Does a Currency Union Need a Capital Market Union?

Joseba Martinez, Thomas Philippon, and Markus Sihvonen*

July 2021

Abstract

We compare risk sharing in response to demand and supply shocks in four types of currency unions: segmented markets; a money market union; a capital market union; and complete financial markets. We show that a money market union is efficient at sharing domestic demand shocks (deleveraging, fiscal consolidation), while a capital market union is necessary to share supply shocks (productivity and quality shocks).

Keywords: risk sharing, currency union, banking union, capital market union, incomplete markets.

JEL: F45, E44, F36

*LBS and CEPR; NYU, CEPR and NBER; Bank of Finland. Acknowledgments: we thank Patrick Bolton, Stijn Claessens, Giovanni Dell'Ariccia, Jordi Gali, Pierre-Olivier Gourinchas, Matthias Kaldorf, Philip Lane, Tommaso Monacelli, Federica Romei and Iván Werning for helpful discussions; as well as participants in seminars at ESSIM, IMF ARC, SEDAM, the Annual Research Conference of the Banco de España, the Annual Macroprudential Conference in Stockholm, IMF/Central Bank of Ireland The Euro at 20 conference, ADEMU, CEBRA AM, NYU, LBS and Bank of Finland. We thank the Bank of Finland and AQRAMI for generous financial support. Corresponding author: Joseba Martinez, jmartinez@london.edu, London Business School, Regents Park, London, NW1 4SA, United Kingdom.

Failures of risk sharing lie at the heart of many economic crises, including the one that threatened the survival of the Eurozone in the early 2010s. A comparison of macro-economic dynamics in Europe to those of the United States during that period reveals the importance of risk sharing. Private leverage cycles are volatile and heterogeneous across U.S. states, just as they are across E.U. countries. They affect output and employment in similar ways. In Europe, however, private leverage cycles are amplified by sudden stops and spreads in banks' funding costs between countries. As spreads widen, weaker countries sink deeper into recession. These are clear signs of inefficient risk sharing.

The creation of *a banking union* has been a deliberate response to these issues. Focusing on banks is a natural step because banks intermediate most of European financial flows. The funding cost of banks has a direct impact on the credit conditions of households and firms. The main purpose of the banking union is to guarantee that funding conditions remain the same across regions within Europe and that they are not directly affected by domestic sovereign risk. There is broad agreement that some form of banking union is necessary to ensure the stability of the Eurozone, even as disagreement persists about its required features, such as deposit insurance, bail-ins, and the funding of resolution.

A central goal of the banking union is to ensure that funding costs in the money market remain equal across regions within Europe. We call this feature of a banking union *a money market union;* we argue that such a money market union is essential to facilitating risk sharing in a currency union. Banks, money market funds and non-financial corporations rely on money markets for their short-term funding, and money market rates serve as benchmark for other markets as well as operational targets for central banks. Monetary policy transmission breaks down when money market funding costs diverge in a currency union.

A money market union is narrower than a full banking union. For insurance, deposit-taking and loan-making remain highly segmented in the Eurozone and a banking union might encourage cross-border activities. It would then entail the sharing of loan default risk at the same time as a diversification of banks' funding costs. This outcome, however, can at best be a long term goal. In the meantime, the stabilization of money market funding costs has been achieved by a mixture of monetary policy and joint backstops for bank's recovery and resolution. The distinction between money markets and long-term capital markets is also crucial when discussing sovereign risk. For instance, all proposals for orderly sovereign debt restructuring involve tools to stabilize money markets.

A second key policy initiative in Europe is the building of *a capital market union*. Such a union can improve risk sharing via financial markets - i.e., equity and fixed income flows apart from cross-border bank flows. The asymmetries in COVID-19 related shocks has resulted in further pleas to both strengthen the banking union and advance in setting up the capital market union. However, there is no clear agreement, and little academic analysis, of the gains from adding a capital market union to a banking or money market union. This paper attempts to bridge that gap.

We model a currency union with nominal (wage) rigidities under four degrees of financial integration: (i) segmented markets as observed during the Eurozone crisis; (ii) a money market union where risk-free interest rates are equalized across regions; (iii) a capital market union with optimal cross-border equity holdings; and (iv) a complete markets economy. We then ask how these model economies respond to two types of shocks: domestic-demand shocks (triggered by public or private deleveraging) and other shocks (TFP shocks, quality shocks, and foreign demand shocks).

