Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles

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Motivation

As the title indicates the motivation for this study are the well documented puzzles of asset pricing.
Motivation: Equity Premium Puzzle

- Using standard power utility function, aggregate consumption growth is too smooth and not sufficiently correlated with the magnitude of the equity sharpe ratio for reasonable values of the risk aversion coefficient.
Motivation: Equity Premium Puzzle

- Using standard power utility function, aggregate consumption growth is too smooth and not sufficiently correlated with the magnitude of the equity sharpe ratio for reasonable values of the risk aversion coefficient.

\[
\frac{E_t(R_{t+1})}{\sigma_t(R_{t+1})} = -\frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})}\rho_t(M_{t+1}, R_{t+1})
\]

\[
M_{t+1} = \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}
\]
Motivation: Risk Free Rate Puzzle

- Do a log-linearization

\[ Er_{t+1}^f = -\ln \delta + \gamma E(\Delta c_{t+1}) - \frac{\gamma^2}{2} \sigma^2(\Delta c_{t+1}) \]
Motivation: Risk Free Rate Puzzle

Do a log-linearization

\[ Er^f_{t+1} = -\ln \delta + \gamma E(\Delta c_{t+1}) - \frac{\gamma^2}{2}\sigma^2(\Delta c_{t+1}) \]

Again given the arguments stated earlier, the model has difficulty explaining the low historical value of the risk free rate.
Motivation: Equity Volatility Puzzle

Equity returns are too volatile given the joint distribution of equity, dividends and aggregate consumption growth to be explained by a power utility representative agent with reasonable risk aversion.
Motivation: Addressing the Challenge

- Consider the following model
Motivation: Addressing the Challenge

- Consider the following model

- The representative agent has Epstein-Zin preferences.
Motivation: Addressing the Challenge

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- The representative agent has Epstein-Zin preferences.

- The growth rate of consumption and dividends contain a persistent predictable component.
Motivation: Addressing the Challenge

- Consider the following model

- The representative agent has Epstein-Zin preferences.

- The growth rate of consumption and dividends contain a persistent predictable component.

- Heteroscedasticity or time-varying economic uncertainty is introduced into the dynamics of the growth rates.
**Notation**

$G_{t+1}$: Aggregate gross growth rate of consumption.

$R_{a,t+1}$: Gross return on asset that returns aggregate consumption as its dividends.

$R_{m,t+1}$: Gross return market portfolio.

$g_{t+1} = \ln G_{t+1}$, $r_{a,t+1} = \ln R_{a,t+1}$, $r_{m,t+1} = \ln R_{m,t+1}$.

$g_{d,t+1}$: log dividend growth rate.

$\delta$: Time discount factor.

$\gamma$: parameter of risk aversion.

$\psi$: Parameter of the elasticity of intertemporal substitution (EIS).

$z_t = \frac{P_t}{C_t}$: Log price-consumption ratio.
We have Epstein-Zin preferences

\[
E_t \left[ \delta^\theta G_{t+1}^{-\theta/\psi} R_{a,t+1}^{-(1-\theta)} R_{i,t+1} \right] = 1
\]

Recall the log-linearization results

\[
r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}
\]

\[
m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1}
\]
Case I: Time-varying Growth rates

Model the consumption and dividend growth rates as

\[ g_{t+1} = \mu + x_t + \sigma \eta_{t+1} \]
\[ g_{d,t+1} = \mu_d + \phi x_t + \varphi_d \sigma u_{t+1} \]

where \( x_t \) is the persistent predictable component

\[ x_{t+1} = \rho x_t + \varphi_e \sigma e_{t+1} \]

\( e_{t+1}, u_{t+1} \) and \( \eta_{t+1} \) are i.i.d \( \mathcal{N}(0, 1) \) \( \phi \) is the leverage ratio on expected consumption growth, \( \varphi_e = 0 \) gives the usual i.i.d rates.
Case I: Time-varying Growth rates

- Intuition: Solve the Euler equation for $z_t$ and $z_{m,t}$

$$z_t = A_0 + A_1 x_t \quad z_{m,t} = A_{0,m} + A_{1,m} x_t$$

The factor loadings are given as

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho} \quad A_{1,m} = \frac{\phi - \frac{1}{\psi}}{1 - \kappa_{1,m} \rho}$$

