The Cross-Section and Time-Series of Stock and Bond Returns

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Abstract

We propose an arbitrage-free stochastic discount factor model that jointly prices the cross-section of expected returns of stock portfolios sorted along the book-to-market dimension, the cross-section of government bonds sorted by maturity, the dynamics of bond yields, and time series variation in expected stock and bond returns. It features three priced risk factors, all of which are yields. The first priced factor is the Cochrane-Piazzesi (2005, CP) factor, a linear combination of bond yields of maturities one- through five-years and a strong predictor of future bond returns. Differential exposure to innovations in the CP factor alone accounts for most of the spread in average returns between value and growth stock portfolios. This new finding suggests a tight link between the pricing of stocks and bonds. The second factor is the dividend yield and prices the mean of stock returns. The level of the term structure is the third factor and prices the cross-section of bond returns. With two additional risk price parameters, the model also replicates the dynamics of bond yields as well as the time-series predictability of stock and bond returns. Finally, the model also does a good job pricing corporate bond portfolios.

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The goal of this paper is to study a model that jointly accounts for stock and bond returns. While tremendous progress has been made in past decades in the pricing of government bonds and the cross-section of stock portfolios separately, the bond-based pricing model does not price the stock portfolios and the stock-based pricing model does not price bond portfolios. This is puzzling, at least from the perspective of a complete (financial) markets model, where the same stochastic discount factor ought to price both stock and bond returns. Yields are a natural starting point to look for the sources of risk that are priced in the cross-section of stock and bond returns. They reflect the state variables of the economy which govern the dynamics of the investment opportunity set. Risky assets provide compensation for bearing the risk reflected in the innovations in these state variables. Our main finding is that three priced yield factors, two government bond bond yields and one dividend yield, are sufficient to reduce account for the excess returns on book-to-market-sorted stock portfolios, the aggregate stock market, and maturity-sorted government bond portfolios. The mean absolute pricing error (MAPE) across these portfolios is 0.4% per year.

Section 1 spells out the theoretical motivation for our focus on yields. Based on this theoretical motivation, Section 2, explores the ability of yield factors to account for stock returns. In a standard Fama-MacBeth exercise, we find that the Cochrane and Piazzesi (2005) (henceforth CP) factor accounts for 90% of the difference between the average returns on value and growth stock portfolios and generates a MAPE of less than 40 basis points per year. The CP factor is the fitted value of a regression of excess bond returns on the lagged one-year bond yield and the two-through five-year forwards rates. It is the best-known predictor of future bond returns. This close connection between a factor that captures predictability in bond returns and a factor that captures the variation in the cross-section of value returns suggests the existence of a unified pricing model for bond and stock returns. While it prices the difference in portfolio returns, the CP factor fails to capture the average risk premium. Adding a second yield-based factor, the dividend yield on the aggregate stock market, turns out to solve this problem. Finally, we know from the state-of-the-art term structure model of Cochrane and Piazzesi (2008) that it takes one yield-based factor to explain cross-sectional variation in average returns on maturity-sorted bond portfolios. That factor is the level of the term structure. We indeed price the average returns on five CRSP maturity-sorted government bond portfolios with the level factor, generating a cross-sectional $R^2$ around 80% and a MAPE below 10 basis points per year.

These first empirical results suggest a parsimonious unified model that can explain both the cross-section of stock and bond returns. Section 3 develops such a model. The SDF takes the same form as in the affine term structure literature. The state dynamics are governed by five factors: CP, level, slope, and curvature of the term structure, and the dividend yield on the aggregate stock market. We show that three non-zero prices of risk suffice to reduce the mean absolute pricing error (MAPE) on ten book-to-market sorted stock return portfolios, the aggregate market return,
and five maturity-sorted government bond portfolios to 40 basis points per year. The price of CP risk plays the role of pricing the difference between value and growth stock returns, the price of dividend yield risk prices the level of stock returns, and the price of level risk plays the role of pricing the level and cross-sectional variation in bond risk premia. With only one non-zero coefficient in the dynamics of the price of risk vector, this SDF model also generates small pricing errors on bond yields. Finally, the model matches the observed predictability of aggregate stock returns by the dividend yield. The last two properties make it truly a model that captures not only the cross-section but also the time-series of expected bond and stock returns. In Section 4 we benchmark our results against the three-factor model of Fama and French (1992b, 1993). We also study several other sets of test assets, including corporate bond portfolios and additional stock portfolios. In almost all cases, our three factor model generates smaller pricing errors than the Fama-French model.

Several interesting questions remain: What economic risk does the CP factor capture? What explains why value stocks have higher exposure to the CP factor than growth stocks? We take a first step to answer these questions by studying the empirical link between CP, the business cycle, and value and growth returns. The CP factor displays interesting cyclical behavior. The CP factor is typically very low at the beginning of a recession (or a period of economic turmoil such as the summer of 1998), it increases during the recession, and peaks at the end of a recession. Hence, bond risk premia increase dramatically from a very low (often negative) value to a very high value over the course of a recession. The rewards to a value minus growth strategy also differ dramatically over time. When CP innovations are in their highest quartile of sample observations, the value spread is 13.6% per year, while it is -1.5% when CP is in the lowest quartile of its sample distribution. Times of high CP occur at the end of recession, when good times (good investment opportunities) are around the corner. Hence, value stocks have high returns exactly when the marginal value of wealth is low. That makes them risky and command a value risk premium.

This paper relates to several strands of the literature. The last twenty years have seen dramatic improvements in economists’ understanding of what determines differences in yields (Cox, Ingersoll, and Ross (1985), Duffie and Kan (1996), Dai and Singleton (2000, 2002)) and returns on bonds, as well as what determines heterogeneity in stock returns which differ by characteristics such as size and book-to-market value (Fama and French (1992a)). Yet, these two literatures have developed largely separately and employ largely different asset pricing factors. This is curious from the perspective of a complete markets model because both stock and bond prices equal the expected present discounted value of future cash-flows, discounted by the same stochastic discount factor. This paper contributes to both literatures and helps to bridge the gap.

On the theory side, representative-agent endowment models have been developed that are successful in accounting for many of the features of both stocks and bonds. Examples are the
external habit model of Campbell and Cochrane (1999), whose implications for bonds were studied by Wachter (2006) and whose implications for the cross-section of stocks were studied separately by Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2006). Likewise, the implications of the long-run risk model of Bansal and Yaron (2004) for the term structure of interest rates were studied by Piazzesi and Schneider (2006), while Bansal, Dittmar, and Lundblad (2005) and Bansal, Dittmar, and Kiku (2007) separately study the implications for the cross-section of equity portfolios. In more recent work, Bansal and Shaliastovich (2007) study some of the joint properties of bond yields and stock returns, although not the ones we will focus on. Other recent work that ties stock and bond markets together in a general equilibrium model is Bekaert, Engstrom, and Grenadier (2005), Bekaert, Engstrom, and Xing (2008), Lettau and Wachter (2007), Campbell, Sunderam, and Viceira (2008), and Gabaix (2008).

On the empirical side, the nominal short rate or the yield spread is routinely used either as a predictor of the aggregate return on the stock market or as a conditioning variable in an estimation of a conditional beta model of the cross-section of stock returns. Chen, Roll, and Ross (1986) were the first to study the connection between stock returns and bond yields. Ferson and Harvey (1991) study stock and bond returns’ sensitivity to aggregate state variables, among which the slope of the yield curve. They conclude that time variation in equity risk premia is important for understanding the cross-sectional variation in size and industry equity portfolios and interest rate risk premia are important for understanding the cross-sectional variation in bond return portfolios. Fama and French (1993) find that three factors (market, size, and book-to-market) account for the variation in stock returns and two bond factors (excess return on long-term bond and a default spread) explain the variation in government and corporate bonds. They find that all of their stocks load in the same way on the term structure factors. Ang and Bekaert (2007) find some predictability of nominal short rates for future aggregate stock returns. Brennan, Wang, and Xia (2004) write down an intertemporal-CAPM model where the real rate, expected inflation, and the Sharpe ratio move around the investment opportunity set. They show that this model prices the cross-section of stocks. Similarly, Petkova (2006) studies the connection between the Fama-French factors and innovations in state variables such as the default spread, the dividend-price ratio, the yield spread, and the short rate. Our focus is on the joint pricing of stock- and bond returns, as well as the implication for the term structure of interest rates. Baker and Wurgler (2007) show that government bonds comove most strongly with “bond-like stocks,” which are stocks of large, mature, low-volatility, profitable, dividend-paying firms that are neither high growth nor distressed. They propose that stocks and bonds are driven by a common sentiment indicator. Finally, Lustig, Van Nieuwerburgh, and Verdelhan (2008) price both nominal bond yields and the aggregate stock market return in a no-arbitrage model in order to measure the wealth-consumption ratio in the data; they do not study the cross-section of bond nor stock returns.
1 Motivation

In this section, we argue that it is natural to use yields to price the cross-section of risky asset returns. Yields reflect the state variables $X_t$ that drive investment opportunities in the economy. The innovations to these state variables $\varepsilon_{t+1}$ are key sources of risk entering in the risk premium of risky assets. We consider a generic set-up where some $N$-dimensional state vector $X_t$ is Markovian. It drives the conditional mean and volatility of aggregate real consumption growth, inflation, and real dividend growth (lowercase letters denote logs):

\[
\Delta c_{t+1} = \mu_c + g_c(X_t) + \sigma_c(X_t)\eta_{t+1},
\]
\[
\pi_{t+1} = \bar{\pi} + g_p(X_t) + \sigma_p(X_t)z_{t+1},
\]
\[
\Delta d_{t+1}^i = \mu_d^i + g_d^i(X_t) + \sigma_d^i(X_t)\eta^i_{t+1},
\]

for some generic functions $g_c(\cdot), g_p(\cdot), g_d^i(\cdot)$ and $\sigma_c(\cdot), \sigma_p(\cdot), \sigma_d^i(\cdot)$ that map the state variable $X_t$ into the real line or the positive part of the real line, respectively.

