Debt Constrained Asset Markets

Timothy J. Kehoe and David K. Levine
Review of Economic Studies, 1993

Presented By: Michelle Zemel
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Motivation

- Complete Markets $\Rightarrow$ Perfect Risk Sharing
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- In the data, however, changes in individual consumption are imperfectly correlated with those in aggregate consumption
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Motivation

- Full enforcement

Full enforcement means that if an agent has an incentive to default, there will be no default at any point in time. To relax the assumption of full enforcement, a punishment for default is introduced.

1) Seizure of public assets (collateral)
2) Permanent exclusion from intertemporal trade (but not from spot market)
Motivation

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- ⇒ No default at any point in time if an agent has incentive to do so
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- Relax the assumption of full enforcement, replace it with a punishment for default
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- Relax the assumption of full enforcement, replace it with a punishment for default
- In this model, punishment for default is
  1) seizure of public assets (collateral)
  2) permanent exclusion from intertemporal trade (but not from spot market)
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Why does this lead to Equilibria with Partial Insurance?

- Even with the punishment in place, an agent might still have incentive to default at a certain state.
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- However, in this model all information is publicly held and common knowledge.
- No one would agree to enter into a contract in which the other party has an incentive to default.
- This limits the feasible allocations, can lead to an allocation which is not first-best, i.e. not full risk sharing.
Motivation

Relationship to Incomplete Markets Model

- The incomplete markets literature (Mankiw 1986, etc.) also generate the partial insurance result.
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- Situations that are too expensive to insure
Main Findings

- Model can generate the partial insurance result
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- Interest rates may be lower than in the constrained equilibrium (less borrowing demand)
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Characterization of Equilibria and Allocations in the Constrained Model

- Efficiency
- Analogues of Social Welfare Theorems
Trade

- Intertemporal and Spot Markets
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- A-D Economy (All intertemporal trades are made at time 0)
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- Markets are Complete
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- There are two types of endowments, private goods and public assets.
- Private endowments, $\tilde{x}^i(\eta)$, cannot be seized from the agent.
- Public endowment, $\tilde{w}^i(\eta)$ represents assets that are available for consumption but can change hands, such as land and property.
Default

- Punishment for Default
Default

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- Central Agency
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- Default Punishment: seizure of assets and exclusion from intertemporal markets
Preferences

- Economy is populated by agents with V-NM utility functions
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The preferences of agent $i$ are given by the von Neumann–Morgenstern utility function

$$U_i(x^i) = (1 - \delta) \sum_{s \in S} \delta^{i(s)-1} \pi_s u_i(x^i_s, \eta_s).$$

(A.1) $u_i(\cdot, \eta)$ is continuous, concave, strictly quasi-concave, and strictly monotonically increasing. 
For $x^i_s > 0$, $u_i(x^i_s, \eta)$ is continuously differentiable.
Allocation Constraints

Which Allocations are Feasible?

- Standard Notion of Social Feasibility:

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Allocation Constraints

Which Allocations are Feasible?
- Standard Notion of Social Feasibility:

An allocation is \textit{socially feasible} if for each state history $s$

\begin{equation}
\sum_{i=1}^{m} x_{s}^{i} \leq \tilde{x}(\eta_{s}).
\end{equation}
Allocation Constraints

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- Individual Rationality Constraint
Allocation Constraints

Which Allocations are Feasible?

- Individual Rationality Constraint
- This constraint will rule out the allocations with default (despite punishment)
Allocation Constraints

Individual Rationality Constraint

we define the indirect utility function \( v_i(p_s, y_s^i, \eta_s) \) as the solution to

\[
\max u_i(x_s^i, \eta_s)
\]

subject to

\[
p_s \cdot x_s^i \leq y_s^i.
\]

We say that the allocation-price pair \((x, p)\) is spot market supporting if \((SS)\)

\[
u_i(x_s^i, \eta_s) = v_i(p_s, p_s \cdot x_s^i, \eta_s)
\]

for each state history \(s\) and agent \(i\). In other words, no agent has any incentive to recontract in spot markets.