We take a resolutely macro-economic perspective in modeling the money market and capital market unions; our models emphasize economic function rather than industry classifications. We study the risk-sharing possibilities afforded by an *ideal* money market union: in our model, a money market union is an institution that guarantees that risk-free interest rates remain the same in all regions irrespective of the shocks that hit these regions. In an ideal money market union risk-free rates depend neither on the health of the domestic sovereign – a no-doom-loop condition – nor on the perceived strength of local banks – a no-sudden-stop condition. Of course, risky rates can differ across regions with different economic conditions, but this divergence corresponds to the efficient pricing of credit risk, not to divergence in money markets funding costs.

We model a capital market union as a market structure that allows frictionless sharing of risk to the market value of private capital. In our model claims to the value of capital most closely resemble traded corporate equity. In reality, the trading of private credit instruments (corporate bonds, securitized loans, etc) plays a crucial role in most proposals related to the capital market union in the EU context. However, we can study an ideal capital market union without taking a stand on the details of risky debt versus equity. The key point is that negative shocks cause equity and risky debt to fall in value. We could allow firms to issue debt and equity, or we could model a repackaging of such claims, without changing our macro-economic insights. In other words, we can assume a form of Modigliani-Miller theorem (Modigliani and Miller, 1958) at the firm level and study the macroeconomic consequences of risk sharing across countries. The money market union does not allow for ex-post adjustments to the value of claims. A capital market union, on the other hand, allows risk transfers, either via default on risky debt, or via changes in the market value of equity.

It is important to note that we consider a particular, plausible, implementation of complete markets. Each country in the currency union is populated by borrowers and savers; complete markets means throughout that the marginal utilities of consumption of *savers* are equalized across borders, because a borrowing constraint restricts borrowers' ability to engage in risk sharing.

	Definition	Demand Shocks	Supply Shocks
Segmented Markets (SM)	$R_{j,t} \neq \bar{R}_t$ for some (j,t)	worse than MMU	worse than MMU
Money Market Union (MMU)	$R_{j,t} = \bar{R}_t$ for all (j,t)	= COMP	worse than CMU
Capital Market Union (CMU)	Foreign equity share φ	= COMP	= COMP
Complete Markets (COMP)	Backus-Smith condition	= COMP	= COMP

Table 1: Summary of Results

We then ask whether such money market and capital market unions can replicate a complete markets economy, and we show that the answer depends on the types of shocks under consideration. We find that a money market union is enough to deal with (de)leveraging shocks, both public and private. However, a capital market union is necessary to attain (or approximate) the complete markets outcome when there are supply shocks.

For deleveraging shocks we find that the money market union provides the same level of risk sharing as a complete markets economy. Deleveraging has real consequences: it creates an aggregate drag on the economy, and it affects output and employment. One of our main findings is that borrowing and lending across regions allows an efficient sharing of the burden of adjustment created by the deleveraging.

This result is based on a surprising symmetry in the demand effects induced by deleveraging. In our model, deleveraging causes a recession and therefore initially lowers the labor income of savers. However, the lower debt burden of borrowers leads to higher demand in the future, which increases the future income of savers. How do these two effects add up? We show that in the benchmark small country model with Cole-Obstfeld preferences these two effects exactly offset each other so that neither the net present value of savers' nominal income nor their nominal consumption expenditure changes. We show that this result holds approximately in more general models. However, it crucially requires that a money market union-type institution guarantees that funding costs are equalized across regions.

We find that a capital market union is necessary for the efficient sharing of other shocks (supply shocks). These shocks have a first order effect on the market values of assets and can only be shared with cross-border claims on private capital. This also underscores the limitations of a money market union: it cannot share supply shocks. Table 1 summarizes our results.

The structure of this paper is the following. Section 1 introduces the basic model structure. Section 2 studies the risk sharing properties of a money market union and section 3 those of capital market union. Finally, the appendix provides additional numerical results, robustness checks and discussion about model assumptions.

Related Literature Our paper is related to various lines of research in international macroeconomics as well as studies of the causes and consequences of the Eurozone crisis.

The optimal currency area pioneered by Mundell (1961) recognized the importance of a risk sharing mechanism. Kenen (1969) argued that such risk sharing should be organized through inter-regional fiscal transfers. However, Mundell (1973) notes that sophisticated financial markets might provide full insurance.

