Online Appendix for “On Reaching for Yield and the Coexistence of Bubbles and Negative Bubbles”

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Abstract

To economize on space, all the proofs of our paper “On Reaching for Yield and the Coexistence of Bubbles and Negative Bubbles” have been relegated to this appendix.

Proof of Proposition 1

The participation constraint of the intermediary is binding because otherwise the intermediary can increase its expected profit by slightly reducing \( \rho_I \). Thus, \( \rho_I^* \) is given by the solution to

\[
E(\hat{x}) + (1 - E(\hat{x}))[\theta \rho_I + (1 - \theta) \frac{\rho_C E[\max(C - \hat{x}I, 0)]}{(1 - E(\hat{x}))I}] = \bar{u}. \tag{1}
\]

Solving for \( \rho_I^* \) gives us Eq. (11).

From Eq. (7) we can solve for \( C \) which gives us

\[
C = D - L(\rho_R) + L(\rho_M). \tag{2}
\]

Substituting \( \rho_I^* \) and \( C \) in the intermediary’s objective function gives us the following unconstrained maximization problem:

\[
\max_{\rho_R, \rho_M} \Pi = \pi - E[\Psi|e = e_H] \tag{3}
\]
where \( \pi \) is given by Eq. (8) and \( \Psi \) is given by Eq. (4).

Assuming that \( \Pi \) is quasiconcave in \( \| \cdot \| \) for \( \| \cdot \| = \| \cdot \| \) the first-order condition (FOC) with respect to \( \rho_M \) is given by

\[
\frac{\partial \Pi}{\partial \rho_M} = \theta L (\rho_M) - \theta \rho_C \Pr [\tilde{x}I \leq C] L' (\rho_M) + \theta \rho_M L' (\rho_M) - \rho^p_M \Pr [\tilde{x}I > C] L' (\rho_M) - \theta I (1 - E (\tilde{x})) \frac{\partial \rho^*_I}{\partial \rho_M} = 0. \tag{4}
\]

Noting that \( \frac{\partial \rho^*_I}{\partial \rho_M} = (1 - \theta) \rho_C \Pr [\tilde{x}I \leq C] L' (\rho_M) / \theta I (1 - E (\tilde{x})) \) and solving for \( \rho_M \) after some simplification we get Eq. (9).

Next, taking the FOC with respect to \( \rho_R \), we get

\[
\frac{\partial \Pi}{\partial \rho_R} = \theta p L (\rho_R) - \theta \rho_C \Pr [\tilde{x}I \leq C] L' (\rho_R) + \theta \rho_R L' (\rho_R) - \rho^p_R \Pr [\tilde{x}I > C + L (\rho_M)] L' (\rho_R) - \rho^p_R \Pr [\tilde{x}I > C + L (\rho_M)] L' (\rho_R) - \theta I (1 - E (\tilde{x})) \frac{\partial \rho^*_I}{\partial \rho_R} = 0. \tag{5}
\]

Noting that \( \frac{\partial \rho^*_I}{\partial \rho_R} = (1 - \theta) \rho_C \Pr [\tilde{x}I \leq C] L' (\rho_R) / \theta I (1 - E (\tilde{x})) \) and solving for \( \rho_R \) after some simplification we get Eq. (10).

Finally, substituting \( \rho^*_R \) and \( \rho^*_M \) in Eq. (2) we get \( C^* \) as given by Eq. (12).

Q.E.D.

**Proof of Corollary 1**

From the FOC (4), if we solve for \( \rho^*_M \) directly without exploiting the definition of \( \eta_M \) we get the following expression for the return on medium-risk loans:

\[
\rho^*_M = \frac{\rho_C}{\theta} - \frac{L (\rho_M)}{L' (\rho_M)} + \frac{(\rho^p_M - \rho_C) \Pr (\tilde{x}I \geq C^*)}{\theta}. \tag{6}
\]

Taking the partial derivative of the above expression w.r.t. \( \theta \) we get:

\[
\frac{\partial \rho^*_M}{\partial \theta} = - \rho_C + \frac{(\rho^p_M - \rho_C) \Pr (\tilde{x}I \geq C^*)}{\theta^2} < 0 \tag{7}
\]

since \( \rho^p_M > \rho_M > \rho_C \), which proves the risk effect for the medium-risk loan.

