Bank Capital and Dividend Externalities*

Viral V. Acharya, Hanh Le, and Hyun Song Shin

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Abstract

We present a model in which by paying out dividends or remaining undercapitalized, a bank transfers value to its shareholders away from creditors, among whom are other banks. This way, one bank’s payout and recapitalization policy affects the equity value and risk of default of other banks. When such externalities are strong and bank franchise values are not too low, the private equilibrium can feature excessive dividends and inefficient recapitalization relative to a coordinated policy that maximizes the combined equity value of banks. We relate the model’s implication to observed bank behaviour during the crisis of 2007-09.

*Viral Acharya (vacharya@stern.nyu.edu) is with New York University, Hanh Le (hanhle@uic.edu) is with the University of Illinois at Chicago, Hyun Song Shin (hsshin@princeton.edu) is with the Bank of International Settlements and Princeton University. We are grateful to David Hirshleifer and Philip Strahan (the editors) and two anonymous referees for comments and suggestions that substantially improved our paper. We thank participants at the 2014 Western Finance Association Meetings and the 2014 Financial Intermediation Research Society Meetings for helpful discussions and Aaditya Iyer and Kate Waldock for excellent research assistance.
1 Introduction

As the financial system’s capital was being depleted, many banks continued to pay dividends well into the depth of the 2007-2009 financial crisis. Large bank holding companies such as Bank of America, Citigroup and JP Morgan maintained a smooth dividend behavior, while securities companies such as Lehman Brothers and Merrill Lynch even increased their dividends as losses were accumulating. This behavior represents a type of risk-shifting that favors equity holders over debt holders (as in the seminal work of Jensen and Meckling, 1976).

We present a simple model where, in the presence of risky debt, risk shifting incentives can motivate the payment of dividends as observed during the crisis. In our model, the bank has assets in place that generate some cash flow in the current period and some uncertain cash flow in the next period. At any given point in time, the bank has a franchise value (say, the present value of all its future cash flows) that is largely determined by the relationship with its customers and counterparties. The bank can pay dividends out of the current cash flow, and carry the remaining cash to the next period. The bank has to fulfill non-negotiable debt obligations in the next period out of the next period’s cash flow and cash savings from the current period. If debt obligations are not satisfied, the bank is in default. In this case, equity holders receive no value from the next period cash flow, and furthermore, the bank’s franchise value is lost, for example, through a disorderly liquidation or transfer to another bank or government.

Given this setup, the optimal policy that maximizes the total equity and debt value of the bank is to pay zero dividends. This is because paying dividends will decrease the
total value of the bank by increasing the probability that its franchise value will be lost. However, we show that in practice when the dividend policy of a bank is set to maximize only its equity holders’ value, the observed dividend policy reflects a tradeoff between (i) paying out to equity holders the available cash today rather than transferring it to creditors in default states in the future; and (ii) saving the equity’s option on the franchise value since dividend payout raises the likelihood of default and thus foregoing this value. When debt is risky (i.e., when bank leverage is sufficiently high), the optimal dividend policy depends on the bank’s franchise value. If the franchise value exceeds a critical threshold, the effect in (ii) dominates and it becomes optimal for the bank not to pay any dividends. However, if the franchise value is below the critical threshold such that risk-shifting benefits in (i) become dominant, then the bank would pay out all available cash as dividends.

The key question is why banks do not find ex-ante mechanisms to guard against such ex-post agency problems, e.g., by including dividend cutoff or earnings retention covenants in bank debt contracts. To understand this, we next introduce financial contracts between two banks, $A$ and $B$, and analyze how the dividend policy of one bank creates externalities on the other bank. The connection between the two banks takes the form of a contingent contract, such as an interest rate or a credit default swap, under which, in the next period, bank $A$ has to pay bank $B$ an amount in one state of the world and vice versa in the other state. We show that bank $B$’s dividends increase the probability that $B$ will default on its debt obligations to bank $A$, thereby exerting negative externalities on bank $A$’s equity value. In this setup, we show the
complementarity of dividend policies of the two banks. Specifically, when bank $B$ pays out all available cash as dividends to its equity holders, bank $A$ is more likely to pay out maximum cash flows since the threshold franchise value under which bank $A$ will pay maximum dividend is higher when $B$ pays maximum dividends.

Based on this result, we characterize the Nash equilibrium as a two-by-two game that at high franchise values features no dividend payouts, at low franchise values full dividend payouts, and for intermediate franchise values multiple equilibria. In this last case, payoff spillover in the banks’ dividend policies – the fact that dividend payments by one bank increases the incentives of the other bank to pay out dividends – interferes with the Nash responses. And as a result, the equilibrium featuring zero dividends by both banks yields the same private benefits as one featuring maximum dividends. We refine these multiple equilibria using global game techniques. Specifically, we assume each bank receives a noisy signal of the other bank’s franchise value and define a unique switching point based on this signal above which both banks pay zero dividends, and below which both banks pay maximum dividends.1

We next characterize the dividend policy that is coordinated and maximizes the joint equity value of the two banks, where each bank’s dividend externalities are internalized. We show that when externalities are big relative to the private benefits of paying dividends (i.e., when banks’ franchise values are not too low), the Nash equilibrium features excessive dividends relative to this policy. Similar to the debt overhang problem described by Myers (1977), banks do not have an incentive to curb excessive dividends.

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1The solution method does not rely on the iterative dominance argument of Morris and Shin (1998, 2003), but focuses on showing uniqueness of equilibrium, as first examined by Goldstein and Pauzner (2005).
dividends, as the benefits accrue only to their creditors who in turn are their inter-connected entities. The interesting features unique to the two-bank model are that (i) interconnectedness can give rise to strategic complementarities in dividend decisions; and (ii) the socially optimal outcome can be obtained purely by coordination among shareholders of interconnected banks. Our model applies generally to all firms in financial distress, but is more relevant in the context of bank holding companies, especially broker-dealer firms, as they have high levels of leverage and interconnectedness.

The novelty of our paper is in modeling this interaction of two agency problems - an agency friction between interconnected banks superimposed over an agency friction between the banks and their outside creditors. While risk-shifting under the one-bank model can be efficiently priced or contracted away (using covenants, e.g.) by individual bank creditors, restricting dividends is desirable in the case of interconnected banks over and above the shareholder-creditor conflict. In particular, cutting dividends can make even equity claims of both banks more valuable, but this is an externality that they do not internalize. While outside creditors are also hurt by this agency friction between interconnected banks, they can limit such risk-shifting by only using exclusive contracts (e.g., covenants that are tied not just to the behavior of banks they lend to, but also of other banks).2

We extend our model to incorporate negative dividends, which we interpret as equity issuance. We show that the result on dividend excessiveness in our benchmark model

\footnote{Note, however, that the presence of explicit or implicit government guarantees can imply that even in the one-bank case, creditors may not have incentives to limit shareholder dividends. In effect, the shareholder-creditor conflict in the presence of government guarantee transforms into a conflict between the taxpayer and the bank as a whole.}
generalizes to under-capitalization of banks in this extension. By issuing equity, a bank increases the value of claims of its creditors, among whom are shareholders of its interconnected banks. Banks pay the cost of equity issuance but do not internalize this positive externality, in turn not issuing the socially optimal level of equity.

The rest of the paper is structured as follows. Section 2 summarizes the related literature. Section 3 presents the striking patterns of financial firms’ dividend policies during the 2007-2009 financial crisis. Sections 4 and 5 present the theoretical analysis on banks’ dividend policies. Section 6 extends the benchmark model to take into account equity issuance decisions. Section 7 discusses the ex-ante contracting problem, how our model is related to the dividend and equity issuance policies of banks during the crisis of 2007-09, and draws policy conclusions. Section 8 concludes.

2 Related Literature

Our paper is related to at least three strands of the literature. The first strand is the large literature on corporate dividend policies. Allen and Michaely (2003), Frankfurter and Wood (2006), Baker (2009), and DeAngelo, DeAngelo and Skinner (2009) provide excellent summaries of this literature. Existing theories propose two main reasons why firms pay dividends: (1) to resolve agency conflicts between managers (the agent) and shareholders (the principal); and (2) to signal firms’ quality in the presence of asymmetric information. (2) offers an alternative explanation for why banks might have been reluctant to cut dividends well into the financial crisis - to signal their quality during a time of uncertainty. However, the fact that some banks increased
their dividend payments appears more consistent with a risk-shifting explanation, as proposed in our paper.

The second strand of related literature studies suboptimal dividend and capital policies as a result of shareholder-debtholder conflict of interests.\(^3\) Black (1976) and Smith and Warner (1979) were the first in this literature that focuses on dividends. Black (1976) points out an extreme example of this conflict, saying “there is no easier way for a company to escape the burden of a debt than to pay out all of its assets in the form of a dividend, and leave the creditors holding an empty shell”. Similar to our one-bank model, Fan and Sundaresan (2000) and DeMarzo and Fishman (2007) analyze the trade-off between paying out dividends and foregoing of the firm’s continuation value. In Fan and Sundaresan (2007), when cash-flow based covenants are in place, not all cash is paid out as dividends, as doing so might violate these covenants, resulting in a loss of the firm’s continuation value. De Marzo and Fishman (2007) study this payoff in the context of optimal security design, where the firm’s agent makes the dividend decision based on the presence of an outstanding risky debt and a line of credit. As the agent’s continuation value is decreasing in the probability of defaulting on debt obligations, he has no incentive to divert cash to shareholders before their debt is serviced.

Shareholders’ aversion to raising capital in our model can be related to the debt overhang problem in Myers (1977). As in Myers (1977), shareholders dislike raising equity as doing so would benefit creditors at their own expense. Admati et al. (2015)

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\(^3\)This strand of literature belongs to the broader literature on risk-shifting (see, for example, Jensen and Meckling (1976), and Galai and Masulis (1976)): the same debtholder-shareholder tension that can affect payout and capital policies as in our paper can also lead to substitution from safer to riskier assets. However, due to lack of detailed asset-level data, asset substitution is difficult to detect as a manifestation of risk-shifting.
show that conflict of interest between the borrower and the creditor can lead to inefficient recapitalization, e.g. shareholders’ reluctance to reduce leverage via issuing equity. Our paper contributes to this second strand of literature by studying the debt-equity agency problem in the form of conflicts of interest across banks in an interconnected system, where one bank’s creditors are other banks’ equity holders. Our new insight is that coordination among bank equity holders can help preserve system-wide capital and stability.

Finally, our paper is related to studies examining suboptimal equilibria arising from externalities in a financial network setting. Bhattacharya and Gale (1987) argue that banks’ ability to borrow from each other creates a moral hazard problem where banks free ride on liquidity and under-invest in liquid assets. Unlike their model where the inefficiency arises because other banks are likely to honor a bank’s borrowing requests, in our model it results from a bank’s failure to honor its debt obligation. As a result, our model delivers the opposite result. In Bhattacharya and Gale (1987), the greater the credibility of payments on interbank claims, the stronger the incentive to free ride and the stronger the moral hazard of insurance provided by these claims. Our model, on the other hand, suggests that the more credible interbank claims, the lower the incentive to shift risks.

Other related papers in this last strand of literature include Zawadowski (2013), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), and Admati et al. (2013). In Zawadowski (2013), individual banks do not have incentives to purchase insurance on counterparty default due to costs of equity and the low probability of counterparty
default. This underinsurance problem is exacerbated by the fact that banks ignore the externalities their own failures impose on their counterparties, the counterparties of their counterparties, and so on. Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) argue that the financial network that arises in equilibrium is inefficient due to the fact that banks only internalize the externalities of their risk taking actions on their immediate counterparties but not on the rest of the network. While these papers focus on the decision to purchase counterparty default insurance (Zawadowski, 2013) and endogenous network formation (Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015), we focus on the dividend and equity issuance decisions, forms of risk-shifting that have not been analyzed in the financial network literature.

The closest paper to ours is Admati et al. (2013), which points out (but does not formally model) that banks do not internalize the negative externalities their distress impose on the financial system (and consequently tax payers’ money) and thus choose socially excessive leverage. We explicitly model bank interconnectedness and show that such externalities can also be costly from the point of view of the banks’ shareholders. Unlike in their model where issuing equity is never desirable by shareholders, it is so under our model when the incentive to preserve the franchise value is strong, i.e, when the franchise value is sufficiently large. In turn, dividend payouts and under-capitalization arise when bank franchise values are sufficiently eroded.
3 Dividend Payments During 2007-2009

To illustrate dividend patterns over the 2007-2009 financial crisis, we focus on a group of ten US banks and securities firms consisting of the largest commercial banks and the largest securities firms, some of which were to converted to bank holding companies in September 2008. The ten firms are Bank of America, Citigroup, JP Morgan Chase, Wells Fargo, Wachovia, Washington Mutual, Goldman Sachs, Morgan Stanley, Merrill Lynch and Lehman Brothers. We do not include Bear Stearns, as it has a relatively short run of data due to its takeover by JP Morgan in March 2008.

(Figure 1a) shows the cumulative losses of the ten firms beginning in 2007Q3, where losses in each quarter are indicated by the respective segment in the bar chart. The losses are raw numbers, not normalized by size. We can see the comparatively large size of Wachovia’s losses, relative to even the large losses suffered by Citigroup.

