Monetary Easing, Leveraged Payouts and Lack of Investment*

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November 10, 2019

Abstract

This paper studies a model in which a low monetary policy rate lowers the cost of capital for entrepreneurs, potentially spurring productive investment. Low interest rates, however, also induce entrepreneurs to lever up so as to increase payouts to equity. Whereas such leveraged payouts privately benefit entrepreneurs, they come at the social cost of reducing their incentives thereby lowering productivity and discouraging investment. If leverage is unregulated (for example, due to the presence of a shadow-banking system), then the optimal monetary policy seeks to contain such socially costly leveraged payouts by stimulating investment in response to adverse shocks only up to a level below the first-best. The optimal monetary policy may even consist of "leaning against the wind," i.e., not stimulating the economy at all, in order to fully contain leveraged payouts and maintain productive efficiency.

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Introduction

The Federal Reserve has kept its policy rates at low levels since the 2008 crisis. The financial structure of corporations in the United States (US) has experienced three remarkable evolutions over the period.[1] First, corporate leverage has significantly risen. Aggregate corporate debt to GDP has reached historically high levels, exceeding in particular those prevailing just before the global financial crisis. The share of corporate credit originated by non-banks—the so-called shadow-banking system—is also at an all-time high. Second, this high leverage has been coincident with significantly negative net equity issuances due to higher share buybacks than ever in the past. Share buyback activity for the S&P 500 Index exceeded for example $800 billion in 2018. Third, fixed business investment since the crisis remains below historical trends to date despite robust corporate profits and favorable tax reforms.

The evolution of the US leveraged-loan market epitomizes these trends. This segment has doubled in size since 2010. Outstanding volumes now approach that of the high-yield bond market. The share of banks in their financing has plummeted to 8%. Nearly 70% of the proceeds fund "shareholder enhancements" such as dividends and buybacks, leveraged buyouts, or mergers and acquisitions.

This paper offers a parsimonious model in which a low monetary policy rate leads to large leveraged payouts by firms that have a detrimental impact on capital expenditures, thereby leading to business investments that are too low from a social perspective. This adverse effect of low rates occurs only when the public sector is unable to regulate private leverage; conversely, an appropriate prudential regulation on leverage in combination with a low

[1] These evolutions are described in details in, e.g., IMF (2017,2019) or Furman (2015).

monetary policy rate can restore the first-best investment level. Thus we offer an equilibrium relationship between several salient features of the current corporate credit cycle: the significant involvement of a large unregulated shadow-banking sector, historically unprecedented levels of leveraged payouts, and disappointing capital expenditures.

Gist of the argument. Suppose that an agent who values consumption at two dates 0 and 1 is endowed with an investment technology that converts date-0 consumption units into date-1 units with decreasing marginal returns to scale. The agent is price-taker in a bond market. As the required return on bonds decreases, the agent (i) invests more in her technology until its marginal return equates the return on bonds, and (ii) borrows more against the resulting date-1 output until so does her marginal rate of inter-temporal substitution. We deem such borrowing for consumption against future output a "leveraged payout." A natural interpretation of this trade is indeed that the agent sets up a corporation that operates her investment, and that this corporation issues bonds, using the proceeds either to buy back shares from her or to pay her a special dividend.

Suppose now that the output from investment increases in costly private effort by the agent. Such moral hazard introduces a tension between investment and leveraged payouts as the interest rate decreases. On the one hand, the agent would like to enter into more leveraged payouts to front-load consumption. On the other hand, borrowing more against date-1 output reduces her incentives to increase this output, thereby making investment less profitable and thus smaller. The agent sets her leverage at the level that optimally trades off consumption-smoothing and incentives. Very much like there is a trade-off between eliciting incentives and smoothing consumption across states of nature in the canonical moral hazard model of Holmström

(1979), there is a tension here between producing an output and borrowing against it.

Such agents in our setup are entrepreneurs facing a (real) interest rate controlled by a benevolent central bank. The central bank aims at stimulating investment with a low interest rate in an economy in which rigid prices fail to send the proper signals to entrepreneurs to invest. Whereas such monetary easing would seamlessly work in the absence of moral hazard, the above mentioned moral-hazard problem creates a wedge between privately and socially optimal leverage as well as investment decisions by entrepreneurs. In the face of a lower rate, entrepreneurs optimally enter into more leveraged payouts at the expense of effort and investment. Whereas reduced effort and investment are deadweight social losses, entrepreneurs' private benefits from leveraged payouts at a distorted rate are a social wash because they must be paid for by other agents—in the form of taxes in our setup.

In sum, our parsimonious model offers a clear connection between monetary easing and the rise of corporate leverage and leveraged payouts at the expense of capital expenditures and productivity. It has noteworthy implications for financial regulation and optimal monetary policy.

Implications for financial regulation. We show that the central bank can implement the first-best despite moral hazard if it has a free hand at regulating corporate leverage. We view the difference between a setting in which it can do so and one in which entrepreneurs lever up as they see fit as a stylized parallel between an economy in which corporate credit originates from regulated banks and one in which it also stems from non banks—the "shadow-banking" sector. We show that monetary easing entails more leveraged payouts at the expense of productive investment in the latter situation than in the former. Accordingly, our theory suggests that the existence of a large shadow-banking system may dramatically affect the transmission of monetary policy. Interestingly, as mentioned above, non banks have played an unprecedented central role in the US corporate credit boom that followed the 2008 crisis. Leveraged payouts during this boom have reached record high volumes whereas business investment has remained disappointing.

Implications for optimal monetary policy. We show that when it cannot regulate leverage, the central bank optimally targets a strictly smaller investment level than when it can regulate leverage. Stimulating investment with low rates comes at the cost of inducing leveraged payouts, which reduce entrepreneurs' incentives and thus productive efficiency. A smaller investment target compared to the first-best optimally trades off scale and productive efficiency. If the pass-through from monetary policy to investment level is rather muted, as observed recently,[2] then the optimal monetary policy may even consist of "leaning against the wind," i.e., not stimulating the economy at all, in order to fully contain leveraged payouts and maintain productive efficiency. This result is akin to Stein (2012), who argues that in the presence of some unchecked credit growth in the shadow-banking system, a monetary policy that leans against the wind can be optimal as it raises the cost of borrowing in all "cracks" of the financial sector.

