left( H - \frac{H^2}{2} \right) E_{S,t+j} [x_{t+j}] \equiv hE_{S,t+j} [x_{t+j}]\). Define \(\hat{x}_t \equiv E_{S,t}[x_t]\) as outsiders’ estimate of the latent variable \(x_t\) conditional on the information available at time \(t\). Since \(s_t = q_t + \epsilon_t\) and \(q_t = Hx_t\), sales are an imperfect (noisy) measure of the latent variable \(x_t\), as is clear from the following “measurement equation”:

\[
s_t = Hx_t + \epsilon_t \quad \text{with} \quad \epsilon_t \sim N(0,R)
\]

(7)

Outsiders also know the variance \(R\) of the noise, \(\epsilon_t\), and the parameters \(A, B\) and \(Q\) of the “state equation”:

\[
x_t = Ax_{t-1} + B + w_{t-1} \quad \text{with} \quad w_t \sim N(0,Q) \text{ for all } t
\]

(8)

Outsiders now solve what is known as a “filtering” problem. Using the Kalman filter (see appendix), the measurement equation can be combined with the state equation to make inferences about \(x_t\) on the basis of current and past observations of \(s_t\). This allows outsiders to form an estimate of actual income \(\pi_t\). While the measurement equation is usually exogenously given, our Kalman filter has the novel feature that the constant slope coefficient \(H\) in the measurement equation is set endogenously by insiders.

The Kalman estimator \(\hat{x}_t\) is unbiased (see Chui and Chen (1991) page 40). The Kalman filter is optimal (“best”) in the sense that it minimizes the mean square error (Gelb (1974)). The solution is formulated in terms of the steady state or “limiting” Kalman filter which is the estimator \(\hat{x}_t\) for \(x_t\) that is obtained after a sufficient number of measurements \(s_t\) have taken
place over time for the estimator to reach a steady state. The steady-state estimator allows us to analyze the long-run behavior of reported income and payout and is given by (Chui and Chen (1991), p78):

\[ \hat{x}_t = (A\hat{x}_{t-1} + B)\lambda + Ks_t \] (9)

where \( \lambda \) and \( K \) are as defined in the proposition. \( K \) is called the “Kalman gain” and it plays a crucial role in the updating process. Substituting \( \hat{x}_{t-1} \) in (9) by its estimate, one obtains after repeated substitution:

\[
\hat{x}_t = B\lambda \left[ 1 + \lambda A + \lambda^2 A^2 + \lambda^3 A^3 + \ldots \right] + K \left[ s_t + \lambda As_{t-1} + \lambda^2 A^2 s_{t-2} + \lambda^3 A^3 s_{t-3} + \ldots \right]
\]

Thus, outsiders’ estimate of current actual income is not only determined by their observation of current sales but also by the whole history of past sales. Hence, insiders’ optimization problem is no longer static but inter-temporal and dynamic. Indeed, the current production decision not only affects insiders’ expectations about current but also future income.

Substituting outsiders’ beliefs \( E_{S,t+j}[\pi(q_{t+j})] = h\hat{x}_{t+j} \) into insiders’ objective function, insiders optimize:

\[
M_t = \max_{q_{t+j} : j=0,\ldots,\infty} E_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \pi(q_{t+j}) - \theta h\hat{x}_{t+j} \right) \right]
\]

which gives the following first-order condition:

\[
\frac{\partial M_t}{\partial q_t} = 1 - \frac{q_t}{x_t} - \theta hK - \theta hK\beta\lambda A - \theta hK(\beta\lambda A)^2 - \theta hK(\beta\lambda A)^3 - \ldots = 0 \quad (12)
\]

Or equivalently:

\[
q_t = \left[ \frac{1 - \theta hK}{1 - \beta\lambda A} \right] x_t \quad (13)
\]

\[^{14}\text{If the disturbances (}\epsilon_t \text{ and } w_t\text{) and the initial state (}x_0\text{) are normally distributed then the Kalman filter is unbiased. When the normality assumption is dropped unbiasedness may no longer hold, but the Kalman filter still minimizes the mean square error within the class of all linear estimators. Under mild conditions (see footnote 29 in the appendix) the Kalman filter converges to its steady state. Convergence is of geometric order and therefore fast.}\]
Outsiders’ conjectured output policy \( q_t = Hx_t \) is a fixed point if and only if:

\[
H = 1 - \frac{\theta hK}{1 - \beta \lambda A} \tag{14}
\]

At the fixed point, outsiders’ expectations are rational given insiders’ output policy, and insiders’ output policy is optimal given outsiders’ expectations. Rearranging (14) gives equation (19) in proposition 2, which pins down the equilibrium value for \( H \). Note that the right hand of equation (14) is less than (or equal to) one, and therefore insiders underproduce (i.e. \( H \leq 1 \)) compared to what is first best. The results are summarized in the following proposition.

**Proposition 2** The insiders’ optimal production plan is given by:

\[
q_t = Hx_t = Hq^o_t \quad \text{for all } t \tag{15}
\]

Payout to outside shareholders equals a fraction \( \theta \) of reported income: \( d_t = \theta \hat{\pi}_t \) where

\[
\hat{\pi}_t = \left( H - \frac{H^2}{2} \right) \hat{x}_t \equiv h\hat{x}_t, \tag{16}
\]

and where \( \hat{x}_t = (A\hat{x}_{t-1} + B) \lambda + K s_t \)

\[
= \frac{\lambda B}{1 - \lambda A} + K \sum_{j=0}^{\infty} (\lambda A)^j s_{t-j}. \tag{17}
\]

\( H \) is the positive root to the equation:

\[
f(H) \equiv H^2 K(\frac{\theta}{2} - \beta A) + H [\beta A(1 + K) - 1 - \theta K] + 1 - \beta A = 0 \tag{19}
\]

with \( K \equiv \frac{HP}{H^2P + R} \), \( \lambda \equiv (1 - KH) \) and \( P \) is the positive root of the equation:

\[
P = A^2 P - \frac{A^2 H^2 P^2}{H^2P + R} + Q. \tag{20}
\]

The error of outsiders’ income estimate \( (\pi_t - \hat{\pi}_t) \) is normally distributed with mean zero (i.e., \( E_{S,t}[\pi_t - \hat{\pi}_t] = 0 \)) and variance \( \hat{\sigma}^2 \equiv E_{S,t}[(\pi_t - \hat{\pi}_t)^2] = h^2 P \).

### 2.1 Production Policy

We know from Proposition 2 that insiders’ optimal production policy is given by \( q_t = Hx_t \) where \( H \) is the solution to equation (19). There exists a unique positive (real) root for \( H \).
which lies in the interval $(0, 1]$. We therefore obtain the following corollary.

**Corollary 2** If outsiders indirectly infer income from sales $(s_t)$ then insiders underproduce (i.e., $q_t = H x_t = H q^o_t \leq q^0_t$).

Insiders underproduce because outsiders do not observe $x_t$ directly but estimate its value indirectly from sales. This gives insiders an incentive to manipulate sales (engage in “signal-jamming”) in an attempt to “fool” outsiders. In particular, insiders trade off the benefit from lowering outsiders’ expectations about income against the cost of underproduction. Insiders’ first-order condition (12) shows that a marginal decrease in current output (and therefore expected sales) lowers outsiders’ beliefs about current income by $hK$, and about income $j$ periods from now by $hK(\lambda A)^j$. Therefore, a marginal cut in output benefits insiders. Insiders keep cutting output up to the point where the marginal cost of cutting (in terms of realized income) equals the marginal benefit (in terms of lowering outsiders’ expectations).