(Figure 1b) shows the cumulative dividends of the ten banks over the two-year period from 2007Q1 to 2008Q4. The first striking feature is that dividend payments by these banks and securities firms continued well into the depth of the crisis in 2008.\footnote{Similarly, Shin (2016a) and Shin (2016b) note the significant dividend payout by banks in a sample of Euro and advanced economies over the 2007-2014 period. Shin (2016a) and Shin (2016b) estimate that retained earnings of these banks would have been at least about 50% higher in 2014, “had the banks chosen to plough back the profits into their own funds rather than paying them out as dividends”.

The bars associated with the large commercial bank holding companies such as Bank of America, Citigroup and JP Morgan Chase show evenly spaced segments corresponding to the respective quarter, indicating that these banks maintained a smooth dividend payment schedule in spite of the crisis. For some other firms, such as Merrill Lynch, the dividend payments increased in the latter half of 2008.
Noticeably, Lehman Brothers and Merrill Lynch increased their dividend payments in 2008\(^5\), while Wachovia and Washington Mutual decreased their dividends drastically in the third quarter of 2008. All four of these firms share the common feature that they either failed outright or were taken over in anticipation of financial distress.

The dividend behavior of these four institutions can be better seen in (Figure 1c), which charts dividend payments of the ten banks where the amounts are normalized so that the dividend payment in 2007Q1 is set equal to 1.

Dividend payments of the ten banks did not change much during 2007, but then diverged sharply during 2008. There are four outliers both on the high side and the low side. On the high side are the two securities firms, Lehman Brothers and Merrill Lynch, which reached levels of dividend payments that are double that of 2007Q1. As is well known, Lehman Brothers filed for bankruptcy on September 15th 2008, while Merrill Lynch agreed to be taken over by Bank of America shortly before that.

The two outliers on the low side are Washington Mutual and Wachovia, which reduced their dividend payments drastically in 2008Q2 and 2008Q3, respectively. Wachovia agreed to be taken over by Wells Fargo in October 2008, while Washing Mutual was seized by its regulator, the Office of Thrift Supervision in September and placed in receivership of the FDIC.

Another way to present a bank’s dividend behavior is to normalize its dividend payment by its book value of equity. Figure (Figure 1d) plots this ratio for the ten banks in our sample. All ratios are normalized such that the ratio in Quarter 1, 2007

is set equal to 1.

The divergent dividend behavior of the four outliers is further highlighted in Figure 2 where cumulative losses for each bank are plotted alongside its quarterly dividend payments. Again, what is striking is the contrast between the two (former) brokers (Lehman Brothers and Merrill Lynch) and the two commercial banks (Washington Mutual and Wachovia). The first two charts show Lehman Brothers’ and Merrill Lynch’s increased dividend payments despite growing losses. In contrast, charts for Washington Mutual and Wachovia show the two curves sloping in opposite directions indicating that dividends were being curtailed as the financial crisis gathered pace.

While dividend payments were eroding capital, banks did not seem to recapitalize timely and adequately. Figure 3 shows the total amount of capital raised to combat losses by the ten banks from 2007Q3 to 2008Q4. Two observations are worthy of note. First, for most banks, most of the capital raised was in the form of preferred stocks and debt instead of common equity. Second, while all commercial banks had positive net issuance of common stocks, three out of the four investment banks, namely Lehman Brothers, Goldman Sachs and Morgan Stanley had negative net common stock issuance, indicating that total repurchases of common stocks exceeded total issuance over the said period. An exception is Merrill Lynch, whose common stock issuance was large relative to the total amount of capital raised.

The important question is whether the mix of capital raised sufficiently restored banks’ common equity base and kept leverage from rising in the presence of mounting
losses. Figures 4 and 5 reveal that the answer is in the negative. Figure 4 plots book leverage ratios for the ten banks in our sample from 2007Q1 to 2008Q4. The plot shows that relative to commercial banks and bank holding companies, investment banks entered the financial crisis with extremely thin capital buffers. As the crisis progressed, banks’ leverage rose steadily until later quarters when book leverage ratios for some banks started to go down (Lehman Brothers, Washington Mutual, Morgan Stanley and Goldman Sachs). Strikingly, despite the amount of common stock issuance, Merrill Lynch’s leverage rose from 26.5 in 2007Q1 to 45.77 in 2008Q2, only to fall down to 29.44 in 2008Q3 before peaking at 58.57 in 2008Q4.

Book leverage ratios are based on accounting numbers which do not reflect market expectations of future losses. Therefore, in Figure 5 we present banks’ quasi-market leverage ratios. The figure shows a remarkable increase in quasi-market leverage over time for all the ten banks, reflecting their failure to recapitalize.

Why did banks continue to pay dividends during the crisis, even when losses were accumulating and despite failure to recapitalize? And, what determines the difference in dividend payments among different banks? The following section presents an ex-post risk-shifting model that might provide answers to these questions. We argue that when leverage is high enough that value transfers from debt holders to equity holders become substantial, banks have an incentive to pay dividends if their franchise

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7Quasi-market leverage is defined as quasi-market value of assets over market value of common equity. Quasi-market value of assets is defined as book value of assets minus book value of common equity plus market value of common equity, where market value of common equity is number of common shares outstanding times price per share.

8Note that both measures of leverage may understate the true leverage over this time period due to questionable accounting maneuvers. One salient example is Lehman Brothers’ use of Repo 105 to artificially move $50 billion in liabilities off the balance sheet in 2008Q2.
values are sufficiently low. The divergent dividend behavior of securities firms and commercial banks can also be explained by our model. More specifically, securities firms’ franchise values appear to have been hit harder by the crisis, given that these values are largely made up of flight-prone client relationships as opposed to the more illiquid loans in the case of commercial banks. According to our model, this might have led to a higher probability of risk-shifting via dividends in securities firms than in commercial banks. The section then analyzes the implication risk-shifting via dividends has on interconnected banks such as broker-dealers, providing yet another rationale for such firms’ greater incentives to pay dividends: interconnected firms ignore the negative externalities of dividend payouts on each others’ franchise values. We then characterize under what circumstances individually chosen dividend policies are excessive relative to coordinated policies and hence regulation restricting dividends can improve outcomes.

4 Single Bank Model

We first lay out a model of dividend policy for a single bank. Then, we introduce a second bank that is financially linked to the first bank and study the externalities in their dividend policies. The model relies partly on the structure in Acharya, Davydenko and Streubulaev (2012).

There are two dates - date 0 and date 1. Consider a bank at date 0 with cash assets of \( c > 0 \) and non-cash assets \( y \) (such as loans and securities) that are due at date 1 and take realizations in the interval \([y, \bar{y}]\) with density \( h(y)\), where \( 0 < y < \bar{y} \).

The bank finances the assets with liabilities \( \ell \) that are due at \( t = 1 \). Assume that
\( \ell \in (y, \hat{y}) \), so that the probability of bank default is non-zero but strictly below 1. There is no possibility of renegotiating this debt in the case of default and the bank cannot issue capital at \( t = 0 \) or \( t = 1 \) against its future value. In other words, the debt contract is hard and the payment of \( \ell \) must be met at \( t = 1 \) using the bank’s cash savings and realized value of assets. The book option value of equity of the bank (BE) at date 0 is defined as:

\[
BE = \mathbb{E} (\max \{0, y - \hat{y}\})
\]  

(1)

where \( \hat{y} \) the threshold value of asset realization when the bank just meets its liabilities \( \ell \). In other words, \( \hat{y} \) satisfies \( c + \hat{y} = \ell \). The book option value of equity is the fair value of the call option on the bank’s portfolio.

An alternative notion of equity for the bank is its market capitalization, or market equity, which reflects the price of its shares. Market equity and the book option value of equity will diverge since market equity reflects the discounted value of future cash flows, as well as the snapshot of the bank’s portfolio. We assume that if the bank survives after date 1, the expected value of its future profit is given by \( V > 0 \). The franchise value \( V \) depends on the market-implied discount rates for future cash flows, as well as expected future cash flows themselves.

Incorporating the franchise value, the market equity of the bank is given by

\[
ME = \mathbb{E} (\max \{0, y - \hat{y}\}) + \mathbb{P}(y \geq \hat{y}) \cdot V
\]  

(2)

where \( \mathbb{P}(y \geq \hat{y}) \) is the probability of bank solvency.

Our focus will be on the bank’s dividend policy at date 0. The bank can pay a dividend \( d \), up to its starting cash balance of \( c \). As a benchmark, consider the first best
dividend - the one that maximizes the total value of the bank (the value of debt plus the value of equity). Denote by \( \hat{y}(d) \) the default threshold of the bank’s non-cash assets when the bank has paid dividend of \( d \). In other words, \( \hat{y} \) is the solvency threshold of \( y \):

\[
\hat{y}(d) \equiv \ell + d - c
\]  

(3)

The bank is solvent at date 1 if and only if \( y \geq \hat{y} \). The bank’s total value consisting of the value of claims of all stakeholders is the sum of dividends paid at date 0, expected assets, plus the expected franchise value:

\[
d + E(y + c - d) + \Pr(y \geq \hat{y}(d)) \cdot V
\]

(4)

The dividend \( d \) only affects (4) through the probability of solvency of the bank. Since the default threshold \( \hat{y} \) is increasing in \( d \), the second term in (4) is strictly decreasing in the dividend. Thus, as long as the bank has positive franchise value \( V \), the value-maximizing dividend policy is to pay none. The intuition for the first best policy is straightforward. In the absence of the bank’s franchise value, a dividend only affects the distribution of payoffs between equity holders and creditors and does not matter for the bank’s total value. However, when the bank has a positive franchise value, paying dividends reduces the bank’s expected franchise value.

Now consider the “second best” dividend policy, that maximizes the shareholder’s payoff. The shareholder’s payoff is given by the sum of the dividend \( d \) and the ex-dividend market value of equity. In other words, the shareholder’s payoff considered
as a function of $d$ is given by

\[
U(d) = d + E(y - \hat{y} + V | y \geq \hat{y}) \cdot Pr(y \geq \hat{y})
\]

\[
= d + E(y - \hat{y} | y \geq \hat{y}) \cdot Pr(y \geq \hat{y}) + Pr(y \geq \hat{y}) \cdot V
\]

(5)

We proceed to analyze the second best dividend policy, and contrast it with the first best. For algebraic tractability, we impose a parametric form on the density $h(.)$, and assume that $y$ is uniformly distributed over the interval $[\underline{y}, \bar{y}]$. Hence, $h(y) = 1/\bar{y} - y$. Then, (5) can be written as

\[
U(d) = d + \frac{(\bar{y} - \hat{y})^2}{2(\bar{y} - y)} + \frac{\bar{y} - \hat{y}}{\bar{y} - y} \cdot V
\]

\[
= d + \frac{(\bar{y} + c - \ell - d)^2}{2(\bar{y} - y)} + \frac{(\bar{y} + c - \ell - d)}{\bar{y} - y} \cdot V
\]

(6)

The shareholder chooses $d$ to maximize (6). The choice reflects the tradeoff between having one dollar of cash in hand today (the first term) versus the the ex dividend market equity of the bank (sum of second and third terms). The derivative $U'(d)$ thus gives the sensitivity of the cum-dividend share price of the bank with respect to the dividend $d$. Although $U(d)$ is a quadratic function of $d$, we see from (6) that $U(d)$ is a convex function of $d$ since the squared $d^2$ term enters with a positive sign. Hence, the first-order condition will not give us the optimum. Instead, given the convexity of the objective function, the optimal dividend policy will be a bang-bang solution, where either no dividends are paid or all cash is paid out in dividends. We summarize this feature in terms of the following Lemma.

**Lemma 1** The dividend policy of the bank that maximizes shareholder payoff is either maximum dividends $d = c$ or no dividends $d = 0$. 

Note that this bang-bang solution does not arise from the assumption of uniform cash flow distribution. Rather, it relies on the assumption that equity holders have an embedded option, and that the choice of dividends is analogous to choosing the strike price of this option. Because the option value is convex in its strike price, so long as the choice of dividends at date $t = 0$ does not affect this distribution in a continuous manner, a corner solution is obtained.

This result implies there are cases under which second best dividends are excessive relative to the first best. From now on we will focus on and refer to the second best dividend policy as the “optimal” dividend policy. To distinguish the second-best policy from the first best, we refer to the first-best dividend policy as the “socially optimal” dividend policy.

4.1 Franchise Value and Optimal Dividend

The fact that the bank either pays maximum or minimum dividends simplifies our analysis greatly, and we can focus on how the bank’s franchise value $V$ affects the bank’s dividend policy. Denote by $U(d, V)$ the shareholder’s payoff function (the cum-dividend price of shares) when dividends $d$ are paid and when the franchise value conditional on survival is $V$. From the bang-bang nature of the solution, we need only compare $U(0, V)$ and $U(c, V)$ in finding the optimal $d$. Define the payoff difference $W(V)$ as

$$W(V) \equiv U(0, V) - U(c, V)$$

(7)

$W(V)$ is the payoff advantage of paying zero dividends relative to paying maximum dividends, expressed as a function of the franchise value $V$. Then, the optimal dividend
policy as a function of the franchise value $V$ is given by

$$d(V) = \begin{cases} 
0 & \text{if } W(V) \geq 0 \\
c & \text{if } W(V) < 0 
\end{cases}$$

(8)

From our expression for $U$ in (6), we have

$$W(V) = U(0, V) - U(c, V)$$

$$= \frac{c^2 - 2c (\ell - y)}{2 (\bar{y} - y)} + \frac{c}{\bar{y} - y} \cdot V$$

(9)

which is an increasing linear function of $V$ with slope $c/(\bar{y} - y)$. Thus, there is a threshold $V^*$ of the franchise value such that the bank pays maximum dividends when $V < V^*$, but pays no dividends when $V \geq V^*$. The intuition is that when the franchise value is high, the value to the shareholders of remaining solvent is high, and the solvency probability can be raised by retaining cash rather than paying out cash as dividends.