Next note that \( \theta \Pr (\tilde{x}I \geq C) / \partial I < 0 \), i.e. an increase in financial intermediary’s liquidity (investment funds) lowers the probability of liquidity shortfalls since \( C = D - L_R - L_M \). Then taking the partial derivative of (6) w.r.t. \( 1 - F (C) = \Pr (\tilde{x}I \geq C) \) we get:

\[
\frac{\partial \rho^*_M}{\partial [1 - F (C)]} = \frac{\rho^p_M - \rho_C}{\theta} > 0. \tag{8}
\]

Hence \( \frac{\partial \rho^*_M}{\partial [1 - F (C)]} \frac{\partial [1 - F (C)]}{\partial I} < 0 \), which proves the liquidity effect for the medium-risk loan.
Similarly, from the FOC (5), if we solve for \( \rho^*_R \) directly without exploiting the definition of \( \eta_R \) we get the following expression for the return on risky loans:

\[
\rho^*_R = -\frac{L'(\rho_R)}{L'(\rho_R)} + \rho_C \Pr (\tilde{x}I \leq C) + \rho_M^p \Pr (C < \tilde{x}I \leq C + L_M) + \rho^p_R \Pr (\tilde{x}I > C + L_M).
\]

Taking the partial derivative of the above expression w.r.t. \( \theta \) we get:

\[
\frac{\partial \rho^*_R}{\partial \theta} = -\rho_C \frac{\Pr (\tilde{x}I \leq C) + \rho_M^p \Pr (C < \tilde{x}I \leq C + L_M) + \rho^p_R \Pr (\tilde{x}I > C + L_M)}{(\theta p)^2} < 0,
\]

which proves that an increase in macroeconomic risk, \( 1 - \theta \), increases the equilibrium lending rate for the risky project, ceteris paribus.

Similarly, taking the partial derivative of Eq. (9) w.r.t. \( p \) we get:

\[
\frac{\partial \rho^*_R}{\partial p} = -\frac{\theta \rho_C \Pr (\tilde{x}I \leq C) + \rho_M^p \Pr (C < \tilde{x}I \leq C + L_M) + \rho^p_R \Pr (\tilde{x}I > C + L_M)}{(\theta p)^2} < 0,
\]

which proves that an increase in specific risk, \( 1 - p \), increases the equilibrium lending rate for the risky project, ceteris paribus.

Finally, taking the partial derivative of Eq. (9) w.r.t. \( I \) we get:

\[
\frac{\partial \rho^*_R}{\partial I} = \frac{f \left( \frac{C}{I} \right) L'(\rho_M) + L(\rho_R)}{I^2} \left[ \frac{\rho_C}{\theta p} - \frac{\rho_M^p}{\theta p} \right] + \frac{f \left( \frac{C + L(\rho_M)}{I} \right) L'(\rho_R)}{I^2} \left[ \frac{\rho_M^p}{\theta p} - \frac{\rho^p_R}{\theta p} \right] < 0,
\]

since \( \rho^p_R > \rho_M^p > \rho_C \), which proves the liquidity effect for the risky loan. Q.E.D.

Proof of Proposition 2

Let \( \mu_1, \mu_2, \mu_3 \) denote the Lagrange multipliers for constraints (14), (15), and (16). Taking the FOC with respect to \( b_M \) the following condition is satisfied at every \( L_R \):

\[
\frac{1}{\int_{I} \int_{z} [(1 - \phi) v' (b) + \phi v' (b - \psi)] f (x) j (I) dxdI} = \mu_1,
\]

where \( j (I) \) is the density function of intermediary’s liquidity, \( I \). Since the RHS in Eq. (13) is constant it follows that \( v' (\cdot) \) on the LHS is constant. If the manager is strictly risk averse (so that \( v' (w) \) is strictly decreasing in \( w \)), the implication of condition (13) is that \( b_M \) is constant.

