Life-Cycle Portfolio Choice with Additive Habit Formation Preferences and Uninsurable Labor Income Risk

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Existing finding vs findings in this paper

- Life cycle models: examine the determinants of household (HH) portfolio choice & wealth accumulation. And their time-varying dynamics.
- Younger HH’s portfolio choice in life cycle literature is counterpart to Equity premium puzzle in equity return literature.
- Standard models predict “Young HHs invest exclusively in equity”, which is at odds with data.
- This paper’s finding conforms to the empirical observations, “Young HHs hold portfolio in more conservative way than the older do.”
Model and Economy

- Agent: Single representative agent
- Preference: Additive internal habit, habit level is a fraction of previous period consumption
- Endowment or income stream: uninsurable labor income subject to transitory/permanent shocks during working periods, & became constant during retirement periods
- Assets or Claims tradable: A ‘Bond’ with constant return & an risky ‘Stock’ with stochastic return
- Constraints: No borrowing/short sales
- Exogenous variable transition: Markovian
- Key characterization: admissible habit-wealth region, which depends on worst-possible income path, and habit strength. (Not the probability of the worst-possible income path realization)
Model

• Why habit preference? Effective CRRA is function of habit and it increases in habit (while it decrease in current period consumption). When the available wealth for consumption is limited (i.e. HH can not consume that much in current period) but habit level is high, the effective CRRA is really high so that high risk aversion derives the wealth fraction allocated in stock decreased.

• Why age matters under habit? HH gets more cautious in increasing her consumption at early period when she is young since from that point she needs to maintain high level of habit until retirement point.
HH’s problem

\[
V_t(z_t) = \max_{c_t,s_t,b_t} \frac{(c_t - h_t)^{1-\gamma}}{1-\gamma} + \beta E_t \left[ V_{t+1}(z_{t+1}) \left( \frac{P_{t+1}}{P_t} \right)^{1-\gamma} \right]
\]

for \( t = t_0, \ldots, T \), s.t. (7), (6), and (5)

\[
V_{T+1} = \lambda \left( \frac{w_{T+1}}{\lambda} \right)^{1-\gamma} \frac{1}{1-\gamma}
\]

\( w_{T+1} = s_T(1 + r_{s,T+1}) + b_T(1 + r_b) \)

\[
X_t = C_t + B_t + S_t
\]

\( B_t \geq 0, S_t \geq 0 \)

\( C_t > H_t, \ t = t_0, \ldots, T \) \hspace{1cm} (5)

\[
\frac{X_t}{P_t} \equiv x_t = [s_{t-1}(1 + r_{s,t}) + b_{t-1}(1 + r_b)] \times \frac{P_{t-1}}{P_t} + y_t
\]

\( H_t \equiv h_t = \delta \frac{C_{t-1}}{P_t} = \delta c_{t-1} \times \frac{P_{t-1}}{P_t} \) \hspace{1cm} (6)

\( H_0 = H^0 \)

\[
Y_t = f(t)P_t \eta_t \quad \quad X_t = S_{t-1}(1 + r_{s,t}) + B_{t-1}(1 + r_b) + Y_t
\]

\( P_t = P_{t-1} \kappa_t \)
Characterizing a feasible set of endogenous variable pair, \((X_t, H_t)\)

- \(X_t = C_t + B_t + S_t \geq H_t\) for \(0 < \delta < 1\)
- Does this enough to deliver feasibility of consumption plan? Yes or No. It depends on the habit persistence, minimum support value of asset return distribution and labor income distribution together with their correlation.
- So we need this

\[
0 < \sum_{k=1}^{\tau} \frac{Y_{t+k}}{(1 + r_b)^k} + X_t - H_t \left[ \sum_{k=0}^{\tau} \left( \frac{\delta}{1 + r_b} \right)^k \right], \quad \tau = 0, \ldots, T - t, \quad (11)
\]

- Normalizing the terms by permanent income \(P_t\) gives

\[
0 < y_{t', t + \tau} + x_t - h_t \Delta_\tau, \quad \tau = 0, \ldots, T - t. \quad (12)
\]

\[
\Delta_\tau = \sum_{k=0}^{\tau} \left( \frac{\delta}{1 + r_b} \right)^k = \frac{1 - \left( \frac{\delta}{1 + r_b} \right)^{\tau+1}}{1 - \frac{\delta}{1 + r_b}}
\]
Feasibility Constraints (12) binds when habit persistence is high and lower bound of on labor income is low.