This result is in line with models of Keeley (1999), Fan and Sundaresan (2000), and DeMarzo and Fishman (2007), where a high continuation value deters the transfer of value to shareholders.\(^9\)

The threshold value $V^*$ solves $W(V^*) = 0$. From (9), we have

$$V^* = \ell - \frac{c}{2} - \bar{y}$$

(10)

We summarize our result as follows.

**Proposition 1** For $V^* = \ell - \frac{c}{2} - \bar{y}$, the optimal dividend policy is given by

$$d(V) = \begin{cases} 
0 & \text{if } V \geq V^* \\
c & \text{if } V < V^* 
\end{cases}$$

(11)

\(^9\)DeMarzo and Fishman (2007) and Fan and Sundaresan (2000) analyze the role of franchise values in the debtholder - shareholder agency problem as in our model, while Keeley (1990) focuses on bank risk taking under mispriced deposit insurance.
Risk-shifting via dividends is more pronounced in banks with high liabilities $\ell$. For two banks with the same book option value of equity, it is therefore the more highly leveraged bank that is more likely to engage in risk-shifting. In addition, risk-shifting is more likely to happen in bad times than good ($\frac{\partial V^*}{\partial c} < 0$ and $\frac{\partial V^*}{\partial y} < 0$).

In the next section, we show that in an interconnected system where banks are contingent debtors of one another, paying dividends creates negative externalities on bank franchise values that are not internalized by the dividend paying bank. This results in a suboptimal decentralized equilibrium in which banks pay out excessive dividends.

5 Two Bank Model

We now turn to the main model of our paper, where there are interconnected banks. We consider two banks linked in a simple way through an over-the-counter swap that depending on the state of the world, will make one bank a creditor of the other.

We denote the two banks as $A$ and $B$. The set-up for each bank is identical to the one above for an isolated bank but we denote the parameters for each bank by means of the subscripts $\{a,b\}$ for banks $A$ and $B$. Thus each bank $i$ is characterized by $(c_i, d_i, \ell_i, y_i, V_i, h_i)$, where $i \in \{a,b\}$. For notational economy, we consider the symmetric case where the support for $t = 1$ realizations of non-cash assets is the same for both banks, and given by $[y, \bar{y}]$.

The two banks have a hard financial contract linking them, that generates a claim and corresponding obligation at $t = 1$. Whether a bank has a claim on, or link to, another bank depends on a state of the world whose realization is independent of the
realization of the non-cash assets of the two banks. Furthermore, we assume that the non-cash asset realizations of the two banks are independent.

In state \( A \), bank \( A \) owes bank \( B \) an amount \( s_a > 0 \); in state \( B \), bank \( B \) owes bank \( A \) an amount \( s_b > 0 \). States \( A \) and \( B \) have probabilities \( p \) and \( (1 - p) \), respectively. There is no other linkage between the banks. We analyze state \( A \) and state \( B \) in terms of possible outcomes for the two banks and then compute market equity values at \( t = 0 \).

In state \( A \), bank \( A \)'s total debt is \( \ell_a + s_a \). Thus, it can avoid default only if \( y_a + c_a - d_a > \ell_a + s_a \). Therefore, its default threshold is given by:

\[
\hat{y}_a \equiv \ell_a + s_a + d_a - c_a \quad (12)
\]

The default point for bank \( B \) in state \( B \) is determined analogously. As before, we assume that default points for both banks lie within the support of their non-cash asset realization, i.e., \( y < \ell_i + s_i - c_i < \ell_i + s_i < \bar{y} \).

The new element in the two-bank case is that the default point of bank \( A \) in state \( B \) depends on the possibility of default by bank \( B \) on its financial contract with bank \( A \). To see this, consider state \( B \) from the standpoint of bank \( A \).

If \( y_b > \hat{y}_b \), then \( B \) makes the full payment of \( s_b \) to bank \( A \), whose cash flow is now given by \( y_a + s_b \). Hence, \( A \)'s default threshold in this case is given by:

\[
\hat{y}_a^{ND} \equiv \ell_a + d_a - c_a - s_b \quad (13)
\]

where the superscript “\( ND \)” indicates the default point for bank \( A \) when bank \( B \) does not default on its obligations.

However, if \( y_b < \hat{y}_b \), then \( B \) defaults. We assume that in this case, \( A \)'s financial
claim ranks *pari passu* with the other outstanding debt of bank $B$. Thus, $A$ recovers the pro-rata share of its claim from $B$’s remaining assets amounting to

$$s_b^D \equiv \frac{s_b}{\ell_b + s_b} \cdot (y_b + c_b - d_b)$$

Then, $A$’s default point is now higher than in the case of no default by $B$, given by:

$$\hat{y}_a^D \equiv \ell_a + d_a - c_a - s_b^D$$

where the superscript “$D$” indicates that the default point of bank $A$ when bank $B$ defaults on its contract.

The distinction between $\hat{y}_a^{\text{ND}}$ and $\hat{y}_a^D$ makes it clear that in some states of the world when $B$ owes $A$ but $B$’s cash flow realization is poor, $A$’s default likelihood goes up. As such, $A$’s default likelihood is increasing not just in its own dividends but also in dividends of $B$ since the more $B$ has paid out in dividends, the less it has available to pay $A$ as its creditor. This dependence in payoffs generates a spillover effect of dividend policy that ties together the interests of the banks. We examine this interaction of dividends and default likelihoods of the two banks and study its implications for their optimal dividend policies.

Consider the payoff of bank $A$’s shareholders at $t = 0$. This payoff is the cum-dividend share price of bank $A$, which is given by the sum of four terms:

$$U_a(d_a, d_b, V_a) = d_a + p \int_{\hat{y}_a(d_a)}^{\hat{y}} [y_a - \hat{y}_a(d_a) + V_a] h_a(y_a) dy_a$$

$$+ (1 - p) \int_{\hat{y}_a(d_a)}^{\hat{y}} \int_{\hat{y}_a^{\text{ND}}(d_a)}^{\hat{y}} [y_a - \hat{y}_a^{\text{ND}}(d_a) + V_a] h_a(y_a) dy_a h_b(y_b) dy_b$$

$$+ (1 - p) \int_{\hat{y}_b(d_b)}^{\hat{y}} \int_{\hat{y}_a(d_a, d_b)}^{\hat{y}} [y_a - \hat{y}_a^D(d_a, d_b) + V_a] h_a(y_a) dy_a h_b(y_b) dy_b$$

$$+ (1 - p) \int_{\hat{y}_b(d_b)}^{\hat{y}} \int_{\hat{y}_a(d_a, d_b)}^{\hat{y}} [y_a - \hat{y}_a^D(d_a, d_b) + V_a] h_a(y_a) dy_a h_b(y_b) dy_b$$

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Each of the four terms has a simple interpretation:

- The first term, $d_a$, is the dividend paid out at $t = 0$ to A’s equityholders.
- The second term captures the payoff in state $A$ when $A$’s cash flow is sufficiently high to pay all of its creditors including $B$.
- The third term captures the outcome in state $B$ when $B$ does not default on $A$ and $A$’s cash flow is high enough to avoid default.
- Finally, the fourth term captures the outcome in state $B$ when $B$ defaults on $A$ and yet $A$’s cash flow is high enough to avoid default.

Note that the payoff function for $A$’s shareholders is written explicitly as a function of both dividends $(d_a, d_b)$, thereby stressing the dependence of the default thresholds on the two dividend policies. The fourth term is the key to understanding the interaction between the two dividend policies. In Appendix A, we show that $\frac{\partial^2 U_a(d_a, d_b)}{\partial d_a^2} > 0$ so that the shareholder’s payoff is convex in the dividend, just as in the single bank case. Then, as with the single bank case, the optimal solution is a bang-bang solution of either no dividends or maximum dividends.

The negative externality of each bank’s dividend payout on the other bank can be characterized in terms of the partial derivative $\frac{\partial U_a}{\partial d_b}$:

$$
\frac{\partial U_a}{\partial d_b} = -(1 - p) \int_{\hat{y}_b}^{y_b} \left[ V_a h_a(\hat{y}_a^D) + \frac{s_b}{(\ell_b + s_b)} Pr[y_a > \hat{y}_a^D] h_b(y_b) dy_b \right]
$$

which is always negative and where we have used the fact that $\frac{\partial \hat{y}_a^D}{\partial d_b} = \frac{s_b}{\ell_b + s_b}$. Note that this result is not reliant on the assumption of cash flows having a uniform distribution.
The intuition is clear. An increase in dividend payout of bank $B$ reduces the cash it has available for servicing its debt, including that due to bank $A$ in state $B$. This increases the default risk of bank $A$, causing it to lose its franchise value more often and bank $A$’s equity holders also to lose their cash flow more often to creditors. We summarize this finding in terms of the following Lemma.

**Lemma 2 (Negative externality of dividend payout)** All else equal, an increase in dividend payout of bank $B$ lowers the equity value of bank $A$. Formally, $\partial U_A / \partial d_B < 0$ and $\partial U_B / \partial d_A < 0$.

In order to characterize the equilibrium dividend policies of the two banks, consider the payoff advantage to bank $A$ of paying zero dividend over paying the maximum dividend of $c_a$ as follows:

$$W_A (d_b, V) = U_A (0, d_b, V) - U_A (c_a, d_b, V) \quad (16)$$

Since $y$ is uniformly distributed over $[\underline{y}, \bar{y}]$, we can write $W_A (d_b, V)$ as:

$$W_A (d_b, V) = Z + \frac{c_a}{\bar{y} - \underline{y}} \cdot V_a \quad (17)$$

where

$$Z = -c_a + \frac{c_a}{\bar{y} - \underline{y}} \left\{ \frac{c_a}{2} + \bar{y} - \ell_a - s_ap \right. + s_b(1 - p) \left( 1 + \frac{\hat{y}_b - \underline{y}}{\bar{y} - \underline{y}} \left[ \frac{y + c_b - d_b + \ell_b + s_b}{2(\ell_b + s_b)} - 1 \right] \right\} \quad (18)$$

We note the close similarity in the functional form for $W_A (d_b, V)$ as compared to the single-bank case. Comparing (17) with (9), we note that in both cases, the payoff advantage to bank $A$ of paying zero dividends is an increasing linear function of $V_A$, with slope $c_a / (\bar{y} - \underline{y})$. Then, just as in the single-bank case, the optimal dividend policy of bank $A$ takes the form of a bang-bang solution where bank $A$ either pays zero...
dividends or pays out all its cash as dividends, depending on its franchise value $V_a$. Denote by $V_a^* (d_b)$ the value of $V_a$ that solves:

$$W_a (d_b, V) = 0 \quad (19)$$

Then, the optimal dividend of bank $A$ is given by

$$d_a (V_a) = \begin{cases} 
0 & \text{if } V_a \geq V_a^* (d_b) \\
c_a & \text{if } V_a < V_a^* (d_b)
\end{cases} \quad (20)$$

The form of the optimal dividend policy is similar to the single-bank case, but the new element is that the switching point $V_a^* (d_b)$ depends on the dividend policy of bank $B$. Given the bang-bang nature of the optimal dividends, we can restrict the action space of the banks to the pair of actions “pay no dividends” and “pay maximum dividends”, and the strategic interaction can be formalized as a $2 \times 2$ game parameterized by the franchise values of the banks. The payoffs for bank $A$ (choosing rows) can then be written as:

$$\begin{array}{ccc}
\text{Bank A} & \text{pay dividend} & \text{not pay dividend} \\
\text{pay dividend} & U_a (c_a, c_b, V_a) & U_a (c_a, 0, V_a) \\
\text{not pay dividend} & U_a (0, c_b, V_a) & U_a (0, 0, V_a)
\end{array} \quad (21)$$

There is an analogous payoff matrix for bank $B$. We first characterize the Nash equilibria associated with this $2 \times 2$ game, which we call uncoordinated dividend policies. We then show that these policies are excessive relative to the coordinated ones.
5.1 Nash Equilibria

Recall our notation $V_a^*(d_b)$ for the threshold value of $V_a$ that determines bank A’s optimal dividend policy. We noted that bank A’s optimal threshold depends on bank B’s dividend $d_b$. However, given the bang-bang solution for both banks’ dividend policies, we need only consider the extreme values for $d_b$, namely $d_b = 0$ and $d_b = c_b$.