The unconditional long-run mean for $q_t$ under the first-best and actual production policies are, respectively, $E[q^o_t] = E[x_t] = B/(1 - A)$ and $E[q_t] = HE[x_t] = BH/(1 - A)$. Lost output, in turn, translates into a loss of income. The unconditional mean income under the first-best and actual production policies are, respectively, $E[\pi^o_t] = \frac{1}{2} E[x_t]$ and $E[\pi_t] = hE[x_t]$.

The following corollary then explains the effect of asymmetric information on the production decision.

---

15 $f(0) = -1 + \beta A < 0$ and $f(1) = \frac{\theta K}{\lambda} \geq 0$. Therefore, $\exists H \in (0, 1]$ for which $f(H) = 0$. Since $\theta$, $A$, $\lambda$, and $\beta$ all fall in the $[0, 1]$ interval, an exhaustive numerical grid evaluation can be executed for all possible parameter combinations. Numerical checks reveal that $H$ is the unique positive root.

16 Note that outsiders are not fooled by insiders’ signal-jamming. In equilibrium, outsiders correctly anticipate this manipulation and incorporate it into their expectations. Nevertheless, insiders are “trapped” into behaving myopically. The situation is analogous to what happens in a prisoner’s dilemma. The preferred cooperative equilibrium would be efficient production by insiders and no conjecture of manipulation by outsiders. This can, however, not be sustained as a Nash equilibrium (as in Stein (1989)) because insiders have an incentive to underproduce whenever outsiders believe the efficient production policy is being adopted.
Corollary 3 The noisier the link between the latent variable \((x_t)\) and its observable proxy \((s_t)\), the weaker insiders’ incentive to manipulate the proxy by underproducing. In particular, insiders’ production decision converges to the first-best one as the variance of measurement errors becomes infinitely large \((R \to \infty)\) or as uncertainty with respect to the latent variable \(x_t\) decreases \((Q \to 0)\), i.e., \(\lim_{Q \to 0} H = \lim_{R \to \infty} H = 1\). Conversely, the more precise the link between \(s_t\) and \(x_t\), the higher the incentive to underproduce. The lower bound for \(H\) is achieved for the limiting cases \(Q \to \infty\) and \(R \to 0\), i.e., \(\lim_{Q \to \infty} H = \lim_{R \to 0} H = 1 - \frac{\theta}{2-\theta}\).

When \(x_t\) becomes deterministic \((Q = 0)\) then the estimation error with respect to \(x_t\), goes to zero (i.e., \(P \to 0\)). This means that the Kalman gain coefficient \(K\) becomes zero too (there is no learning). But if there is no learning \((K = 0\) and \(\lambda = 1\)) then insiders’ output decision \(q_t\) no longer affects outsiders’ estimate of the cost variable, as illustrated by equation (9). As a result the production policy becomes efficient (i.e., \(H = 1\) and \(q_t = x_t\)).

Similarly, if there are measurement errors then the link between sales and the latent cost variable becomes noisy. This mitigates the under-investment problem, because the noise obscures insiders’ actions and therefore their incentive to cut production.

In the absence of measurement errors \((R = 0)\) the link between sales \(s_t\) and the contemporaneous level of the latent variable \(x_t\) becomes deterministic. Outsiders know for sure that an increase in sales results from a fall in marginal costs. Therefore, when observing higher sales, outsiders want higher payout. In an attempt to manage outsiders’ expectations downwards, insiders underproduce. If \(R = 0\) then we get the efficient outcome \((H = 1)\) only if insiders get all the income \((\theta = 0)\); otherwise we get under-investment \((H < 1)\). As the insiders’ stake of income goes to zero \((\theta \to 1)\) also production goes to zero (i.e., \(H \to 0\)). Both outsiders and insiders get nothing, even though the firm could be highly profitable! This result is in sharp contrast with the symmetric information case where the efficient outcome is obtained no matter how small the insiders’ share of the income. Thus, for firms where insiders have

\[\text{For } R = 0 \text{ we get } P = Q, K = 1/H \text{ and } \lambda = 0. \text{ Therefore, from Proposition } 2 \text{ it follows that } \hat{x}_t = s_t/H \text{ and } s_t = Hx_t. \text{ Consequently, } \hat{x}_t = x_t.\]
a very small ownership stake (e.g. public firms with a highly dispersed ownership structure), asymmetric information and the resulting indirect inference-making process by outsiders could undermine the firm’s very existence, an issue we return to in section 3.

Figure 1 illustrates the effect of the key model parameters \((R, Q, A \text{ and } \theta)\) on production efficiency. Efficiency is measured with respect to two different variables: the unconditional mean output \((E[q_t])\), and unconditional mean income \((E[\pi_t])\). The degree of efficiency is determined by comparing the actual outcome with the first-best outcome, i.e., \(E[q_t]/E[q^*_t] = H\) (dashed line), and \(E[\pi_t]/E[\pi^*_t] = 2h\) (solid line).

The figure shows that the efficiency loss is larger with respect to output than income because the loss in revenues due to underproduction is to some extent offset by lower costs of production. Panel A and B confirm that full efficiency is achieved as \(R\) moves towards \(\infty\) and for \(Q = 0\). Panel C shows that a higher autocorrelation in marginal costs substantially reduces efficiency because it allows outsiders to infer more information about the latent cost variable from sales and therefore gives insiders stronger incentives to distort production.

Finally, panel D shows that production is fully efficient if outsiders have no real stake in the firm’s income (i.e., \(\theta = 0\)). Efficiency severely declines as outsiders’ stake increases. For \(\theta = 1\), insiders achieve only 28% of the first-best output level. However, one can show that as \(Q/R \to 0\) incentives are fully restored, and the first-best outcome can be achieved even for \(\theta = 1\). This confirms that the root cause of underproduction is the process of indirect inference and not the outside ownership stake per se. The firm’s ownership structure serves, however, as a transmission mechanism through which inefficiencies can be amplified.

2.2 The time-series properties of income

Proposition 2 also allows us to derive the time-series properties of income:

**Proposition 3** The firm’s “actual income” is \(\pi_t = hx_t\). The firm’s “reported income”, \(\hat{\pi}_t(=\)
The proposition characterizes three types of income: the “income target” ($\pi_t^*$), “reported income” ($\hat{\pi}_t$) and “actual income” ($\hat{\pi}_t$). Reported income follows a target that is determined by the contemporaneous level of sales. However, as equation (21) shows, the reported income only gradually adjusts to changes in sales because the SOA coefficient $(1 - \lambda A)$ is less than unity. This leads to income smoothing in the sense that the effect on reported income of a shock to sales is distributed over time. In particular, a dollar increase in sales leads to an immediate increase in reported income of only $hK$. The lagged incremental effects in subsequent periods are given by $hK\lambda A$, $hK(\lambda A)^2$, $hK(\lambda A)^3$, ... The long-run effect of a dollar increase in sales on reported income equals $hK \sum_{j=0}^{\infty} (\lambda A)^j = \frac{hK}{1 - \lambda A}$, which is the slope coefficient $\gamma^*_1$ of the income target $\pi_t^*$ (see equation (23)). In contrast, with symmetric information, the impact of a shock to sales is fully impounded into reported income immediately.