We assume the existence of a unique stochastic discount factor (SDF). Both complete markets models and endogenously incomplete markets models satisfy this assumption. Most utility functions lead to a log stochastic discount factor of the form:

\[
m_{t+1} = f(X_t, \varepsilon_{t+1}, \eta_{t+1}),
\]

(1)

where $f(\cdot)$ is a generic real-valued function of the state, the innovations in the state, and unexpected consumption growth. The nominal log SDF satisfies $m^s_{t+1} = m_{t+1} - \pi_{t+1}$. This class includes the standard consumption-CAPM, external habit preferences, long-run risk preferences, just to name a few. In these models, $f$ measures be the log inter-temporal marginal rate of substitution of the representative agent. Typically, $f$ is additively separable in its arguments, but we do not impose this.

The price of a $\tau$-period nominal zero-coupon bond satisfies:

\[
P^s_t(\tau) = E_t \left[ e^{m^s_{t+1} + \cdots + m^s_{t+\tau}} \right].
\]

The corresponding nominal bond yield is $y^s_t(\tau) = -\log(P^s_t(\tau))/\tau$. By the Markovian nature of $X$ and the law of iterated expectations, bond yields are (potentially non-linear) functions of those state variables in $X_t$ that affect the dynamics of real consumption growth and inflation. Innovations in yields reflect innovations in these state variables (the relevant partition of $\varepsilon$).

Just like bond yields, the log dividend-price ratio on a stock is a “yield,” which reflects information about the state variables $X$. To see this, note that the price of a stock at time $t$ is the
sum of the prices of the claims to the dividend in each of periods \( t + j, j \geq 1 \). These are the prices of the equity strips. The price-dividend ratio of each strip is a function of \( X_t \) and so is the price-dividend ratio of their sum. Hence, price-dividend ratios reflect those state variables in \( X_t \) that affect the dynamics of real consumption growth (through the SDF) and real dividend growth. Innovations to the dividend yield reflects innovations to those state variables.

The risk premium on any risky asset \( i \) is the conditional covariance of innovations to \( m \) and innovations to asset \( i \)'s return. Both innovations depend on \( \varepsilon_{t+1} \) shocks, but also on unexpected consumption growth, unexpected inflation, and unexpected dividend growth. Only the orthogonal component of these shocks, \((\eta_{t+1}, u_{t+1}^i, z_{t+1}) \perp \varepsilon_{t+1})\), is not reflected in either bond yields or the dividend-price ratio. Hence, as long as this orthogonal component is not too highly priced, yields (bond yields and dividend yields combined) should be very informative for risk premia on risky assets. Below, we explore this intuition, and indeed find that we are able to account for most of the cross-sectional variation in both stock and bond returns with only three yield factors.

2 Preliminary Empirical Analysis

2.1 The State Vector and Its Innovations

Section 1 motivates us to explore the link between the returns on stocks and bonds on the one hand and yields on the other hand. It is well-known from the term structure literature that three factors –level, slope, and curvature– account for over 99% of the variation in government bond yields of different maturities. Since Cochrane and Piazzesi (2005), it is also understood that a fourth factor is needed to adequately describe the dynamics of bond returns. We will refer to this factor as the \( CP \) factor, and detail its construction below. Finally, we add the dividend yield on the aggregate stock market, henceforth \( DP \), to the state vector. The dividend yield potentially reflects additional state variables that matter for expected returns or expected dividend growth and that are not reflected or hard to extract from bond yields. In summary, we model the monthly dynamics of the state vector \( X \) as:

\[
X_{t+1} = \mu_X + \Gamma X_t + \varepsilon^X_{t+1},
\]

with Gaussian innovations \( \varepsilon^X_{t+1} \sim N(0, \Sigma_X) \), i.i.d. The state vector contains five yield variables: the \( CP \) factor, the level factor, the slope factor, the curvature factor, and the log dividend yield on the aggregate stock market portfolio \( (DP) \) in order of appearance. The first four factors have been suggested by Cochrane and Piazzesi (2008) to price the nominal term structure of interest rates. The only restriction on the companion matrix \( \Gamma \) is that the DP factor is modeled as an AR(1): \( \Gamma_{(1:4,5)} = 0_{4 \times 1} \). More on this in Section 3.3. This VAR is estimated by OLS and the innovations
$\varepsilon^X$ are readily extracted.

We construct the $CP$ factor following the procedure outlined in Cochrane and Piazzesi (2005). We use monthly Fama-Bliss yield data for nominal government bonds of maturities one- through five-years. These data are available from June 1953 until December 2008. We construct one- through five-year forward rates from the 1- through 5-years bond prices. We then regress the equally-weighted average of the one-year excess return on bonds of maturities of two, three, four, and five years on a constant, the one-year yield, and the two- through five-year forward rates. The yields are one-period lagged relative to the return on the left-hand side. The $CP$ factor is the fitted value of this predictive regression. The $R^2$ of the regression on our sample is 20.4%. This $R^2$ is roughly twice that of the yield spread, another well-known bond return predictor. We repeat the exercise at quarterly and annual frequency, where the $R^2$ rises to 24.6% with quarterly and 27.8% with annual time-series.

We form level, slope, and curvature as the first three principal components of the Fama-Bliss yields. As an aside and unlike Cochrane and Piazzesi (2008), we do not orthogonalize level, slope, and curvature to CP. The advantage of not orthogonalizing the factors is that their residuals are less highly correlated. Innovations of $CP$ are the first element of $\varepsilon$, which we label $\varepsilon^{CP}$. Since the $CP$ factor captures the bond risk premium, $\varepsilon^{CP}$ can be interpreted as innovations in the bond risk premium. The innovations to level, slope, curvature and $DP$ are labeled $\varepsilon^L$, $\varepsilon^S$, $\varepsilon^C$, and $\varepsilon^{DP}$. Figure 1 plots the $CP$ factor (top panel) and its innovation $\varepsilon^{CP}$ (bottom panel).

As a preliminary exercise to our main estimation in Section 3, we first show that $\varepsilon^{CP}$ prices the cross-section of stock portfolios sorted along the value dimension (Section ), and that $\varepsilon^L$ prices the cross-section of bond returns sorted along the maturity dimension (Section ).

2.2 Pricing the Cross-Section of Stocks with a Bond Factor

We form log excess returns corrected for a Jensen term:

$$r_{i,t+1}^{e} \equiv r_{i,t+1} - y_{t}^{S}(1) + .5Var[r_{i,t+1}],$$

for each of $i \in \{1, \cdots, N\}$ stock portfolio returns. We focus on the cross-section of 10 value-weighted decile portfolios sorted alongside the book-to-market dimension in this section, but consider other portfolios later. The value spread, measured as $r_{10,t+1}^{e} - r_{1,t+1}^{e}$, is 5.12% per year. We use a standard two-step estimation procedure; see Fama and MacBeth (1973) or Lettau and Ludvigson (2001). In

1Our level, slope, and curvature innovations have a correlation with CP innovations of 0.08, -0.17, and -0.83 compared to -0.87, 0.74, 0.07 and in Cochrane and Piazzesi (2008)'s approach. In both approaches the correlation between $CP$ and $DP$ innovations is -0.03. We verified that orthogonalizing level, slope, and curvature to CP instead does not substantially affect our results.

2Lowercase letters denote logs. All our results are quantitatively similar for gross excess returns and for gross real returns.
the first step, we run \( N \) univariate time-series regressions of returns on the \( \varepsilon^{CP} \) factor a constant in order to estimate the factor beta. In the second step we run one univariate cross-sectional regression of the \( N \) average returns on the betas from the first step and a constant in order to estimate the market price of risk.