Note for $\psi > 1$ $A_1$ is positive. Higher expected growth leads to a higher wealth-to-consumption ratio. Also $A_{1,m} > A_1$ so the effect is larger for the dividend claim.
Case I: Time-varying Growth rates

- Innovations to SDF

\[ m_{t+1} - E_t m_{t+1} = \left[ -\frac{\theta}{\psi} + \theta - 1 \right] \sigma \eta_{t+1} \]

\[ - (1 - \theta) \left[ \kappa_1 \left( 1 - \frac{1}{\psi} \right) \frac{\varphi e}{1 - \kappa_1 \rho} \right] \sigma e_{t+1} \]

\[ = \lambda_{m,\eta} \sigma \eta_{t+1} - \lambda_{m,e} \sigma e_{t+1} \]
Case I: Time-varying Growth rates

- Innovations to SDF

\[ m_{t+1} - E_t m_{t+1} = \left[ -\frac{\theta}{\psi} + \theta - 1 \right] \sigma \eta_{t+1} \]

\[ - (1 - \theta) \left[ \kappa_1 \left( 1 - \frac{1}{\psi} \right) \frac{\varphi_e}{1 - \kappa_1 \rho} \right] \sigma e_{t+1} \]

\[ = \lambda_{m,\eta} \sigma \eta_{t+1} - \lambda_{m,e} \sigma e_{t+1} \]

- Given the assumption of homoscedacity the equity premium is

\[ E(r_{m,t+1} - r_{f,t}) = \beta_{m,e} \lambda_{m,e} \sigma^2 - 0.5 \text{var}(r_{m,t}) \]

Since \( \lambda_{m,e} \) is increasing in \( \rho \) so is the equity premium.
Case II: Introducing Heteroscedasticity

Earlier we had constant conditional premium and volatility. To make up for it we introduce heteroscedasticity.
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Earlier we had constant conditional premium and volatility. To make up for it we introduce heteroscedasticity

\[ x_{t+1} = \rho x_t + \varphi e \sigma_t e_{t+1} \]
\[ g_{t+1} = \mu + x_t + \sigma_t \eta_{t+1} \]
\[ g_{d,t+1} = \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1} \]
\[ \sigma_{t+1}^2 = \sigma^2 + \nu_1 (\sigma_t^2 - \sigma^2) + \sigma_\omega \omega_{t+1} \]

where \( \sigma_t \) is a stochastic volatility component and \( e_{t+1}, u_{t+1}, \eta_{t+1} \) and \( \omega_{t+1} \) are i.i.d. \( \mathcal{N}(0, 1) \)
Case II: Introducing Heteroscedasticity

Can show that

\[ z_t = A_0 + A_1 x_t + A_2 \sigma_t^2 \]

where

\[ A_2 = \frac{0.5 \left[ (\theta - \frac{\theta}{\psi})^2 + (\theta A_1 \kappa_1 \varphi_e)^2 \right]}{\theta (1 - \kappa_1 \nu_1)} \]

For \( \gamma, \psi > 1 \) \( \theta < 0 \) so a rise in volatility lowers the log price-consumption ratio. This will help us capture the negative correlation between \( z_{m,t} \) and consumption volatility.
Case II: Introducing Heteroscedasicity

Similarly

\[ m_{t+1} - E_t(m_{t+1}) = \lambda_{m,\eta} \sigma_t \eta_{t+1} - \lambda_{m,e} \sigma_t e_{t+1} - \lambda_{m,\omega} \sigma_\omega \omega_{t+1} \]

\[ E_t(r_{m,t+1} - r_{f,t}) = \beta_{m,e} \lambda_{m,e} \sigma_t^2 + \beta_{m,\omega} \lambda_{m,\omega} \sigma_\omega^2 - 0.5 \text{var}(r_{m,t}) \]

As such this model is able to produce higher equity premia. Furthermore, the negative correlation between return innovations and innovations in market volatility is also captured.

\[ \text{cov}_t((r_{m,t+1} - E_t r_{m,t+1}), \text{var}_{t+1}(r_{m,t+2}) - E_t[\text{var}_{t+1}(r_{m,t+1})]) \]

\[ = \beta_{m,\omega} (\beta_{m,e}^2 + \phi_d^2) \sigma_\omega^2 < 0, \quad \beta_{m,\omega} = \kappa_1 A_{2,m} < 0 \]