Intuitively, reported income only partially adjusts to a contemporaneous shock in sales because in the short run outsiders cannot distinguish between a transitory measurement error and a persistent shock to the latent cost variable. However, as subsequent sales are observed the transitory or persistent nature of the shock is gradually revealed. Reported income can therefore also be expressed as a distributed lag model in which it is a function of current and past sales, by repeated backward substitution of equation (22):

$$\hat{\pi}_t = \frac{h\lambda B}{1 - \lambda A} + Kh \sum_{j=0}^{\infty} (\lambda A)^j s_{t-j}.$$  

The following corollary then summarizes how asymmetric information affects income:
Corollary 4 Measurement errors create asymmetric information, which in turn leads to smoothing of reported income. A lower degree of information asymmetry (i.e., $R$ falls relative to $Q$) leads to less smoothing. In the limit (i.e., $R = 0$ or $Q \to \infty$) both reported income and target income coincide with actual income at all times (i.e., $\pi_t = \hat{\pi}_t = \pi^*_{t}$ for all $t$).

No financial smoothing whatsoever occurs when $R = 0$ because in that case all information asymmetry is eliminated. In the absence of measurement errors, it is possible to infer the marginal cost variable $x_t$ with 100% accuracy from the observed sales figure $s_t$. The same result obtains when $Q \to \infty$ because in that case measurement errors are negligibly small compared to the variance of the latent cost variable. This important result confirms again that asymmetric information and not uncertainty per se is the root cause of income smoothing. The corollary also confirms that as the degree of information asymmetry goes to zero, our rational expectations equilibrium converges to the simple sharing rule that prevails under symmetric information. Indeed: $\lim_{R \to 0} d_t = \theta \lim_{R \to 0} \hat{\pi}_t = \theta \pi_t$. Finally, given that (i) reported income is smooth relative to actual income and (ii) payout is based on reported income, it follows that insiders soak up the variation. We address this issue next in Section 2.3 where we discuss payout.

2.3 Payout Policy

Since the payout to outsiders is given by $d_t = \theta \hat{\pi}_t$, it follows that the firm’s payout policy to outsiders is described by the target adjustment model for $\hat{\pi}_t$ in (22):

$$d_t = \lambda A d_{t-1} + \theta h K s_t + \theta h \lambda B .$$  \hspace{1cm} (25)

The payout model is similar to the well known Lintner (1956) dividend model. The key difference is that in Lintner (1956) the payout target is determined by the firm’s net income, whereas in our model the target is a function of sales because net income is not directly observed by outsiders. Payout in our model is not smoothed relative to reported income but
3 Robustness, extensions and discussion

3.1 Independent audited disclosure and ownership structure

Our analysis in section 2 showed that the firm’s production policy becomes increasingly more inefficient as insiders’ real ownership stake \((1 - \varphi\alpha)\) decreases. This could pose serious problems for public firms, which often have a small inside equity base. Our model predicts that under-investment could become so severe that firms stop producing altogether, even if they are inherently profitable. It may therefore come as no surprise that mechanisms have been developed to reduce the degree of information asymmetry. In particular, publicly traded companies (unlike private firms) are subject to stringent disclosure requirements. The traditional argument put forward to justify disclosure is often that of investor protection. The general underlying idea is that outside investors need to be protected from fraud or conflicts of interests by insiders (usually managers). Audited disclosure is generally believed to benefit outsiders by curtailing insiders’ ability to exploit their informational advantage and to extract informational rents.

Our paper shows that the case for audited accounting information rests not only on investor protection. Our model shows that asymmetric information is problematic even if insider trading is precluded and outsiders’ property rights are 100% guaranteed (i.e., \(\alpha = 1\)). Moreover, disclosure is not necessarily a win/lose situation for outsiders/insiders. In our setting, eliminating information asymmetry would be welcomed by outsiders and insiders alike. In other words, disclosure (assuming it can be achieved in a relatively costless fashion) is a win-win

\(^{18}\)Payout smoothing in the strict Lintner sense obtains, e.g., if insiders are risk-averse and subject to habit formation. Lambrecht and Myers (2012) show that insiders of this type smooth payout relative to income by borrowing and lending.
situation for all parties involved.

Formally, in proposition 2, we showed that, on the basis of current and past sales, outsiders calculate an income estimate \( \hat{\pi}_t \). The error of outsiders’ estimate, \( \pi_t - \hat{\pi}_t \), is normally distributed with zero mean and variance \( \hat{\sigma}^2 \). Suppose now that, in addition to the sales data, auditors provide each period an independent estimate \( y_t \) of income where \( y_t \sim N(\pi_t, \sigma^2) \). Importantly, auditors provide their assessment after \( \epsilon_t \) and \( w_{t-1} \) are realized. The auditors’ estimate is unbiased (i.e., \( E_t[y_t] = \pi_t \)) but subject to some random error \( (y_t - \pi_t) \). Insiders nor auditors have control over the error, and the error is independent across periods. In summary, on the basis of the full sales history \( I_t \) outsiders construct a prior distribution of current income that is given by \( N(\hat{\pi}_t, \hat{\sigma}^2) \). Auditors then provide an independent estimate \( y_t \), which outsiders know is drawn from a distribution \( N(\pi_t, \sigma^2) \). As will become clear, auditor independence (i.e. insiders cannot influence auditors’ perception of income) is key.

Using simple Bayesian updating, it follows that the outsiders’ estimate of income conditional on \( y_t \) and on the sales history \( I_t \) is given by:

\[
\kappa y_t + (1 - \kappa) \hat{\pi}_t \quad \text{where} \quad \kappa = \frac{\hat{\sigma}^2}{\hat{\sigma}^2 + \sigma^2}.
\]  

(26)

The parameter \( \kappa \) can be interpreted as a parameter that reflects the quality of the additional information provided. A value of \( \kappa \) close to 0 means that the audited disclosure is highly unreliable and carries little weight in influencing outsiders’ beliefs about income.

How does the provision of information by independent auditors influence insiders’ deci-

\footnote{It might be possible for outsiders to refine the estimate of the latent cost variable \( x_t \) by using the entire history of auditors’ income estimates. We ignore this possibility, and assume that all relevant accounting information is encapsulated in the auditors’ most recent income estimate.}

21
Insiders’ optimization problem can now be formulated as:

\[
M_t = \max_{q_{t+j}:j=0,\infty} \quad E_t \left[ \sum_{j=0}^{\infty} \beta^j (\pi(q_{t+j}) - \varphi \alpha \kappa E_{t+j}(y_{t+j}) - \varphi \alpha(1 - \kappa) E_{S,t+j} [\pi(q_{t+j})]) \right] \\
= \max_{q_{t+j}:j=0,\infty} \quad E_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \pi(q_{t+j}) (1 - \varphi \alpha \kappa) - \varphi \alpha(1 - \kappa) E_{S,t+j} [\pi(q_{t+j})] \right) \right] \\
= (1 - \varphi \alpha \kappa) \max_{q_{t+j}:j=0,\infty} \quad E_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \pi(q_{t+j}) - G(\varphi, \alpha, \kappa) E_{S,t+j} [\pi(q_{t+j})] \right) \right] 
\]

(27)

where \( G(\varphi, \alpha, \kappa) \equiv \frac{\varphi \alpha(1-\kappa)}{1-\varphi \alpha \kappa} \equiv \frac{\varphi(1-\kappa)}{1-\varphi \kappa} \), and where we made use of the fact that the auditors’ estimate is unbiased at all times, i.e., \( E_{t+j}(y_{t+j}) = \pi(q_{t+j}) \) for all \( j \), irrespective of insiders’ decision rule for \( q_{t+j} \). In other words, insiders cannot distort auditors’ estimate (the release of the accounting information by independent auditors occurs after income is realized).