Figure 2 shows the results from the first step. Each bar represents the slope coefficient (beta) of a time-series regression of a portfolio return \( r_{t+1}^{i,e} \) on the contemporaneous \( \varepsilon_{t+1}^{CP} \) factor. The data are monthly from June 1952 until December 2008. The book-to-market ratio increases from the left to the right. There is a clear pattern in these betas: they are much larger for value than for growth portfolios. The beta of growth stocks is essentially zero, whereas the beta of the highest book-to-market decile is 0.7. This difference is statistically different from zero with a t-stat of 3.9.

In a second stage, we run a cross-sectional regression of average excess stock returns on the first-stage beta and a constant. We find a market price of risk for \( \tilde{\lambda} = \varepsilon^{CP} \) shocks of 0.56 with a t-stat of 8.85. The positive price of risk suggests that positive innovations in \( CP \) denote good times. With a positive risk price, assets that have a high exposure to \( CP \) shocks have high expected returns because their returns are high exactly when times are good (and the marginal utility of the agent who prices the assets is low). Value stocks are such assets, growth stocks are not, hence the value premium. This one factor model is able to price the cross-section of book-to-market decile portfolios very well. The mean absolute pricing error (MAPE) is 33 basis points per year and the root mean squared pricing error (RMSE) is 49 basis points per year. The \( R^2 \) of the cross-sectional \( R^2 \) is 90.7%. Figure 3 plots average realized excess returns against predicted excess returns, formed as \( \alpha + \beta r^{\tilde{\lambda}} \), where \( \alpha \) is the second-stage regression intercept. It confirms visually that a large fraction of the cross-sectional variation in stock returns is explained by their exposure to innovations in the bond risk premium, as proxied by \( \varepsilon^{CP} \). Combining results from the two stages, value firms (BM10) and growth firms (BM1) have a differential exposure to \( CP \) of .74 in the monthly data. With a market price of risk of 0.56, this translates into an annual value premium of 5%, which is essentially the entire value premium observed in the data.

In sum, the exposure to \( CP \) innovations can explain the spread in average excess returns between value and growth stocks, but misses the level of the risk premia. We will introduce such a market level factor, namely \( \varepsilon^{DP} \), in Section 3. Nevertheless, the small pricing errors show that a single bond factor, linked to the bond risk premium, can explain most of the variation around a common market return. We consider the regression analysis of this section as preliminary evidence. Section 3 explores the links between stock returns, bond returns, and bond yields in much more detail.
2.3 CP and the State of the Economy

The previous analysis suggests a key role for the $CP$ factor as a predictor of bond returns and an explanatory variable for the cross-section of value and growth returns. However, it leaves open a few important questions. What source of economic risk does the CP factor capture? Why are value stocks more exposed to this risk? The answer to these questions is important because there is not a lot of evidence yet on the link between the value premium and economic risk. Lakonishok, Schleifer, and Vishny (1994) study the timing of value-growth returns and conclude that “It is impossible to conclude from this that value strategies did particularly badly in recessions, when the marginal utility of consumption is especially high.” In this section, we take a fresh look at the timing of value-growth returns in the data. We find that value-growth returns are especially high at the end of a recession, when CP is high, investment opportunities are great, and marginal utility of consumption is low. That makes them risky, and suggests a risk-based explanation of the value premium.

The $CP$ factor is clearly related to the state of the economy. Figure 1 plots the $CP$ factor (top panel) against the NBER recession dates (grey bars). The $CP$ factor tends to be low at the beginning of a recession, increase substantially during a recession, and peak at the end of a recession. During the average recession in the sample, $CP$ increases by 128 basis points (1.11 standard deviations), typically from a negative to a positive value. The fact that the CP factor peaks at the end of a recession (the through of the business cycle), when good economic times are around the corner and marginal utility growth for the representative investor is presumably low, can explain the positive price of CP risk we found above.

Returns on value and growth stocks are also linked to the cycle. An investment strategy which is long value and short growth stocks has low returns when entered at the start of a recession and high returns when entered at the end of a recession. Average monthly holding period returns on the value-minus-growth strategy are only 2.39% during NBER recessions (15% of the months in the sample) but 5.85% the rest of the time. So, the fortunes of a value-growth strategy reverse over the course of a recession.

Not all periods with a high (low) $CP$ factor occur at the end (beginning) of recessions. However, several of these low-to-high swings in the CP factor are also associated with times of economic uncertainty, such as 1987 (stock market crash), 1998 (Asian and Russian crisis, LCTM), or 2003 (jobless recovery). An even starker difference between the returns on value and growth stock emerges when we condition on contemporaneous $CP$ innovations. Figure 4 divides the sample into four quartiles depending on the $\epsilon_{CP}$ innovation. The top panel is for the months with the highest 25% of $CP$ innovations, the second panel from the top for the 50-75 percentile, the third panel for the 12-50 percentile, and the bottom panel for the lowest 25 percent $CP$ innovations. It shows that the value spread is highest when CP innovations are highest. The value spread declines
monotonically from 13.6% per annum in the top quartile to 8.9% in the second quartile, to -0.6% in the third quartile, and to -1.45% in the bottom quartile. This strong relationship between realized value returns and realized $CP$ innovations, the positive price of risk for $CP$ innovations, and the fact that the $CP$ factor is high at the end of a recession suggest the plausibility of a risk-based explanation. Value stock returns are high exactly when there is good news about future excess bond returns (improved investment opportunities), which are times of low marginal utility. That makes value stocks risky assets, because they have high returns exactly when the investor needs it the least.

[Figure 4 about here.]

2.4 Pricing the Cross-Section of Bonds with a Bond Factor

While the $CP$ factor is both a strong predictor of future bond returns and its innovations a strong explanatory variable of differences between average returns of value and growth stocks, it is well known that innovations in $CP$ are not a good explanatory variable of differences between average returns on long- and short-term bonds. Cochrane and Piazzesi (2008) show that a different bond factor, the level of the term structure, has significant ability to explain the cross-section of bond returns. We use the innovations in our level factor $\varepsilon^L$ to confirm this result. We use CRSP excess bond returns on portfolios of maturities 1, 2, 5, 7, and 10 years. Average excess returns (including Jensen adjustment) are 1.12%, 1.32%, 1.69%, 2.00%, and 1.63%, respectively per year. The sample is monthly from June 1952 until December 2008. Figure 5 shows the level betas of excess bond returns on the 5 maturity-sorted bond portfolios. They are decreasing from short to long maturities and negative. The market price of risk for level innovations is estimated to be -0.03 with a t-stat of -3.06. Annualized MAPE and RMSE are 11 and 15 basis points per year, respectively. The cross-sectional $R^2$ is 75.7%. The increasingly negative betas combined with the negative market price of risk allow this one-factor model to capture 0.70% of the observed 0.51% spread between the excess return on the 10-year and the 1-year government bonds. Despite the fact that the model can account for the spread between long-term and short-term bonds, it misses out on the level. The intercept from the cross-sectional regression is 0.99% per year with a t-stat of 4.83. We will address this in the next section.

[Figure 5 about here.]
3 A Unified Model of Stock and Bond Returns

The previous analysis suggests that two term structure variables, a proxy for innovations to the bond risk premium $\varepsilon^{CP}$ and innovations to the level factor $\varepsilon^L$, can help explain both the cross-sectional variation in book-to-market stock returns and maturity-sorted bond returns. These results suggest that it may be possible to find a parsimonious stochastic-discount factor (SDF) model in which only yield factors are priced that can jointly account for the cross-section of stock and bond returns. Four remarks are in order. First, such a model needs to additionally match the *levels* of the risk premia on stocks and bonds, a shortcoming of the univariate analysis in the previous section. Second, the multivariate nature of such a unified model introduces new challenges due to correlations between the factors and the returns on the various classes of assets. We explain this in detail below. Third, we broaden the model beyond unconditional pricing errors to also accommodate time-variation in expected stock returns driven by the dividend yield. Fourth, we derive the no-arbitrage term structure implied by the model, estimate its parameters, and show that it matches the observed dynamics of bond yields. In short, it provides a unified pricing framework for stocks and bonds. For brevity, this section focuses on the 10 book-to-market sorted stock portfolios and on returns sampled at monthly frequency between 1952 and 2008. The next section investigates robustness to different test assets, sampling frequency, and sample periods.