Next, taking the FOC with respect to \( b_R \) the following condition is satisfied at every \( L_R \):

\[
\frac{1}{\int_{I} \int_{z} [(1 - \phi) v' (b) + \phi v' (b - \psi)] f (x) j (I) dxdI} = \mu_1 + \mu_2 \left[ \frac{1 - g \left( L(\rho_R) | e^L \right)}{g \left( L(\rho_R) | e^H \right)} \right],
\]

(14)
where \( g(L(\rho_R) | e) > 0 \) is the density function of loans conditional on effort. As is common in the literature, we then invoke the monotone likelihood ratio property (MLRP), i.e., \( g(L(\rho_R) | e_L) / g(L(\rho_R) | e_H) \) is decreasing in \( L_R \). This means that, as risky loans increase, the likelihood of getting a given level of risky loans and profits if effort is \( e_H \), relative to the likelihood if effort is \( e_L \), must increase. Hence, an increase in \( L_R \) increases the right-hand side (RHS) of Eq. (14). It follows that the left-hand side (LHS) is increasing in \( L_R \) and the denominator of the LHS is decreasing in \( L_R \). The denominator of LHS will be decreasing in \( L_R \) if, and only if, \( v'(\cdot) \) is decreasing in \( L_R \). Note, however, that \( \psi = \min(\psi, \gamma \Psi) \) is increasing in \( L_R \). This is clear once we rewrite Eq. (4) by substituting for \( C = I - L_R - L_M \) to get the following expression for \( \Psi \):

\[
\Psi = \left\{ \begin{array}{ll}
\rho_M^{\beta} [L_R + L_M - I (1 - x)] & \text{if } 0 < \ell \leq L_M \\
\rho_M^{\beta} L_M + \rho_M^{\beta} [L_R - I (1 - x)] & \text{if } \ell > L_M
\end{array} \right.
\tag{15}
\]

Since \( \psi \) is increasing in \( L_R \) and given that \( v'' < 0 \), it follows that the denominator of the LHS is decreasing in \( L_R \) if, and only if, managerial bonuses, \( b_R \), are monotonically increasing in \( L_R \).

Next, taking the FOC with respect to \( \psi \), the following condition is satisfied for every \( L_R \):

\[
\int \int_{I \times x} \left[ 1 - \mu_1 \psi' (b - \psi) - \mu_2 \psi' (b - \psi) \left( 1 - \frac{g(L(\rho_R) | e_L)}{g(L(\rho_R) | e_H)} \right) \right] \phi g(L(\rho_R) | e) f(x) j(I) \, dx \, dI \\
= \mu_3.
\tag{16}
\]

Because constraint (16) is binding, it follows that \( \mu_3 > 0 \). Thus, the following condition is satisfied:

\[
\left[ 1 - \mu_1 \psi' (\cdot) - \mu_2 \psi' (\cdot) \left( 1 - \frac{g(L(\rho_R) | e_L)}{g(L(\rho_R) | e_H)} \right) \right] > 0.
\tag{17}
\]

Finally, taking the FOC with respect to \( \phi \), the following condition is satisfied for every \( L_R \):

\[
\int \int_{I \times x} \psi g(L(\rho_R) | e_H) f(x) j(I) \, dx \, dI - z \\
+ \mu_1 \int \int_{I \times x} [-v(b) + v(b - \psi)] g(L(\rho_R) | e_H) f(x) j(I) \, dx \, dI \\
+ \mu_2 \int \int_{I \times x} [-v(b) + v(b - \psi)] [g(L(\rho_R) | e_H) - g(L(\rho_R) | e_L)] f(x) j(I) \, dx \, dI \\
+ (\mu_4 - \mu_5) = 0,
\tag{18}
\]
where \( \mu_4 \) and \( \mu_5 \) correspond to the Lagrange multipliers for the constraints \( \phi \geq 0 \) and \( \phi \leq 1 \), respectively. An audit takes place if, and only if,

\[
k(\ell) = \int_{I} \int_{x} \psi g(L(\rho_{R})|e^H) f(x) j(I) dx dI - z
\]

\[
+ \mu_1 \int_{I} \int_{x} [-v(b) + v(b - \psi)] g(L(\rho_{R})|e^H) f(x) j(I) dx dI
\]

\[
+ \mu_2 \int_{I} \int_{x} [-v(b) + v(b - \psi)] [g(L(\rho_{R})|e^H) - g(L(\rho_{R})|e^L)] f(x) j(I) dx dI > 0.
\]

This is because, if \( k(\ell) > 0 \), it implies that \( \mu_5 > \mu_4 \). But \( \mu_5 > \mu_4 \) if, and only if, the constraint \( \phi \leq 1 \) is binding as a binding constraint implies that \( \mu_5 > 0 \) but \( \mu_4 = 0 \). This would be the case if, and only if, \( \phi = 1 \). It follows that \( \phi = 1 \) if \( k(\ell) > 0 \) and \( \phi = 0 \) otherwise. Let \( \ell^* \) denote the threshold such that \( k(\ell^*) = 0 \). To prove that it is optimal to audit if, and only if, \( \ell > \ell^* \), it would suffice to show that \( k'(\ell) \) is strictly increasing in \( \ell \).