An allocation-price pair \((x, p)\) is said to be (interim) individually rational if, for each state history \(s\) and agent \(i\),

\[
(\text{IR}) (1 - \delta)\sum_{\sigma \preceq s} \delta^{t(\sigma) - t(s)} \pi_\sigma u_i(x_\sigma^i, \eta_\sigma) \geq (1 - \delta)\sum_{\sigma \preceq s} \delta^{t(\sigma) - t(s)} \pi_\sigma v_i(p_\sigma, p_\sigma \cdot \tilde{x}_i(\eta_\sigma), \eta_\sigma).
\]
A constrained transfer equilibrium is a triple \((x, p, q)\) with the allocation \(x \in l_\infty\), spot prices \(p \in \mathbb{R}_\infty\), and intertemporal prices \(q \in l_\infty^x\) such that the allocation is socially feasible (SF); it exhausts the value of the social endowment,

\[(E.1)\quad \sum_{i=1}^{m} q(x^i) = q(\bar{x});\]

each agent's allocation \(x^i\) maximizes utility subject to a budget constraint and interim individual rationality constraints,

\[(E.2)\quad \max U_i(z^i)\]

subject to

\[q(z^i) \leq q(x^i)\]

\[(1 - \delta) \sum_{\sigma \in S} \delta^{t(\sigma) - t(s)} \pi_\sigma u_i(z^{\sigma}_s, \eta_\sigma) \geq (1 - \delta) \sum_{\sigma \in S} \delta^{t(\sigma) - t(s)} \pi_\sigma v_i(p_\sigma, p_\sigma \cdot \bar{x}(\eta_\sigma), \eta_\sigma), \quad s \in S;\]

and the spot prices \(p\) are consistent with the intertemporal prices \(q\) in the sense that

\[(E.3)\quad x, z \in l_\infty, \quad \text{and } p_s \cdot x_s = p_s \cdot z_s \text{ for all } s \in S \text{ imply } q(x) = q(z).\]
Example 1

Setting

- State Space: 3 states, $\eta_0 = 3$
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- Private endowments vary with the state, no assets
- $\bar{x}_1(1) = \bar{x}_2(2) = \bar{x}_h$ and $\bar{x}_1(2) = \bar{x}_2(1) = \bar{x}_l$
Example 1

Solving for the Unique Unconstrained Equilibrium (First Best Allocation)
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\[ x_{jt}^i = \frac{(x_h + x_l)}{2} \quad i = 1,2 \quad j = 1,2 \quad t = 1,2,\ldots \]
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Solving for the Unique Unconstrained Equilibrium (First Best Allocation)

- $x_{jt}^i = \frac{(\bar{x}_h + \bar{x}_l)}{2}$ $i = 1,2$ $j = 1,2$ $t = 1,2,....$
- $q_{jt} = \delta^{t-1}$ $j = 1,2$ $t = 1,2,....$
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- \( x_{jt}^i = \frac{(\bar{x}_h + \bar{x}_l)}{2} \quad i = 1, 2 \quad j = 1, 2 \quad t = 1, 2, \ldots \)
- \( q_{jt} = \delta^{t-1} \quad j = 1, 2 \quad t = 1, 2, \ldots \)
- Constant interest rate = \( \frac{q_{jt}}{q_{jt+1}} - 1 \)
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- Is the Unconstrained Allocation Feasible?
- Suppose \((\bar{x}_h, \bar{x}_l) = (15, 4)\)
- Let \(\delta = \frac{1}{2}\)
- Agent i’s utility in state i is \(\log\left(\frac{19}{2}\right)\)
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- Is the Unconstrained Allocation Feasible?
- Suppose $(\bar{x}_h, \bar{x}_l) = (15, 4)$
- Let $\delta = \frac{1}{2}$
- Agent $i$’s utility in state $i$ is $\log\left(\frac{19}{2}\right)$
- In state 1, when agent 1’s endowment is $\bar{x}_h$, then his utility of autarky is $\frac{2}{3} \log(15) + \frac{1}{3} \log(4)$
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- but \(\frac{2}{3}\log(15) + \frac{1}{3}\log(4) > \log\left(\frac{19}{2}\right)\), so agent 1 has an incentive to default in state 1
- The unconstrained equilibrium violates (IR), thus constrained equilibrium will not be first best
Example 1

Constrained Equilibrium Allocations and Prices?