Related literature

Our paper revisits the notion of "malinvestment" that has been prominent in Austrian economics (Hayek, 1931, for example). Malinvestment refers to the possibility that distortion of the real interest rate due to monetary easing

[2]Besides Furman (2015), see also the evidence presented for the United States by Wang (2019), who documents a weak pass-through of monetary policy to bank lending rates for the past two decades, especially so at low interest rates. See also the discussion and references in Wang (2019) for similar evidence of a weak pass-through of negative interest rates to the real economy in case of Europe and Japan.

subsidizes activities that are not socially desirable (but become privately profitable) at the expense of preferable investments. We are the first, to our knowledge, to connect the current fierce debate on the social optimality of leveraged share buybacks to this old idea of malinvestment.

Our paper also relates to two more recent strands of literature.

First, Bolton et al. (2016), Martinez-Miera and Repullo (2017) or Boissay et al. (2016) offer like us models in which a low cost of funds may be detrimental to incentives in the private sector. Whereas a low cost of capital is due to positive supply shocks in their setups, it stems from an optimal monetary policy decision aimed at stimulating the economy after a negative shock in our setup.

Second, we argue in this paper that this relation between cost of capital and incentives explains why low policy rates may fail to stimulate investment. Several recent contributions suggest alternative causes for this failure of monetary easing to spur investment. Brunnermeier and Koby (2018) show that this may stem from eroded lending margins in an environment of imperfectly competitive banks. Coimbra and Rey (2017) study a model in which the financial sector is comprised of institutions with varying risk appetites. Starting from a low interest rate, further monetary easing may increase financial instability, thereby creating a trade-off with the need to stimulate the economy. A distinctive feature of our approach is that we *jointly* explain low investment and high leveraged payouts by corporates.

The paper is organized as follows. As a stepping stone, Section 1 presents a partial-equilibrium model of optimal investment and consumption-smoothing in the presence of moral hazard. Section 2 embeds it in a full-fledged equilibrium model to determine the optimal monetary policy and derives the main results. Section 3 presents concluding remarks.

1 Cost of capital, investment, and leveraged payouts

Consider an economy with a single consumption good and two dates indexed by $t \in \{0; 1\}$. An entrepreneur has access to an investment technology that transforms I date-0 consumption units into a number of date-1 units equal to $f(I)$ with probability e, and to zero with the complementary probability, where f satisfies the Inada conditions. The entrepreneur controls e, the probability of success of her investment, at a private cost $e^2 f(I)/(2\pi)$ that is subtracted from her date-0 utility over consumption, where $\pi \in (0, 1)$.[3] The entrepreneur is risk-neutral over consumption at dates 0 and 1 and does not discount date-1 consumption at date 0. She has a large date-0 endowment of the consumption good $W > 0$. She can trade securities with risk-neutral counterparties that require a gross expected return $r > 0$ between dates 0 and 1.

The rest of this section solves for the entrepreneur's utility-maximization problem, discussing in turn the cases in which the entrepreneur's cost of capital r is larger or smaller than her (unit) discount rate.

Suppose first that $r \geq 1$. The entrepreneur in this case uses her own date-0 resources to fund the investment I in technology f, and uses the residual $(W - I)$ to purchase securities earning the expected return r. She selects the investment level I and the effort level e that solve

$$\max_{e,I} \left\{ \left(e - \frac{e^2}{2\pi} \right) f(I) + r(W - I) \right\} \tag{1}$$

[3]The linearity of effort cost with respect to output size plays no other role than simplifying the algebra.

maximized at $\left(\hat{e},\hat{I}\right)$ such that

$$\hat{e}=\pi,\ \frac{\pi}{2}f'(\hat{I})=r. \tag{2}$$

In the case $r\geq 1$, the probability of success π does not depend on the cost of capital r. Both investment $\hat{I}$ and expected output $\pi f(\hat{I})$ decrease with respect to r.

Leveraged payouts. Consider now the case in which $r<1$. Given her unit discount factor, the entrepreneur would like to borrow at the rate r against the date-1 consumption that she can generate out of the technology f. Such borrowing is akin to a leveraged payout, whereby the entrepreneur sets up a firm that runs the investment in the technology f at date 0, and then lets this firm borrow against its expected future cash flows to buy back shares from the entrepreneur or pay her a special dividend.[4]

More precisely, the timing is as follows. The entrepreneur announces an investment level I, an effort level e, and a leverage $(1-x)$ against her output, where $x\in[0,1]$ is the fraction of the output against which she does not borrow — her "skin in the game." Investors buy her bonds. The entrepreneur consumes the proceeds and then exerts private effort. The entrepreneur selects (e,I,x) that maximizes her expected utility subject to the effort level e being incentive-compatible. Formally, she solves

$$\max_{e,I,x}\left\{\frac{(1-x)ef(I)}{r}+W-I+\left(xe-\frac{e^2}{2\pi}\right)f(I)\right\} \tag{3}$$

[4]Dividends and share buybacks are equivalent in this environment that abstracts from any informational or differential tax considerations relating to the two forms of shareholders payouts.

s.t.

$$e = \arg\max_{y} \left\{ xy - \frac{y^2}{2\pi} \right\}. \tag{4}$$

Date-0 consumption is the sum of the endowment net of investment $(W - I)$ and of the present expected value of the fraction $(1 - x)$ of output against which the entrepreneur borrows at the rate r. Date-1 expected consumption is the expected retained output $xef(I)$. Condition (4) is the incentive-compatibility constraint, stating that the announced effort e must maximize the entrepreneur's date-1 consumption net of effort costs. Simple algebra[5] yields the respective first-order conditions with respect to e, x, I:

$$e = \frac{\pi}{2 - r}, \tag{5}$$

$$x = \frac{1}{2 - r}, \tag{6}$$

$$\frac{\pi f'(I)}{2(2 - r)} = r. \tag{7}$$

They imply that in the case with $r < 1$, a lower cost of capital r induces an increase in leveraged payouts (a lower value of the skin in the game x). Furthermore, since a lower r induces both a lower probability of success $e = \pi/(2 - r)$ and a higher investment $I = f'^{-1}(2r(2 - r)/\pi)$, the overall impact of a reduction in r on expected output $ef(I)$ is ambiguous. Suppose for example that $f(I) = I^{1/\gamma}$, where $\gamma > 1$. We show in the appendix that the expected output increases in r for $r \in [2/(\gamma + 1), 1]$, and decreases otherwise.