Comparing the optimization problem (27) with the original one we solved in (6), one can see that both problems are essentially the same, except for the fact that the outside ownership parameter \( \theta \) in (6) has been replaced by the governance index \( G(\varphi, \alpha, \kappa) \) in (27). This means that the solution for \( q_{t+j} \) can be obtained by merely replacing \( \theta \) by \( G(\varphi, \alpha, \kappa) \) in the solution we previously obtained.

\( G(\varphi, \alpha, \kappa) \) ranges across the \([0,1]\) interval and can be interpreted as an (inverse) governance index that crucially depends on the outsiders’ ownership stake \( (\varphi) \), the degree of investor protection \( (\alpha) \) and on the quality of audited disclosure \( (\kappa) \). If \( \kappa = 0 \) (i.e., \( G = \theta \)) then the independently provided accounting information is completely unreliable and discarded by outsiders. In that case the optimization problem and its solution coincide exactly with the ones presented in section 2. If \( \kappa = 1 \) (i.e., \( G = 0 \)) then the independently provided accounting information is perfectly reliable. All information asymmetry is resolved and we get the first-best outcome that was presented in section 1. Since \( \frac{\partial G(\theta,\kappa)}{\partial \kappa} \leq 0 \), it follows that:

**Corollary 5** Higher quality audited disclosure \( (\kappa) \) improves the firm’s operating efficiency.
3.2 Accounting quality, stock market size and growth

In this section we examine the model’s implications for corporate investment (and economic growth more generally) by analyzing the initial decision to set up the firm.

Assume that an investment cost \( E \) is required to establish the firm at time \( t = 0 \). The financing is raised from inside and outside equity. To abstract from adverse selection issues (see Myers and Majluf (1984)) we assume that insiders have access to an unbiased estimate for \( x_0 \) at time zero (i.e., \( \hat{x}_0 = x_0 \)). As a result insiders and outsiders attach the same value \( V(x_0; \theta, \kappa) \) to the firm when the firm is founded, as given in the following proposition.

**Proposition 4** The value of the firm at time \( t = 0 \) is given by:

\[
V_0(x_0; \theta, \kappa) = \frac{h}{(1 - \beta A)} \left( x_0 + \frac{B\beta}{1 - \beta} \right)
\]  

(28)

where the determinant \( h \) of the production policy (\( h \equiv H - \frac{H^2}{2} \)) is obtained as described in proposition 2 but by replacing \( \theta \) by \( G(\theta; \kappa) \) in equation (19).

We know that the firm value monotonically declines in the real ownership stake \( \theta(\equiv \alpha \varphi) \) and that the first-best firm value is achieved when the outside ownership stake is zero (i.e., \( \theta = 0 \)). Assuming the investment in the firm happens on a now-or-never basis at \( t = 0 \), the first-best investment decision is given by the following criterion: invest if and only if \( V(x_0; \theta = 0, \kappa) \geq E \).

Note that the accounting quality \( \kappa \) does not influence the investment decision when \( \theta = 0 \), because without outside investors audited disclosure becomes superfluous.

Assume next, without loss of generality, that insiders have no money to contribute and need to raise the full amount \( E \) from outsiders. Assume further that the quality of audited disclosure (\( \kappa \)) is exogenously given, but that the real ownership stake \( \theta \) can be chosen. The decision problem is therefore to identify the lowest value for \( \theta \) that allows insiders to raise enough outside equity, \( S_0 \), to cover the investment cost (i.e., \( S_0(x_0; \theta, \kappa) = E \)).
Since $\hat{x}_0 = x_0$, the initial inside ($M_0$) and outside ($S_0$) equity values are:

\[
M_0 = V_0(x_0; \theta, \kappa) - \theta E_{S_0}[V_0(\hat{x}_0; \theta, \kappa)] = (1 - \theta)V_0(x_0; \theta, \kappa) \tag{29}
\]

\[
S_0 = \theta E_{S_0}[V_0(\hat{x}_0; \theta, \kappa)] = \theta V_0(x_0; \theta, \kappa) \tag{30}
\]

The (constrained) optimal value for $\theta$ is therefore the solution to:

\[
\theta^o = \min \{\theta | \theta V_0(x_0; \theta, \kappa) = E\} \tag{31}
\]

The solution is illustrated in Figure 2. Panel A plots the total firm value $V_0(x_0; \theta, \kappa)$ as a function of outsiders’ real ownership $\theta$ for three different levels of disclosure quality ($\kappa$). In line with our earlier results, total firm value declines monotonically with respect to $\theta$. The loss can be substantial: the first-best firm value equals 1900 (i.e., for $\theta = 0$), whereas the firm value under 100% outside ownership equals a mere 920 (i.e., for $\theta = 1$). High quality audited disclosure ($\kappa = 0.9$) can, however, significantly mitigate the value loss. For example for $\kappa = 0.9$ the loss in value appears to be less than 1% for as long as insiders own a majority stake. In the absence of audited disclosure or when audited disclosure is completely useless (i.e., $\kappa = 0$), significant value losses kick in at much lower outside ownership levels. For example, at $\theta = 0.5$ about 10% of the first-best value is lost.

Panel B shows the total outside equity value as a function of the outside ownership stake for three different levels of disclosure quality. The curves resemble “outside equity Laffer curves”\(^{20}\) The outside equity value $\theta V_0(x_0; \theta, \kappa)$ is an inverted U-shaped function of $\theta$ that reaches a unique maximum. This maximum changes significantly according to the quality of the audited disclosure, and equals about 1550, 1200 and 1020 for high quality, low quality and no audited disclosure, respectively. No investment would take place in the absence of audited disclosure, because the amount of outside equity that can be raised is inadequate to finance the investment cost (which equals $E = 1100$). Investment would take place in the two cases

\(^{20}\)The traditional Laffer curve is a graphical representation of the relation between government revenue raised by taxation and all possible rates of taxation. The curve resembles an inverted U-shaped function that reaches a maximum at an interior rate of taxation.
where accounting information is audited, and about $\theta^o = 58\%$ ($\theta^o' = 63\%$) of shares would end up in outsiders’ hands with high (low) quality audited disclosure.

Our results provide theoretical support for a number of empirical studies that have found a positive link between economic growth, stock market size, stock market capitalizations, and quality of accounting information. The standard explanation for this result is that higher quality accounting information provides better investor protection. While higher investor protection (i.e., higher $\alpha$) also leads to higher stock market valuations in our model, audited disclosure does not as such improve investor protection in our model. Instead, *independent* audited disclosure reduces the inefficiencies from indirect inference because insiders are less concerned about the effect of their actions on outsiders’ expectations. Our model therefore highlights an important role of independent audited disclosure and monitoring that has hitherto not been recognized in the literature.

### 3.3 Forced disclosure and the “big bath”

Insiders’ payout policy guarantees that the capital market constraint is satisfied at all times, i.e., $S_t \geq \varphi \alpha E_t[V_t|I_t]$. But will insiders be willing to adhere to this payout policy under all circumstances? Insiders’ participation constraint is satisfied if they are better off paying out than triggering collective action. Collective action implies that stockholders “open up” the firm and uncover its true value ($V_t$). It is reasonable (although not necessary) to assume that collective action also imposes a cost upon insiders. Without loss of generality assume that these costs are proportional to the firm value and given by $C_t = cV_t$.