- Candidate Allocation: $\bar{x}_1(1) = \bar{x}_2(2) = 10$ and $\bar{x}_1(2) = \bar{x}_2(1) = 9$
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Constrained Equilibrium Allocations and Prices?

- Candidate Allocation: $\bar{x}_1(1) = \bar{x}_2(2) = 10$ and $\bar{x}_1(2) = \bar{x}_2(1) = 9$

- SF is clearly satisfied

Agent $i$'s utility in state $i$ is $\frac{2}{3} \log(10) + \frac{1}{3} \log(9)$.

Agent $i$'s utility of autarky in state $i$ is $\frac{2}{3} \log(15) + \frac{1}{3} \log(4)$.

But these two quantities are equal since $10^{\frac{2}{3}} 9^{\frac{1}{3}} = 15^{\frac{2}{3}} 4^{\frac{1}{3}}$. This allocation satisfies (IR).
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But these two quantities are equal since $10 \times 9 = 15 \times 4$. This allocation satisfies (IR).

Use first order conditions to obtain prices (such that the allocational is a optimal given prices)

$q_t = (\frac{5}{9}, -1)$

This is an equilibrium.
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- But these two quantities are equal since \( 10^{\frac{2}{3}} \cdot 9^{\frac{1}{3}} = 15^{\frac{2}{3}} \cdot 4^{\frac{1}{3}} \). This allocation satisfies (IR).
- Use first order conditions to obtain prices (such that the allocational is a optimal given prices)
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- But these two quantities are equal since \( 10^2 9 = 15^2 4 \). This allocation satisfies (IR).
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- But these two quantities are equal since \( 10^{\frac{2}{3}} 9 = 15^{\frac{2}{3}} 4 \). This allocation satisfies (IR).
- Use first order conditions to obtain prices (such that the allocational is a optimal given prices)
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- This is an equilibrium
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  - Agent i’s utility of autarky in state i is $\frac{2}{3} \log(15) + \frac{1}{3} \log(4)$.
  - But these two quantities are equal since $10^29 = 15^24$. This allocation satisfies (IR).
- Use first order conditions to obtain prices (such that the allocational is a optimal given prices)
  - $q_t = (\frac{5}{9})^{t-1}$
- This is an equilibrium
Example 1

Constrainted vs. Unconstrained Equilibrium

- Unconstrained Equilibrium $\Rightarrow$ First Best Allocation $\Rightarrow$ Perfect Risk Sharing

\[ q_{jt} = \delta_t - 1 \quad \Rightarrow \quad r = \left( \frac{1}{2} \right) \left( -1 \right) - 1 = -1 = 1 \]

\[ q_{jt} = \left( \frac{5}{9} \right) t - 1 \quad \Rightarrow \quad r = \left( \frac{5}{9} \right) \left( -1 \right) - 1 = \frac{9}{5} - 1 = \frac{4}{5} \]
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**Constrainted vs. Unconstrained Equilibrium**

- Unconstrained Equilibrium $\Rightarrow$ First Best Allocation $\Rightarrow$ Perfect Risk Sharing
- Constrained Equilibrium $\Rightarrow$ Only Partial Insurance (consumption volatility is reduced, but not eliminated)
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Constrained vs. Unconstrained Equilibrium

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- Constrained Equilibrium $\Rightarrow$ Only Partial Insurance
  (consumption volatility is reduced, but not eliminated)
- Interest Rate differs in the two equilibria:

\[
q_{jt} = \frac{\delta_t}{t - 1} \Rightarrow r = \left(\frac{1}{2}\right) \left(\frac{5}{9t - 1}\right) = \frac{9}{5t - 1} = \frac{4}{5}
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- Constrained Equilibrium $\Rightarrow$ Only Partial Insurance (consumption volatility is reduced, but not eliminated)
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Constrainted vs. Unconstrainted Equilibrium