This is usually referred to as the volatility feedback effect.
Data

- Data runs from 1928 to 1998.
- Consumption is measured by real nondurables and services, CPI is used as the deflator.
- Dividends are from CRSP value-weighted return.
- Model is calibrated to match growth rate and market data.
- Simulations are conducted to test the ability of the model to replicate observed features of the data and avoid the issue of biased estimates.
Data

\[ \rho = 0.970 \quad \text{Stationarity} \]
\[ \sigma = 0.0078 \quad \text{chosen to match uncond. variance} \]
\[ \text{and autocorr. of } g_t \]
\[ \phi_e = 0.044 \quad \text{chosen to match uncond. variance} \]
\[ \text{and autocorr. of } g_t \]
\[ \phi = 3 \quad \text{chosen to match levered effect of dividends} \]
\[ \phi_d = 4.5 \quad \text{chosen so } \phi_d \sigma = 0.0351 \text{ in order to match} \]
\[ \text{uncond. var. of } g_{d,t} \text{ and } \text{cor}(g_d, g) \]
Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles

Table I

Annualized Time-Averaged Growth Rates

The model parameters are based on the process given in equation (4). The parameters are $\mu = \mu_d = 0.0015$, $\rho = 0.979$, $\sigma = 0.0078$, $\phi = 3$, $\varphi_c = 0.044$, and $\varphi_d = 4.5$. The statistics for the data are based on annual observations from 1929 to 1998. Consumption is real nondurables and services (BEA); dividends are from the CRSP value-weighted return. The expression $AC(j)$ is the $j^{th}$ autocorrelation, $VR(j)$ is the $j^{th}$ variance ratio, and $corr$ denotes the correlation. Standard errors are Newey and West (1987) corrected using 10 lags. The statistics for the model are based on 1,000 simulations each with 840 monthly observations that are time-aggregated to an annual frequency. The mean displays the mean across the simulations. The 95% and 5% columns display the estimated percentiles of the simulated distribution. The $p$-val column denotes the number of times in the simulation the parameter of interest was larger than the corresponding estimate in the data. The Pop column refers to population value.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>SE</td>
</tr>
<tr>
<td>$\sigma(g)$</td>
<td>2.93 (0.69)</td>
<td></td>
</tr>
<tr>
<td>$AC(1)$</td>
<td>0.49 (0.14)</td>
<td></td>
</tr>
<tr>
<td>$AC(2)$</td>
<td>0.15 (0.22)</td>
<td></td>
</tr>
<tr>
<td>$AC(5)$</td>
<td>-0.08 (0.10)</td>
<td></td>
</tr>
<tr>
<td>$AC(10)$</td>
<td>0.05 (0.09)</td>
<td></td>
</tr>
<tr>
<td>$VR(2)$</td>
<td>1.61 (0.34)</td>
<td></td>
</tr>
<tr>
<td>$VR(5)$</td>
<td>2.01 (1.23)</td>
<td></td>
</tr>
<tr>
<td>$VR(10)$</td>
<td>1.57 (2.07)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(g_d)$</td>
<td>11.49 (1.98)</td>
<td></td>
</tr>
<tr>
<td>$AC(1)$</td>
<td>0.21 (0.13)</td>
<td></td>
</tr>
<tr>
<td>$corr(g, g_d)$</td>
<td>0.55 (0.34)</td>
<td></td>
</tr>
</tbody>
</table>
Results: Case I

- Data does not support the idea of i.i.d. growth rates. \( VR(2) \) is different from 0.
- Recall given stationarity

\[
VR(2) = \frac{\text{var}(r_t + r_{t-1})}{2\text{var}(r_t)} = \frac{2\text{var}(r_t) + 2\text{cov}(r_t, r_{t-1})}{2\text{var}(r_t)} = 1 + \rho(1)
\]

There seems to be support for this in the data. Note, the above relation holds under stationarity but it does not imply it. Under the assumption of i.i.d.

\[
VR(q) = 1 \quad \forall q
\]

- The model seems to be able to capture some of the features of the data.
## Results: Asset Pricing Implications in Case I