3.1 Setup

The dynamics of the state vector were given by equation (2). Even though we use five factors, only three of them will carry a non-zero price of risk in our preferred specification. The other factors are only relevant for the expectations’ formation of future state variables. The stochastic discount factor is modeled as in the affine term structure literature:

$$M_{t+1} = \exp \left( -y^S_t(1) - \frac{1}{2} \Lambda_t' \Sigma X \Lambda_t - \Lambda_t' \varepsilon X_{t+1} \right),$$

with $y^S_t(1)$ the 1-month short rate and market prices of risk that are affine in the state $\Lambda_t = \Lambda_0 + \Lambda_1 X_t$.

The no-arbitrage condition for each asset is given by:

$$\log E_t [M_{t+1} R_{t+1}] = 0,$$

which implies, with \( r_{e}^{t+1} = E_t[r_{e}^{t+1}] + \varepsilon_{e}^{R,t+1} \):

\[
E_t \left[ r_{e}^{t+1} \right] = -cov_t \left( r_{e}^{t+1}, m_{t+1} \right) = cov \left( \varepsilon_{R}^{t+1}, \varepsilon_{m}^{X,t} \right) \Lambda_t = \Sigma_{XR} (\Lambda_0 + \Lambda_1 X_t) .
\] (4)

Unconditional expected excess returns are computed by taking the unconditional expectation of (4):

\[
E \left[ r_{e}^{t+1} \right] = \Sigma_{XR} (\Lambda_0 + \Lambda_1 E \left[ X_t \right]) .
\]

In what follows, we first show that a parsimonious model is able to price the cross-section of unconditional expected stock and bond returns. We then show that this does not come at the expense of pricing the time series and cross-section of bond yields. Our test assets are the 10 portfolios sorted on their book-to-market ratio, the value-weighted market return from CRSP, and bond returns with maturities 1, 2, 5, 7, and 10 years from CRSP. Our estimation procedure estimates the dynamics of the state by OLS in a first step. It then finds the market price of risk coefficients \( \Lambda_0 \) that minimize the pricing errors on the test assets in a second step. All 16 moments are weighted equally. In a third step, we estimate the parameters in \( \Lambda_1 \) to fit bond yields and time variation in expected returns.

### 3.2 The Cross-section of Unconditional Expected Returns

We now focus on the cross-section of unconditional, i.e., average, expected stock and bond returns. We switch off all time variation in risk prices \( \Lambda_1 = 0 \) and optimize over a subset of \( \Lambda_0 \). As a point of reference, we start by pricing the 16 securities with a risk-neutral SDF \( M_{t+1} = \exp \left\{ -y_{t}^{R} \right\} \). That is, all prices of risk are zero: \( \Lambda_0 = 0 \). The first column of Table I labeled RN SDF, thus recovers the risk premia we want to explain. It shows that the risk premium spread on value minus growth (BM10-BM1) is 5.1% per year. Bond risk premia are not monotone in maturity and smaller than equity risk premia. The MAPE across all securities equals 5.5%.

Our first candidate SDF to explain the stock and bond risk premia is the one proposed by Cochrane and Piazzesi (2008). This is a natural candidate because their model does a great job pricing the term structure of nominal bond yields. As they do, we only allow the market price of level risk to be non-zero. This implies one non-zero element in \( \Lambda_0 \). The second column of Table I shows that the best-fitting SDF, labeled SDF2, is unable to jointly explain the cross-section of stock and bond returns. The MAPE is 2.68%. All pricing errors on the stock portfolios are large and positive, while all pricing errors on the bond portfolios are large and negative. The reason that this bond pricing model does not do better for the bond return portfolios is that the risk premia on

\(^4\)Return innovations \( \varepsilon_{R}^{t+1} \) are formed by regressing log excess stock returns on the lagged log dividend yield and log excess bond returns on the lagged CP factor. See below.
stock portfolios are larger in magnitude and therefore receive most attention in the optimization. Consequently, the estimation concentrates its efforts on reducing the pricing errors of stocks. To illustrate that this bond SDF is able to price the cross-section of bonds, we estimate the same model by minimizing only the bond pricing errors (the first five moments in the table). The third column of Table 1 indeed confirms that the bond pricing errors are small in this case. However, this SDF is not able to price the cross-section of expected stock returns nor the expected return on the aggregate stock market. The MAPE increases to 4.82%.

Our second candidate SDF, labeled SDF3, is a version of the canonical equity pricing model, the Capital Asset Pricing Model (CAPM). The only non-zero price of risk is the one corresponding to the dividend yield. The fourth column of Table 1 reports pricing errors for the CAPM. This SDF3 model is again unable to jointly price stock and bond returns. The MAPE is 1.50%. One valuable feature is that pricing errors of book-to-market portfolio returns go through zero. This means that the model gets the level of all expected stock returns right. However, their pattern clearly shows the value spread. Pricing errors on bond portfolios are sizeable as well and are all positive.

Having concluded that both the bond-based SDF and the stock-based SDF offer crucial ingredients to price bond returns and stock returns, but that neither is able to satisfactorily price both cross-sections, we now turn to our unified model. We allow the price of risks of the CP factor, the level factor, and the dividend yield to be non-zero. For pricing purposes, this is a three-factor model. The three elements in $\Lambda_0$ are estimated to minimize the 16 pricing errors. As before, there is no time variation in the prices of risk: $\Lambda_1 = 0$. We label this model $KLN$; the fifth column shows the results. The model succeeds in eliminating the value spread: The spread between the extreme portfolios is only 25bp per year. We also price the market portfolio well, and the bond pricing errors are much lower than in the other models SDF2 and SDF3. The MAPE falls to a low 40bp per year. This is essentially the same magnitude pricing error we found in the univariate analysis of Section 2 for the 10 book-to-market stock portfolios. But now, we also price bonds, and with one set of risk prices. In sum, using one SDF with three price of risk parameters only, we are able to match the level and the pattern in stock and bond risk premia jointly. Finally, the price of CP risk is estimated to be positive, while the price of level factor risk and dividend-yield risk are negative. These are the signs predicted by theory.

[Table 1 about here.]

---

5 We also considered a model in which the dividend yield is replaced by the aggregate stock market return. The pricing errors are very similar to the results for SDF3.

6 For our model, we calculated (asymptotic) standard errors on the $\Lambda_0$ estimates using GMM with an identity weighting matrix. They are 33.87 for the CP factor price, 9.38 for the level factor price, and 1.44 for the DP factor price. Hence, the first two risk prices are statistically different from zero, whereas the last one is not.
of risk estimates $\Lambda_0$ govern how much each risk factor contributes to risk premium of that test asset. Figure 6 shows that risk premium decomposition into risk compensation for exposure to the $CP$ factor, the level factor, and the $DP$ factor. The decomposition is computed as $\Sigma' X R \Lambda_0$, which is a five-by-one vector containing three non-zero elements. The top panel is for the market portfolio (first bar) and the five bond portfolios (last five bars) whereas the bottom panel is for the book-to-market decile portfolios. For the decile portfolios, the graph shows that the spread between value and growth risk premia comes about through differential compensation for $CP$ risk exposure. The level of the risk premia is accounted for by compensation for market risk ($DP$). Likewise, the market equity risk premium is accounted for by exposure to $DP$ shocks.

For bonds in the top panel, compensation for level risk and compensation for $CP$ risk work in the opposite direction. This is an effect that is absent in the univariate analysis. Even though $CP$ and level innovations are not that highly correlated (0.08), longer-term bond returns have high exposure to both. This illustrates that going from a univariate to a multivariate analysis is non-trivial. Appendix A works out the details of the argument. Its main insight is that even if the $CP$ factor were the perfect pricing factor for the decile stock portfolios, the level factor were the perfect pricing factor for the bond portfolios, and the $CP$ and the level factor were completely uncorrelated with each other, the two-factor model would fail to price stock and bond portfolios jointly. The issue is that the bond portfolio returns are exposed to the $CP$ factor while the stock returns have little exposure to the level factor. In that situation, as the appendix shows, the joint pricing model only works if the average bond portfolio returns are proportional to $CP$. This is exactly what we find: the exposures of bonds to $CP$ are virtually linear in maturity and with almost the same slope as level. This proportionality is what enables us construct a joint pricing model of stocks and bonds.

[Figure 6 about here.]

### 3.3 Implications for the Yield Curve and Return Predictability

#### 3.3.1 Prices of risk and estimation procedure

The previous section showed that a parsimonious SDF model was able to price unconditional excess returns on both book-to-market stock portfolios and maturity-sorted bond portfolios. An important question is how well the same SDF model does in pricing the term structure of nominal bond yields and in matching return predictability of both bonds and stocks. We now show that our model does well along these additional dimensions.

It is well-known that three factors (level, slope, and curvature) are necessary to adequately describe bond yield dynamics, e.g., Dai and Singleton (2000, 2002). While the slope and curvature factor were unnecessary to price average returns, as shown above, we included them in the
state vector \( X \) because they are necessary to adequately describe yields. They enter through the expectations formation in the state variable dynamics in equation (2).