- Unconstrainted Equilibrium ⇒ First Best Allocation ⇒ Perfect Risk Sharing
- Constrained Equilibrium ⇒ Only Partial Insurance (consumption volatility is reduced, but not eliminated)
- Interest Rate differs in the two equilibria:
  - Unconstrainted Equilibrium:
    \[ q_{jt} = \delta^{t-1} \Rightarrow r = \left(\frac{1}{2}\right)^{(1) - 1} - 1 = 2 - 1 = 1 \]
  - Constrained Equilibrium:
    \[ q_{jt} = \left(\frac{5}{9}\right)^{t-1} \Rightarrow r = \left(\frac{5}{9}\right)^{(1) - 1} - 1 = \frac{9}{5} - 1 = \frac{4}{5} \]
Example 1

Can we force a first best allocation?
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- If the two agents are patient enough, then the threat of exclusion from intertemporal trade is enough to force the first best allocation as an equilibrium.
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- If the two agents are patient enough, then the threat of exclusion from intertemporal trade is enough to force the first best allocation as an equilibrium.
- Calculate the value of $\delta$ for which agents will have no incentive to default when in their high state.
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Can we force a first best allocation?

- If the two agents are patient enough, then the threat of exclusion from intertemporal trade is enough to force the first best allocation as an equilibrium
- Calculate the value of $\delta$ for which agents will have no incentive to default when in their high state

$$\log \frac{19}{2} = (1 - \tilde{\delta}) \sum_{t=1}^{\infty} \tilde{\delta}^{2t-2} \log 15 + (1 - \tilde{\delta}) \sum_{t=1}^{\infty} \tilde{\delta}^{2t-1} \log 4.$$  

It is

$$\tilde{\delta} = \frac{\log 15 + \log 2 - \log 19}{\log 19 - \log 4 - \log 2} = 0.52805.$$  

For all $\delta$ such that $\tilde{\delta} \leq \delta < 1$, the unconstrained equilibrium is the unique equilibrium of the debt constrained economy.
Efficiency Concepts

An allocation-price pair \((x, p)\) is said to be admissible if it satisfies (SF), (IR), and (SS). An allocation-price pair is said to be efficient if it is admissible and cannot be dominated by any other admissible allocation-price pair. An allocation-price pair is said to be conditionally efficient (conditional on price vector \(p\)) if it is admissible and cannot be Pareto dominated by any allocation \(\tilde{x}\) such that \((\tilde{x}, p)\) satisfies (IR) and \(\sum m_i \tilde{x}_i \leq \sum m_i \bar{x}_i(\eta)\) for all \(s\).
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An allocation-price pair \((x,p)\) is said to be *admissible* if it satisfies (SF), (IR), and (SS).
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- An allocation-price pair is said to be *efficient* if it is admissible and cannot be dominated by any other admissible allocation-price pair
- An allocation-price pair is said to be *conditionally efficient* (conditional on price vector \(p\)) if it admissible and cannot be Pareto dominated by any allocation \(\tilde{x}\) such that \((\tilde{x}, p)\) satisfies (IR) and \(\sum_{i=1}^{m} p_s \tilde{x}^i_s \leq p_s \bar{x}(\eta_s)\) for all \(s\)
Efficiency Concepts

\[ j = 1 \implies \text{Efficiency and Conditional Efficiency are equivalent} \]
Efficiency Concepts

- $j = 1 \Rightarrow$ Efficiency and Conditional Efficiency are equivalent.
- Conditional efficiency is stronger than efficiency: $\tilde{x}$ need not be admissible, it can violate (SS) or (SF).
Efficiency Concepts

- \( j = 1 \Rightarrow \) Efficiency and Conditional Efficiency are equivalent
- Conditional efficiency is stronger than efficiency: \( \tilde{x} \) need not be admissible, it can violate (SS) or (SF)
- Conditional efficiency is weaker than efficiency: an admissible allocation-price pair at prices other than \( p \) might Pareto dominate \((x,p)\)
Propositions

Proposition 1. Efficient allocation-price pairs exist.