The following proposition collects the above results.

Proposition 1. ***(Cost of capital, investment, and leveraged payouts)*** *Let $\overline{r}(r) = \min\{r; 1\}$. The entrepreneur chooses skin in the game x,*

[5] See proof of Proposition 1 in the Appendix.

effort e, and investment I, such that

$$x = \frac{1}{2-\overline{r}(r)}, e = \pi x = \frac{\pi}{2-\overline{r}(r)}, \frac{\pi f'(I)}{2(2-\overline{r}(r))} = r. \tag{8}$$

Thus,

- *For $r \in (1, +\infty)$, a reduction in the cost of capital r is irrelevant for corporate leverage, payout policy, and incentives. It spurs investment and expected output.*
- *For $r \leq 1$, a reduction in the cost of capital r spurs leveraged payouts that reduce the entrepreneur's incentives and thus degrade asset quality; investment is less sensitive to r than in the case $r > 1$.*

Proof. See the appendix. ■

The entrepreneur's linear preferences induce a sharp difference between the two cases discussed in Proposition 1. This permits a clear and simple exposition of the important intuition behind our results.[6] In the case $r > 1$, fluctuations in the cost of capital only affect corporate investment I. When $r < 1$, by contrast, the cost of capital affects corporate leverage as well, even though the entrepreneur has all the internal liquidity W needed for investment. Leveraged payouts reduce incentives and thus shift the entire production function downwards.

The next section embeds this partial-equilibrium model with exogenous cost of capital into a model in which a central bank controls the real rate and thus firms' cost of capital in the presence of nominal rigidities. The central bank seeks to maximize a standard social welfare function, and sets its policy rate so as to mitigate the distortions induced by sticky prices.

[6]The broad qualitative insights would carry over under strict concavity.

2 Investment, leveraged payouts, and optimal monetary policy

2.1 Setup

Time is discrete. There is a single consumption good that serves as numéraire. There are two types of private agents, workers and entrepreneurs, and a public sector.

Workers. At each date, a unit mass of workers are born and live for two dates. They derive utility from consumption only when old, and are risk-neutral over consumption at this date. Each worker supplies inelastically one unit of labor when young in a competitive labor market. Each worker also owns a technology that transforms l units of labor into $g(l)$ contemporaneous units of the consumption good.

Entrepreneurs. At each date, a unit mass of entrepreneurs are born and live for two dates. Entrepreneurs are essentially identical to that in the previous section. They are risk-neutral over consumption at each date and do not discount future consumption. They are born with a large endowment W of the numéraire good.[7] Each entrepreneur born at date t is endowed with a technology that transforms l units of labor at date t into $f(l)$ consumption units at the next date $t+1$ with probability e, and zero units with the complementary probability.[8] Entrepreneurs control the probability of success e at a private cost $e^2 f(l)/(2\pi)$ that is subtracted from their utility when young.

The technology f features a lag between production and delivery of con-

[7]We could endogenize this endowment as labor income at some additional complexity and without gaining insights.

[8]The joint distribution of entrepreneurs' outcomes is immaterial.

sumption services. This technology thus stands in our stylized model for the most interest-sensitive sectors of the economy such as durable-good, housing or capital-good sectors. We accordingly deem technology f the *capital-good sector*, and technology g the *consumption-good sector*.[9] We also term *investment* the resources spent to produce the capital good.

The functions f and g satisfy the Inada conditions and f is twice continuously differentiable.

Public sector. The public sector does not consume. It maximizes the sum of the utilities of agents in the private sector, discounting that of future generations with a factor arbitrarily close to 1.

Bond market. There is a competitive market for one-period bonds denominated in the numéraire good.

Monetary policy. The public sector announces at each date a rate of return at which it is willing to trade arbitrary quantities of bonds.

Fiscal policy. The public sector can tax workers as it sees fit. It can in particular apply lump-sum taxes. However, it cannot tax entrepreneurs. This latter assumption is made stark in order to yield a simple and clear exposition of our results. As detailed below, all that matters is that the public sector does not have a free hand at regulating entrepreneurs' behavior with appropriate tax schemes. In particular, it cannot use taxation as a substitute for prudential regulation. One possible reason entrepreneurs cannot be taxed is that they can operate in a different jurisdiction.

Relationship to new Keynesian models. This setup can be described as a much simplified version of a new Keynesian model in which money serves only as a unit of account ("cashless economy") and monetary policy consists

[9] A full-fledged model of f as a capital-good technology would require that the date-t investment be combined with labor at date $t+1$ in order to generate consumption. This would complicate the analysis without adding substantial insights.

of enforcing the short-term nominal interest rate. Such monetary policy has real effects in the presence of nominal rigidities. We entirely focus on these real effects; in particular, we fully abstract from price-level determination by assuming extreme nominal rigidities in the form of a fixed price level for the consumption good. This will enable us in what follows to tractably introduce ingredients that are typically absent from mainstream monetary models.[10]

2.2 Steady state

We first study steady states in which the public sector announces a constant gross interest rate r at each date. We suppose that the public sector offsets its net position in the bond market at each date with a lump-sum tax or rebate on current old workers. We denote $w \geq 0$ the steady-state wage, and $l \in [0, 1]$ the steady-state quantity of labor used by entrepreneurs. The steady state associated with the policy rate r can then be characterized as follows.

Entrepreneurs. Up to the change of variable $I = wl$, each entrepreneur's problem is identical to that in Section 1. As in Section 1, we denote x the skin in the game of an entrepreneur and $\overline{r}(r) = \min\{r; 1\}$. Each entrepreneur's objective is then

$$\max_{e,l,x} \left\{ (1 + r - \overline{r}(r)) \left[\frac{(1-x)ef(l)}{r} + W - wl \right] + \left(xe - \frac{e^2}{2\pi} \right) f(l) \right\} \quad (9)$$

s.t.

$$e = \arg\max_{y} \left\{ xy - \frac{y^2}{2\pi} \right\}. \quad (10)$$

[10] In somewhat related setups, Benmelech and Bergman (2012), Caballero and Simsek (2019) or Farhi and Tirole (2012) also abstract from price-level determination as we do. Their focus is, however, on the financial-stability implications of monetary policy.