Our SDF model implies an essentially affine term structure of yields (Duffee 2002). We model the nominal short rate as

\[
y_t^s(1) = \xi_0 + \xi'_1 X_t.
\]

Then, the SDF implies that the price of a nominal bond of maturity \( \tau \) is exponentially affine in the state variables \( X \):

\[
P_t^s(\tau) = \exp \{A^s(\tau) + B^{s'}(\tau)X_t\}.
\]

By no-arbitrage, we have:

\[
P_t^s(\tau) = E_t [M_{t+1}P_{t+1}^s(\tau - 1)]
\]

\[
= \exp \left\{ -\xi_0 - \xi'_1 X_t + A^s(\tau - 1) + B^{s'}(\tau - 1)\mu_X + B^{s'}(\tau - 1)\Gamma X_t - \Lambda'_1 \Sigma X B^s(\tau - 1) + \frac{1}{2} B^{s'}(\tau - 1) \Sigma X B^s(\tau - 1) \right\},
\]

which implies that \( A^s(\tau) \) and \( B^{s}(\tau) \) follow from the recursions:

\[
A^s(\tau) = -\xi_0 + A^s(\tau - 1) + B^{s'}(\tau - 1)\mu_X - \Lambda'_1 \Sigma X B^s(\tau - 1) + \frac{1}{2} B^{s'}(\tau - 1) \Sigma X B^s(\tau - 1),
\]

\[
B^{s'}(\tau) = -\xi'_1 + B^{s'}(\tau - 1)\Gamma - B^{s'}(\tau - 1) \Sigma X \Lambda_1,
\]

with initial conditions \( A^s(0) = 0 \) and \( B^{s}(0) = 0_{5 \times 1} \).

Matching yield dynamics and return predictability facts requires freeing up parameters in the matrix \( \Lambda_1 \) which governs the time variation in the market price of risk. In staying with parsimony, we only freely estimate two parameters in \( \Lambda_1 \). First, to match the predictability of bond returns and the yield dynamics, we allow \( \Lambda_{1(2,1)} \) to be non-zero, following Cochrane and Piazzesi (2008). Second, we choose \( \Lambda_{1(5,5)} \) to match the predictability of the aggregate stock market return by the dividend yield \( d_p \). That is, geometric excess returns (including Jensen adjustment) on the aggregate stock market, \( r_{t+1}^{e,m} \), are modeled as:

\[
r_{t+1}^{e,m} = r_{t+1}^m - y_t^s(1) + \frac{1}{2} \text{var}_t [r_{t+1}^m] = \delta_0 + \delta_1 d_p + \varepsilon_{t+1}^m,
\]

with Gaussian innovations \( \varepsilon_{t+1}^m \sim \mathcal{N}(0, \sigma^2_m) \) and covariance matrix \( \text{cov} (\varepsilon_{t+1}^m, \varepsilon_{t+1}^X) = \sigma_{Xm}^7 \).

Several other elements in \( \Lambda_1 \) are non-zero in order to impose the following restrictions. First, we want to avoid that the \( CP \) ratio forecasts future stock returns. The element \( \Lambda_{1(5,1)} \) is chosen in such

\footnote{We will assume that the Jensen adjustment \( \frac{1}{2} \text{var}_t (r_{t+1}) \) is a constant. This would arise under homoscedasticity assumptions on return innovations. This adjustment is tiny in practise.}
a way the conditional expectation of aggregate excess stock market returns does not depend on CP: $\Lambda_{1(5, 1)} = -\Lambda_{1(2, 1)} \frac{\sigma_{Xm(2)}}{\sigma_{Xm(5)}}$. We can relax this assumption, but it does not significantly improve the model fit. Second, we want the dynamics of government bond yields not to depend on the dividend yield. After all, it is not possible to exactly recover the dividend yield by combining nominal bond yields. Likewise, the dividend yield should not predict future bond returns. The following parameter restrictions are sufficient to implement this second set of restrictions:

\[
\begin{align*}
\Gamma_{(1:4, 5)} &= 0_{4 \times 1}, \\
\xi_{1(5)} &= 0, \\
\zeta_{(1:4, 5)} &= 0_{4 \times 1},
\end{align*}
\]

with $\zeta = \Sigma \Lambda_1$. We choose $\Lambda_{1(1:4, 5)}$ to ensure $\zeta_{(1:4, 5)} = 0_{4 \times 1}$. This implies $\Sigma_{(1:4, :)} \Lambda_{1(:, 5)} = 0_{4 \times 1}$. Hence, we need an element in the null space of $\Sigma_{(1:4, :)}$ that we scale to ensure that $\Lambda_{1(5, 5)}$ still matches the market return predictability by the dividend yield. More formally, if $z \equiv \text{null} \left( \Sigma_{(1:4, :)} \right)$, then the entries in the last column of $\lambda_1$ satisfy

\[
\Lambda_{1(:, 5)} = z \frac{\Lambda_{1(5, 5)}}{z(5)} \tag{6}
\]

In summary, the matrix $\Lambda_1$ takes the following structure:

\[
\Lambda_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & \Lambda_{1(1, 5)} \\
\Lambda_{1(2, 1)} & 0 & 0 & 0 & \Lambda_{1(2, 5)} \\
0 & 0 & 0 & 0 & \Lambda_{1(3, 5)} \\
0 & 0 & 0 & 0 & \Lambda_{1(4, 5)} \\
\Lambda_{1(5, 1)} & 0 & 0 & 0 & \Lambda_{1(5, 5)}
\end{bmatrix}
\]

subject to the restriction in equation (6).

We estimate the parameters in three steps. The first two steps are the same as in the unconditional analysis described above. The third step estimates $\xi_0$, $\xi_1$, and the parameters of $\Lambda_1$ to match nominal bond yields. The entries in last column of $\Lambda_1$ are not estimated but are chosen directly to (i) match exactly the predictability of aggregate stock market returns by the dividend yield and (ii) to satisfy equation (6). In particular, the third step estimates $\xi_0$, the four non-zero elements in $\xi_1$, and the parameter $\Lambda_{1(2, 1)}$ to minimize the distance between the five annual yields from the Fama-Bliss data set (maturities 1 through 5 years) and the corresponding yields implied
by the SDF:

$$\min_{\xi_0, \xi_1, \Lambda_{1(2,1)}} \left\{ \sum_{n=1}^{5} \sum_{t=1}^{T} \left( y_t^{S}(12 \times n) + \frac{A_t^{S}(12 \times n)}{12 \times n} + \frac{1}{12 \times n} B_t^{S}(12 \times n) X_t \right)^2 \right\},$$

where the factor “12” is introduced because we have a monthly model and we price annual yields. In optimizing over $\Lambda_1$, we modify $\Lambda_0$ to $\bar{\Lambda}_0 \equiv \Lambda_0 - \Lambda_1 E[X_t]$ to ensure that the unconditional asset pricing results from the previous section are unaffected. In related work, Adrian and Moench (2008) match bond returns first and then study the implications for bond yields.

### 3.3.2 Yield curve dynamics

The model’s SDF does a very nice job matching yields. The annualized standard deviation of the pricing errors of yields equal 14bp, 9bp, 6bp, 11bp, 15bp for 1-5 year yields. Figure 7 plots this model’s implications for the yields of maturities one through five years. The parameter $\Lambda_{1(2,1)}$ is estimated to be negative, consistent with the results in Cochrane and Piazzesi (2008). A negative sign means that an increase in $CP$ leads to higher bond risk premia because the price of level risk is negative and a higher $CP$ makes it more negative.

[Figure 7 about here.]

### 3.3.3 Stock and bond return predictability

While the model matches the predictability of aggregate stock market returns by the dividend yield exactly, as explained above, it \textit{implies} the predictive coefficient for (i) the ten book-to-market portfolios using the dividend yield and (ii) long-term bond returns using the $CP$ factor. In this section, we show that the implied predictive coefficient closely line up with the predictive coefficients we estimate in the data directly.

We first compute the predictive coefficient of the dividend yield for the ten stock portfolios. In the model, this predictive coefficient follows from the fifth element of $\sigma_{X_i}'\Lambda_1$, where $\sigma_{X_i}$ is the correlation between return innovations on portfolio $i$ and the state vector. Figure 8 displays the model-implied predictive coefficient versus the coefficient we compute in the data directly. The latter is computed for each of the portfolios as we do in equation (5) for the aggregate stock market. It shows that we match the predictability of all ten book-to-market portfolios.

[Figure 8 about here.]