### Table II

**Asset Pricing Implications—Case I**

This table provides information regarding the model without fluctuating economic uncertainty (i.e., Case I, where $a_0 = 0$). All entries are based on $\delta = 0.998$. In Panel A the parameter configuration follows that in Table I, that is, $\mu = \mu_d = 0.0015$, $\rho = 0.979$, $\sigma = 0.0078$, $\phi = 3$, $\phi_e = 0.044$, and $\psi_d = 4.5$. Panels B and C describe the changes in the relevant parameters. The expressions $E(R_m-R_f)$ and $E(R_f)$ are, respectively, the annualized equity premium and mean risk-free rate. The expressions $\sigma(R_m), \sigma(R_f),$ and $\sigma(p-d)$ are the annualized volatilities of the market return, risk-free rate, and the log price-dividend, respectively.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\psi$</th>
<th>$E(R_m-R_f)$</th>
<th>$E(R_f)$</th>
<th>$\sigma(R_m)$</th>
<th>$\sigma(R_f)$</th>
<th>$\sigma(p-d)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: $\phi = 3.0$, $\rho = 0.979$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.5</td>
<td>0.5</td>
<td>0.55</td>
<td>4.80</td>
<td>13.11</td>
<td>1.17</td>
<td>0.07</td>
</tr>
<tr>
<td>7.5</td>
<td>1.5</td>
<td>2.71</td>
<td>1.61</td>
<td>16.21</td>
<td>0.39</td>
<td>0.16</td>
</tr>
<tr>
<td>10.0</td>
<td>0.5</td>
<td>1.19</td>
<td>4.89</td>
<td>13.11</td>
<td>1.17</td>
<td>0.07</td>
</tr>
<tr>
<td>10.0</td>
<td>1.5</td>
<td>4.20</td>
<td>1.34</td>
<td>16.21</td>
<td>0.39</td>
<td>0.16</td>
</tr>
<tr>
<td>Panel B: $\phi = 3.5$, $\rho = 0.979$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.5</td>
<td>0.5</td>
<td>1.11</td>
<td>4.80</td>
<td>14.17</td>
<td>1.17</td>
<td>0.10</td>
</tr>
<tr>
<td>7.5</td>
<td>1.5</td>
<td>3.29</td>
<td>1.61</td>
<td>18.23</td>
<td>0.39</td>
<td>0.19</td>
</tr>
<tr>
<td>10.0</td>
<td>0.5</td>
<td>2.07</td>
<td>4.89</td>
<td>14.17</td>
<td>1.17</td>
<td>0.10</td>
</tr>
<tr>
<td>10.0</td>
<td>1.5</td>
<td>5.10</td>
<td>1.34</td>
<td>18.23</td>
<td>0.39</td>
<td>0.19</td>
</tr>
<tr>
<td>Panel C: $\phi = 3.0$, $\rho = \phi_e = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.5</td>
<td>0.5</td>
<td>-0.74</td>
<td>4.02</td>
<td>12.15</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>7.5</td>
<td>1.5</td>
<td>-0.74</td>
<td>1.93</td>
<td>12.15</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>10.0</td>
<td>0.5</td>
<td>-0.74</td>
<td>3.75</td>
<td>12.15</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>10.0</td>
<td>1.5</td>
<td>-0.74</td>
<td>1.78</td>
<td>12.15</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Evidence of Heteroscedasity

Consider the absolute value of residuals (realized volatility) obtained from an AR(5) formulation of consumption growth, Panel A. Also consider using price-dividend to predict the residuals, Panel B.
Consider the absolute value of residuals (realized volatility) obtained from an AR(5) formulation of consumption growth, Panel A. Also consider using price-dividend to predict the residuals, Panel B. There seems to be evidence of time-varying volatility in consumption growth rate.

### Table III

**Properties of Consumption Volatility**

The entries in Panel A are the variance ratios ($VR(j)$) for $|\epsilon_{g,t}|$, which is the absolute value of the residual from the regression $g_t = \sum_{j=1}^{5} A_j g_{t-j} + \epsilon_{g,t}$, where $g_t$ denotes annual consumption growth rate. Panel B provides regression results for $|\epsilon_{g,t+j}| = \alpha + B(j)(p_t - d_t) + v_{t+j}$, and $j$ indicates the forecast horizon in years. The statistics are based on annual observations from 1929 to 1998 of real nondurables and services consumption (BEA). The price-dividend ratio is based on the CRSP value-weighted return. Standard errors are Newey and West (1987) corrected using 10 lags.