Panel A, $\delta=0.4$, $\min y_t=0.7 \times E_y$

Panel B, $\delta=0.8$, $\min y_t=0.5 \times E_y$
Transform cash-on-hand to cash-on-hand in excess of admissible boundary

\[ \tau^* = \arg \max_{\tau \in [0, \ldots, T-t+1]} \{ h_t \Delta_{t+\tau} - y_{t, t+\tau} \}. \]  

\[ \hat{x}_t = x_t - (h_t \Delta_{\tau^*} - y_{t, t+\tau^*}), \; \hat{x}_t > 0. \]  

Transformation of cash on hand

![Diagram showing transformation of cash on hand](image-url)
Calibration

- HH’s life span: Age 25 to 75, Retirement begins after age 65
- Age-income profile for the high school grads cohort, f(t) from Cocco et al. (2005)
- $B=0.95$, $\gamma=5$, $0\leq \delta \leq 0.8$
- For the baseline calibration, no bequest motive ($\lambda=0$) assumed
- Distribution of income shock:
  - Symmetric: For transitory shock, $\eta$, 2-state Markov process mean of 1, matching STDEV of 27%. For permanent shock, $\kappa$, which has mean of 1, matching 10% STDEV
  - Asymmetric (crucial for feasibility constraint): Augment the above 2-state Markov process by introducing a third, worst-possible state value which is zero or low income. Probability for zero income is set 0.5% and the one for low income 1.3%, respectively
- Stock returns: i.i.d. Markov process with 2 symmetric outcomes around mean of 6%, calibrated to match STDEV of 15%
- Bond return: A constant, 1%
- For each calibration, simulate 10,000 HHs cross-section. And compute wealth, consumption and stock fraction in portfolio
- $H_0=0.5* f(t_0)* P(t_0)* \delta$
- For symmetric income shock, initial wealth set to be zero, while for asymmetric case, 80% of first period permanent income
Result:
Savings (as a fraction of $x-h$) and Equity fraction

Figure 3
Policy functions for asymmetric distribution of income shock (with zero income), $\gamma=5$, $\delta=0.6$ for the first period of the model
• As HH is closer to boundary, saving rate approaches 1

• For fixed excess cash-on-hand, saving rate increase in habit

• As HH is closer to boundary, stock fraction decreases to zero since stock is risky so that it can not ensure that consumptions in the future stay above habit

• For fixed excess cash-on-hand, stock fraction decreases in habit

• Habit and worst-possible outcome of labor income derives that young HH with low financial wealth gets conservative in portfolio choice

• That finding can be more apparent when using asymmetric labor income transitory shock since it aggravates the worst-possible scenario consumption plan
Figure 4
Portfolio policy functions for asymmetric income shock (zero income), γ=5, δ=0.6 at ages 25 and 64 and evaluated at h=0.3
For this level of habit, the correspondence between cash on hand x and its transformed value \( \hat{x} \) shown on the graph is given by: \( x \approx \hat{x} + 0.73 \).
Figure 5
Symmetric distribution of income shock for various values of $\delta$, $\gamma=5$
Financial wealth is the total holdings of bond and stock. Consumption, income, and financial wealth are in thousands of dollars (inflation adjusted with 1992 as a base year).
Symmetric shock

- High habit tightens the feasibility constraints gives more savings motive. Consumption is more monotonically increasing in time. Therefore it increases wealth accumulation through the life cycle

- Higher habit reduces stock fraction

- Stock allocation is humped-shaped since habit and worst – possible labor income shock play for young HH’s portfolio choice, and decreasing human capital and high habit play for old HH’s portfolio choice

- One problem for high level of habit is: Need high $\delta>0.8$ to generate high equity premium and low risk free, while it brings a way more dramatic wealth accumulation pattern, which is at odds with empirical observation
  - need heterogeneous agent setting??
Asymmetric shock

- Worst-possible outcome (either zero or low) occurs with low probability from Carroll (1992)’s study. 0.5% for zero income and 1.3% for 10% of permanent income

- $\delta=0.6$, initial wealth = 80% of initial permanent income

- Compare to CRRA with zero-income state case to see the impact from habit distinctively → non-zero initial wealth offsets the possible impact from zero-income state
Figure 6
Portfolio choice with asymmetric distribution of income shock, $\gamma=5$
Solid line ("Symmetric") corresponds to benchmark. Zero income $\eta = 0$ occurs with probability 0.5%.
Low income $\eta = 0.1$ occurs with probability of 1.3%. "EU" denotes expected utility (CRRA) case with $\delta = 0$ and zero-income state.
Conclusion

• Penalizing HH for not conforming habit level under severely low labor income state helps explain more conservative portfolio choice and higher saving rates for young HHs, which have not been explained correctly under CRRA setting

• But high habit level implies counterfactually high wealth accumulation
Short Discussion

• Habit parameter does not have to be constant throughout the life cycle.

• High habit parameter and high present value of human capital usually make borrowing in reality easier and flexible as long as agent has been built up good credit history. So why not setting the maximum borrowing amount to be a function of level of habit and age of the agent. Then this might work as a offsetting term for the caution from worst-possible labor income shock.