---

8 The point estimates from the third estimation step are: $\hat{\Lambda}_{1(2,1)} = -1.3066, \hat{\Lambda}_{1(5,1)} = 25.2$ (implied), $\hat{\Lambda}_{1(1,5)} = -3.2$ (implied), $\hat{\Lambda}_{1(2,5)} = 3.2$ (implied), $\hat{\Lambda}_{1(3,5)} = 0.5$ (implied), $\hat{\Lambda}_{1(4,5)} = -4.9$ (implied), $\hat{\Lambda}_{1(5,5)} = -4.8, \xi_0 = 0.0044, \xi_{1(1)} = -0.1105, \xi_{1(2)} = 0.0513, \xi_{1(3)} = -0.0235, \xi_{1(4)} = -0.2091, \xi_{1(5)} = 0$ (implied).
Our model also matches the bond return forecasting power of $CP$. More specifically, we simulate 1,000 time series of our model of 678 observations, which is the length of our sample. For each of the samples, we construct annual returns of 2-year,...,5-year bonds in excess of the 1-year rate. We construct the $CP$ factor for each of the samples and regress the bond returns on the $CP$ factor. This delivers 1,000 predictive coefficients for each maturity. Figure 9 reports the predictive coefficients, averaged across the 1,000 sample paths, alongside the estimates in the data. The model closely replicates the predictability the predictability of long-term bond returns. The simulated slope coefficients are within one standard deviation from the observed slope coefficients. The model simulation also does a good job matching the $R^2$ of these predictability regressions; they are about one standard deviation away from the observed ones.\footnote{Predicting excess bond returns with the first element of the state vector (as opposed to with the re-estimated CP variables) in each run of the simulation delivers similar results. The reason is that the implied CP has a correlation of 90\% with the first element of the VAR.} In sum, our model matches the cross-section and time-series of stock and bond returns using three priced factors and two factors driving variation in risk premia.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure9}
\caption{Figure 9 about here.}
\end{figure}

4 Extensions

We consider several extensions and robustness checks in this section. First, we compare our results to the canonical Fama-French model (Fama and French (1992b, 1993)). Second, we study additional stock and bond portfolios. Third, we study several subsamples and redo the estimation of our main specification at quarterly and annual frequencies.

4.1 Comparison to Fama-French

A first natural point of comparison is to verify how our model does relative to the three-factor model of Fama and French (1992b, 1993). The latter offers a well-known benchmark for pricing the cross-section of stocks. We ask how well it prices the cross-section of book-to-market stocks and government bonds over our monthly sample from June 1952 until December 2008. We use the market return (MKT), the size (SMB), and the value factor (HML) as pricing factors and price the same 16 test assets. The SDF of the Fama-French model takes the same form as equation (3) but the innovations $\varepsilon^X$ are the innovations in MKT, SMB, and HML.\footnote{As for all other returns, innovations to these factors are formed by regressing MKT, SMB, and HML on lagged $dp$; see equation (5).} The last column of Table 1 contains the pricing errors in the Fama-French models. The MAPE is 57 basis points per year, substantially above the 40 basis points of our model in the fifth column. The slightly worse fit in
the last column is due to higher pricing errors on the bond portfolios. This is consistent with the findings in Fama and French (1993) who introduce additional pricing factors beyond MKT, SMB, and HML to price bonds. Our results suggest that three yield-based factors suffice.

4.2 Other Test Assets

Given that we found a unified SDF that does a good job pricing the cross-section and time-series of book-to-market sorted stock and maturity-sorted bond returns, a natural question that arises is whether the same SDF model also prices other stock or bond portfolios. In addition, studying more test assets allows us to address the Lewellen, Shanken, and Nagel (2009) critique. They argue that explanatory power of many risk-based models for the cross-section of (size and) value stocks may be poorly summarized by the cross-sectional $R^2$. As long as the proposed factor is (weakly) correlated with SMB or HML, the proposed factor is likely to produce betas that line up with expected returns. One of their proposed remedies is to use more test assets in the evaluation of asset pricing models. Our benchmark model added maturity-sorted government bond portfolios to the cross-section of book-to-market stock portfolios to address this concern. In addition, we study several other sets of test assets here. We start by adding corporate bond portfolios. Then we study replacing ten decile book-to-market portfolios by ten size decile portfolios, 25 size and book-to-market portfolios, and ten earnings-price portfolios.

4.2.1 Adding Corporate Bond Portfolios

One asset class that deserves particular attention is corporate bonds. After all, at the firm level, stocks and corporate bonds are both claims on the firm’s cash flows albeit with different priority structure. We ask whether, at the aggregate level, our SDF model is able to price portfolios or corporate bonds sorted by ratings class. Fama and French (1993) also include a set of corporate bond portfolios in their analysis but end up concluding that a separate credit risk factor is needed to price these portfolios. Instead, we find that the same three factors we used so far are also able to price the cross-section of corporate bond portfolios.

We use data from Citi’s Yield Book for four investment-grade portfolios: AAA, AA, A, and BBB. Return data for these portfolios are available monthly from January 1980 until December 2008. Their annualized excess returns are listed in the first column of Table 2. For the sample 1980-2008, there is almost no spread in average returns between the various corporate bond portfolios. This lack of spread is almost entirely due to the credit crisis in the Fall of 2008. For example, in October 2008, BBB bonds suffered a -10.7% return (a seven-standard deviation drop) whereas AAA bonds only suffered a -1.5% drop. For the sample up to December 2006, there is a 60bp spread per year between BBB and AAA. However, even though there is not much of a spread in the full sample, there is a lot of action in the time series of these corporate bond returns, and therefore
in the conditional covariance with the stochastic discount factor. This makes it non-trivial for the model to generate no spread between the corporate bond returns.

In a first exercise, we calculate Euler equations errors for these four portfolios, using our SDF model presented in Section 3.2. That is, we do not re-estimate the market price of risk parameters $\Lambda_0$, but simply evaluate the euler equations for the corporate bond portfolios. The resulting annualized pricing errors are listed in the second column of Table 2. The model does a reasonable job pricing the corporate bonds: pricing errors are about 1.3% per year, compared to excess returns of about 3.5% per year. The mean absolute pricing error among all 20 assets (ten BM portfolios, the market portfolio, five Treasury bond portfolios, and four corporate bond portfolios) is 58 basis points per year, compared to 40 basis points per year without the corporate bond portfolios.

Equally interesting is to re-estimate the market price of risk parameters of the SDF model when the corporate bond portfolios are included in the set of test assets. However, we do not allow for additional priced factors; the $CP$, the level, and the $DP$ factor are the only priced factors. The third column of Table 2 shows the results. Specifically, the corporate bond pricing errors are down to below 60 basis points per year on average. The overall MAPE on all 20 assets is 49 basis points per year, a mere 9 basis points above the MAPE when corporate bonds were not considered and 9 basis points less than when the corporates were not included in the estimation. There is no monotone pattern in the pricing errors on the corporate bonds, suggesting our model is successful in generating no spread between the excess returns. Finally, comparing Columns 2 and 3, the point estimates for the market prices of risk $\Lambda_0$ in Panel B are very similar for the models with or without corporate bonds. For comparison, the last column reports results for the Fama-French three-factor model. Its pricing errors are higher than in our three-factor model; the MAPE is 77 basis points. Average pricing errors on the corporate bond portfolios are around 1.05% per year, and monotonically declining in credit quality.

[Table 2 about here.]

Next, we verify that the implied Treasury yields of the extended model of Column 3 still match the observed one- through five-years Fama-Bliss yields. That is, we repeat the second-stage of the estimation and estimate one parameter in the $\Lambda_1$ matrix in order to match the Treasury yield dynamics. We calculate pricing errors on the yields. The annualized standard deviation of the pricing errors of yields equal 18bp, 9bp, 12bp, 21bp, and 27bp for 1-5 year yields. This is almost identical to the case where no corporate bonds were included. The match between model and data looks indistinguishable from that in figure 7. Finally, we verify that the time-series return predictability results remain unchanged as well.
4.2.2 Different Stock Portfolios

Table 3 shows the results of three exercises where we replace the ten book-to-market sorted portfolios by other sets of stock portfolios. In the first three columns we use ten market capitalization-sorted portfolios alongside the bond portfolios and the market. The first column shows the risk premia to be explained (risk neutral SDF). Small firms (S1) have about 3.5% higher risk premia than large stocks (S10). Our model in the second column manages to bring the overall mean absolute pricing error down from 5.8% per year to 0.52% per year, comparable to the 40 basis points we obtained with the book-to-market portfolios. This MAPE is comparable to that in the Fama-French model in the third column. The Fama-French model does better eliminating the spread between small and large stocks, whereas our model does better pricing the bond portfolios alongside the size portfolios. The next three columns use earnings-price-sorted stock portfolios and are organized the same way. High earnings-price portfolios have average risk premia that are 4.4% higher per year than low earnings-price portfolios. Our model reduces this spread in risk premia to less than 1% per year, while continuing to price the bonds reasonably well. The MAPE is 78 basis points per year compared to 91 in the Fama-French model. The last three columns use the five-by-five market capitalization- and book-to-market-sorted portfolios. Our three-factor model manages to bring the overall mean absolute pricing error down from 7.29% per year to 1.37% per year. This is again comparable to the three-factor Fama-French model. Relative to the FF model, ours reduces the pricing errors on the S1B1 portfolio but makes a larger error on the S1B4 and S1B5 portfolios. In all size quintiles but the first, our model is successful at eliminating most of the value spread, just like the FF model. Finally, the market price of risk estimates $\Lambda_0$ for the three additional sets of test assets are similar to those we found for the book-to-market portfolios in Table 1.