Taking the derivative of \( k(\ell) \) with respect to \( \ell \) after some simplification we get

\[
k'(\ell) = \int_{I} \int_{x} \left[ 1 - \mu_1 \psi' \langle b - \psi \rangle - \mu_2 \psi' \langle b - \psi \rangle \left( 1 - \frac{g(L(\rho_{R})|e^L)}{g(L(\rho_{R})|e^H)} \right) \right] g(L(\rho_{R})|e^H) \psi' \langle \ell \rangle dFdJ,
\]

where \( F \) and \( J \) represent the distribution functions of \( x \) and \( I \), respectively. Because \( \psi'(\ell) > 0 \) and given condition (17), it follows that \( k'(\ell) > 0 \).

Similarly, taking the derivative of \( k(\ell) \) with respect to \( \ell \) after some simplification we get

\[
k'(\ell) = \int_{I} \int_{x} \left[ 1 - \mu_1 \psi' \langle b - \psi \rangle - \mu_2 \psi' \langle b - \psi \rangle \left( 1 - \frac{g(L(\rho_{R})|e^L)}{g(L(\rho_{R})|e^H)} \right) \right] g(L(\rho_{R})|e^H) \psi' \langle \Psi \rangle dFdJ,
\]

where \( \psi'(\Psi) > 0 \). It follows that an audit will take place if the cost of covering total liquidity shortfall is high enough, which will be the case if \( \ell \) is high enough.

Q.E.D.

**Proof of Proposition 3**

As before the participation constraint is binding from which we can solve for \( \rho_{ia}^{\pi} \). Also from the budget constraint, we have \( C = I - L(\rho_R) - L(\rho_M) \). Substituting \( \rho_{ia}^{\pi} \) and \( C \) in \( \pi \) we need to solve for an unconstrained maximization problem. Taking the FOC with respect to \( \rho_i \) and solving for \( \rho_{ia}^{\pi} \) and \( \rho_{ia}^{\pi} \) we get

\[
\rho_{ia}^{\pi} = \rho_G \Pr \left[ (\hat{z}I \leq I) | e^H \right] + \rho_{ia}^{\pi} \Pr \left[ (\hat{z}I > C^*) | e^H \right] + \frac{\partial E[b+z|e=e^H]}{\partial \psi_{ai}^{\pi} | e=H} \theta_L(\rho_{ai}^{\pi}), \quad (22)
\]
\[ \rho_R^n = \frac{\rho_C \Pr[\{\hat{x} I \leq C\} | e^H] + \rho_M^n \Pr[(C < \hat{x} I \leq C + L_M) | e^H] + \rho_R^n \Pr[(\hat{x} I > C + L_M) | e^H]}{\theta_{\rho} \left(1 - \frac{1}{\eta_R}\right)} \]

\[ \frac{\partial E[b + z|e = e^H]}{\partial \rho_i} + \frac{\partial E[L(\rho_R)|e = e^H]}{\partial \rho_R} \text{,} \]

where \( \eta_M = -\rho_M L'(\rho_M)/L_M \geq 0 \) and \( \eta_R = -\rho_R \frac{\partial E[L(\rho_R)|e = e^H]}{E[L(\rho_R)|e = e^H]} \). In the case of symmetric information, \( \hat{E}[L(\rho_R)|e = e^H] = L(\rho_R)|e = e^H \) since risky loans are non-stochastic. It follows that \( \eta_R = \eta_R \) with symmetric information. Noting that the first term on the RHS of Eqs. (22) and (23) is \( \rho_M^n \) and \( \rho_R^n \) respectively we get expressions (22) and (23). Next note that

\[ \frac{\partial E[b + z|e = e^H]}{\partial \rho_i} = \frac{\partial E[b + z|e = e^H]}{\partial L_i} \frac{\partial L_i}{\partial \rho_i} < 0 \]

for \( i = R, M \) since bonuses, \( b \), are increasing in loan volume; audit costs (\( z \)) are increasing in loan volume (of both medium-risk and risky loans) since an increase in loan volume increases the probability of liquidity shortfalls thereby increasing the expected audit costs (\( z \)); while \( \partial L_i/\partial \rho_i < 0 \). Finally noting that \( L'(\rho_M) < 0 \) and \( \partial E[L_R|e = e^H]/\partial \rho_R < 0 \) it follows that the second term on the RHS of (24) is positive and thus \( \rho_M^n > \rho_M^* \) and \( \rho_R^n > \rho_R^* \). Q.E.D.