The following preliminary result is important for our argument:

**Lemma 3** $V_a^*(0) < V_a^*(c_b)$ and $V_b^*(0) < V_b^*(c_a)$

In other words, the optimal threshold point for bank A’s dividend policy is lower when bank B is paying no dividends. Bank A refrains from paying dividends for a greater range of franchise values when bank B also refrains from paying dividends. In this sense, the two banks’ decisions to refrain from paying dividends are mutually reinforcing. The proof of this lemma is given in Appendix B. A direct corollary of the lemma is that we have multiple Nash equilibria when the franchise values $(V_a, V_b)$ of the two banks fall in an intermediate range.

**Proposition 2** Nash equilibrium dividend policies are given as follows.

1. When $V_a > V_a^*(0)$ and $V_b > V_b^*(0)$, the action pair $(d_a, d_b) = (0, 0)$ is a Nash equilibrium.

2. When $V_a > V_a^*(c_b)$ and $V_b < V_b^*(0)$, the action pair $(d_a, d_b) = (0, c_b)$ is a Nash equilibrium.

3. When $V_a < V_a^*(0)$ and $V_b > V_b^*(c_a)$, the action pair $(d_a, d_b) = (c_a, 0)$ is a Nash equilibrium.

4. When $V_a < V_a^*(c_b)$ and $V_b < V_b^*(c_a)$, the action pair $(d_a, d_b) = (c_a, c_b)$ is a Nash equilibrium.
Clauses 1 and 4 give rise to the cases of interest. The cases covered in 1 and 4 have a non-empty intersection, and so imply that we have multiple equilibria in the dividend policies of the banks. Whenever \((V_a, V_b)\) are such that

\[ V^*_a(0) < V_a < V^*_a(c_b) \quad \text{and} \quad V^*_b(0) < V_b < V^*_b(c_a) \]  

then both \((d_a, d_b) = (0, 0)\) and \((d_a, d_b) = (c_a, c_b)\) are Nash equilibria. The reason for the multiplicity arises from the payoff spillovers of the dividend policies of the two banks. The more dividends are paid out by one bank, the greater is the incentive of the other bank to pay out dividends. We present a global game refinement of this multiplicity in Appendix D.

Figure 6 characterizes the region of multiple equilibria as the box in the middle. We call these equilibrium outcomes of “uncoordinated dividend policies” to indicate that they are chosen as individual best responses to the other bank’s choice.

### 5.2 Excessive Dividends under Uncoordinated Policies

Given the negative externality to paying dividends, uncoordinated dividend policies can be excessive even relative to the policies that maximize the joint market equity values of the two banks. We noted earlier that when the interests of the creditor are taken into account, the dividends that maximize bank shareholders’ value are excessive relative to those that maximize the overall bank value. To show the excessive nature of dividends even for joint market equity value maximization, consider a dividend policy \((d_a, d_b)\) that maximizes the joint equity value of the two banks, \(U_a(d_a, d_b) + U_b(d_a, d_b)\).

We call these policies the coordinated ones. Then, we obtain that
Proposition 3 (Excessive Dividends) Uncoordinated dividend policies can be excessive compared to the coordinated one.

A proof and an illustration of this Proposition is provided in Appendix C.

6 Model with Equity Issuance

In this section, we extend our benchmark model to allow for negative dividends, which we interpret as equity issuance. The domain for \( d_i \) now becomes \((-\infty < d_i \leq c_i)\).

We also assume that whenever the bank issues equity \( (d_i \leq 0) \), it incurs a cost equal to \(-K_i d_i^2\). We show that the dividend excessiveness result from the benchmark model implies suboptimal capitalization by interconnected banks. In this setting, as the single-bank case yields the same insights on risk-shifting as those obtained from the multiple-bank case, we only present multiple-bank results. With this additional ingredient, bank \( A \)'s equity value becomes:

\[
U_a(d_a, d_b, V_a) = d_a - \frac{1}{2} K_a d_a^2 I_{d_a \leq 0} + p \int_{y_a(d_a)}^{\hat{y}_a} [y_a - \hat{y}_a(d_a) + V_a] h_a(y_a) dy_a
\]

\[
+ (1-p) \int_{y_b(d_b)}^{\hat{y}_b} \left[ \int_{\hat{y}_a(d_a)}^{\hat{y}_a} [y_a - \hat{y}_a^{ND}(d_a) + V_a] h_a(y_a) dy_a \right] h_b(y_b) dy_b
\]

\[
+ (1-p) \int_{y_b(d_b)}^{\hat{y}_b} \left[ \int_{\hat{y}_a(d_a, d_b)}^{\hat{y}_a} [y_a - \hat{y}_a^{PD}(d_a, d_b) + V_a] h_a(y_a, d_a) dy_a \right] h_b(y_b) dy_b
\]

Where \( I_{d_a \leq 0} \) is an indicator function, taking the value of 0 if \( d_a > 0 \) and 1 if \( d_a \leq 0 \).

We first show conditions under which it is optimal for the bank to pay dividends or issue equity, and solve for the optimal equity issuance in the latter case. We then discuss conditions under which banks’ optimal equity issuance policies are strategic complements or substitutes. Next, we show that equity issuance by one bank creates
a positive externality on the other bank. Finally, we show that the Nash equilibrium equity issuance outcome features undercapitalization relative to the first best outcome that maximizes the combined shareholder value of both banks.

6.1 Equity Issuance Decision

The optimal dividend/equity issuance decision is summarized in Proposition 4 below:

**Proposition 4** There exists a threshold continuation value \( V_a^* \):

- above which it is optimal for bank \( A \) to issue equity, in which case the optimal amount of equity to be issued is:
  \[
  d_a^* = \frac{1}{K_a(\bar{y} - y) - 1} \left\{ \frac{ps_a + l_a - c_a - V_a - y}{2(\bar{y} - y)(l_b + s_b)} (l_b + s_b)^2 + (c_b - d_b + y)^2 + 2(d_b - c_b - \bar{y})(l_b + s_b) \right\}
  \]

- below which it is optimal for the bank to pay out maximum dividends: \( d_a^* = c_a \).

Furthermore, the threshold continuation value \( V_a^* \) only depends on the cost of equity issuance when this cost and the riskiness of the \( t = 1 \) non-cash asset value are sufficiently low. In particular, there exists a threshold value of \( K_a(\bar{y} - y) \):

- above which \( V_a^*(1) = X_a - c_a \),
- under which
  \[
  V_a^*(2) = X_a - c_a + c_a \left[ K_a(\bar{y} - y) - 1 \right] \left[ (K_a(\bar{y} - y) - 1) - \sqrt{(K_a(\bar{y} - y) - 1)^2 - 1} \right]
  \]
  where \( X_a - c_a < V_a^*(2) < V_a^*(1) \) and:

\[
X_a = ps_a + l_a - y + \frac{(1 - p)s_b}{2(\bar{y} - y)(l_b + s_b)} \left[ (l_b + s_b)^2 + (c_b - d_b + y)^2 + 2(d_b - c_b - \bar{y})(l_b + s_b) \right].
\]

We present the proof in Appendix E. The proposition suggests that, similar to our benchmark case, risk-shifting via dividend payments occurs at low continuation values. The difference with the benchmark analysis is that at high continuation values, it is optimal for the bank to issue equity. The optimal amount of equity issuance is the result of trading off the cost of equity issuance \( K_a \) against the benefit of preserving the bank’s continuation value by reducing the probability of default.
6.2 Complementarity/ Substitutability in Equity Issuance Decisions

We now analyze how the optimal equity issuance decision of one bank is affected by that of its interconnected bank.

When bank $A$ decides to issue equity, we obtain that the sign of the equity issuance policy of $A$ in response to that of $B$, $\frac{d(d_A)}{d(d_A)}$, is the same as the sign of $\frac{\partial^2 E_a(d_A, d_B)}{\partial d_a \partial d_b}$. This cross-partial term can be derived to be as follows:

$$\frac{\partial^2 E_a(d_a, d_b)}{\partial d_a \partial d_b} = (1 - p) \frac{s_b}{l_b + s_b} \int_{\tilde{y}}^{\hat{y}_b} \left[ h_a(\tilde{y}_a^D) - V_a h_a'(\tilde{y}_a^D) \right] h_b(y_b) dy_b$$

(24)

where we have used the fact that $\frac{\partial y_a^D}{\partial d_b} = \frac{s_b}{l_b + s_b}$, and the assumption that both $y_a$ and $y_b$ are uniformly distributed over $[y, \hat{y}]$.

This condition suggests that two distinct effects drive the interaction in banks’ equity issuance decisions: (i) Lower equity issuance of $B$ increases $A$’s benefits from risk-shifting due to an increase in $B$’s default risk and thereby an increase in $A$’s overall default risk. (ii) Lower equity issuance of $B$ increases the amount of cash flows $A$ has to generate to avoid default. This in turn affects $A$’s probability of keeping its franchise value, thereby affecting its expected benefits of equity issuance derived from franchise value protection.

How banks’ equity issuance interacts is unambiguous under effect (i): low equity issuance of $B$ makes it more attractive for $A$ to risk-shift and issue less equity (Strategic complementarity). The effect of $B$’s equity issuance decision on that of $A$ under (ii) depends on how likely $A$ is able to generate higher cash flows in $B$’s default states. Specifically, if cash flows are uniformly distributed, the probability that $A$ generates
enough cash flows to avoid default in $B$’s default states does not depend on the default threshold. As a result, $B$’s equity issuance decision does not affect $A$’s expected benefits of protecting its franchise value and effect (ii) disappears.

When cash flows are not uniformly distributed, we obtain two cases. On the one hand, if $\int_{\hat{y}_a}^{\hat{y}_b} [h'_{\hat{a}} (\hat{y}_a)] h_b(y_b) dy_b < 0$, $A$ is less likely to generate enough cash flows to meet the higher default threshold ($\hat{y}_a^D$) resulting from $B$’s lower equity issuance. As a result, the expected benefits from saving the franchise value is lower and $A$ finds it optimal to decrease equity issuance when $B$’s equity issuance is low (Strategic complementarity).

On the other hand if $\int_{\hat{y}_a}^{\hat{y}_b} [h'_{\hat{a}} (\hat{y}_a)] h_b(y_b) dy_b > 0$, $A$ is more likely to generate sufficient cash flows that meet the higher default threshold. In this case, the expected benefits from saving the franchise value is higher and $A$ finds it optimal to increase equity issuance when $B$’s equity issuance is low (Strategic substitutability).\(^{10}\)

Overall, the above analysis suggests the following:

- when $\int_{\hat{y}_a}^{\hat{y}_b} [h'_{\hat{a}} (\hat{y}_a)] h_b(y_b) dy_b \leq 0$, banks’ equity issuance decisions are strategic complements.\(^{11}\)

- when $\int_{\hat{y}_a}^{\hat{y}_b} [h'_{\hat{a}} (\hat{y}_a)] h_b(y_b) dy_b > 0$, banks’ equity issuance are strategic complements under effect (i) and strategic substitutes under effect (ii).\(^{12}\) Which effect dominates depends on $A$’s franchise value.

\(^{10}\)This analysis can be generalized to understand how the probability of issuing equity of one bank depends on the amount of equity issuance of its counterparty.

\(^{11}\)This condition holds, for example, when $\hat{y}_a^D$ is to the right of the mean of a well-behaved bell-shaped distribution, indicating $A$ has high default risk in $B$’s default states.

\(^{12}\)This condition holds, for example, when $\hat{y}_a^D$ is to the left of the mean of a well-behaved bell-shaped distribution, indicating $A$ has low default risk in $B$’s default states.
1. If A’s franchise value is sufficiently low such that $V_a < \frac{\int_{y_a}^{y_b} [h_a(\hat{y}_a^D)] h_b(y_b) dy_b}{\int_{y_a}^{y_b} [h'_a(\hat{y}_a^D)] h_b(y_b) dy_b}$, then A’s desire to protect its franchise value is low and hence effect (i) dominates: banks’ equity issuance decisions are strategic complements.

2. If A’s franchise value is sufficiently high such that $V_a > \frac{\int_{y_a}^{y_b} [h_a(\hat{y}_a^D)] h_b(y_b) dy_b}{\int_{y_a}^{y_b} [h'_a(\hat{y}_a^D)] h_b(y_b) dy_b}$, then A’s desire to protect its franchise value is high and hence effect (ii) dominates: banks’ equity issuance decisions are strategic substitutes.

### 6.3 Positive Externalities of Banks’ Equity Issuance

The effect of bank B’s equity issuance on bank A’s equity value is:

$$\frac{\partial U_a}{\partial d_b} = -(1 - p) \int_{y_a}^{y_b} \left[ V_a h_a(\hat{y}_a^D) + \frac{s_b}{\ell_b + s_b} Pr[y_a > \hat{y}_a^D] h_b(y_b) dy_b \right] < 0,$$

That is, the higher the amount of equity issued by bank B, the higher the value of bank A. In other words, equity issuance policies are positive externalities.

### 6.4 Undercapitalization under Uncoordinated Policies

We can now show that:

**Proposition 5** When banks’ franchise values are high such that it is optimal for them to issue equity, uncoordinated equity issuance decisions result in undercapitalization relative to coordinated ones. That is,

$$d^{FB}_a < d^*_a, \quad d^{FB}_b < d^*_b,$$

where $d^*_a$ and $d^*_b$ are Nash outcomes and $d^{FB}_a$ and $d^{FB}_b$ are first best outcomes.