| Horizon | Panel A: Variance Ratios | Panel B: Predicting $|\epsilon_{g,t+j}|$ |
|---------|--------------------------|-----------------------------------|
|         | $VR(j)$ | SE | $B(j)$ | SE | $R^2$ |
| 2       | 0.95    | (0.38) | -0.11 | (0.04) | 0.06 |
| 5       | 1.26    | (1.09) | -0.10 | (0.05) | 0.04 |
| 10      | 1.75    | (2.46) | -0.08 | (0.08) | 0.03 |
Table IV

Asset Pricing Implications—Case II

The entries are model population values of asset prices. The model incorporates fluctuating economic uncertainty (i.e., Case II) using the process in equation (8). In addition to the parameter values given in Panel A of Table II ($\delta = 0.998$, $\mu = \mu_d = 0.0015$, $\rho = 0.979$, $\alpha = 0.0078$, $\phi = 3$, $\varphi_e = 0.044$, and $\varphi_d = 4.5$), the parameters of the stochastic volatility process are $\nu_1 = 0.987$ and $\sigma_w = 0.23 \times 10^{-5}$. The predictable variation of realized volatility is 5.5%. The expressions $E(R_m - R_f)$ and $E(R_f)$ are, respectively, the annualized equity premium and mean risk-free rate. The expressions $\sigma(R_m)$, $\sigma(R_f)$, and $\sigma(p - d)$ are the annualized volatilities of the market return, risk-free rate, and the log price-dividend, respectively. The expressions $AC1(p - d)$ and $AC2(p - d)$ denote, respectively, the first and second autocorrelation. Standard errors are Newey and West (1987) corrected using 10 lags.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>SE</td>
</tr>
<tr>
<td>Returns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(r_m - r_f)$</td>
<td>6.33</td>
<td>(2.15)</td>
</tr>
<tr>
<td>$E(r_f)$</td>
<td>0.86</td>
<td>(0.42)</td>
</tr>
<tr>
<td>$\sigma(r_m)$</td>
<td>19.42</td>
<td>(3.07)</td>
</tr>
<tr>
<td>$\sigma(r_f)$</td>
<td>0.97</td>
<td>(0.28)</td>
</tr>
<tr>
<td>Price Dividend</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(\exp(p - d))$</td>
<td>26.56</td>
<td>(2.53)</td>
</tr>
<tr>
<td>$\sigma(p - d)$</td>
<td>0.29</td>
<td>(0.04)</td>
</tr>
<tr>
<td>$AC1(p - d)$</td>
<td>0.81</td>
<td>(0.09)</td>
</tr>
<tr>
<td>$AC2(p - d)$</td>
<td>0.64</td>
<td>(0.15)</td>
</tr>
</tbody>
</table>
Further Implications: Volatility of the SDF

In the case with heteroscedasticity the Sharpe-ratio is time-varying. Furthermore, the maximal Sharpe-ratio is known to be a function of the volatility of the SDF. So what determines this volatility.

Table V

<table>
<thead>
<tr>
<th>Decomposing the Variance of the Pricing Kernel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entries are the relative variance of different shocks to the variance of the pricing kernel. The entries are based on the model configuration described in Table IV with $\gamma = 10$. The volatility of the maximal Sharpe ratio is annualized in order to make it comparable to the Sharpe ratio on annualized returns.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Relative Variance of Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatility of Pricing Kernel</strong></td>
</tr>
<tr>
<td>Independent Consumption</td>
</tr>
<tr>
<td>Expected Growth Rate</td>
</tr>
<tr>
<td>Fluctuating Economic Uncertainty</td>
</tr>
<tr>
<td>0.73</td>
</tr>
<tr>
<td>14%</td>
</tr>
<tr>
<td>47%</td>
</tr>
<tr>
<td>39%</td>
</tr>
</tbody>
</table>
Further Implications: Predictability

Studying predictability of excess returns and consumption growth rates by price-dividend ratios and the predictability of price-dividend ratios by consumption volatility.
Further Implications: Predictability

The model is able to capture the predictability of excess returns, price-dividend and low or no predictability of consumption growth rates.