4.3 Different Samples and Frequencies

4.3.1 Subsample Analysis

We investigate the robustness of our main result in Table 1 by studying two subsamples. If we start the analysis in 1963, an often-used starting point for cross-sectional equity analysis (e.g., Fama and French (1993)), we find very similar results. The left columns of Table 4 shows a MAPE of 40bp, identical to what we found for the full sample. Our model improves relative to the Fama-French three-factor model, which has a pricing error of 72 basis points. There are no monotone patterns in the pricing errors on bonds or book-to-market decile portfolios. In the right columns, we investigate the results in the second half of our sample, 1980-2008. Mean absolute pricing errors rise to 59 basis points but the risk premia to be explained in this subsample are higher as well. In this subsample,
the MAPE under the Fama-French model is 106 basis points. Panel B of Table 4 shows that the price of risk estimates are similar to the ones from the benchmark estimation. These subsample results use yield factors which are estimated over the entire sample. In unreported results, we have re-estimated the state vector (e.g., the $CP$ factor) over the sub-sample in question. The results are similar.

4.3.2 Quarterly and Annual Results

To be completed.

5 Conclusion

We document that differential exposure to innovations in the Cochrane-Piazzesi (CP) factor, a strong predictor of future excess bond returns and hence a proxy for the bond risk premium, can help explain why value stocks have higher average returns than value stocks. This connection between a bond market variable and the cross-section of stocks leads us to develop a unified pricing model for stocks and bonds. We propose a no-arbitrage stochastic discount factor model that combines the canonical three-factor affine term structure model (level, slope, and curvature) with the CP factor, and a dividend yield factor. Only three factors need to have non-zero prices of risk to account for average returns on the book-to-market decile portfolios, the market portfolio, and maturity-sorted government bond portfolios. The price of market risk captures the common level of equity risk premia, the price of level risk captures the cross-sectional spread between long-term and short-term bond returns, and the price of CP risk captures the cross-sectional spread between value and growth stock returns. This model also captures the dynamics of bond yields and expected excess stock returns well when the market prices of risk change with the CP factor.

It also replicates risk premia on investment-grade corporate bond portfolios.

We document that value stocks have much higher excess returns than growth stocks when innovations in the $CP$ factor are above average. The value premium is 13.6% per annum in the top quartile and -1.5% in the bottom quartile of $CP$ innovations. This pattern in the timing of value minus growth returns is intriguing. We find some evidence that the $CP$ factor increases in times of turbulence, such as recessions. The $CP$ factor seems to peak at the end of a recession when good times are around the corner. If such times are times of low marginal utility for investors, value stocks are risky because they have high returns when investors need them the least.

In future work we plan to study equilibrium asset pricing models that can quantitatively generate the links between returns on book-to-market stocks and bonds that we have documented
Such a model would allow for an interpretation of the $CP$ factor in terms of macroeconomic sources of risk affecting the nominal term structure in general and bond risk premia in particular. Such a model would also trace back the value-growth premium to differential exposure of their cash-flow processes to these same sources of macroeconomic risk. Such differential exposure of cash-flows may in turn be linked to irreversibility constraints in investment, as in Zhang (2005), or to differences in production technologies, as in Papanikolaou (2007) and Panageas and Yu (2006).
References


A How Pricing Stocks and Bonds Jointly Can Go Wrong

Consider two factors $F_i$, $i = 1, 2$, with innovations $\eta_{t+1}^i$. We normalize $\sigma(\eta_{t+1}^i) = 1$. We also have two cross-sections of test assets with excess, geometric returns $r_{t+1}^{ki}$, $i = 1, 2$ and $k = 1, \ldots, K_i$, with innovations $\varepsilon_{t+1}^{ki}$. We assume that these returns include the Jensen’s correction term. Suppose that both cross-sections exhibit a one-factor pricing structure:

$$E(r_{t+1}^{ki}) = \text{cov}(\varepsilon_{t+1}^{ki}, \eta_{t+1}^i)\lambda_i, \ i = 1, 2.$$ 

The first factor perfectly prices the first set of test assets, whereas the second factor prices the second set of test assets. This does not imply that there exists a single SDF that prices both sets of assets.

Consider the following model of returns for both sets of test assets:

$$\varepsilon_{t+1}^{ki} = E(r_{t+1}^{ki})\eta_{t+1}^i, \ \varepsilon_{t+1}^{k2} = E(r_{t+1}^{k2})\eta_{t+1}^2 + \alpha_2\eta_{t+1}^3,$$

with $\text{cov}(\eta_{t+1}^2, \eta_{t+1}^3) = 0$. It implies:

$$\text{cov}(\varepsilon_{t+1}^{ki}, \eta_{t+1}^i) = E(r_{t+1}^{ki})\text{var}(\eta_{t+1}^i) = E(r_{t+1}^{ki}),$$

and hence $\lambda_i = 1, \ i = 1, 2$. Suppose, however, that $\text{cov}(\eta_{t+1}^1, \eta_{t+1}^3) \neq 0$. Let $\text{cov}(\eta_{t+1}^1, \eta_{t+1}^3) = \rho = \text{corr}(\eta_{t+1}^1, \eta_{t+1}^3)$. Then we have:

$$\text{cov}(\varepsilon_{t+1}^{k1}, \eta_{t+1}^1) = E(r_{t+1}^{k1}) \rho, \ \text{cov}(\varepsilon_{t+1}^{k1}, \eta_{t+1}^2) = E(r_{t+1}^{k1}),$$

$$\text{cov}(\varepsilon_{t+1}^{k2}, \eta_{t+1}^1) = (r_{t+1}^{k2}) \rho + \alpha_2\text{cov}(\eta_{t+1}^1, \eta_{t+1}^3), \ \text{cov}(\varepsilon_{t+1}^{k2}, \eta_{t+1}^2) = E(r_{t+1}^{k2}).$$

If $\alpha_2$ is not proportional to $E(r_{t+1}^{k2})$, then there exist no $\Lambda_1$ and $\Lambda_2$ such that:

$$E(r_{t+1}^{ki}) = \text{cov}(\varepsilon_{t+1}^{ki}, \eta_{t+1}^i)\Lambda_1 + \text{cov}(\varepsilon_{t+1}^{ki}, \eta_{t+1}^2)\Lambda_2.$$ 

However, if there is proportionality and $\alpha_2 = \alpha E(r_{t+1}^{k2})$, then we have:

$$\text{cov}(\varepsilon_{t+1}^{k2}, \eta_{t+1}^1) = E(r_{t+1}^{k2})(\rho + \alpha\text{cov}(\eta_{t+1}^1, \eta_{t+1}^3)) = E(r_{t+1}^{k2})\xi,$$

and $\Lambda_1$ and $\Lambda_2$ are given by:

$$\Lambda_1 = \frac{1 - \rho}{1 - \xi \rho}, \ \text{and} \ \Lambda_2 = \frac{1 - \xi}{1 - \xi \rho}.$$
Table 1: Unified SDF Model for Stocks and Bonds - Pricing Errors

Panel A of this table reports pricing errors on 10 book-to-market sorted stock portfolios, the value-weighted market portfolio, and five bond portfolios of maturities 1, 2, 5, 7, and 10 years. They are expressed in percent per year. Each column corresponds to a different stochastic discount factor (SDF) model. The first column contains the risk-neutral SDF and therefore reports the average pricing errors that are to be explained. The second column presents the results for a bond pricing model, where only the level factor is priced (SDF2). In the third column, we only use the bond returns as moments to estimate the same SDF as in the second column. The SDF model of the fourth column only allows the innovations to the dividend-price ratio on the aggregate market portfolio to be priced, and therefore is a CAPM-like model (SDF3). The fifth column presents our SDF model with three priced factors (KLN). The last column refers to the three factor model of Fama and French (1992). The last row of Panel A reports the mean absolute pricing error across all 16 securities (MAPE). Panel B reports the estimates of the prices of risk. The first five columns report market prices of risk $\Lambda_0$ for (a subset) of the following pricing factors: $\varepsilon^{CP}$ ($CP$), $\varepsilon^L$ (Level), and $\varepsilon^{DP}$ ($DP$). In the last column, the pricing factors are the innovations in the excess market return (MKT), in the size factor (SMB), and in the value factor (HML), where innovations are computed as the residuals of a regression of these factors on the lagged dividend-price ratio on the market. In all specifications, we set $\Lambda_1 = 0$. The data are monthly from June 1952 through December 2008.