“Forced disclosure” pricks the bubble that has been building up over time and brings outsiders’ beliefs about the firm value back to reality, i.e., $E_t[V_t|I_t] = V_t$. A sufficient (but not necessary) condition for insiders to keep paying out according to outsiders’ expectations is:

$$M_t = V_t - \varphi \alpha E_t[V_t|I_t] \geq V_t - \varphi \alpha V_t - cV_t \iff V_t \geq \frac{\varphi \alpha}{\alpha \varphi + c} E_t[V_t|I_t] \quad (32)$$
Insiders have an incentive to trigger collective action if the firm’s actual value \( V_t \) drops sufficiently below what outsiders believe the firm to be worth \( E_t(V_t|I_t) \). This situation arises if outsiders’ beliefs about the latent cost variable (as reflected by \( \hat{x}_t \)) are overoptimistic due to measurement errors.

How can one reduce the likelihood of costly forced disclosure? Since a lower nominal outside ownership stake \( (\varphi) \) and a lower degree of investor protection \( (\alpha) \) relax insiders’ participation constraint, one obvious solution is to reduce either of these two (or a combination of both). Unfortunately, this also reduces the firm’s capacity to raise outside equity. Therefore, firms that rely heavily on outside equity (e.g. public firms) adopt more efficient (in terms of cost and speed) disclosure mechanisms such as voluntary audited disclosure. While “big baths” do occur in reality, they rarely result from a very costly forced disclosure process but they are much more likely to happen through the process of regular voluntary audited disclosures, which we discussed previously.

### 3.4 Stock-based compensation

Stein (1989) argues that stock-based compensation induces insiders to inflate income. How does stock-based compensation affect insiders’ production incentives in our setting where market pressures apply not only with respect to the current stock price but also with respect to future payout? To explore this question we now consider the scenario where insiders get each period a fraction \( \delta \) of the existing outside equity. Insiders get the shares cum dividend and must sell them in the market upon receipt (in contrast to their existing stockholding \( 1 - \varphi \) which they are not allowed to sell).\(^{22}\) Outsiders know that their equityholding will be diluted

\(^{21}\)Calculating the exact condition under which insiders optimally exercise their option to trigger collective action by outsiders is beyond the scope of this paper.

\(^{22}\)It is not crucial for the analysis that shares are sold immediately. The key restriction is that insiders do not have discretion regarding the timing of the sale, as this would introduce an adverse selection and an optimal stopping problem.
each period by a fraction $1 - \delta$, and take this into account when pricing the outside equity, $S_t$. Managers’ optimization problem is now given by:

$$
M_t = \max_{q_{t+j}, j=0,\infty} E_t \left[ \sum_{j=0}^{\infty} \beta^j (\pi(q_{t+j}) - \theta E_{S,t+j} [\pi(q_{t+j})] + \delta S_{t+j}) \right]
$$

(33)

Solving this problem gives the following proposition:

**Proposition 5** If insiders get each period the cash equivalent of a fraction $\delta$ of the outside equity then their optimal production decision is given by $q_t = H x_t$ where $H$ is the solution to:

$$
H = 1 - \frac{hK\theta \left(1 - \frac{\delta}{1 - \beta(1 - \delta)A}\right)}{1 - \beta\lambda A}
$$

(34)

The value of the outside equity (cum dividend) at time $t$ is:

$$
S_t = \phi \alpha E_{S,t} \left[ \sum_{i=0}^{\infty} \beta^i (1 - \delta)^i \pi(q_{t+j}) \right] = \frac{\theta h}{1 - \beta A(1 - \delta)} \left( \hat{x}_t + \frac{B\beta(1 - \delta)}{1 - \beta(1 - \delta)} \right)
$$

(35)

Stock based compensation mitigates, but does not eliminate the underinvestment problem except if outsiders in effect own 100% of the firm (i.e. $\delta = 1$).

Equation (34) shows that increasing stock-based compensation is similar to reducing $\theta$, outsiders’ stake in the firm. From (34) it is clear that $H = 1$ if $\delta = 1$, i.e., the efficient outcome is achieved if outside equityholders get 100% diluted each period.

Unlike Stein (1989) insiders do not have an incentive to inflate income in the presence of stock compensation because market pressures do not only apply to the current stock price but also to future payout. By inflating income insiders not only inflate the current stock price, but also outsiders’ expectations regarding future dividend payout. Therefore insiders’ immediate gain with respect to their stock-based compensation is more than offset by the loss from paying higher future dividends (unless insiders own 100% of the firm).

How then can incentives to inflate income arise? High powered compensation mechanisms (such as stock options, or other contracts that are convex in reported income) that lever
up the effect of income changes may be a possible explanation. Giving insiders a tenure of limited duration may also encourage them to inflate income because they escape the market discipline with respect to future dividend payout once they are retired and they leave it to their successors to meet the raised expectations. Similarly, incentives to inflate income may arise in the run-up to an anticipated cash offer that allows insiders to cash in their shares and flee.

4 Empirical implications

Our paper provides empirical implications for a variety of literatures in financial economics.

4.1 Time-series and cross-sectional properties of corporate income

Our theory of intertemporal income smoothing yields rich and testable implications on the time-series properties of reported income and payout to outsiders. First, “reported income” is smooth compared to “actual income” because the former is based on outsiders’ expectations whereas the latter corresponds to actual cash flow realizations.

Second, reported income follows inter-temporally a target adjustment model. The “income target” is a linear, increasing function of sales, so that when there is a shock to sales (and therefore to the income target), reported income adjusts towards the new target, but adjustment is partial and distributed over time because outsiders only gradually learn whether a shock to sales is due to measurement error or due to a shift in the firm’s cost structure.

Third, the current level of reported income can be expressed as a distributed lag model of current and past sales, where the weights on sales decline as we move further in the past. Since payout to outsiders is a fraction of reported income, the current payout also has a target adjustment model where current payout depends on current sales and previous period’s
payout, similar to the Lintner (1956) dividend model.

In terms of cross-sectional analysis, our model predicts that the speed of adjustment towards the income target should decrease with the degree of information asymmetry between inside and outside investors and with the degree of persistence (autocorrelation) in income.

4.2 Real smoothing

Our model predicts that if insiders face capital market pressure then asymmetric information and the resulting inference process lead to underproduction by firms.

There is convincing empirical evidence that firms engage in real smoothing, and are prepared to sacrifice value in order to meet earnings targets. Baber, Fairfield, and Haggard (1991) find that firms cut R&D spending to avoid reporting losses. Survey-based evidence by Graham, Harvey, and Rajgopal (2005) indicates that: (i) insiders (managers) always try to meet outsiders’ earnings per share (EPS) expectations at all costs to avoid serious repercussions; and, (ii) many managers under-invest by postponing or forgoing positive NPV projects to smooth earnings and therefore engage in real smoothing. Roychowdhury (2006) finds that firms discount product prices to boost sales and thereby meet analyst earnings forecasts. Bhojraj, Hribar, Picconi, and McInnis (2009) find evidence suggesting that firms which cut discretionary expenditures and/or manage accruals to achieve the latest analyst forecast benchmark achieve a short-run stock price benefit, but destroy long-run firm value. Finally, Daniel, Denis, and Naveen (2012) analyze situations in which the firm’s cash flow from operations is insufficient to meet its expected levels of dividends and investment. They find that among dividend-paying firms with a cash flow shortfall, over two-thirds reduce investment (relative to median industry levels). These investment cutbacks are economically significant - they constitute 65% of the shortfall.
4.3 Corporate ownership structure

(i) Our model predicts that the degree of income smoothing should increase in the cross-section of firms as outside ownership increases. Kamin and Ronen (1978) and Amihud, Kamin, and Ronen (1983) show that owner-controlled firms do not smooth as much as manager-controlled firms. Prencipe, Bar-Yosef, Mazzola, and Pozza (2011) also provide direct evidence for this. They find that income smoothing is less likely among family-controlled companies than non-family-controlled companies in a set of Italian firms.