Table VI
Predictability of Returns, Growth Rates, and Price–Dividend Ratios

This table provides evidence on predictability of future excess returns and growth rates by price–dividend ratios, and the predictability of price–dividend ratios by consumption volatility. The entries in Panel A correspond to regressing $r_{t+1} + r_{t+2} + \cdots + r_{t+j} = \alpha(j) + B(j) \log (P_t/D_t) + \nu_{t+j}$, where $r_{t+1}$ is the excess return, and $j$ denotes the forecast horizon in years. The entries in Panel B correspond to regressing $g_{t+1} + g_{t+2} + \cdots + g_{t+j} = \alpha(j) + B(j) \log (P_t/D_t) + \nu_{t+j}$, and $g$ is annualized consumption growth. The entries in Panel C correspond to $\log (P_{t+j}/D_{t+j}) = \alpha(j) + B(j)|\epsilon_{g,t}| + \nu_{t+j}$, where $|\epsilon_{g,t}|$ is the volatility of consumption defined as the absolute value of the residual from regressing $g_t = \sum_{j=1}^{5} A_j g_{t-j} + \epsilon_{g,t}$. The model is based on the process in equation (8), with parameter configuration given in Table IV and $\gamma = 10$. The entries for the model are based on 1,000 simulations each with 840 monthly observations that are time-aggregated to an annual frequency. Standard errors are Newey and West (1987) corrected using 10 lags.

| Variable | Panel A: Excess Returns | | | Panel B: Growth Rates | | | Panel C: Volatility | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | Data | SE | Model | Data | SE | Model | Data | SE | Model |
| $B(1)$ | $-0.08$ | $(0.07)$ | $-0.18$ | $0.04$ | $(0.03)$ | $0.06$ | $-8.78$ | $(3.58)$ | $-3.74$ |
| $B(3)$ | $-0.37$ | $(0.16)$ | $-0.47$ | $0.03$ | $(0.05)$ | $0.12$ | $-8.32$ | $(2.81)$ | $-2.54$ |
| $B(5)$ | $-0.66$ | $(0.21)$ | $-0.66$ | $0.02$ | $(0.04)$ | $0.15$ | $-8.65$ | $(2.67)$ | $-1.56$ |
| $R^2(1)$ | $0.02$ | $(0.04)$ | $0.05$ | $0.13$ | $(0.09)$ | $0.10$ | $0.12$ | $(0.05)$ | $0.14$ |
| $R^2(3)$ | $0.19$ | $(0.13)$ | $0.10$ | $0.02$ | $(0.05)$ | $0.12$ | $0.11$ | $(0.04)$ | $0.08$ |
| $R^2(5)$ | $0.37$ | $(0.15)$ | $0.16$ | $0.01$ | $(0.02)$ | $0.11$ | $0.12$ | $(0.04)$ | $0.05$ |
Based on the idea that news about growth rates and economic uncertainty changes perceptions about future expected growth rates and economic uncertainty, a model that avoids the usual assumption of i.i.d. growth rates and embraces the notion of time-varying expected growth rates and consumption volatility (economic uncertainty) is posited.
Although, numerically implemented the properties of the model have to a great extent also been formulated using the log-linearization approach of Campbell and Shiller.

The focus has been to reconcile the findings in data, including those dubbed asset pricing puzzles, with a theoretical model capable of reproducing the different well-recognized aspects of asset pricing data.
Conclusion

- Employing a model that includes Epstein-Zin preferences and incorporates fluctuating expected growth rates and volatility or uncertainty for that matter it has been shown that the model is capable of reproducing
  - a high equity premium
  - a low risk free rate
  - time-varying volatilities
  - and the predictability of returns
Discussion

- Does data validate the model?
- They state that it is difficult to determine whether or not growth rates are i.i.d.
- They consider variance ratios as a test.
- This is like saying that a process where the variables have the same mean and variance is stationary.
- Nonetheless, their result are very interesting and truly indicate that a model with i.i.d. growth rates is not enough.
In terms of predictability it would be interesting to combine their model with that of Lettau and Ludvigson (2001).

Recall that by decomposing total wealth into human wealth and asset wealth and deriving a co-integrating relationship they were able to show that deviations from the co-integrating relationship was able to forecast returns in short to medium term horizons.
The model and results in this paper are very interesting.

But we need to find a way to distinguish between models given data.

Also theoretical reasonings combined with empirical verifications as in Lettau and Ludvigson (2001) can only improve and reinforce the results of a model.