Expression (9) for entrepreneurs' surplus subsumes (1) and (3). From Proposition 1, each entrepreneur chooses e, x, and l such that

$$x = \frac{1}{2 - \overline{r}(r)}, e = \pi x, \frac{\pi f'(l)}{2(2 - \overline{r}(r))} = rw. \tag{11}$$

Furthermore, taking into account that $x = 1$ whenever $r \geq 1$, one can write an entrepreneur's net position in the bond market when young as:

$$\mathbb{1}_{\{r \geq 1\}}(W - wl) - \frac{(1 - x)ef(l)}{r}. \tag{12}$$

Workers. Young workers' income is comprised of labor income in the capital-good sector wl, labor income in the consumption-good sector $w(1-l)$, and profits from the consumption-good sector $g(1-l) - w(1-l)$. These latter profits are maximum when

$$g'(1 - l) = w. \tag{13}$$

Since they consume only when old, workers invest the resulting total income

$$g(1 - l) + wl \tag{14}$$

in the bond market thereby receiving a pre-tax income

$$r[g(1 - l) + wl] \tag{15}$$

when old.

Public finances. The government stands ready to trade bonds at the announced rate r. It balances its budget by rebating a lump-sum (possibly negative) to old workers.

Equations (11) and (13) uniquely determine the steady-state values of (x, e, l, w) for a given interest rate r. In turn, the surplus of a given cohort, for such an interest rate, is:

$$\underbrace{(1 + r - \overline{r}(r)) \left(\frac{(1-x)ef(l)}{r} + W - wl \right) + \left(xe - \frac{e^2}{2\pi} \right) f(l)}_{\text{Entrepreneurs' surplus}}$$

$$+ \quad \underbrace{rwl + rg(1-l)}_{\text{Old workers' pre-tax income}} \tag{16}$$

$$+ \underbrace{(1-r) \left[\mathbb{1}_{\{r \geq 1\}} (W - wl) - \frac{(1-x)ef(l)}{r} + g(1-l) + wl \right]}_{\text{Rebate to old workers}} \tag{17}$$

$$= W + \left(e - \frac{e^2}{2\pi} \right) f(l) + g(1-l). \tag{18}$$

The entrepreneurs' surplus is given by expression (9) and old workers' pre-tax income by (15). The net demand for government bonds at each date is the sum of workers' savings (14) and entrepreneurs' net position in the bond market (12). The government rebates to old workers this amount net of the repayment of bonds issued in the previous period (equal to r times this net demand).[11]

The social costs of leveraged payouts. An important remark is in order before solving for the optimal steady-state interest rate. Note from expression (18) that the interest rate r affects social surplus only indirectly through its impact on the values of e and l that entrepreneurs choose in equilibrium. Entrepreneurs' surplus by contrast also directly depends on r from (10). In particular, when $r < 1$, entrepreneurs directly benefit from lower interest

[11]We assume for brevity throughout the paper that old workers' consumption (16)+(17) is always positive in the relevant range of the interest rate. It is easy to see that this is so as long as workers earn a sufficiently large amount of total income at each date.

rates through higher leveraged payouts. Proposition 1 describes how they optimally trade off the benefits from such leveraged payouts with the negative impact of reduced incentives on their expected output. Expression (18) shows that this trade-off is privately but, however, not socially optimal. Weaker incentives leading to a reduced expected output are social costs whereas early payouts for consumption are only transfers from old workers towards young entrepreneurs that are neutral given the assumed social welfare function. If some agents (entrepreneurs) benefit from transferring consumption across dates at a rate different from one, then other agents (workers here) have to pay for it. The impossibility to tax entrepreneurs implies indeed that their leveraged payouts must be financed by taxes on old workers. In short, leveraged payouts are in this model a form of inefficient rent extraction by entrepreneurs that is detrimental both to savers, as it redistributes resources away from them, and to social welfare, as it results in a reduced expected output. Notice that if their gains from leveraged payouts were compensated for by a lump-sum tax on entrepreneurs, then this would eliminate the welfare-neutral redistribution from workers to entrepreneurs, yet this would leave unchanged the incentive-based distortion in output.

We now solve for the optimal steady-state interest rate. Expression (18) implies that the public sector optimally seeks to implement effort and investment (e^*, l^*) such that $e^* = \pi$ and $\pi f'(l^*)/2 = g'(1 - l^*)$. Given that profit maximization implies

$$g'(1 - l) = w \tag{19}$$

in the consumption-good sector and

$$\frac{\pi f'(l)}{2(2-\overline{r}(r))} = rw \tag{20}$$

in the capital-good one, the public sector can reach (e^*, l^*) by setting the rate $r^* = 1$. The optimality of an interest rate equal to the (unit) growth rate of the population of course relates to the "golden rule" maximizing steady-state utility in overlapping-generations models. Note that at this unit optimal rate, inflows and outflows in the bond market exactly offset each other so that the net rebate to old workers is zero.

2.3 Monetary easing

Suppose now that one cohort of workers — the one born at date 0, say — has a less productive technology than that of its predecessors and successors. Unlike the other cohorts, their technology transforms y units of labor into $\rho g(y)$ contemporaneous units of the consumption good, where $\rho \in (0, 1)$. We study in turn the implications of such time-varying productivity for optimal policy and welfare in three different contexts with incremental frictions:

1. The wage w is flexible.
2. The wage is downward rigid and the public sector can regulate private leverage.
3. The wage is downward rigid and the public sector cannot regulate private leverage.

2.3.1 Flexible-wage benchmark

Proposition 2. ***(Laissez-faire is optimal when the wage is flexible)*** *If the wage is flexible, the public sector implements the first-best by setting the interest rate at the steady-state level $r^* = 1$ at each date. At this rate, there is no need to regulate leverage.*

The cohort born at date -1 subsidizes that born at date 0. There are no other transfers across cohorts.