The proof of this proposition is given in Appendix F.

### 7 Discussion and Policy Implications

In this section, we first discuss how our model complements existing literature in explaining the observed excessive dividends and under-capitalization of banks during the
recent financial crisis (Sections 7.1 and 7.2) and in providing a new contagion channel (Section 7.3). We then propose potential reasons why ex-ante contracting (or the lack of which) fails to correct such suboptimal policies (Section 7.4). Lastly, we provide policy implications of our model (Section 7.5).

7.1 Bank Dividend Payouts During 2007-2009

Our result is helpful in understanding the divergent dividend patterns documented in section 3. All the financial firms mentioned had very high leverage ratios coming into the crisis. This, coupled with the fact that current and expected future cash flows were low means that these firms, according to the model, had substantial benefits from risk-shifting via a maximum dividend policy. In fact, consistent with the model’s result that the probability of dividend payment is increasing in leverage, Lehman Brothers, being the most highly levered bank, increased its dividends remarkably in the period leading up to its failure. On the other hand, commercial banks, which were not as highly levered as securities firms due to their tighter capital requirements, either smoothed out or decreased their dividends throughout the crisis.

High leverage, however, should not result in risk-shifting if the franchise value of the bank is high enough. According to our model, a bank tends to risk-shift by paying dividends only when a bad shock depresses its franchise value to a sufficiently low level. While banks’ franchise values were depressed during the crisis, there are reasons to believe that they were more depressed for securities firms compared to commercial banks, resulting in the observed dividend behavior. While commercial banks’ clients, whose relationship with the bank are commonly formed through illiquid loan contracts,
are not very likely to “run” during bad times, securities firms’ customers may have found it easier to exit relationships.\textsuperscript{13}

Importantly, the divergent behaviour in dividend payments between investment banks and commercial banks can also be partially explained by our two-bank model, which argues that excessive dividends can result from strategic complementarity of (uncoordinated) dividend policies between interconnected banks. Compared to commercial banks, investment banks and broker-dealers are more highly interconnected to each other and to the rest of the financial system via proprietary trading and other securities market activities.

One significant activity where the interconnectedness of investment banks and broker-dealers manifests itself is in OTC derivatives contracts.\textsuperscript{14} According to BIS (2009), exposure of dealers to OTC derivatives contracts in December 2008 amounted to $33,889 billion, with interest rates derivatives and credit default swaps (CDS) being the two most significant components ($18,420 bil and $5,652 bil, respectively). Breakdowns of

\textsuperscript{13}In fact, the crisis of 2007-2009 revealed that a major class of securities firms’ clients - hedge funds relying on prime brokerage services- are prepared to shift their cash and securities to safer institutions when signs of distress occur (see Duffie, 2010). The collapse of Bear Stearns and Lehman Brothers prompted large flows of hedge fund client assets out of Morgan Stanley and Goldman Sachs (those with historically the largest share of the prime brokerage business), and into commercial banks that were perceived, at the time, as the most creditworthy, such as Credit Suisse, JP Morgan, and Deutsche Bank. According to Global Custodian magazine, 44 percent of hedge funds reduced balances with Goldman and 70 percent backed out from Morgan Stanley. Since prime brokerage is a high profit margin activity, that involves the bank lending cash and securities to hedge funds and providing custody and other businesses, the loss of relationships with hedge fund clients may have caused a significant decline in franchise values of many securities firms and potentially an increase in franchise values of several creditworthy commercial banks.

\textsuperscript{14}Duffie (2010) states: “At least one of the two counterparties of most OTC derivatives is a dealer. It would be uncommon, for example, for a hedge fund to trade directly with, say, an insurance company. Instead, the hedge fund and the insurance company would normally trade with dealers. Dealers themselves frequently trade with other dealers. Further, when offsetting a prior OTC derivatives position, it is common for market participants to avoid negotiating the cancellation of the original derivatives contract. Instead, a new derivatives contract that offsets the bulk of the risk of the original position is frequently arranged with the same or another dealer. As a result, dealers accumulate large OTC derivatives exposures, often with other dealers”.
counterparty types show the significant interconnectedness of investment banks with other investment banks and financial institutions. Out of the $18,420 bil ($5,652 bil) exposure to interest rates derivatives contracts (CDS contracts), $6,629 bil ($3,177 bil) were between two dealers, $10,731 bil ($2,377 bil) were between a dealer and another financial institution, and only $1,061 bil ($98 bil) were between a dealer and a non-financial customer.\textsuperscript{15} Our two-bank model appears particularly relevant for such highly interconnected parts of the financial sector.

### 7.2 Bank Equity Issuance

The failure or near failure of many large financial institutions during the 2007-2009 financial crisis indicates that these institutions were significantly undercapitalized given the amount of risk they took. Nonetheless, private forces did not bring about a timely recapitalization (See Section 3 and the evidence in Acharya, Gujral, Kulkarni, and Shin (2012).) Bill Dudley’s (2009) speech noted the resistance to equity issuance by banks and government-sponsored enterprises, whose executives told regulators “repeatedly over the past 18 months” that “now is not a good time to raise capital”.

Similar to Admati et al (2013) and Calomiris and Herring (2013), we argue that debt overhang is a plausible reason for banks’ reluctance to raise optimal levels of equity capital. An implication unique to our model is that inefficiencies are more pronounced for banks that are more interconnected due to externalities that cannot be internalized.

This may offer one explanation for the observation made by Haldane (2009) that more

\textsuperscript{15}Table 1 in Appendix G documents the derivatives exposures of 9 out of 10 banks in our sample as of quarter 1 of 2008. The table reveals significantly higher derivatives exposures of investment banks relative to those of commercial banks.
systemically important banks tend to have lower capital buffers.\footnote{An explanation offered by Haldane (2009) is that larger, more systemic banks can afford lower capital buffers thanks to implicit guarantee by the government.}

\section{7.3 Financial Contagion}

Our model also complements the literature on direct channels of contagion (see Allen and Gale, 2000, and Zawadowski, 2013) and suggests that risk-shifting from shareholders to creditors can contribute to contagion in an interconnected financial network. A distressed bank has incentives to pay out dividends, thereby increasing the probability of its default, leading to erosion in asset values of its counterparty which is an otherwise healthy bank. Duarte and Kolasinski (2014) empirically examine possible channels of contagion across broker-dealers and dealer banks during the 2007-2009 financial crisis. They conclude that the direct channel (franchise value erosion via counterparty risk exposure) accounted for almost all of the contagion. The indirect channel (system wide illiquidity shock caused by the failure of a large institution) accounted for a mere 5% of all contagion prior to government intervention in the fall of 2008, and disappeared post-intervention.

\section{7.4 Ex-ante Contracting}

A natural question arises as to why creditors and shareholders of banks do not always find mechanisms ex ante to limit the inefficiencies in their dividend or equity issuance decisions. One reason we proposed is that banks do not fully internalize the externalities they create on their counterparties. Other reasons, which we discuss below, involve creditors’ lack of monitoring incentives and difficulties in writing complete ex-ante con-
Creditor contracts can write covenants limiting dividends in the bank’s stressed states. In practice, such covenants can be suboptimally few and ineffective for the following reasons. First, bank creditors do not have strong incentives to write such covenants in the presence of government’s explicit and implicit guarantees on bank debt (Becht, Bolton, and Roell (2011)). Second, when banks are interconnected, optimal contracting on dividend is practically impossible. The policies of one bank depends on those of its counterparties, and in turn on the policies of the counterparties of its counterparties. In other words, efficient contracts written would have to be informationally dependent across firms. In practice, contracts and covenants tend to be non-exclusive: one party in a contractual relationship cannot constrain its counterparties’ policies that are dependent on third parties, leading to inefficient outcomes.17

Third, dividend-restricting covenants may also be ineffective because creditors and shareholders may underestimate the probability of stress, which later gets realized and leads to risk-shifting. Finally, covenants mandating equity issuance do not exist in practice possibly due to the negative information conveyed by this decision a la Myers and Majluf (1984). Covenants limiting leverage can in principle encourage equity issuance. However, such covenants are not effective in practice as the true economic leverage of banks, masked by accounting maneuvers and banks’ complexity, is often hard to gauge. Even if it were possible to effectively contract on equity issuance, these ex-ante contracts would be ineffective for the exclusivity reasons discussed above.

17See Bisin and Gottardi (1999), Bisin and Guaitoli (2004), Bisin, Geanakoplos, Gottardi, Minelli and Polemarchakis (2011), Dubey, Geanaoplos, and Shubik (2005), Acharya and Bisin (2014) for the literature on non-exclusive contracts.
7.5 Policy Implications

Our model provides theoretical rationale for the use of dividend restrictions as part of the U.S Prompt Corrective Action (PCA) procedure. Introduced in 1991 following the banking crisis in the 1980’s, PCA was an early intervention mechanism intended to provide swift measures to turn around troubled financial institutions. Among different measures are mandatory limits to dividends and compensation to senior managers of banks that are under-capitalized.\textsuperscript{18}

Our model confirms the relevance of PCA in crisis periods when banks usually find their franchise values depressed and themselves pushed into the risk-shifting/dividend paying regions. Risk-shifting by individual banks, however, is not sufficient for dividend restrictions aiming at protecting bank creditors to be necessary.\textsuperscript{19} These restrictions are necessary only if individual banks’ dividend policies have negative externalities on the rest of the financial system and the economy. As we have shown, limiting dividend payments can help preserve bank capital and be desirable even from the perspective of combined banks’ shareholders in a setting with interconnected banks.

Along the same lines, our model supports the Basel III requirement that banks must maintain a capital conservation buffer consisting solely of Tier I capital (and accounting for 2.5% of the banks’ risk-weighted assets). Building this buffer may involve issuing new equity, reductions in discretionary earnings distribution, dividend payments, and salaries and bonuses. Basel III also suggests that regulators forbid banks from dis-

\textsuperscript{18}Compensation to senior managers of banks can be thought of as dividends paid to internal capital, and hence is applicable to our model here.

\textsuperscript{19}Dividend restrictions are originally designed under PCA to limit losses to the FDIC insurance fund.

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tributing capital when banks have depleted their capital buffers.

In this regard, the policy implications of our paper are in line with those from Admati et al. (2013) which advocates government control of banks’ equity payout and equity issuance. Admati et al. (2013) argue that higher capital means that equity holders and bank managers have more “skin in the game” and thereby are less inclined to take excessive risk. This rationale is consistent with our first theoretical result that banks’ incentives to transfer value away from creditors to shareholders, in particular, by paying out dividends, increase with leverage.

Analyzing the payout decision from a different angle, Acharya, Mehran and Thakor (forthcoming) reach the same conclusion as ours that part of bank capital should only be available to equity holders when banks perform well. Acharya, Mehran and Thakor (forthcoming) argue that banks tend to fund themselves with excessive leverage in anticipation of correlated failures and government bail-out of bank creditors. Consequently, optimal regulation features a contingent rule, in which part of bank capital is unavailable to creditors upon failure and available to shareholders only in the good states.

While the recommendations by Admati et al. (2013) and Acharya, Mehran and Thakor (forthcoming) are based on the argument of moral hazard from government bailouts, the novel element central to our analysis is the presence of bank interconnectedness and externalities. In our model, dividend and capital regulation arises not only from the desire to curb an agency problem between shareholders and creditors of individual banks, but also from a failure to coordinate among shareholders of in-
terconnected banks. Without such coordination, individual banks do not internalize the externalities they impose on their interconnected banks, and hence their dividend and capital policies can be inefficient relative to the optimal policies for the combined banks’ shareholders.

Our model also speaks to the social benefits of clearing-house arrangements of derivative contracts. Under these arrangements, banks internalize costs of their default on each other by all putting upfront margins and capital. Ex-post, when one is in trouble, this upfront capital can be used for co-insurance. Dividend policy can be understood in light of this general principle. Originally, clearinghouses of commercial banks were formed mainly to deal with information-based contagion. Clearinghouses for derivatives, on the other hand, are intended to deal with counterparty risk and interconnectedness issues (Duffie and Zhu, 2011). The key insight is that in each of these cases, one bank’s equity is effectively - and in part - a debt claim on other banks. Hence, insights from agency problems between equity and debt of each bank carry over to conflicts of interest across inter-connected banks.

8 Conclusion

Why did banks continue to pay dividends well into the 2007-2009 financial crisis? We argue in this paper that a combination of risk-shifting incentives and low franchise values

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20Collateralization of derivatives contracts were weak in the run-up to the financial crisis. Singh (2010) reports that Goldman Sachs, Citi, JP Morgan, Bank of America, and Morgan Stanley were jointly carrying almost $500 billion in residual OTC derivatives payables as of December 2009. Singh (2010) mentions two main reasons for this under-collateralization. First, these large financial institutions were viewed as privileged and safe clients. Second, “dealers have agreed, based on the bilateral-nature of the contracts, not to mandate adequate collateral for dealer to dealer positions. In fact, dealers typically post no initial margin to each other for these contracts”.

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can lead to such a striking dividend pattern. Interestingly, when banks are contingent creditors of each other, dividend payouts by one bank may exert negative externalities on the other banks’ equity. Because individual banks do not internalize these negative externalities, their uncoordinated dividend policies can be excessive. Similarly, our model with equity issuance can help partially explain why banks did not recapitalize adequately and timely. These banks are highly interconnected so that raising capital would create positive externalities on other banks that they do not fully internalize.