<table>
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<th>KLN</th>
<th>FF</th>
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Panel B: Prices of Risk Estimates

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<td>SMB</td>
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<td>HML</td>
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</table>
Table 2: Unified SDF Model for Stocks, Treasuries, and Corporate Bonds

Panel A of this table reports pricing errors on 10 book-to-market-sorted stock portfolios, the value-weighted market portfolio, five Treasury bond portfolios of maturities 1, 2, 5, 7, and 10 years, and four corporate bond portfolios sorted by S&P credit rating (AAA, AA, A, and BBB). They are expressed in percent per year. Each column corresponds to a different SDF model, as described in the text. The last row reports the mean absolute pricing error across all 20 securities (MAPE). Panel B reports the estimates of the prices of risk. In all specifications, we set $\Lambda_1 = 0$. The estimation period for stocks and Treasury bonds is June 1952 through December 2008, while the corporate bond portfolio data are available only from January 1980 until December 2008.

<table>
<thead>
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<th>RN SDF</th>
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<th>KLN SDF</th>
<th>FF SDF</th>
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<td>-0.19</td>
<td>0.81</td>
</tr>
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<td>10-yr</td>
<td>1.62</td>
<td>0.17</td>
<td>0.77</td>
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</tr>
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<td>0.32</td>
<td>-0.65</td>
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<td>-1.63</td>
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<td>0.51</td>
<td>-0.32</td>
</tr>
<tr>
<td>BM9</td>
<td>9.69</td>
<td>0.69</td>
<td>0.91</td>
<td>1.10</td>
</tr>
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</tr>
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<td><strong>MAPE</strong></td>
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<td>0.49</td>
<td>0.77</td>
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<table>
<thead>
<tr>
<th></th>
<th>CP/Market</th>
<th>Level/SMB</th>
<th>DP/HML</th>
</tr>
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<td><strong>Panel B: Prices of Risk Estimates</strong></td>
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Table 3: Other Stock Portfolios - Pricing Errors

This table reports robustness with respect to different stock market portfolios, listed in the first row. Panel A of this table reports pricing errors (in % per year) on various stock portfolios, the value-weighted market portfolio, and five bond portfolios of maturities 1, 2, 5, 7, and 10 years. Each column corresponds to a different stochastic discount factor (SDF) model. The first column contains the risk-neutral SDF. The second column presents our SDF model with three priced factors (KLN). The third column refers to the three factor model of Fama and French (FF). The last row of Panel A reports the mean absolute pricing error across all securities (MAPE).

Panel B reports the estimates of the prices of risk. The data are monthly from June 1952 through December 2008.

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<th>Assets</th>
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<th>10 Earnings-Price Portfolios</th>
<th>25 Size and Value Portfolios</th>
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<td>Assets RN SDF KLN FF</td>
<td>Assets RN SDF KLN FF</td>
</tr>
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<td>1.11 0.13 0.85</td>
<td>1.11 -0.31 1.04</td>
<td>1.11 -0.96 0.96</td>
</tr>
<tr>
<td>2-yr</td>
<td>1.31 0.05 0.95</td>
<td>1.31 -0.41 1.25</td>
<td>1.31 -0.99 1.12</td>
</tr>
<tr>
<td>5-yr</td>
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<td>1.69 -0.07 1.73</td>
<td>1.69 -0.13 1.49</td>
</tr>
<tr>
<td>7-yr</td>
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<td>1.99 0.51 2.13</td>
<td>1.99 0.82 1.83</td>
</tr>
<tr>
<td>10-yr</td>
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<td>1.62 0.31 1.55</td>
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<td>6.00 -0.66 0.27</td>
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Panel B: Market Prices of Risk

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Table 4: Other Sample periods - Pricing Errors

This table reports robustness with respect to different sample periods. It is otherwise identical to Table 1. The data are monthly from January 1963 through December 2008 in the left columns and from December 1978 until December 2008 in the right columns.

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<td>FF kernel</td>
<td>RN kernel</td>
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<td><strong>Panel A: Pricing Errors (in % per year)</strong></td>
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<tr>
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<td>3.82</td>
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<tr>
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<tr>
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<td>6.74</td>
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<tr>
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<td><strong>Panel B: Market Prices of Risk</strong></td>
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<td>CP/Market</td>
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<td>DP/HML</td>
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Figure 1: The Cochrane-Piazzesi Factor and Its Innovation

The top panel plots the Cochrane-Piazzesi factor, a linear combination of the one-year nominal yield and 2- through 5-year nominal forward rates that best forecasts the one-year excess nominal bond return. The bottom panel plots the innovations in the CP factor, extracted from the VAR described in equation (??) of the main text. Both variables are measured against the right axis. The grey bars are NBER recessions. The data are monthly from June 1952 until December 2008.
Figure 2: Exposure of 10 Book-to-Market Portfolio Excess Returns to Innovations in the $CP$ Factor

The figure plots the factor exposures (betas) of the 10 book-to-market portfolio returns $r_{t+1}^{i,c} \equiv r_{t+1}^{i} - y_t^\delta(1) + 0.5 \times \text{Var}[r_{t+1}^i]$ to the contemporaneous $e_{t+1}^{CP}$ factor. Each bar denotes the slope coefficient of a time-series regression of one portfolio return on the $e_{t+1}^{CP}$ factor. The left bar is for growth stocks (lowest book-to-market decile), the right bar for value stocks (highest book-to-market decile). The data run from June 1952 until December 2008.
Figure 3: Average Realized vs Predicted Returns: CP Innovations Factor

Scatter diagram of average realized versus predicted excess returns. The predicted excess return is generated as the fitted value $\alpha + \beta_i \bar{\lambda}$ of a cross-sectional regression of average excess returns on betas $\beta_i$. The intercept is $\alpha$, the slope $\bar{\lambda}$. The test assets are the 10 book-to-market portfolio returns. The solid line is the 45 degree line. The data run from June 1952 until December 2008.
The various panels denote average yearly returns on the 10 book-to-market portfolios by $\varepsilon_{CP}$ quartile. The top panel is for the months with the highest 25% of CP innovations, the second panel from the top for the 50-75 percentile, the third panel for the 12-50 percentile, and the bottom panel for the lowest 25 percent CP innovations.
Figure 5: Exposure of Bond Portfolio Excess Returns to Level Innovations

The figure plots the factor exposures (betas) of five CRSP bond portfolio returns (1-yr, 2-yr, 5-yr, 7-yr, and 10-yr maturities) to the contemporaneous level factor innovations $\varepsilon^L$, measured from the VAR in equation (2). Each bar denotes the slope coefficient of a time-series regression of one portfolio return on the level innovation. The left bar is for the shortest maturity bond, the right bar for longest maturity bonds. The data are monthly from June 1952 until December 2008.
Figure 6: Exposure of Bond Portfolio Excess Returns to Level Innovations

The figure plots the risk premium decomposition into risk compensation for exposure to the CP factor, the level factor, and the DP factor. The top panel is for the market portfolio (first set of three bars) and the five bond portfolios (last five sets of bars) whereas the bottom panel is for the book-to-market decile portfolios. The three bars for each asset are computed as $\sum X_k \Lambda_0$, which is a five-by-one vector containing three non-zero elements. The data are monthly from June 1952 until December 2008.
Figure 7: SDF Model-Implied Yields

This figure plots annual yields on nominal bonds of maturities 1- through 5 years as implied by stochastic discount factor model estimated in Section 3.3. The data are Fama-Bliss yields on nominal bonds of maturities 1- through 5 years.
Figure 8: Implied Predictive Coefficients of the ten Book-to-Market Portfolios

The figure plots the predictive coefficients implied by the SDF model and the ones implied by running a predictive regression in the data. The returns are on the ten book-to-market portfolios and the log dividend yield on the aggregate stock market is used to predict future returns. The data run from June 1952 until December 2008.
Figure 9: Implied Predictive Coefficients of Bond Returns

The figure plots the predictive coefficients implied by the SDF model and the ones implied by running a predictive regression in the data. The returns are on annual bond returns and the CP factor is used to predict future returns. To compute the predictive coefficients in the model, we simulate 1,000 samples of the same length as our data and estimate the CP factor and the predictive regressions in each of these samples. We report the average, across 1,000 samples, predictive coefficient. The data run from June 1952 until December 2008.