Proof. Let us introduce the notation $\rho_t = 1 + (\rho - 1)\mathbb{1}_{\{t=0\}}$. We also use the subscripted notation $(e_t, x_t, l_t, w_t, r_t)$ to denote the values of (e, x, l, w, r) for the cohort born at date t out of the steady state.

The social welfare function assigns the same weight to every unit of consumption no matter who consumes it and when, as well as to private costs of effort no matter when they are incurred. The first-best is thus reached when the output of cohort t net of effort costs

$$\left(e_t - \frac{e_t^2}{2\pi}\right) f(l_t) + \rho_t g(1 - l_t) \tag{21}$$

is maximum for all t, or

$$e_t = \pi, \rho_t g'(1 - l_t) = \pi f'(l_t)/2. \tag{22}$$

With a flexible wage, setting $r_t = 1$ for all t implements the first-best. This induces $x_t = 1$. Profit maximization in both sectors and labor-market clearing then imply

$$e_t = \pi, \tag{23}$$

$$\rho_t g'(1 - l_t) = w_t = r_t w_t = \pi f'(l_t)/2, \tag{24}$$

which characterizes the first-best from (22).

The proof that the only net transfer across cohorts is that from the date-(-1) cohort towards the date-0 one is in the appendix. ■

When the wage is flexible, the steady-state unit interest rate $r^* = 1$ is unsurprisingly optimal at all dates even in the presence of time-varying productivity. From (24), the date-0 wage adjusts to a level $w_0 < w^*$ such that the employment level in the capital-good sector l_0 is above l^*. For the remainder of the paper, we respectively denote l_ρ and w_ρ this first-best date-0 employment level and the associated market wage in this case of a flexible wage.

Time-varying productivity only has a redistributive effect across the cohorts born at -1 and 0 that is immaterial given our social welfare function. The savings of agents born at date 0 and thus facing a less productive economy do not suffice to repay the bonds of old date-(-1) agents that are due at date 0, and so these latter old agents must pay a tax. Workers born at date 0 conversely receive a matching rebate once old at date 1, as savings from date-1 born agents are back to the higher steady-state value.[12]

2.3.2 Rigid wage and regulated leverage

We introduce for the remainder of the paper an additional friction in this economy in the form of a rigid wage:

Assumption. ***(Downward-rigid wage)*** *The wage cannot be smaller than the steady-state wage w^* at date 0.*

In other words, we suppose that the wage is too downward rigid to track the transitory negative productivity shock that hits the date-0 cohort, and

[12] A public sector averse to intergenerational inequality (unlike ours) could of course smooth these transfers in an international capital market.

that the public sector cannot regulate it in the short run.[13]

In preparation for our main result, we first suppose here that the public sector not only sets the interest rate at each date and taxes workers, but can also control entrepreneurs' leverage. The following proposition shows that in this case, the combination of a reduction in the date-0 interest rate and of a prudential regulation enforcing that entrepreneurs do not borrow at this date implements the first-best, albeit through higher transfers from cohort -1 to cohort 0 than under a flexible wage.

Proposition 3. ***(Monetary easing and prudential regulation implement the first-best)*** *The public sector implements the first-best outcome with the following policy:*

- *It sets $r^* = 1$ at all dates other than 0 and thus need not regulate leverage at these dates.*
- *It sets $r_\rho = w_\rho / w^* < 1$ at date 0 and imposes $x_0 = 1$ to young date-0 entrepreneurs.*

The cohort born at date -1 subsidizes that born at date 0, more so than under flexible wage. There are no other transfers across cohorts.

Proof. First-order conditions for profit maximization (19) and (20) show that the capital-good sector is interest-rate sensitive whereas the consumption-good sector is not. The public sector can accordingly make up for the absence of appropriate price signals in the date-0 labor market by distorting the date-

[13] We could also assume a partial wage adjustment without affecting the analysis. Note also that the analysis would be similar if the date-0 productivity shock was permanent ("secular stagnation"). All that would matter in this case would be the number of periods it takes for the wage to adjust to the level that is optimal given the productivity shock.

0 capital market. By setting the date-0 policy rate at

$$r_\rho = \frac{w_\rho}{w^*} < 1, \tag{25}$$

and imposing $x_0 = 1$ at date 0, the public sector implements the flexible-wage outcome in the labor market. Entrepreneurs hire up to the optimal level l_ρ since they face under this policy the same first-order condition as when the wage is flexible and $r^* = 1$:

$$\frac{\pi}{2} f'(l_\rho) = r_\rho w^* = w_\rho. \tag{26}$$

Each worker accommodates by applying in her own firm the residual quantity of labor that she cannot sell on the labor market at the disequilibrium wage w^*. She does so at a marginal return below wage ($\rho g'(1 - l_\rho) = w_\rho < w^*$), and produces at the socially optimal level by doing so. ■

Note that the combination of date-0 monetary easing and leverage regulation maximizes the social welfare function, but that it implies more subsidy to date-0 entrepreneurs from date-(-1) workers. This owes to the fact that such young entrepreneurs, facing a rate $r_\rho < 1$, prefer to consume their endowment when young rather than save it, and the public sector must make up for this lower demand for bonds with higher date-0 taxes on old workers.

2.3.3 Rigid wage and unregulated leverage

Suppose now that the public sector no longer has the ability to regulate entrepreneurs' leverage. This corresponds to an economy in which a significant fraction of credit activity can be considered to take place in an unregulated shadow-banking system. The following proposition, where we employ the subscript u to denote outcomes under the rigid-wage and unregulated-leverage case, shows that monetary easing in this case not only induces leveraged payouts but also a lack of investment that puts the first-best out of reach.