**Proof of Proposition 4**

We can rewrite the manager’s problem as follows:

\[ \max_{\rho_R, \rho_M, x} \int \int v \left(\left[b(L(\rho_R)) + b_M - \psi' \right] | e = e^H \right) f(x) g(L_R | e = e^H) \, dx \, dL_R - e^H \]

subject to

\[ L(\rho_R) + L(\rho_M) + C = I, \]

and

\[ L(\rho_M) \geq L_M^k \forall \theta^k. \]

where \( L_M^k \) is decreasing in \( \theta^k \). Taking the FOC with respect to \( \rho_R \) we get

\[ \int \int v' \left( \left[b'_R(L_R) L'_R (r_{RR}^*) - \frac{\partial \psi}{\partial \rho_R} \right] f(x) g(L_R | e = e^H) \, dx \, dL_R - \lambda_1 L'_R = 0 \]

where \( \lambda_1 \) is the Lagrange multiplier for constraint (26). Taking the FOC with respect to \( \rho_M \) we get

\[ \int \int v' \left[ -\frac{\partial \psi}{\partial \rho_M} \right] f(x) g(L_R | e = e^H) \, dx \, dL_R - \lambda_1 L'_M + \lambda_2 L'_M = 0 \]
where $\lambda_2$ is the Lagrange multiplier for constraint (27). Finally, taking the FOC with respect to $C$ we get

$$
\int \int_{L_R} v'(\cdot) \left[ -\frac{\partial \tilde{\psi}}{\partial C} \right] f(x) g \left( L_R | e = e^H \right) dx dL_R - \lambda_1 = 0. \quad (30)
$$

The first term in FOC (29) is positive. This is because $\frac{\partial \tilde{\psi}}{\partial P_M} = \frac{\partial \tilde{\psi}}{\partial L_M} L'_M$. An increase in $L_M$ reduces cash holdings and hence increases $\tilde{\psi}$. It follows that $\partial \tilde{\psi} / \partial L_M > 0$. Given that $L'_M < 0$ it follows that $\partial \tilde{\psi} / \partial P_M < 0$ and thus the first term is positive. Since the budget constraint (26) is binding, the Lagrange multiplier, $\lambda_1 > 0$. Since $L'_M < 0$, the second term in (29) is also positive. It follows that in order for the FOC to be satisfied, $\lambda_2 > 0$. This implies that the second constraint (27) is also binding and hence $L(P_M) = L^k_M \forall^k$.

In FOC (30) the first term is positive since $v'(\cdot) > 0$ and $\partial \tilde{\psi} / \partial C < 0$ given that an increase in cash holdings lowers the penalty costs. It follows from Eq. (30) that $\lambda_1$ is given by:

$$
\lambda_1 = \int \int_{L_R} v'(\cdot) \left[ -\frac{\partial \tilde{\psi}}{\partial C} \right] f(x) g \left( L_R | e = e^H \right) dx dL_R > 0. \quad (31)
$$

Substituting Eq. (31) in FOC (28) we get the following condition:

$$
\int \int_{L_R} v'(\cdot) \left[ b'_R (L_R) L'_R (p_R^* - \frac{\partial \tilde{\psi}}{\partial R}) - \frac{\partial \tilde{\psi}}{\partial R} \right] f(x) g \left( L_R | e = e^H \right) dx dL_R = \left[ \int \int_{L_R} v'(\cdot) \left[ -\frac{\partial \tilde{\psi}}{\partial C} \right] f(x) g \left( L_R | e = e^H \right) dx dL_R \right] L'_R. \quad (32)
$$

Eq. (32) says that at the optimum the manager chooses the volume of risky loans, $L_R$, such that the net marginal benefit of an incremental loan (given by the LHS of Eq. (32)) just equals the marginal costs (given by the RHS of Eq. (32)). In other words, at the optimum the net marginal benefit of issuing risky loans (given by the expected increase in managerial commissions minus the expected increase in penalties) just equals the marginal cost (since the same amount could have been invested in liquid cash reserves thereby reducing the expected penalty cost for the manager).