(ii) In our model underproduction is more severe the smaller is the inside ownership and this results in an “outside equity Laffer curve”. Morck, Shleifer, and Vishny (1988) document a non-monotonic relation between Tobin’s Q and managerial stock ownership, and McConnell and Servaes (1990) report an "inverted-U" or "hump-shaped" relation between Q and managerial ownership. Numerous successors investigate the ownership-performance relation using different data, various measures of performance and ownership structure, and alternative empirical methods. The standard interpretation of the hump-shaped performance-ownership relation is that incentive alignment effects dominate for low inside ownership but, as managerial ownership increases, these incentive benefits eventually are overtaken on the margin by the cost of an increased managerial ability to pursue non-value-maximizing activities without being disciplined by shareholders. Our paper provides a new explanation for the non-monotonic relation between Tobin’s Q and managerial stock ownership.

(iii) According to our model firms that do not have access to independent and high quality auditors can issue less outside equity. Our model therefore predicts that inside ownership stakes should be greater in countries with weaker quality of accounting information, which appears consistent with the widespread phenomenon of greater private and family firms in such countries.

\[\text{Note that the firm’s replacement value is a constant in our model. Therefore, Tobin’s Q is the outside equity value scaled down by a constant. Figure 2 would in essence be unaltered if the Y-axis represented Tobin’s Q rather than the market value of the outside equity.}\]
4.4 Public versus private firms

Public (private) firms tend to have a high outside (inside) ownership. Our model therefore has a number implications for the behavior of public versus private firms.

(i) The model’s main prediction is that public firms underproduce and that their output is less sensitive to economic shocks. Asker, Farre-Mensa, and Ljungqvist (2012) evaluate differences in investment behavior between stock market listed and privately held firms in the U.S. Listed firms invest less and are less responsive to changes in investment opportunities compared to matched private firms, especially in industries in which stock prices are particularly sensitive to current earnings. They show that the observed patterns are consistent with theoretical models emphasizing the role of managerial myopia. Their result is consistent with what is predicted by our model, in that firms with a higher outside ownership produce less and production is less sensitive to changes in the marginal cost variable. This result follows from the fact that insiders become increasingly concerned about “ratcheting up” outsiders’ expectations as outsiders’ stake in the firm increases.

(ii) Since smoother income leads to smoother payout, one would expect, all else equal, that public firms also smooth payout more than private firms. This implication is consistent with Michaely and Roberts (2012) who show that private firms smooth dividends less than their public counterparts.

Our model shows that as real ownership of outside shareholders approaches 100% the existence of the firm is in doubt. How can public firms with a low ownership stake then exist? Our model argues that:

(iii) Income figures that are independently provided by auditors improve production efficiency because it reduces insiders’ incentives to manipulate income through their production policy. Thus, all else equal higher quality accounting information should increase firm productivity, stock market capitalization, and, more generally, economic growth (as confirmed, for instance, by Rajan and Zingales, 1998).
(iv) It seems implausible that the existence of auditing alone resolves all inefficiencies. A standard approach in finance research suggests that managers are paid based on a stock-based incentive compensation in which the share price captures all relevant information for future cash flows. Such schemes resolve the problems occurring under fixed compensation and missing incentives, which yield inefficient decisions. Our model (see section 3.4 on stock-based compensation) shows that stock-based compensation can act as a substitute for inside ownership, and that it is not merely a desirable but necessary component for the proper functioning of public corporations when asymmetric information is prevalent. Our prediction is supported by the widespread use of stock based compensation in public corporations. Gao, Lemmon, and Li (2000) compare CEO compensation in public and private firms and find that public-firm pay - but not private-firm pay - is sensitive to measureable performance variables such as stock prices and profitability. When a firm goes public, pay becomes more performance-sensitive.

5 Related literature

While our model relies on insights of Stein (1989) and Jin and Myers (2006), there are several important differences. In Stein (1989), myopic managerial behavior takes the form of an attempt to inflate earnings so as to boost stock prices. In contrast, in our model, insiders are not directly concerned about stock prices, but fear intervention by outsiders when their expectations are not met; as a result, myopic behavior by insiders takes the form of managing earnings downward and underproducing so as not to set outsiders’ expectations about future income too high. Further, in Stein (1989) the time-series properties of observed earnings and unmanipulated earnings are essentially the same (the difference between the two happens to be constant at all times, allowing original earnings to be reconstructed from observed earnings). In contrast, in our model reported income is smooth compared to actual income and follows a simple target adjustment model that can be linked to the underlying economic fundamentals...
in a very transparent and empirically testable fashion.\[24\]

Jin and Myers (2006) also differs from our model in a number of fundamental ways. While in their model the actual income process is completely exogenous, in our model income is endogenously determined through insiders’ output decision. This allows us to identify the effect of asymmetric information on insiders’ production decisions (real smoothing). Also, in Jin and Myers (2006) outsiders base their income estimates at each moment in time on their initial prior information and they do not learn about the evolution of the latent income component. As a result, there is no *intertemporal* smoothing in their model. In our model outsiders observe sales, a noisy proxy for output, which allows them to update their expectations regarding the latent marginal cost variable.

Our underinvestment result is reminiscent of Bebchuk and Stole (1993) and Kanodia and Mukherji (1996) but based on different economic premises. Unlike our model where insiders have an infinite horizon, Bebchuk and Stole (1993) examine managerial investment decisions in the presence of short-term managerial objectives. They show that when investors cannot observe the level of investment in the long-run project underinvestment is induced, but when investors can observe investment but not its productivity, overinvestment occurs. In Kanodia and Mukherji (1996), assets in place are assumed to have a cash flow structure that is correlated over time, and accounting only reports total output from assets in place and investment, without separating investment. In these respects, their model is somewhat similar to ours. The crucial difference is that in our setup, insiders maximize their own share of income from the firm and care about setting shareholder expectations as this affects the future payouts insiders must meet in order to avoid shareholder interventions; in contrast, in Kanodia and Mukherji, insiders maximize current shareholder welfare, so that their investment decision is

\[24\] Another difference is that in Stein (1989) stock prices are strong-form efficient at all times because outsiders can reconstruct the original earnings stream from the observed earnings. In contrast, stock prices are unbiased but only semi-strong efficient in our model because outsiders constantly learn and update their expectations on the basis of observable signals that act as a noisy proxy for the unobserved output variables seen only by the insiders.
dependent on share price, but this in turn also depends on inference about the investment decision.

Our paper is also related to the literature on ratcheting which posits that current performance acts as a partial basis for setting future targets. In particular, our model solution shares similarities with Weitzman (1980) who postulates that this period’s target is a weighted average of last period’s performance and last period’s target. As a result the target can be formulated as a distributed lag of current and past realizations. While Weitzman (1980) assumes the ratcheting principle and its distributed lag structure, ratcheting and its specific structure are endogenously derived in our paper\footnote{Ratcheting in differing contexts is also endogenously obtained in Freixas et al. (1985) and Baron et al. (1987), Laffont and Tirole (1988), Indjejikian and Nanda (1999) among others, but the structure of the ratcheting effect is very different to ours, and less tractable (especially if these models had to be extended to a corporation’s infinite horizon setting).}. Long-term relationships between insiders and outsiders are also subject to “career concerns” (e.g., Gibbons and Murphy (1992), Holmström (1999)). Concern for one’s future tends to encourage effort early in a relationship, offsetting the potential negative impact of the ratchet effect. The ratchet effect could possibly also be mitigated if information regarding the latent variable could be inferred from performance from other firms in the industry. An analysis of these issues is beyond the scope of this paper.