Our model generates two main testable hypotheses as follows. First, during financial crises, banks that have higher leverage or lower franchise values are more likely to risk-shift via dividend payments and are reluctant to issue equity. Second, banks that are more connected with each other have dividend policies and capital issuance decisions that are more likely to exhibit strategic complementarities. Although the first hypothesis has been discussed under existing literature, the second hypothesis is novel.

Our arguments call for policy measures in coordinating dividend payments during bad times, where continuation values of some banks can be sufficiently low and risk-shifting incentives can be substantial. If low franchise value banks can agree not to pay dividends and recapitalize themselves adequately, then the franchise values of their counterparty banks are less likely to be lost, capital is better preserved in the system, and as a consequence the total value of the financial sector could be higher.
References


Figure 1a: Cumulative Losses, 2007Q3 - 2008Q4

This figure plots the cumulative losses for the ten banks included in our study, over the period from Quarter 3, 2007 to Quarter 4, 2008. All numbers are in billions of US dollars.
Figure 1b: Cumulative Dividends, 2007Q1 - 2008Q4

This figure plots the cumulative dividends paid by the ten banks included in our sample, over the period from Quarter 1, 2007 to Quarter 4, 2008. All numbers are in billions of US dollars.
Figure 1c: Dividend Payments \( (2007Q1 = 1) \)

This figure plots dividend payments of the ten banks included in our sample, where all amounts are normalized so that the dividend payment in Quarter 1, 2007 is set to be equal to 1.
Figure 1d: Dividends as a Percentage of Book Equity (2007Q1 = 1)

This figure plots the ratio of dividends over book value of equity for the ten banks included in our sample. All ratios are normalized such that the ratio in Quarter 1, 2007 is set equal to 1.
Figure 2: Dividend Payments and Cumulative Losses

These figures plot cumulative losses alongside quarterly dividends for Lehman Brothers, Merrill Lynch, Washington Mutual, and Wachovia. All numbers are in billions of US dollars.
Figure 3: Banks’ Capital Issuance by Type

This figure plots the amount of capital raised by the ten banks included in our sample from Quarter 3 of 2007 to Quarter 4 of 2008. Net common stock issuance is common stock issued minus common stock repurchased. Net preferred stock issuance is preferred stock issued minus preferred stock redemptions. Net long term debt issuance is long term debt issued minus long term debt repayments. All numbers are in billions of US dollars. Data source: Cash flow statements from banks’ 10-Q and 10-K reports.
Figure 4: Book Leverage

This figure plots the book leverage ratios of the ten banks included in our sample from Quarter 1 of 2007 to Quarter 4 of 2008. Book leverage is defined as total book value of assets (ATQ) divided by total book value of common equity (CEQQ). Data source: Compustat.
Figure 5: Quasi-Market Leverage

This figure plots the quasi-market leverage ratios of the ten banks included in our sample from Quarter 1 of 2007 to Quarter 4 of 2008. Quasi-market leverage is defined as quasi-market value of assets over market value of common equity. Quasi-market value of assets is defined as book value of assets (ATQ) minus book value of common equity (CEQQ) plus market value of common equity, where market value of common equity is number of common shares outstanding (CSHOQ) times price per share (PRCCQ). *Data source:* Compustat and CRSP.
Figure 6: Strategic Complementarity of Dividend Policies

$V_a$ and $V_b$ are the franchise values of banks $A$ and $B$, respectively. $V_i^*$ is the threshold franchise value of bank $i$ ($i \in \{a, b\}$), below which it pays maximum dividends. $d_i$ and $c_i$ are, respectively, the dividend payment and $t = 1$ cash flow of bank $i$. For each cell, the first and second values are the Nash dividend policy of bank $A$ and bank $B$, respectively.

<table>
<thead>
<tr>
<th></th>
<th>$V_b^*$</th>
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<tbody>
<tr>
<td>$(d_a=c_a)$</td>
<td>$(c_a, 0)$</td>
<td>$(0, 0)$</td>
<td>$(0, 0)$</td>
</tr>
<tr>
<td>$(d_a=0)$</td>
<td>$(c_a, c_b)$</td>
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$V_a^*$ | $V_a^*$
| $(d_b=0)$ | $(d_b=c_b)$ |
Figure 7: Excessiveness of Uncoordinated Dividend Policies

$V_a$ and $V_b$ are the franchise values of banks $A$ and $B$, respectively. The shaded regions correspond to different coordinated dividend policies. The lower left region features both banks paying maximum dividends. The upper left region features bank $A$ paying maximum dividends and bank $B$ paying zero dividends. The upper right region features both banks not paying any dividends. The lower right region features bank $A$ paying zero dividends and bank $B$ paying maximum dividends. Not shown in the plot are regions where one of the banks or both banks has very high franchise values, under which the optimal coordinated dividend policies is for both banks to pay zero dividends. The threshold franchise values under which bank $A$ and $B$ pays maximum dividends under the Nash equilibrium, respectively, are represented by the two vertical and the two horizontal lines.
Appendix A  Two-Bank Analysis: Miscellaneous Details

The impact of bank A’s dividend payment on its own equity value is:

\[
\frac{\partial U_a}{\partial d_a} (d_a, d_b, V_a) = 1 - p (V_a h_a (\hat{y}_a) + 1 - H_a (\hat{y}_a)) \\
- (1 - p) (1 - H_b (\hat{y}_b)) (V_a h_a (\hat{y}_a^{ND}) + 1 - H_a (\hat{y}_a^{ND})) \\
- (1 - p) \int_{\hat{y}_b}^{y_b} (V_a h_a (\hat{y}_a^D) + 1 - H_a (\hat{y}_a^D)) h_b (y_b) dy_b
\]

where \( H_i (\hat{y}) \) is defined as \( \int_{\hat{y}}^{y} h_i (y) dy \), the probability that bank i defaults. Note that \( \hat{y}_a \) and \( \hat{y}_a^{ND} \) are functions of \( d_a \), \( \hat{y}_b \) is a function of \( d_b \), and \( \hat{y}_a^D \) is a function of \( y_b, d_a \) and \( d_b \).

The second derivative takes the following form:

\[
\frac{\partial^2 U_a}{\partial d_a^2} (d_a, d_b) = ph_a (\hat{y}_a) + (1 - p) h_a (\hat{y}_a^{ND}) (1 - H_b (\hat{y}_b)) \\
+ (1 - p) \int_{\hat{y}_b}^{y_b} h_a (\hat{y}_a^D) h_b (y_b) dy_b > 0
\]

Appendix B  Proof of Lemma 3

In this section, we will examine how the optimal dividend policy of one bank is influenced by the optimal dividend policy of its counterparty. Specifically, we will explore how the probability of one bank paying maximum dividends, represented by its threshold franchise value, depends on the dividends paid by the other bank.

Note that from (17), we get an expression for the threshold franchise value of bank A in a two-bank case as follows:
\[ V_a^*(d_b) = p s_a + l_a - y - \frac{c_a}{2} + \frac{(1-p)s_b}{2(y-y)} (l_b + s_b) \left( (l_b + s_b)^2 + (c_b - d_b + y)^2 + 2(d_b - c_b - \bar{y})(l_b + s_b) \right) \]

Now we would want to compare \( V_a^*(d_b = 0) \) to \( V_a^*(d_b = c_b) \). From (28), we have that:

\[ \frac{\partial V_a^*(d_b)}{\partial d_b} = \frac{(1-p)s_b}{(\bar{y}-y)(l_b + s_b)} (l_b + s_b - c_b + d_b - y) \]

Using the assumption that \( y < \ell_i + s_i - c_i < \ell_i + s_i < \bar{y} \) and because \( 0 \leq d_b \leq c_a \), we have that \( \frac{\partial V_a^*(d_b)}{\partial d_b} > 0 \). Therefore:

\[ V_a^*(d_b = 0) < V_a^*(d_b = c_b) \]

This result suggests that when two banks are interconnected (e.g., via a contingent contract as described in our model), one bank is more likely to pay maximum dividends when the other bank pays maximum dividend. We call this result strategic complementarity of dividend policies.

**Appendix C  Proof and Illustration of Proposition 3**

Excessive dividends can happen when either of the banks’ franchise value is sufficiently low such that its optimal uncoordinated dividend policy is to pay maximum dividends. Let \( d_a^* \) and \( d_b^* \) be the privately optimal (Nash) dividend policies for banks \( A \) and \( B \), respectively. Assume \( V_a < V_a^* \) such that \( d_a^* = c_a \). We will prove that when the franchise value of \( B \) is big such that dividend externalities created by \( A \)'s dividends are too
large compared to its private gain, it is jointly optimal for bank $A$ to pay less than its privately optimal dividend level.

Assume now that bank $A$ deviates from its privately optimal dividend policy and pays $d_a = d^*_a - \epsilon$, while bank $B$ employs its privately optimal policy $d_b = d^*_b$. The combined equity value of banks $A$ and $B$ in this case will be:

$$U_a(d_a, d_b^*) + U_b(d_a^*, d_b^*)$$

The change in the combined bank value resulting from this deviation is:

$$U_a(d_a, d_b^*) + U_b(d_a^*, d_b^*) - U_a(d_a^*, d_b^*) - U_b(d_a^*, d_b^*)$$

Let $\epsilon \to 0$, we can write this expression as follows:

$$- \frac{\partial U_a}{\partial d_a} \epsilon \frac{\partial U_a}{\partial d_a} - \frac{\partial U_b}{\partial d_a} \epsilon \frac{\partial U_a}{\partial d_a} = - \left( \frac{\partial U_a}{\partial d_a} + \frac{\partial U_b}{\partial d_a} \right) \epsilon \frac{\partial U_a}{\partial d_a}$$

where $dd_a = \epsilon$. When $d_a^* = c_a$, we know from Proposition 3 that $\frac{\partial U_a}{\partial d_a} > 0$. In addition, $\frac{\partial U_b}{\partial d_a}$ is always negative (Lemma 2). The former term represents the private benefits of bank $A$ from dividend payments while the latter represents the negative externalities $A$’s dividends exert on $B$’s equity value. On the one hand, paying out dividends is costly to a bank due to the potential loss of its franchise value, and therefore the private benefits of $A$’s dividend payment are decreasing in its franchise value. On the other hand, $A$’s dividends increase the probability that $B$ loses its franchise value, this negative externality being more pronounced the larger $B$’s franchise value. Therefore, if $B$’s franchise value is big such that the negative externalities are large relative to $A$’s
private benefits from dividend payments, then A’s dividends are excessive relative to coordinated policies. Formally, let $f(V_b) \equiv -\frac{\partial U_b}{\partial d_a} - \frac{\partial U_a}{\partial d_a}$, we have that

$$\frac{\partial f(V_b)}{\partial V_b} = \frac{p(\ell_a + s_a - y)}{(\bar{y} - y)^2} > 0$$

As the joint gain from bank A deviating is increasing in $V_b$, there exists a sufficiently high value of $V_b$ such that $f(V_b) > 0$, and A’s uncoordinated dividend policy is jointly excessive.

Given the corner solution for the optimal dividend policy in our setup, bank A’s dividend policy is jointly excessive if:

$$f'(V_b) \equiv -(U_b(d_a = c_a) - U_b(d_a = 0)) - (U_a(d_a = c_a) - U_a(d_a = 0)) > 0$$

where

$$U_b(d_a = c_a) - U_b(d_a = 0) =$$

$$-p \frac{c_a}{(\bar{y} - y)^2} (V_b - \ell_b - d_b + c_b + s_a)$$

$$(\bar{y} - \ell_b - d_b + c_b + s_a) + \frac{\bar{y}^2 - (\ell_b + d_b - c_b - s_b)^2}{2}$$

$$+ \frac{pc_a}{(\bar{y} - y)^2} \left[ \bar{y} \left( V_b + \frac{\bar{y}}{2} \right) - (V_b + \bar{y}) \left( \ell_b + d_b - c_b - \frac{s_a}{\ell_a + s_a} \left( \frac{c_a}{2} + y \right) \right) + 0.5(\ell_b + d_b - c_b)^2 + (\ell_a + s_a - c_a - y) \frac{s_a}{\ell_a + s_a} \left( 2(\ell_b + d_b - c_b) - \frac{s_a c_a}{\ell_a + s_a} \right) - \frac{s_a}{(\ell_a + s_a)^2} \left( \frac{s_a}{\ell_a + s_a} \left( (\ell_a + s_a - c_a)^2 - y^2 \right) - (\ell_b + d_b - c_b)(c_a - 2(\ell_a + s_a)) \right) + \frac{s_a^2}{3(\ell_a + s_a)^2} \left( 3(\ell_a + s_a)^2 - 3c_a(\ell_a + s_a) + c_a^2 \right) \right]$$

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And $U_a (d_a = c_a) - U_a (d_a = 0)$ is given by:

$$U_a (d_a = c_a) - U_a (d_a = 0) = \frac{c_a}{(\bar{y} - y)^2} \left[ (\bar{y} - y)^2 - p (\bar{y} - y) \left( V_a + \frac{c_a}{2} + \bar{y} - l_a - s_a \right) - (1 - p)(\bar{y} - \hat{y}_b) \left( V_a + \frac{c_a}{2} + \bar{y} - l_a + s_b \right) - (1 - p)(\hat{y}_b - y) \left( V_a + \bar{y} - l_a + \frac{c_a}{2} + s_b \frac{\bar{y} + c_b - d_b + l_b + s_b}{2(l_b + s_b)} \right) \right]$$