The manager reaches for yield if, and only if, his expected utility from reaching for yield exceeds his expected utility from not reaching for yield. More formally, this is true if, and only if, the following expression is positive:

$$
\Delta \Pi_m = \int \int_{L_R} v \left( b (L_R^a) + b_M - \tilde{\psi} | e = e^H \right) f(x) g \left( L_R | e = e^H \right) dx dL_R
$$
\[ - \int_{L_R} v \left( b \left( L_R^{a} \right) + b_M | e = e^H \right) g \left( L_R | e = e^H \right) dL_R, \]  
(33)

where \( L_R^a \) denotes the loan volume when the manager reaches for yield; \( L_R^{na} \) denotes the loan volume when the manager does not reach for yield; and \( \Delta \Pi_m \) denotes the expected utility of the manager from reaching for yield minus the expected utility from not reaching for yield conditional on high effort. In Eq. (33) \( L_R^a > L_R^{na} \) and thus \( \tilde{\psi} > 0 \). If, \( L_R^a = L_R^{na} \), then there’s no agency problem and thus \( \Delta \Pi_m = 0 \). We next show that \( \Delta \Pi_m > 0 \) for sufficiently large \( I \).

Adding and subtracting \( \int_{L_R} v \left( b \left( L_R^{a} \right) + b_M | e = e^H \right) g \left( L_R | e = e^H \right) dL_R \) to Eq. (33) yields

\[ \Delta \Pi_m = \int_{L_R} v \left( b \left( L_R^{a} \right) + b_M | e = e^H \right) g \left( L_R | e = e^H \right) dL_R - c, \]  
(34)

where

\[ c = \int_{L_R} v \left( b \left( L_R^{a} \right) + b_M | e = e^H \right) g \left( L_R | e = e^H \right) dL_R \]  
(35)

\[ - \int_{L_R} v \left( b \left( L_R^{a} \right) + b_M - \tilde{\psi} \right) e = e^H f(x) g \left( L_R | e = e^H \right) dx dL_R \]

The first term in Eq. (34) is positive because \( L_R^a > L_R^{na} \) and \( e' (\cdot) > 0 \). Hence, \( \Delta \Pi_m > 0 \) as long as \( c \) is small enough. It can then be shown that \( c \) is decreasing in \( I \) and, for high enough \( I \), \( \Delta \Pi_m > 0 \). Thus, to prove the proposition it would suffice to show that \( c \) is decreasing in \( I \).

Note that

\[ \int_{L_R} \int_{x} v \left( b \left( L_R^{a} \right) + b_M - \tilde{\psi} \right) e = e^H f(x) g \left( L_R | e = e^H \right) dxdL_R \]  
(36)

\[ = \int_{L_R} \int_{x} (1 - \phi) v \left( b \left( L_R^{a} \right) + b_M | e = e^H \right) f(x) g \left( L_R | e = e^H \right) dxdL_R \]

\[ + \int_{L_R} \int_{x} \phi v \left( b \left( L_R^{a} \right) + b_M - \psi | e = e^H \right) f(x) g \left( L_R | e = e^H \right) dxdL_R, \]

where \( \phi = \Pr (\ell > \ell^*) \).

Substituting Eq. (36) in Eq. (35) and taking the partial derivative of Eq.
Note that in the presence of an agency problem, note that the audit contingency condition (as given by Eq. (19)) is independent of volume. Rates for both medium-risk and risky projects (thereby decreasing investment volume) while tight monetary policy increases the lending rates for both medium-risk and risky projects (thereby increasing investment volume). Hence, in the absence of any agency problem, loose monetary policy lowers the lending rate for both medium-risk and risky projects because bonuses are increasing in loan volume. Further, \( \frac{\partial \phi}{\partial A} > 0 \) and \( \frac{\partial \phi}{\partial R} > 0 \). We know \( \frac{\partial L_R}{\partial A} < 0 \) and \( \frac{\partial L_R}{\partial R} < 0 \) because the loan rate is decreasing in liquidity. Hence, the second term in (37) is negative. Next, note that \( \frac{\partial \phi}{\partial \ell} < 0 \) because the ex ante audit probability is given by \( \phi = \Pr(\ell > \ell^*) \), where \( \ell = \max \{xI - C, 0\} = \max \{L_R + L_M - (1 - x)I, 0\} \). Given that \( C = D - L_R - L_M \), because \( \ell \) is decreasing in \( I \), it follows that the audit probability, \( \phi \), is decreasing in \( I \). Furthermore, \( \{v(b|\cdot) - v(b - \psi|\cdot)\} > 0 \) because \( b > b - \psi \) and because \( v'(\cdot) > 0 \). Hence, the third term in (37) is also negative. Q.E.D.