An early, very comprehensive discussion of the objectives, means and implications of income smoothing can be found in the book by Ronen and Sadan (1981) (which includes references to some of the earliest work on the subject). In Lambert (1984) and Dye (1988) risk-averse managers without access to capital markets want to smooth the firm’s reported income in order to provide themselves with insurance\footnote{Models driven by risk-aversion (or limited liability) of managers naturally lead to considering optimal compensation schemes and how they affect smoothing, but we have excluded this literature on managerial compensation for sake of brevity.}. Fudenberg and Tirole (1995) develop a model where reported income is paid out as dividends and where risk-averse managers enjoy private benefits from running the firm but can be fired after poor performance. They assume that recent income observations are more informative about the prospects of the firm.
than older ones. They show that managers distort reported income to maximize the expected length of their tenure: managers boost (save) income in bad (good) times. Graham (2003) also explains and describes existing evidence that convexity of corporate taxes in firm profits can lead to income smoothing, though it is unclear it should lead to “real” smoothing as in our model.

There are also signaling and information-based models to explain income smoothing. Ronen and Sadan (1981) employ a signaling framework to argue that only firms with good future prospects smooth earnings because borrowing from the future could be disastrous to a poorly performing firm when the problem explodes in the near term. Trueman and Titman (1988) also argue that managers smooth income to convince potential debtholders that income has lower volatility in order to reduce the cost of debt. Smoothing costs arise from higher taxes and auditing costs. Tucker and Zarowin (2006) provide evidence that the change in the current stock price of higher-smoothing firms contains more information about their future earnings than does the change in the stock price of lower-smoothing firms. Our model assumes that there are at least some limits to perfect signaling and is in this sense complementary to these alternative explanations for earnings smoothing.

The basic mechanism in our model can be considered similar to that in a strand of signal-jamming equilibrium models in which the indirect inference process distorts corporate choices. This informational effect is similar to the ones discussed (albeit in different economic settings) in Milgrom and Roberts (1982), Riordan (1985), Gal-Or (1987), Stein (1989), Holmström (1999), and more recently Bagnoli and Watts (2010). The learning process (which we model

\[27\] In a slightly different approach to motivating earnings smoothing, Goel and Thakor (2003) develop a theory in which greater earnings volatility leads to a bigger informational advantage for informed investors over uninformed investors, so that if sufficiently many current shareholders are uninformed and may need to trade in the future for liquidity reasons, they want the manager to smooth reported earnings.

\[28\] While in our model insiders have an incentive not to raise outsiders’ expectations regarding income, opposite incentives arise in Bagnoli and Watts (2010) who examine the interaction between product market competition and financial reporting. They show that Cournot competitors bias their financial reports so as to create the impression that their production costs are lower than they actually are.

35
as a filtering problem) and the resulting intertemporal smoothing are, however, quite different from existing papers.

6 Conclusion

The theory of income smoothing developed in this paper assumes that (i) insiders have information about income that outside shareholders do not, but (ii) outsiders are endowed with property rights that enables them to take collective action against insiders if they do not receive a fair payout that meets their expectations. We showed that insiders try to manage outsiders’ expectations. Furthermore, insiders report income consistent with outsiders’ expectations based on available information rather than the true income. This gives rise to a theory of inter-temporal smoothing – both real and financial – in which observed income and payout adjust partially and over time towards a target and insiders under-invest in production. The primary friction driving the smoothing is information asymmetry as insiders are averse to choosing actions that would unduly raise outsiders’ expectations about future income. Interestingly, this problem is more severe the smaller is the inside ownership and thus should be a greater hindrance to the functioning of publicly (or dispersedly) owned firms. We show that the firm’s outside equity value is an inverted U-shaped function of outsiders’ ownership stake. This “outside equity Laffer curve” shows that the under-investment problem severely limits the firm’s capacity to raise outside equity. However, a disclosure environment with adequate quality of independent auditing can help mitigate the problem, leading to the conclusion that accounting quality can enhance investments, size of public stock markets and economic growth. While this theory of inter-temporal smoothing of income and payout conforms to several existing findings (such as the Lintner (1956) model of payout policy), it also leads to a range of testable empirical implications in the cross-section of firms as information asymmetry and ownership structure are varied. These implications are worthy of empirical investigation.
7 Appendix

Proof of Proposition 1: The firm value is given by:

\[ V_t = E_t \left[ \sum_{j=0}^{\infty} \beta^j \left( q_{t+j} - \frac{q_{t+j}^2}{2x_{t+j}} \right) \right] \]  

(36)

The first-order and second-order conditions with respect to \( q_t \) are, respectively,

\[
\frac{\partial V_t}{\partial q_t} = 1 - \frac{q_t}{x_t} = 0 \quad \text{and} \quad \frac{\partial^2 V_t}{\partial q_t^2} = -\frac{1}{x_t} < 0
\]  

(37)

Solving the first-order condition gives the expressions for \( q_t \) as in the proposition. The second-order condition is always satisfied under the reasonable (see footnote 9) assumption that production costs are positive (i.e. \( x_t > 0 \)).

Proof of Proposition 2: Insiders’ optimization problem can be formulated as:

\[ M_t = \max_{\{q_{t+j}, j=0, \ldots, \infty\}} \{ q_t + \sum_{j=0}^{\infty} \beta^j (\pi(q_{t+j}) - \theta E_{S,t+j}(\pi(q_{t+j})) \} \]  

(38)

We guess the form of the solution and use the method of undetermined coefficients (and subsequently verify our conjecture). The conjectured solution for outsiders’ rational expectations based on the information \( I_t \) is as follows:

\[ E_{S,t} [\pi(q_t)] = b + \sum_{j=0}^{\infty} a_j s_{t-j} \]  

(39)

where the coefficients \( b \) and \( a_j (j = 0, 1, \ldots) \) remain to be determined.

The first-order condition is

\[
\frac{\partial M_t}{\partial q_t} = 1 - \frac{q_t}{x_t} - \theta (a_0 + \beta a_1 + \beta^2 a_2 + \beta^3 a_3 + \ldots) = 0.
\]  

(40)

\[ \iff q_t = \left[ 1 - \theta \sum_{j=0}^{\infty} a_j \beta^j \right] x_t \equiv H x_t. \]  

(41)

Outsiders rationally anticipate this policy and can therefore make inferences about the latent variable \( x_t \) on the basis of their observation of current and past sales \( s_{t-j} \) \((j = 0, 1, \ldots)\). We
know that $s_t = q_t + \epsilon_t$. This measurement equation can be combined with the state equation
(8) to make inferences about $x_t$ on the basis of current and past observations of $s_t$. This, in turn, allows outsiders to form an estimate of realized income $\pi_t$. It can be shown that the Kalman filter is the optimal filter (in terms of minimizing the mean squared error) for the type of problem we are considering (see Chui and Chen (1991)).