As

$$\frac{\partial f' (V_b)}{V_b} = \frac{p c_a s_a (2 (\ell_a + s_a - y) - c_a)}{2 (\bar{y} - y)^2 (\ell_a + s_a)} > 0,$$

A’s uncoordinated dividend policy is excessive when $V_b$ is sufficiently high, i.e., when

$$V_b > \frac{(\bar{y} - y)^2 (2 (\ell_a + s_a))}{p c_a s_a (2 (\ell_a + s_a - y) - c_a)} \left( \gamma + U_a (d_a = c_a) - U_a (d_a = 0) \right)$$

where

$$\gamma = -(\bar{y} - \ell_b - d_b + c_b + s_b)$$

$$+ \frac{p c_a}{(\bar{y} - y)^2} \left[ \bar{y} \left( \frac{y}{2} \right) - (\bar{y} - \ell_b + d_b - c_b - s_b)^2 \right]$$

$$+ \frac{s_a}{(\ell_a + s_a)^2} \left[ \frac{s_a}{\ell_a + s_a} \left( \frac{c_a}{2} + \bar{y} \right) \right]$$

$$+ \frac{0.5}{(\ell_a + s_a)^2} \frac{s_a}{\ell_a + s_a} \left( (\ell_a + s_a - c_a - y) \frac{s_a}{\ell_a + s_a} \left( 2 (\ell_b + d_b - c_b) - \frac{s_a c_a}{\ell_a + s_a} \right) - (\ell_b + d_b - c_b) (c_a - 2 (\ell_a + s_a)) \right)$$

$$- \frac{s_a}{(\ell_a + s_a)^2} \left[ \frac{s_a}{\ell_a + s_a} \left( (\ell_a + s_a - c_a)^2 - y^2 \right) - (\ell_b + d_b - c_b) (c_a - 2 (\ell_a + s_a)) \right]$$

$$+ \frac{s_a^2}{3 (\ell_a + s_a)^2} \left[ 3 (\ell_a + s_a)^2 - 3 c_a (\ell_a + s_a) + c_a^2 \right] \right]$$

Figure 7 illustrates the result of Proposition 3 with an example of two identical banks having contingent financial contracts with each other. We vary each bank’s franchise
value and compare the coordinated dividend policies to the uncoordinated ones. In the figure the four shaded regions correspond to different coordinated dividend policies. The lower left region features both banks paying maximum dividends. The upper left region features bank A paying maximum dividends and bank B paying zero dividends. The upper right region features both banks not paying any dividends. The lower right region features bank A paying zero dividends and bank B paying maximum dividends. The threshold franchise values under which bank A or bank B pays maximum dividends given the other bank pays zero and maximum dividends, respectively, are represented by the set of two vertical and the set of two horizontal lines.

This figure, combined with Figure 6, allows a comparison of dividend policies under coordinated and uncoordinated strategies. As can be seen, when the negative effect one bank’s dividend policies on the other bank’s equity is internalized by the former, the non dividend paying region is much larger under the coordinated strategy, and the dividend paying region much smaller. Not shown in the figure are regions where one of the banks or both banks has very high franchise values, under which the optimal coordinated dividend policies is always for both banks to pay zero dividends.

When both banks have very high (very low) franchise values, the coordinated dividend policy is the same as the uncoordinated one: they both pay zero (maximum) dividends. However, when one bank, say bank A, has a low franchise value and the other bank, say bank B, has a high franchise value, the former pays maximum dividends under the Nash equilibrium, which is excessive under coordinated dividend policies. This is because by paying maximum dividends, it increases the probability that
the other bank defaults on its debt obligation and losses its high franchise value. This negative externality is not internalized by the low franchise bank under uncoordinated dividend policies. In this case, coordination leads to both banks paying zero dividends: the high franchise value does not only provide discipline for bank B but also constrains risk taking incentives of its interconnected bank.

Appendix D  Global Game Refinement

Having identified the multiplicity of equilibria of the $2 \times 2$ game due to the payoff spillovers of bank dividends, we now employ global game techniques to refine the outcome. Following the constructions used in the global game literature (Morris and Shin (1998, 2001, 2003)), we consider the following variation of our model.

First, the game is symmetric in the sense that all parameters are identical across the two banks. Hence, if the franchise values of the two banks are identical $V_a = V_b = V$, the promised payoffs under the OTC contracts are identical, so that $s_a = s_b = s$, and both banks have the same cash holding $c_a = c_b = c$.

However, rather than being a parameter that is common knowledge between the banks, we suppose that $V$ is uniformly distributed on the interval $[0, \bar{V}]$ where $\bar{V}$ is large relative to the threshold points $V_a^*$ and $V_b^*$ for the two banks.

Second, rather than the franchise values of the two banks being common knowledge, assume that each bank observes a slightly noisy signal of the common franchise value. Specifically, bank A observes the realization of the signal $x_a$ given by

$$x_a = V + \varepsilon_a$$  (30)
where $\varepsilon_a$ is a uniformly distributed noise term, taking values in $[-\eta, \eta]$ for small $\eta > 0$. Similarly, bank $B$ observes signal $x_b = V + \varepsilon_b$ where $\varepsilon_b$ is uniformly distributed in $[-\eta, \eta]$. We assume that the realizations of $\varepsilon_a$, $\varepsilon_b$ and $V$ are all mutually independent.

The noisy nature of the signals defines a Bayesian game built around the underlying one shot game, called the \textit{global game} (see Morris and Shin (1998) for details). The \textit{strategy} of a bank maps each realization of its signal $x_i$ to its dividend payment $d_i \in \{0, c_i\}$. An equilibrium is defined as a pair of strategies where the action prescribed given signal realization $x_i$ maximizes bank $i$'s expected payoff conditional on its signal realization given the opponent’s strategy.

A \textit{switching strategy} associated with a switching point $x^*$ is defined as the mapping:

$$
d(x_a) = \begin{cases} 
0 & \text{if } x_a \geq x^* \\
c_a & \text{if } x_a < x^*
\end{cases}
$$

We then have the following result for the global game refinement of our dividend game. Define the function $W(x)$ as

$$
W(x) \equiv \frac{1}{2}W(0, x) + \frac{1}{2}W(c, x)
$$

where $W(d_b, x)$ is the function defined in (16) that gives the payoff advantage to bank $A$ with franchise value $V = x$ of paying zero dividends over paying maximum dividends when bank $B$ pays dividends of $d_b$. The function $W(x)$ defined above has the interpretation of the expected payoff advantage to bank $A$ of paying zero dividends when bank $B$ is randomizing equally between paying zero dividends and paying maximum dividends. Given the symmetry of the payoff parameters, $W(x)$ also applies to bank $B$’s payoff advantage given bank $A$’s dividend policy.
With these preliminary definitions, we have our main result on the global game refinement of equilibrium.

**Proposition 6 (Global Game Refinement)** There is a unique $x^*$ that solves $W(x^*) = 0$. There is an equilibrium of the global game where both banks use the switching strategy around the switching point $x^*$. There is no other equilibrium in switching strategies.

The proposition departs from the iterative dominance argument of Morris and Shin (1998, 2003), and hence is strictly less general. The claim is that when banks are restricted to switching strategies, there is a unique equilibrium. Goldstein and Pauzner (2005) proves a strictly stronger result in the context of a bank run game, where the switching equilibrium is shown to be the unique equilibrium, irrespective of the class of equilibria considered.

The proof of the Proposition follows in three steps, and relies on the well-known result that strategic uncertainty at the switching point is given by a uniform density over the incidence of actions. First, we know from the expression (17) that both $W(0, x)$ and $W(c, x)$ are increasing linear functions of $x$ with slope $c / (\bar{y} - y)$, so that $W(x)$ is also an increasing linear functions of $x$ with slope $c / (\bar{y} - y)$. Since $W(x)$ changes sign from negative to positive as $x$ increases, there is a unique $x^*$ that solves $W(x^*) = 0$.

Next, we show that both banks using the switching strategy around $x^*$ constitutes an equilibrium of the global game. Conditional on bank $A$ observing the signal $x_a = x$, the expected payoff advantage of paying zero dividends over paying maximum dividends is given by

$$\Pr(d_b = 0|x_a = x) \times W(0, x) + \Pr(d_b = c|x_a = x) \times W(c, x) \quad (33)$$
Suppose that both banks employ the switching strategy around the point $x^*$ where $x^*$ solves $W(x^*) = 0$. Then, we have

$$
\Pr (d_b = 0 | x_a = x^*) = \Pr (x_b > x^* | x_a = x^*)
$$
$$
\Pr (d_b = c | x_a = x^*) = \Pr (x_b \leq x^* | x_a = x^*)
$$

Given the symmetric nature of the noisy signals of the two banks, we have

$$
\Pr (x_b > x^* | x_a = x^*) = \Pr (x_b \leq x^* | x_a = x^*) = \frac{1}{2}
$$

Thus, conditional on bank $A$ observing the signal $x_a = x$, the expected payoff advantage of paying zero dividends over paying maximum dividends is given by

$$
\frac{1}{2} W(0, x^*) + \frac{1}{2} W(c, x^*) = 0
$$

so that bank $A$ is indifferent between paying zero dividends and maximum dividends.

For signal $x > x^*$, $\Pr (d_b = 0 | x_a = x) > \frac{1}{2}$, so that bank $A$ strictly prefers to pay zero dividends. Analogously, for signal $x < x^*$, $\Pr (d_b = 0 | x_a = x) < \frac{1}{2}$, so that bank $A$ strictly prefers to pay maximum dividends of $c$. Hence, the switching strategy around $x^*$ is the best response of bank $A$ when bank $B$ itself follows the switching strategy around $x^*$. Given the symmetric nature of the game, an exactly analogous argument shows that the switching strategy around $x^*$ by bank $B$ is the best response when bank $A$ uses the same strategy. This proves that the strategy pair where both banks use switching strategies around $x^*$ is an equilibrium of the global game.

The final part of the proposition claims that there is no other switching equilibrium of the global game. But this claim is immediate from the fact that $x^*$ is the unique
solution to \( W(x) = 0 \). If, contrary to the Proposition there is another switching equilibrium around the point \( x' \), where \( x' \neq x^* \), then we have \( W(x') \neq 0 \) so that the switching strategy around \( x' \) cannot be the best response to the switching strategy around \( x' \) by the other bank. This completes the proof of the proposition.

The global game refinement is preserved in the limiting case where the noisy signal becomes increasingly accurate, since the key feature of the construction is to maintain the joint density of the signal realizations that bank \( A \)'s signal is equally likely to be higher or lower than the realization of bank \( B \)'s signal. This feature of the joint signal realizations does not depend on the support \([-\eta, \eta]\) of the noise in the banks’s signal. Even in the limit as \( \eta \to 0 \), we have the key feature that \( \Pr (x_b > x^* | x_a = x^*) = \Pr (x_b \leq x^* | x_a = x^*) = \frac{1}{2} \).

**Appendix E  Proof of Proposition 4**

As a bank’s equity value function depends on whether it pays dividends or issues equity, we first solve for the locally optimal values of \( d_i \) over different ranges of \( d_i \). We then show that the bank either pays out maximum dividends or issues a finite amount of equity capital, and that the switching point governing this decision is unique.