**Proof of Proposition 5**

Taking the partial derivative of \( \rho^*_C \) with respect to \( \rho_C \), we get

\[
\frac{\partial \rho^*_M}{\partial \rho_C} = \frac{\Pr[(\tilde{z}I \leq C) | e = e H]}{\theta (1 - \frac{1}{\gamma_M})} > 0
\]

and

\[
\frac{\partial \rho^*_R}{\partial \rho_C} = \frac{\Pr[(\tilde{z}I \leq C) | e = e H]}{\theta_p (1 - \frac{1}{\gamma_R})} > 0
\]

which implies that in the absence of any agency problem, loose monetary policy lowers the lending rates for both medium-risk and risky projects (thereby increasing investment volume) while tight monetary policy increases the lending rates for both medium-risk and risky projects (thereby decreasing investment volume).

To show that a change in \( \rho_C \), ceteris paribus, has no effect on managerial behavior in the presence of an agency problem, note that the audit contingency condition (as given by Eq. (19)) is independent of \( \rho_C \). Furthermore, the manager’s problem as given by Eq. (25), (26), and (27) is also independent of \( \rho_C \).
Consequently, the condition which determines whether or not managers will reach for yield (Eq. (33)) is also independent of $\rho_C$ implying that, all other things being equal, a change in $\rho_C$ has no affect on managerial behavior in the presence of an agency problem. Q.E.D.

**Proof of Proposition 6**

An impatient investor $j$ will not run as long as the cost of borrowing, $\rho^j_B$, is lower than the expected opportunity cost of running which is given by the expected payoff forgone at $t = 2$ as a result of investor $j$ liquidating his portfolio at $t = 1$. Hence, an impatient investor $j$ will not run if and only if

$$\rho^j_B < \theta \rho_I + (1 - \theta) \frac{\rho_C E \left[ \max (C - \hat{x} I, 0) \right]}{(1 - E(\hat{x})) I},$$

(41)

where the RHS of the above inequality denotes the expected opportunity cost of running. Then for any given monetary policy, $\rho_C$, an impatient investor will be indifferent between running and not running when

$$\rho^j_B \left( r^j_* \mid \rho_C \right) = \theta \rho_I + (1 - \theta) \frac{\rho_C E \left[ \max (C - \hat{x} I, 0) \right]}{(1 - E(\hat{x})) I},$$

(42)

where $r^j_*$ denotes the threshold risk level such that impatient investors with a risk level above $r^j_*$ run while the others borrow.

We can then rank our investors according to their risk profile such that $I(\bar{r}^j) > I(r^j) > I(\tilde{r}^j)$ where $I(r^j)$ denotes an investor with risk profile $r^j$. Let $\bar{r}$ represent the maximum risk level of an investor. Then the expected number of investors who run is given by

$$E(\hat{x}) \sum_{r^j = \bar{r}_*}^\bar{r} I(r^j).$$

(43)

Since $\rho^j_B$ is increasing in both $r^j$ and $\rho_C$ it follows from Eq. (42) that $\partial r^j_*/\partial \rho_C < 0$, i.e. a monetary tightening decreases the threshold risk profile at which an investor is indifferent between running and borrowing, while a monetary loosening increases the threshold risk profile at which an investor is indifferent between running and borrowing. It follows that $E(\hat{x}) \sum_{r^j = \bar{r}_*}^\bar{r} I(r^j)$ is increasing in $\rho_C$.

Q.E.D.