Proposition 4. ***(Rigid wage and unregulated leverage)***

1. *The optimal interest rates are* $r^* = 1$ *at all dates other than 0 and* $r_u \leq 1$ *at date 0.*

2. *Surplus is strictly lower when leverage is not regulated than when it is because date-0 investment is strictly lower: Entrepreneurs use a quantity of labor* l_u *strictly smaller than the first-best one* l_ρ*.*

3. *The cohort born at date* -1 *subsidizes that born at date 0, more so than under rigid wage and regulated leverage. There are no other transfers across cohorts.*

Proof. See the appendix. ■

From (11), in the absence of leverage regulation, the skin in the game of an entrepreneur x and thus her effort e (strictly) increase in r for $r < 1$. As a result, attempts at spurring investment/employment in the capital-good sector with a reduction in the date-0 interest rate boost leveraged payouts and degrade productive efficiency. This unintended consequence of monetary easing implies that social surplus is maximized at a lower date-0 use of labor in the capital-good sector l_u than in the presence of a prudential regulation imposing $x = 1$: $l_u < l_\rho$. In this sense, lack of investment relative to the first-best is part of a second-best policy in the absence of a strict prudential regulation. Also, monetary easing is more anti-redistributive in the sense that date-0 leveraged payouts by young entrepreneurs lead to an issuance of corporate debt that crowds out public bonds and forces the public sector to raise more taxes on old workers than under regulated leverage.[14]

Should monetary easing be more or less aggressive in the presence of unregulated leverage? Interestingly, investigating whether the optimal date-0 interest rate r_u—the one that leads entrepreneurs to choose l_u—is lower or higher than the optimal date-0 rate in the presence of regulated leverage, r_ρ, reveals an important tradeoff. On the one hand, investment is less sensitive to the interest rate when leverage is unregulated, which implies setting a lower interest rate than in the presence of leverage regulation in order to reach a given target level for l.[15] On the other hand, as we just stated, the target for employment in the capital-good sector should be lower in the absence of leverage regulation—$l_u < l_\rho$, which goes in the direction of setting $r_u > r_\rho$ as less stimulation is needed. The following proposition shows that the dominant effect depends on the size of the shock ρ:

Proposition 5. ***(Optimal interest rate)*** *There exists $\bar{\rho} \in [0, 1)$ such that*

- *If, ceteris paribus, $\rho \geq \bar{\rho}$, then it is optimal to ignore the shock ρ and leave the date-0 interest rate at its steady-state value: $r_u = r^* = 1 > r_\rho$. Investment is strictly below the first-best level but productive efficiency is at the first-best ($l_u < l^*$ but $e^* = \pi$).*

- *If $\bar{\rho} > 0$, then for $\rho \in (0, \bar{\rho})$ the optimal monetary policy is more accommodative than when leverage is regulated: $r_u < r_\rho$. Investment and productive efficiency are both strictly below their first-best levels ($l_u < l^*$ and $e^* < \pi$).*

[14] In the presence of an international capital market, this deficit could of course be financed with debt issuance rather than with current higher taxes on workers.

[15] In the absence of leverage regulation a given investment level l is reached by setting r such that $\pi f'(l) = 2w^* r(2 - r)$ whereas r is such that $\pi f'(l) = 2rw^*$ for $x = 1$.

Proof. See the appendix. ■

Proposition 5 shows how the optimal interest rate trades off productive efficiency e and scale l in the capital sector. If ρ is sufficiently large (the shock is small), it is always optimal to avoid any leveraged payout by leaving the rate at $r^* = 1$, thereby preserving productive efficiency $e^* = \pi$ at the cost of investing at a scale smaller than the first-best. It may be that this policy is actually optimal for all possible shocks (case $\bar{\rho} = 0$). Consider for example the limiting case in which the function f is constant. In this case, a reduction in the interest rate has only an adverse effect on productive efficiency and no impact on scale. It is thus undesirable to cut the interest rate below one no matter the size of the shock.

Stein (2012) argues that in the presence of some unchecked credit growth in the shadow-banking system, a monetary policy that leans against the wind can be optimal as it raises the cost of borrowing in all "cracks" of the financial sector. This resonates with our result that the optimal policy response to sufficiently small productivity shocks—and possibly for all shocks— consists in "leaning against the wind" this way, and setting $r_u = 1$.

The proof of Proposition 5 offers formal examples in which $\bar{\rho}$ is either equal to zero or strictly positive. In this latter case, there is a discontinuity in the stance of monetary policy as ρ becomes smaller than $\bar{\rho}$. It becomes preferable in this case to spur l even though this comes at an important cost for productive efficiency. In this case, there is aggressive monetary easing that still has a limited impact on investment, and generates instead a surge in leveraged payouts, that in turn induce low productive efficiency.

3 Concluding remarks

This paper has shown that a standard hidden-effort problem combined with hidden leverage suffices to explain why ultra-aggressive monetary easing may lead to a surge in corporate borrowing aimed at funding shareholder payouts rather than business investment. Whereas we aimed at delivering these insights in the simplest possible setup, future research could enrich the analysis along several routes discussed below.

Shadow banking. We interpret the respective polar cases of regulated (Section 2.3.2) and unregulated (Section 2.3.3) leverage as respectively the situation in which the financial sector is mostly comprised of banks subject to prudential regulation and that in which a large shadow-banking sector operates. An interesting route for future research consists of studying the intermediate situation in which the regulation of leverage can only be imperfectly enforced, and examining the interplay of such imperfect enforcement with the crowding-out of investment by financial risk-taking highlighted here.[16]

Taxing entrepreneurs. Whereas we assume that entrepreneurs cannot be taxed at all for expositional simplicity, our results rely only on the assumption that the public sector does not have a free hand at taxing them. If entrepreneurial taxation were to be unbridled, then it would be easy to deter socially inefficient leveraged share buybacks, for example through the taxation of date-0 consumption by entrepreneurs or of corporate debt. An interesting route for future research consists in studying the situation in which such taxation is distortive or/and can only be imperfectly enforced.[17]

[16] Plantin (2015) develops a model of leverage regulation under imperfect enforcement.

[17] Landier and Plantin (2017) offer a model of optimal capital taxation under imperfect enforcement.

Zero lower bound and asset purchases. In the face of a zero lower bound (ZLB) on policy rates, the Federal Reserve (and central banks of several developed economies) responded to the 2008 crisis with unconventional policies that include the purchase of private claims such as mortgage-related securities (and corporate debt). Suppose that the public sector is subject to a similar ZLB in our setup: It cannot set the date-0 rate below $r^* = 1$.[18] The public sector can still enter into asset purchases, swapping date-0 entrepreneurs' claims to their date-1 output with public bonds, akin to remunerated excess reserves. Such swaps spur investment at date 0: If the public sector trades $1/r_0$ bonds for each date-1 consumption unit, then this amounts to grant a lower interest rate to date-0 entrepreneurs. Such asset purchases, however, have the same adverse implications for incentives as interest-rate reductions because they reduce entrepreneurs' skin in the game in the very same way as in our case of unregulated leverage.