One can show (see Chui and Chen (1991), p78) that the error of the steady state estimator, $x_t - \hat{x}_t$, is normally distributed with zero mean and variance $P$, i.e., $E_{S,t}[x_t - \hat{x}_t] = 0$ and $E_{S,t}[(x_t - \hat{x}_t)^2] = P$, or $p(x_t|I_t) \sim N(\hat{x}_t, P)$, where $\hat{x}_t$ is given by:

$$\hat{x}_t = A\hat{x}_{t-1} + B + K[s_t - H(A\hat{x}_{t-1} + B)] = (A\hat{x}_{t-1} + B)\lambda + Ks_t$$  \hspace{1cm} (42)

$$\lambda \equiv (1 - KH) \quad \text{and} \quad K \equiv \frac{H/P}{H^2P + R}$$  \hspace{1cm} (43)

and where $P$ is the positive root of the equation (20). $K$ is called the “Kalman gain” and it plays a crucial role in the updating process. Using the conjectured solution for $q_t$ it follows that outsiders’ estimate of income at time $t$ is given by:

$$E_{S,t}[\pi_t] = E_{St} \left[ H\hat{x}_t - \frac{H^2x_t}{2} \right] = \left( H - \frac{H^2}{2} \right) \hat{x}_t$$  \hspace{1cm} (45)

$$= \left( H - \frac{H^2}{2} \right) \left[ \frac{\lambda B}{1 - \lambda A} + K \sum_{j=0}^{\infty} (\lambda A)^j s_{t-j} \right]$$  \hspace{1cm} (46)

$$= b + \sum_{j=0}^{\infty} a_j s_{t-j}$$  \hspace{1cm} (47)

where the last step follows from our original conjecture given by equation (39). This allows

29If there is little prior history regarding sales $s_t$ then $K_t$ itself will vary over time because $P_t$, the variance of the estimation error, initially fluctuates over time. Once a sufficient number of observations have occurred $P_t$, and therefore $K_t$, converge to their stationary level $P$ and $K$. A sufficient condition for the filter to converge is that $\lambda A < 1$. The order of convergence is geometric (see Chui and Chen, 1991, Theorem 6.1 on Page 88).
us to identify the coefficients $b$ and $a_j$:

$$b = \left( H - \frac{H^2}{2} \right) \left[ \frac{\lambda B}{1 - \lambda A} \right]$$

$$a_j = \left( H - \frac{H^2}{2} \right) K (\lambda A)^j$$

(48)

(49)

For this to be a rational expectations equilibrium it has to be that (see equation (41)):

$$H = 1 - \theta \sum_{j=0}^{\infty} a_j \beta^j = 1 - \frac{\theta \left( H - \frac{H^2}{2} \right) K}{1 - \beta \lambda A}$$

(50)

Simplifying gives the condition for $H$ in the proposition. Fixing outsiders’ beliefs (i.e. $E_{S,t}[\pi(q_{t+j})] = \left( H - \frac{H^2}{2} \right) \hat{x}_{t+j} \equiv h\hat{x}_{t+j}$) and solving for insiders’ optimal production it follows from (11)– (13) that insiders’ output strategy is a fixed point. One can also immediately verify that the second order condition for a maximum is satisfied (assuming $x_t$ is positive).

Finally, we calculate the expected value and variance of the estimate’s error: $\pi_t - \hat{\pi}_t$. We make use of the result that the error with respect to the steady state estimator for $x_t$ is normally distributed with zero mean and variance $P$. Hence,

$$E_{S,t}[\pi_t - \hat{\pi}_t] = E_{S,t}[h(x_t - \hat{x}_t)] = 0$$

$$E_{S,t}[(\pi_t - \hat{\pi}_t)^2] = E_{S,t}[h^2(x_t - \hat{x}_t)^2] = h^2 P$$

(51)

(52)

**Proof of Proposition 3**: Actual income under insiders’ production policy is given by:

$$\pi_t = q_t - \frac{q_t^2}{2x_t} = hx_t$$

(53)

We know from the proof of proposition 2 that $\hat{\pi}_t = E_{S,t}[\pi_t] = b + \sum_{j=0}^{\infty} a_j s_{t-j}$ (where the values for $b$ and $a_j$ are defined there). Lagging this expression by one period, it follows that $\hat{\pi}_t - \lambda A \hat{\pi}_{t-1} = hKs_t + h\lambda B$. Substituting this expression into the target adjustment model (21) gives:

$$\lambda A \hat{\pi}_{t-1} + Khs_t + h\lambda B = \hat{\pi}_{t-1} + (1 - \lambda A)\pi_t^* - \hat{\pi}_{t-1} + \lambda A \hat{\pi}_{t-1}$$

(54)

Simplifying and solving for $\pi_t^*$ gives equation (23).
Proof of Proposition 4: Assume that \( \hat{x}_0 \equiv E_{S,0}[x_0] = x_0 \) when the equity is issued. As a result, outsiders and insiders predict the same future path for \( x_t \) at time \( t = 0 \). Indeed, 
\[
E_{S,0}[x_1] = A\hat{x}_0 + B = E_0[x_1]; \quad E_{S,0}[x_2] = A^2\hat{x}_0 + AB + B = E_0[x_2]; \quad E_{S,0}[x_3] = \ldots.
\]
Therefore, insiders and outsiders value the company identically. This firm value is
\[
V_0 = E_0[\sum_{j=0}^{\infty} \beta^j \pi_j] = h x_0 + \beta (hAx_0 + hB) + \beta^2 (hA^2x_0 + hAB + hB) + \ldots \quad (55)
\]
\[
= h x_0 (1 + \beta A + \beta^2 A^2 + \beta^3 A^3 + \ldots) + hB\beta (1 + \beta A + \beta^2 A^2 + \beta^3 A^3 + \ldots)
\]
\[
+ hB\beta^2 (1 + \beta A + \beta^2 A^2 + \ldots) + \frac{hB\beta^3}{1 - \beta A} + \frac{hB\beta^4}{1 - \beta A} + \ldots
\]
\[
= \frac{h}{(1 - \beta A)} \left( x_0 + \frac{B\beta}{1 - \beta} \right). \quad \blacklozenge \quad (56)
\]

Proof of Proposition 5: The valuation equation for \( S_t \) follows immediately from proposition 4 (by substituting \( x_0 \) and \( \beta \) by \( \hat{x}_t \) and \( \beta(1 - \delta) \), respectively). The derivation of the equilibrium value for \( H \) is as given in the proof to proposition 2 but with \( \theta \) replaced by \( \theta \left( 1 - \frac{\delta}{1 - \beta(1 - \delta)\delta} \right) \).

References


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Reported Income on Discretionary Spending Decisions: The Case of Research and Development,”


The figure examines how production efficiency is affected by the variance of measurement errors ($R$), the variance of the latent cost variable $x_t$ ($Q$), the autocorrelation at lag one of the latent cost variable ($A$) and outsiders' real ownership stake ($\theta$). Production efficiency is measured by comparing unconditional mean output ($E(q_t)$) and unconditional mean income ($E(\pi_t)$) relative to their first-best level. The baseline parameter values used to generate the figures in this paper are: $A = 0.9$, $B = 10$, $Q = 5$, $R = 1$, $\beta = 0.95$ and $\theta = 0.8$. 
The figure plots the total initial firm value $V_0$ (panel A) and outside equity value $S_0$ (panel B) as a function of outsiders’ real ownership stake ($\theta$) for three different levels of audited disclosure quality ($\kappa$). The inverted U-shaped curves in panel B are the so-called “outside equity Laffer curves”. The baseline parameter values used to generate the figure are the same as before, i.e., $A = 0.9$, $B = 10$, $Q = 5$, $R = 1$, $\beta = 0.95$ and $\theta = 0.8$. 

$A)$ Total Firm Value as a function of $\theta$ and $\kappa$ 

$B)$ Outside Equity Value as a function of $\theta$ and $\kappa$