**Proof of Proposition 7**

From Proposition 6 we know that the expected number of investors who run is lower in a loose monetary policy regime relative to that in a tight monetary policy regime. Also, in a loose monetary policy regime liquidity is more readily available at lower rates. It follows that in a loose monetary policy regime the expected cost associated with liquidity shortfalls, $E \left[ \Psi \mid e = e^H \right]$, is lower relative to that in a tight monetary policy regime. Furthermore, from the proof of Proposition 4 we know that $\Delta \Pi_m$ is increasing in $I$. Thus, in order to prove
Proposition 7 it would suffice to show that $\Delta \Pi_m$ as given by Eq. (33) is decreasing in $E[\Psi | e = e^H]$ for any given $I$. For brevity, we can rewrite $\Delta \Pi_m$ as follows

$$\Delta \Pi_m = A - B$$  \hspace{1cm} (44)

where

$$A = \int_{L_R} \int x \left( b(L_R^a) + b_M - \tilde{\psi}|e = e^H \right) f(x)g(L_R|e = e^H) \, dx \, dL_R$$  \hspace{1cm} (45)

and

$$B = -\int_{L_R} x \left( b(L_R^a) + b_M|e = e^H \right) g(L_R|e = e^H) \, dL_R$$  \hspace{1cm} (46)

First, we need to show that $A$ is decreasing in $E[\Psi | e = e^H]$. We know that $\tilde{\psi} = \min (\tilde{\psi}, \gamma \Psi)$. It follows that $\partial \tilde{\psi} / \partial \Psi > 0$. Also, from Proposition 2 we know that $b_M$ is constant. Since $v'(\cdot) > 0$, in order to prove that $A$ is decreasing in $E[\Psi | e = e^H]$ we need to show that $\partial b(L_R^a) / \partial \Psi < 0$. Note that

$$\frac{\partial b(L_R^a)}{\partial \Psi} = \frac{\partial b}{\partial L_R^a} \frac{\partial L_R^a}{\partial \Psi}$$  \hspace{1cm} (47)

where $\partial L_R^a / \partial \Psi$ is given by

$$\frac{\partial L_R^a}{\partial \Psi} = \frac{\partial L_R^a}{\partial \rho_R^a} \frac{\partial \rho_R^a}{\partial \Psi} < 0$$  \hspace{1cm} (48)

The above inequality holds because $\partial L_R^a / \partial \rho_R^a < 0$ and $\partial \rho_R^a / \partial \Psi > 0$ (i.e. an increase in the expected penalty cost is reflected in a higher lending rate and vice versa). Since $\partial b / \partial L_R^a > 0$, it follows that $\partial b(L_R^a) / \partial \Psi < 0$.

Next we need to ensure that a change in $A$ associated with a change in $E[\Psi | e = e^H]$ is not outweighed by a change in $B$ associated with a change in $E[\Psi | e = e^H]$ so that overall $\partial \Delta \Pi_m / \partial E[\Psi | e = e^H] < 0$. Since $b_M$ is constant this condition will be satisfied as long as $\left| \frac{\partial b(L_R^a)}{\partial E[\Psi | e = e^H]} \right| > \left| \frac{\partial b(L_R^a)}{\partial E[\Psi | e = e^H]} \right|$. To show that this condition is also satisfied note from Eq. (9) and Eq. (10) that $\partial \rho_L / \partial E[\Psi | e = e^H] > 0$ and thus $\partial C / \partial E[\Psi | e = e^H] > 0$ (i.e. a decrease in expected penalty cost decreases the loan rates thereby increasing risky investments and hence ex ante fewer funds are held in cash reserves). In Proposition 4 we showed that in the presence of an agency problem the minimum possible amount (just enough to satisfy any regulatory constraints) is invested in the medium-risk asset and the rest of the available investment funds are allocated to risky projects. In other words, there is overinvestment in the risky asset in the presence of an agency problem. Hence the result from Proposition 4 implies that $\left| \frac{\partial L_R^a}{\partial E[\Psi | e = e^H]} \right| > \left| \frac{\partial L_R^a}{\partial E[\Psi | e = e^H]} \right|$, which in turn implies that $\left| \frac{\partial b(L_R^a)}{\partial E[\Psi | e = e^H]} \right| > \left| \frac{\partial b(L_R^a)}{\partial E[\Psi | e = e^H]} \right|$ since $b (L_R) > 0$. Thus, $\partial \Delta \Pi_m / \partial E[\Psi | e = e^H] < 0$.  


In summary, from the proof of Proposition 4 we know that $\Delta \Pi_m$ is increasing in $I$. Also Proposition 6 implies that $E[\Psi | e = e^H]$ is lower in a loose monetary policy regime relative to a tight monetary policy regime. Finally, in conjunction with $\partial \Delta \Pi_m / \partial E[\Psi | e = e^H] < 0$ it follows that $I^*_L < I^*_T$. Q.E.D.