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[18] For example, because the private sector can secretly store with a unit gross return.

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Appendix

Proof of Proposition 1

The case $r \geq 1$ is straightforward and derived in the body of the paper. In the case $r < 1$, in order to derive the conditions in (7), notice first that (4) implies $e = \pi x$. Plugging this into (3), the objective becomes

$$\frac{\pi x[2-(2-r)x]}{2r} f(I) + W - I, \tag{27}$$

and first-order conditions with respect to x and I yield the two remaining conditions in (7).

Suppose $f(I) = I^{1/\gamma}$. When $r < 1$, the expected output is

$$ef(I) = \left(\frac{\pi}{2-r}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{2r}\right)^{\frac{1}{\gamma-1}}, \tag{28}$$

and standard derivation yields its variations with respect to r. ■

Proof of Proposition 2

The only result that is not established in the body of the paper regards transfers across cohorts. For any t, in the absence of leverage regulation, the surplus of a date-t cohort is

$$\begin{aligned}
&(1+r_t-\overline{r}(r_t))\left[\frac{(1-x_t)e_tf(l_t)}{r_t}+W-w_tl_t\right]+r_tw_tl_t+r_t\rho_tg(1-l_t)+\\
&\mathbb{1}_{\{r_{t+1}\geq 1\}}(W-w_{t+1}l_{t+1})-\frac{(1-x_{t+1})e_{t+1}f(l_{t+1})}{r_{t+1}}+w_{t+1}l_{t+1}+\rho_{t+1}g(1-l_{t+1})\\
&-r_t\left[w_tl_t+\rho_tg(1-l_t)+\mathbb{1}_{\{r_t\geq 1\}}(W-w_tl_t)-\frac{(1-x_t)e_tf(l_t)}{r_t}\right]
\end{aligned}\tag{29}$$

The first line in (29) is the consumption of entrepreneurs plus old workers' pre-tax income. The next two lines are the lump-sum rebated to old workers, comprised of the net savings of the next cohort (second line) minus the repayment of outstanding bonds to the private sector (third line).

From (29), straightforward computations show that under the optimal policy, the surplus of each cohort born at any date $t \notin \{-1;0\}$ is given by

$$W+\frac{\pi f(l^*)}{2}+g(1-l^*),\tag{30}$$

whereas that of cohort -1 is

$$W+\frac{\pi f(l^*)}{2}+\rho g(1-l_\rho),\tag{31}$$

and that of cohort 0 equals

$$W+\frac{\pi f(l_\rho)}{2}+g(1-l^*).\tag{32}$$

Comparing the surpluses of cohorts 0 and -1 with their respective outputs shows that cohort -1 pays a subsidy equal to $g(1-l^*) - \rho g(1-l_\rho)$ to cohort 0. ■

Proof of Proposition 3

From (29), and accounting for leverage regulation, straightforward computations show that the surplus of each cohort born at any date $t \notin \{-1; 0\}$ is given by

$$W + \frac{\pi f(l^*)}{2} + g(1-l^*), \tag{33}$$

whereas that of cohort -1 is

$$w^* l_\rho + \frac{\pi f(l^*)}{2} + \rho g(1-l_\rho), \tag{34}$$

and that of cohort 0 equals

$$W + \frac{\pi f(l_\rho)}{2} + g(1-l^*) + W - w^* l \rho. \tag{35}$$

Cohort -1 thus pays a subsidy equal to $g(1-l^*) - \rho g(1-l_\rho) + W - w^* l_\rho$ to cohort 0, larger than under flexible wage. This is due to the fact that young date-0 entrepreneurs are unwilling to save $(W - w^* l_\rho)$ at the rate $r_\rho < 1$, and prefer instead to consume this when young. This forces the public sector to collect this additional amount from old date-0 workers. ■

Proof of Proposition 4

Proof of points 1. and 2. Setting $r_t = 1$ maximizes (21) for all $t \neq 0$. Regarding the date-0 cohort, the optimal rate $r \leq 1$ maximizes the output net of effort of the cohort:

$$\Sigma_\rho(r) = \left(e(r) - \frac{e(r)^2}{2\pi}\right) f\left(l(r)\right) + \rho g(1 - l(r)), \tag{36}$$

where relations (11) implicitly define $e(r)$ and $l(r)$, which are obviously differentiable with respect to r, respectively increasing and decreasing. Straightforward computations yield:

$$\Sigma'_\rho(r) = \frac{\pi(1-r)f(l(r))}{(2-r)^3} + \left[\frac{\pi(3-2r)f'(l(r))}{2(2-r)^2} - \rho g'(1 - l(r))\right] l'(r). \tag{37}$$

For r'_ρ such that $l(r'_\rho) = l_\rho$, we have by definition of l_ρ that $\pi f'(l(r'_\rho))/2 = \rho g'(1 - l(r'_\rho))$, which implies

$$\Sigma'_\rho(r'_\rho) = \frac{\pi(1-r'_\rho)f(l(r'_\rho))}{(2-r'_\rho)^3} - \frac{\pi}{2}\left(\frac{1-r'_\rho}{2-r'_\rho}\right)^2 f'(l(r'_\rho))l'(r'_\rho) > 0, \tag{38}$$

implying in turn points 1. and 2. in the proposition ($l_u < l_\rho$).