**Appendix E.1  Locally Optimal \( d_i \)**

When \( 0 \leq d_a \leq c_a \) (dividend payout region), we are back to the benchmark case where the optimal dividend policy is given by (19). When \( -\infty < d_a \leq 0 \) (equity issuance
region), bank A’s equity value becomes:

\[
U_a(d_a, d_b, V_a) = d_a - \frac{1}{2} K_a d_a^2 + p \int_{\hat{y}_{a(d_a)}}^{\bar{y}} [y_a - \hat{y}_a(d_a) + V_a] h_a(y_a) dy_a \\
+ (1 - p) \int_{\hat{y}_{b(d_b)}}^{\bar{y}} \left[ \int_{\hat{y}_{a(d_a)}}^{\bar{y}} [y_a - \hat{y}_a^{ND}(d_a) + V_a] h_a(y_a) dy_a \right] h_b(y_b) dy_b \\
+ (1 - p) \int_{\hat{y}}^{\bar{y}} \left[ \int_{\hat{y}_{b(d_b)}}^{\bar{y}} [y_a - \hat{y}_a^{PD}(d_a, d_b) + V_a] h_a(y_a) dy_a \right] h_b(y_b) dy_b
\]

(35)

Absent coordination, the locally optimal \(d_a\) over the range \(-\infty < d_a \leq 0\) is chosen to maximize (35). The first order condition is:

\[
\frac{\partial U_a}{\partial d_a} = d_a \left( \frac{1}{\bar{y} - \hat{y}} - K_a \right) + \frac{p s_a + l_a - V_a - c_a - y}{\bar{y} - \hat{y}} \\
- \frac{1 - p}{(\bar{y} - \hat{y})^2} \left( \frac{s_b}{2 (l_b + s_b)} \left( (l_b + s_b)^2 - (c_b - d_b + \bar{y})^2 \right) + s_b (\bar{y} - l_b - s_b - d_b + c_b) \right) \\
= 0
\]

(36)

Assuming that the cost of equity issuance is not too low, i.e.

\[
K_a > \frac{1}{\bar{y} - \hat{y}},
\]

(37)

the second order condition is:

\[
\frac{\partial^2 U_a}{(\partial d_a)^2} = \frac{1}{\bar{y} - \hat{y}} - K_a < 0,
\]

and the optimal equity issuance level is an interior solution and is given by the following expression:

\[
d_a^* = \frac{1}{K_a (\bar{y} - \hat{y}) - 1} \begin{cases} 
  p s_a + l_a - c_a - V_a - \frac{y}{\bar{y} - \hat{y}} \\
  + \frac{(1 - p) s_b}{2 (\bar{y} - \hat{y}) (l_b + s_b)} \left( (l_b + s_b)^2 + (c_b - d_b + \bar{y})^2 + 2 (d_b - c_b - \bar{y}) (l_b + s_b) \right)
\end{cases}
\]

(38)
\(d_a^*\) given above is the local optimum over \(-\infty < d_a \leq 0\) if \(d_a^* \leq 0\) or \(V_a \geq V_a^{**}(d_b)\), where:

\[
V_a^{**}(d_b) = ps_a + l_a - c_a - y + \frac{(1 - p) s_b}{2 \left( \bar{y} - y \right) (l_b + s_b)} \left( (l_b + s_b)^2 + (c_b - d_b + \bar{y})^2 + 2 (d_b - c_b - \bar{y}) (l_b + s_b) \right)
\]

If, on the other hand, \(V_a < V_a^{**}\), \(d_a^*\) given by (38) is outside the range of \((-\infty, 0]\), and \(d_a = 0\) will be the optimal solution over \(-\infty < d_a \leq 0\).

### Appendix E.2 Globally Optimal \(d_i\)

Having solved for the local optima, we now find the global optimum under different conditions. Let

\[
X_a = ps_a + l_a - y + \frac{(1 - p) s_b}{2 \left( \bar{y} - y \right) (l_b + s_b)} \left( (l_b + s_b)^2 + (c_b - d_b + \bar{y})^2 + 2 (d_b - c_b - \bar{y}) (l_b + s_b) \right)
\]

We can rewrite \(V_a^{**}(d_b)\) as \(X_a - c_a\), and \(V_a^*(d_b)\) (the switching point for \(V_a\) over \(0 \leq d_a \leq c_a\), from (28)) as \(X_a - \frac{c_a}{2}\). Based on the locally optimal solutions for \(d_a\), we can characterize the globally optimal dividend/capital decision based on different continuation values as follows:

1. When \(V_a \leq X_a - c_a\): \(d_a = 0\) will be the optimal solution over \(-\infty < d_a \leq 0\), and from (19), \(d_a = c_a\) is the optimal solution over \(0 \leq d_a \leq c_a\). Therefore, the global optimum over \(-\infty < d_a \leq c_a\) is \(d_a^* = c_a\).

2. When \(X_a - c_a < V_a < X_a - \frac{c_a}{2}\): (38) is the optimum over \(-\infty < d_a \leq 0\), and from (19), \(d_a = c_a\) is the optimal solution over \(0 \leq d_a \leq c_a\). The globally optimal \(d_a\) will be either the optimal equity issuance given by (38) or \(d_a = c_a\), depending on the bank’s continuation value, analyzed below.
3. When $V_a \geq X_a - \frac{c_a}{2}$: (38) is the optimum over $-\infty < d_a \leq 0$, and from (19), $d_a = 0$ is the optimal solution over $0 \leq d_a \leq c_a$. Therefore, the global optimum over $-\infty < d_a \leq c_a$ is $d_a^* < 0$ given by (38).

We now characterize the optimal dividend/capital decision over the middle range for the bank’s continuation value, i.e. $X_a - c_a < V_a < X_a - \frac{c_a}{2}$. Let:

$$g(V_a) = U_a(d_a^* < 0) - U_a(d_a^* = c_a)$$

We will find values of $V_a \in (Z - c_a, Z - \frac{c_a}{2})$ for which the bank finds it optimal to issue equity: $g(V_a) > 0$. Let:

$$a_1 = \frac{(1-p)s_b}{2(\bar{y} - y)(l_b + s_b)} \left((l_b + s_b)^2 + (c_b - d_b - y)^2 + 2(d_b - c_b - \bar{y})(l_b + s_b)\right),$$

$$a_2 = ps_a + l_a - c_a - y$$

$g(V_a)$ can be simplified as:

$$g(V_a) = -\frac{1}{2(\bar{y} - y)(K_a(\bar{y} - y) - 1)^2} V_a^2 + \frac{1}{\bar{y} - y} \left[ \frac{1}{(K_a(\bar{y} - y) - 1)^2} (a_1 + a_2) + c_a \right] V_a - \frac{1}{2(\bar{y} - y)(K_a(\bar{y} - y) - 1)^2} (a_1 + a_2)^2 - \frac{c_a}{\bar{y} - y} \left( a_2 - \frac{c_a}{2} \right) - \frac{c_a}{\bar{y} - y} a_1$$

which is a quadratic function in $V_a$. The discriminant of this function is:

$$\frac{c_a^2}{(\bar{y} - y)^2(K_a(\bar{y} - y) - 1)^2} \left( (K_a(\bar{y} - y) - 1)^2 - 1 \right)$$

(41)

For this discriminant to be positive, we impose an assumption that $K_a > \frac{c_a}{\bar{y} - y}$. This is a similar but slightly stricter assumption compared to (37). As the quadratic coefficient
is negative, the bank finds it optimal to issue equity \((g(V_0) > 0)\) if \(Z_1 < V_a < Z_2\), where

\[
Z_1 = X_a - c_a + c_a \left( K_a \left( \bar{y} - y \right) - 1 \right) \left( K_a \left( \bar{y} - y \right) - 1 \right) - \sqrt{\left( K_a \left( \bar{y} - y \right) - 1 \right)^2 - 1},
\]

\[
Z_2 = X_a - c_a + c_a \left( K_a \left( \bar{y} - y \right) - 1 \right) \left( K_a \left( \bar{y} - y \right) - 1 \right) + \sqrt{\left( K_a \left( \bar{y} - y \right) - 1 \right)^2 - 1}
\]

where \(X_a\) is defined by (39).

Using the assumption that \(K_a > \frac{2}{\bar{y} - y}\), \(Z_2\) is clearly greater than \(X_a - \frac{c_a}{2}\) and therefore, \(Z_2\) does not fall within the interval \((X_a - c_a, X_a - \frac{c_a}{2})\). On the other hand, \(Z_1\) is greater than \(X_a - c_a\). Whether \(Z_1\) falls within the interval \((X_a - c_a, X_a - \frac{c_a}{2})\) depends on the value of \(K_a(\bar{y} - y)\). It is easy to see that there exists a value of \(K_a(\bar{y} - y)\),

- above which \(Z_1 > X_a - \frac{c_a}{2}\), in which case it is optimal for the bank to pay out maximum dividends when its continuation value falls within the interval \((X_a - c_a, X_a - \frac{c_a}{2})\).

- under which \(Z_1 < X_a - \frac{c_a}{2}\), in which case it is optimal for the bank to pay out maximum dividends when its continuation value falls within the interval \((X_a - c_a, K_1)\), and issue equity when its continuation value is within the interval \((Z_1, X_a - \frac{c_a}{2})\).

This result, coupled with results under parts 1 and 3 of this section, implies there exists a unique switching point \(V_a^*\), above which it is optimal for the bank to issue equity and under which it is optimal for the bank to pay maximum dividends. This concludes the proof of Proposition 4.

Appendix F Proof of Proposition 5

From (38), the reaction functions for the Nash equilibrium can be rewritten as:
\[ d_a^* = \bar{c}_a + \theta_a \left( d_b^* + 2d_b^* \left(l_b + s_b - c_b - y \right) \right) \]  \hspace{1cm} (42)

\[ d_b^* = \bar{c}_b + \theta_b \left( d_a^* + 2d_a^* \left(l_a + s_a - c_a - y \right) \right) \]  \hspace{1cm} (43)

\( \bar{c}_a \) and \( \bar{c}_b \) are functions of \( V_a \) and \( V_b \) respectively, \( \theta_a > 0, \theta_b > 0 \). We have from Section 6.2 that the reaction functions are strictly increasing: \( \frac{\partial d_a^*}{\partial d_b^*} > 0, \frac{\partial d_b^*}{\partial d_a^*} > 0 \). Let us denote the reaction functions under Nash equilibrium by \( G_i(.) \) so that \( d_a^* = G_a(d_b^*) \); \( d_b^* = G_b(d_a^*) \).

Under the first best (FB) case,

\[
\frac{\partial U_a}{\partial d_a} + \frac{\partial U_b}{\partial d_a} = 0
\]  \hspace{1cm} (44)

As \( \frac{\partial U_b}{\partial d_a} < 0 \), we must have that \( \frac{\partial U_a}{\partial d_a} > 0 \). Moreover, (36) can be rewritten as

\[
\frac{\partial U_a}{\partial d_a} = \frac{1 - K_a (\bar{y} - y)}{\bar{y} - y} (d_a - G_a(d_b))
\]  \hspace{1cm} (45)

Therefore

\[
\frac{\partial U_a}{\partial d_a} > 0 \Rightarrow d_a - G_a(d_b) < 0 \Rightarrow d_a^{FB} < G_a(d_b^{FB})
\]  \hspace{1cm} (46)

Similarly, we can show that

\[
d_b^{FB} < G_b(d_a^{FB})
\]

Denote first best reaction functions by \( H(.) \) such that

\[
d_a^{FB} = H_a(d_b^{FB}) \text{ and } d_b^{FB} = H_b(d_a^{FB})
\]

Then

\[
H_a(.) < G_a(.) \text{ and } H_b(.) < G_b(.)
\]
The domain and range of $G(.)$ and $H(.)$ are negative. The claim in Proposition 5 follows almost directly from the fact that the reaction functions under the first best case are always smaller than the reaction functions under the Nash equilibrium. Below is a heuristic proof.

Let $d_a^*, d_b^*$ be the Nash equilibrium. We have that:

$$G_a (d_a^*) = d_b^*, \quad G_b (d_b^*) = d_a^* \Rightarrow G_a (d_a^*) - G_b^{-1} (d_b^*) = 0$$

When $V_a$ and $V_b$ are big enough, we have that

$$G_a (0) = \bar{c}_a < 0, \text{ and that}$$

$$G_b (0) = \bar{c}_b < 0 \Rightarrow G_b^{-1} (0) > 0. \Rightarrow G_a (0) - G_b^{-1} (0) < 0$$

Now let $x \in (d_a^*, 0)$. Assume WLOG that $d_a^*$ is the “first” Nash equilibrium outcome. That is, all other Nash equilibria, if they exist, involve $A$ raising more equity than $d_a^*$. Then we have that:

$$x \in (d_a^*, 0) \Rightarrow G_a (x) - G_b^{-1} (x) < 0 \Rightarrow H_a (x) - H_b^{-1} (x) < 0$$

Denote the first best equity issuance as $d_a^{FB}$. We have that $H_a (d_a^{FB}) - H_b^{-1} (d_a^{FB}) = 0$.

It follows that $d_a^{FB} < d_a^*$: Nash equilibrium equity issuance amount is less than the first best equity issuance amount.\(^{21}\) 

\(^{21}\) We can use a similar proof to show that if $(d_a^*, c_b)$ is Nash equilibrium and first best policy involves $B$ paying out dividends $(d_b^{FB} = c_b)$, then $d_a^{FB} < d_a^*$

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Appendix G  Banks’ Derivatives Exposures

Table 1: Banks’ Derivatives Exposures

Derivatives exposure is computed as the sum of derivatives receivable and derivatives payable divided by book value of equity. Derivatives receivables and payables are netted values subject to SFAS No. 157. Banks are ranked from highest derivatives exposure (1) to lowest derivatives exposure (10). No consistent value for Wells Fargo is found. Source: Banks’ 10-Q reports for the first quarter of 2008.

<table>
<thead>
<tr>
<th>Bank name</th>
<th>Derivatives Exposure</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goldman Sachs</td>
<td>4.55</td>
<td>1</td>
</tr>
<tr>
<td>Merrill Lynch &amp; CO INC</td>
<td>4.54</td>
<td>2</td>
</tr>
<tr>
<td>Morgan Stanley</td>
<td>4.25</td>
<td>3</td>
</tr>
<tr>
<td>Lehman Brothers</td>
<td>3.56</td>
<td>4</td>
</tr>
<tr>
<td>Citigroup</td>
<td>2.09</td>
<td>5</td>
</tr>
<tr>
<td>JP Morgan</td>
<td>1.42</td>
<td>6</td>
</tr>
<tr>
<td>Wachovia</td>
<td>0.61</td>
<td>7</td>
</tr>
<tr>
<td>Bank of America</td>
<td>0.51</td>
<td>8</td>
</tr>
<tr>
<td>Washington Mutual</td>
<td>0.14</td>
<td>9</td>
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