Proposition 2. Suppose that the Markov chain $\pi$ has a single ergodic class and no transient states. Suppose further either (i) that every agent has positive assets or (ii) that $u_i$ is strictly concave. Then there is a discount factor $\delta < 1$ and an allocation-price pair $(x, p)$ with $x$ stationary such that for all $\delta$, $\delta \leq \delta < 1$, $(x, p)$ is efficient, conditionally efficient, and first-best.

Proposition 3. If utility functions are identically homothetic, then an allocation-price pair is efficient if and only if it is conditionally efficient.
First Welfare Theorem
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First Fundamental Theorem of Welfare Economics: If \((x,q)\) is a price equilibrium with transfers, then the allocation \(x\) is efficient.

Proposition 4. (First welfare theorem) If \((x, p, q)\) is a constrained transfer equilibrium, then \((x, p)\) is conditionally efficient.
Second Welfare Theorem

*Second Fundamental Theorem of Welfare Economics:* Any Pareto efficient allocation can be supported as a price quasi equilibrium with transfers.

**Proposition 5.** (Second welfare theorem) Suppose that \((x, p)\) is conditionally efficient. Then there exist prices \(q \in l^*_\infty\) such that \((x, p, q)\) is a constrained quasi-equilibrium.
Example 2

Setting
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- 4 states, $\eta_0 = 1$
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- Transition probabilities: \( \pi(2|1) = \pi(3|1) = \frac{1}{2} \) and 
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- Effectively, this reduces to a one period 2 state economy
Example 2

Preferences
Example 2

Preferences

Each consumer has preferences

\[ U_i(x) = \frac{1}{2} a^i x_2^i - \frac{1}{2} (40)(b_1^i (x_{13}^i)^{-4} + b_2^i (x_{23}^i)^{-4}), \]

where \((a^1, b_1^1, b_2^1) = (1, 1, 243)\) and \((a^2, b_1^2, b_2^2) = (2, 243, 1)\)
Example 2

Endowments

- State 2: $\bar{x}_1^1 = \bar{x}_1^2 = 1, \bar{x}_1 = 4$
- State 3: $(\bar{x}_1^1, \bar{x}_1^2) = (1, 12), (\bar{x}_2^1, \bar{x}_2^2) = (11, 1), (\bar{x}_1, \bar{x}_2) = (13, 13)$
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Efficiency

- Claim: This allocation-price pair is efficient, but not conditionally efficient
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Example 2

Pareto Improvement

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- $(\tilde{x}, p)$ satisfies (IR), neither agent has an incentive to default in either state
- The threat of seizure of the public assets (in both states) allows this
Example 2

What is the Problem?

- $\tilde{x}$, $p)$ satisfies (SF) and (IR) and Pareto dominates $(x,p)$
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  - Agent 2: value of allocation on spot market $= 9.5 + 2.9 = 12.4$
  - Optimal reallocation $= (9.3, 3.1)$
- Notice that this cannot be an equilibrium, the spot markets do not clear
Example 2

What have we seen?
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What have we seen?

- An allocation can be efficient but not conditionally efficient
- Failure of the standard second welfare theorem
- \((x,p)\) was efficient but could not be supported in an equilibrium
- Can also show example of allocation which is conditionally efficient, but not efficient, and which can be supported in an equilibrium
Conclusions

- General equilibrium models with endogeneous debt limits break the link between aggregate and individual consumption.
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- Lustig and Van Nieuwerburgh (2007): an application of this model, relaxation of default punishment
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- Formulate and calibrate the model to generate equity premium, interest rates, and other interesting pricing moments.
- What are you assuming when you use this model? How realistic is the story?
Discussion

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  - Public Goods (Collateral)
  - Exclusion from Intertemporal Trade
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- The assumption that you are entirely unable to borrow against labor income, however, is questionable (but not significant)
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- There are examples of certain markets where permanent exclusion is a reasonable punishment
- For example, personal default on loans (credit, mortgage, bank loans, etc.) is reported in credit history and prevents (at least temporarily) from participating in certain intertemporal markets
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Harder to think of an example for other financial markets, such as stock and bond markets