Proof of point 3. From (29), straightforward computations show that the surplus of each cohort born at any date $t \notin \{-1; 0\}$ is given by

$$W + \frac{\pi f(l^*)}{2} + g(1 - l^*), \tag{39}$$

whereas that of cohort -1 is

$$w^* l_u + \frac{\pi f(l^*)}{2} + \rho g(1 - l_u) - \frac{\pi(1 - r_u)f(l_u)}{r_u(2 - r_u)^2}, \tag{40}$$

and that of cohort 0 equals

$$W + \frac{\pi f(l_\rho)}{2} + g(1 - l^*) + W - w^* l_u + \frac{\pi(1 - r_u)f(l_u)}{r_u(2 - r_u)^2}. \tag{41}$$

Cohort -1 thus pays a subsidy equal to $g(1-l^*) - \rho g(1-l_u) + W - w^* l_u + \pi(1-r_u)f(l_u)/[r_u(2-r_u)^2]$ to cohort 0, larger than under rigid wage and regulated leverage. This is due to the fact that young date-0 entrepreneurs consume an additional $\pi(1-r_u)f(l_u)/[r_u(2-r_u)^2]$ when young borrowed against their date-1 output, which forces the public sector to collect this additional amount from old date-0 workers. ■

Proof of Proposition 5

Step 1. It is optimal to set $r_u = 1$ for ρ sufficiently large.

Differentiating

$$\frac{\pi f'(l(r))}{2(2-\overline{r}(r))} = rw \tag{42}$$

w.r.t. r for $r \in (0,1)$ yields

$$l'(r) = \frac{4w^*(1-r)}{\pi f''(l(r))}, \tag{43}$$

and so one can write

$$\Sigma'_\rho(r) = (1-r)\left[\underbrace{\frac{\pi f(l(r))}{(2-r)^3}}_{A} + \frac{4w^*}{\pi f''(l(r))}\left[\underbrace{\frac{\pi(3-2r)f'(l(r))}{2(2-r)^2} - \rho g'(1-l(r))}_{B}\right]\right] \tag{44}$$

We have $\lim_{r\to 1} l(r) = l^*$, and so for (ρ, r) sufficiently close to $(1,1)$, term B becomes arbitrarily close to 0 from the first-best condition $\pi/2f'(l^*) = g'(1-l^*)$. Term A on the other hand stays bounded away from 0 for (ρ, r) in the neighborhood of $(1,1)$, and thus $\Sigma' > 0$ in this neighborhood. Furthermore, a standard continuity argument implies that $\lim_{\rho\to 1} r_u = 1$. As a result, $\Sigma'(r_u)$ must be strictly positive for ρ sufficiently close to 1, implying that (r_u, l_u) is actually equal to $(1, l^*)$ for ρ sufficiently close to 1.

Step 2. Existence of $\bar{\rho}$.

Let $\underline{r}$ denote the value of r such that (42) yields $l(\underline{r}) = 1$. Let Ω denote the subset of values of $\rho \in (0,1)$ such that the maximum of $\Sigma_\rho(r)$ over $r \in [\underline{r}, 1]$ is interior, that is, such that it is reached at some $r \in (\underline{r}, 1)$. We know from Step 1 that $r_u = 1$ for ρ sufficiently large. This implies that $\Omega \neq (0,1)$, and therefore that $\bar{\rho}$, if it exists, is strictly smaller than 1.

If $\Omega = \emptyset$, this means that $\Sigma_\rho(r)$ is maximum at $r = 1$ for every $\rho \in (0,1)$ because Σ'_ρ is strictly positive in the right-neighborhood of $\underline{r}$ (in turn because $g'(1 - l(r))$ is unbounded in this neighborhood) and thus the maximum of Σ_ρ, if not interior, cannot be at $\underline{r}$. It must therefore be at $r = 1$. This implies that $\bar{\rho}$ exists and is equal to 0 in this case.

Suppose otherwise that $\Omega \neq \emptyset$. We show that Ω must be of the form $(0, \bar{\rho})$. To see this, notice that for any $\rho \in \Omega$, the envelope theorem implies that

$$\frac{d\Sigma_\rho}{d\rho} = g(1 - l_u). \tag{45}$$

The output net of effort costs of the date-0 cohort when the interest rate is $r_u = 1$ reads:

$$\frac{\pi}{2} f(l^*) + \rho g(1 - l^*) \tag{46}$$

because there are no payouts, effort is equal to π, and investment to l^* in this case from relations (11). Expression (46) is linear in ρ, with a slope $g(1 - l^*)$

larger than $g(1-l_u)$ since $l_u > l^*$ when $r_u < 1$ from (42). This means that if $\rho \in \Omega$, then the left-neighborhood of ρ is also within Ω because Σ_ρ admits a local extremum that is strictly larger than its value for r in the neighborhood of 1. This establishes that Ω is an interval of the form $(0, \bar{\rho})$.

Step 3. Examples such that $\bar{\rho} = 0$ and $\bar{\rho} > 0$.

Suppose that $f(l) = \frac{2g'(0.5)}{\pi} l^{\frac{1}{\gamma}}$ for $\gamma > 1$.

We have

$$\frac{\pi f'(l_u)}{2(2-r_u)} = r_u w^*, \tag{47}$$

$$\frac{\pi f'(l_\rho)}{2} = r_\rho w^*, \tag{48}$$

implying

$$r_u(2-r_u) = r_\rho \left(\frac{l_u}{l_\rho}\right)^{\frac{1}{\gamma}-1}. \tag{49}$$

l_u and l_ρ remain bounded and bounded away from 0 for as $\gamma \to 1$ because they are smaller than 1 and larger than l^* which tends to 0.5 as $\gamma \to 1$. Thus, letting $\gamma \to 1$ in (49) yields

$$r_\rho \simeq r_u(2-r_u), \tag{50}$$

and this implies

$$r_u < r_\rho \tag{51}$$

since $r_\rho < 1$. Note that we have actually established that $\lim_{\gamma \to 1} \bar{\rho} = 1$.

We have

$$\Sigma'_{\rho}(r) = (1-r)\left[\frac{2g'(0.5)l^{1/\gamma}}{(2-r)^3} - \frac{2w^*(3-2r)\gamma}{\pi(2-r)^2(\gamma-1)}l + \frac{\rho g'(1-l)\gamma^2 8w^*}{(\gamma-1)\pi^2}l^{2-1/\gamma}\right] \tag{52}$$

There exists l^0 sufficiently small such that the first term dominates the second one for all values of $(r,l) \in [0,1] \times (0,l^0)$ for γ sufficiently large. The third term dominates the second term for γ sufficiently large and all $(r,l) \in [0,1] \times (l^0,1)$. Thus $\Sigma'_{\rho} > 0$ for γ sufficiently large which implies $\bar{\rho} = 0$. ■