Cole and Obstfeld (1991) analyze a two-country, two-good endowment economy with flexible prices and show that adjustments to the terms of trade provide insurance against country specific shocks. Heathcote and Perri (2002) analyze production economies and find that models with asset market segmentation match cross-country correlations better than the complete markets model. Kehoe and Perri (2002) endogenize the incompleteness of markets by introducing enforcement constraints that require each country to prefer the allocation it receives by honoring its liabilities rather than living in autarky from any given time onward.

Obstfeld and Rogoff (1995) introduce nominal rigidities in the style of New Keynesian business cycle models into the open economy framework. Ghironi (2006) provides a discussion of this literature and emphasizes the difficulties in modeling market incompleteness. Gali and Monacelli (2008) circumvent the issue by assuming complete asset markets. This is also the approach followed by Blanchard et al. (2014) who model the Eurozone as a two-country (core and periphery) model.

There is a large literature on risk sharing in currency unions. Bayoumi and Masson (1995) discuss the issue of risk sharing and fiscal transfer before the creation of the Euro, and Asdrubali et al. (1996) provide evidence for the US. The Eurozone crisis spurred interest in this topic. Lane (2012) provides a detailed account of the principal drivers of the Eurozone crisis; the specific role of the boom/bust cycle in capital flows is analyzed by Lane (2013) and Gourinchas and Obstfeld (2012). Martin and Philippon (2017) provide a framework and an identification strategy to study the Eurozone crisis. They decompose each country's dynamics into three components: private leverage cycles, sovereign risks, and sudden stops/banking crises. They find that credit spreads play an important role in exacerbating the Eurozone crisis. We extend their analysis to study analytically what type of market integration is necessary for the efficient sharing of different types of shocks. We also enhance their analysis by modeling aggregate demand spillovers and monetary policy. Bolton and Jeanne (2011) analyze the transmission of sovereign debt crises through the banking systems of financially integrated economies. Hepp and von Hagen (2013) provide evidence from Germany and Afonso and Furceri (2008) from the EMU. Schmitt-Grohe and Uribe (2016) emphasize the role of downward wage rigidity. Farhi and Werning (2017) analyze risk sharing in a currency union in a model with nominal rigidities. They show that fixed exchange rates increase the value of risk sharing and that complete markets do not lead to constrained efficient risk sharing. Using a similar model, Auray and Eyquem (2014) argue that complete markets can lead to lower welfare than financial autarky. Hoffmann et al.

(2018) find that the introduction of the euro led to a more integrated interbank market, yet had little effect on cross-border bank-to-firm lending.

A common thread in both IRBC and NOE research is that the composition of financing flows is not discussed in detail beyond distinguishing between complete markets and non-contingent bond economies, as explained in Devereux and Sutherland (2011b) and Coeurdacier and Rey (2012). The authors provide a simple approximation method for portfolio choice problems in general equilibrium models that are solved using first-order approximations around a non-stochastic steady state. A few papers address specifically one of the enduring puzzles in open economy macroeconomics, the home equity bias puzzle. Coeurdacier and Gourinchas (2016) solve jointly for the optimal equity and bond portfolio in an environment with multiple shocks. In Heathcote and Perri (2013), home bias arises because endogenous international relative price fluctuations make domestic assets a good hedge against labor income risk. Sihvonen (2018) studies the aggregate effects of equity home bias in a model that features nominal rigidities and fixed exchange rates. Fornaro (2018) and Benigno and Romei (2014) study the effect of deleveraging shocks in open economies with nominal rigidities. Fornaro (2018) compares the consequences of a tightening of the exogenous borrowing limit in Bewley economies with and without nominal rigidities and fixed exchange rates. Benigno and Romei (2014) consider a two-country model in which one country is a net debtor and the other is a creditor. They analyze the effect of a tightening in the borrowing limit. The literature on sudden stops in emerging markets (Mendoza and Smith, 2006; Mendoza, 2010; Chari et al., 2005) focuses on the imposition of an external credit constraint. These models are couched in representative agent frameworks and do not account for domestic credit flows. On the other hand, the borrower-saver models, (see e.g. Eggertsson and Krugman, 2012), and more generally the two agent New Keynesian models (Bilbiie, 2008; Debortoli and Gali, 2017) lack the international dimension. Our paper instead presents a model that can account for both domestic and external capital flows, which is important for our results.

Finally, some papers have studied the insurance properties of a riskless bond in partial equilibrium settings or endowment economies. Yaari (1976) shows that a patient consumer can self-insure against transitory income shocks through borrowing and lending. This self-insurance property is generally important in heterogeneous agent models with incomplete markets (see e.g. Aiyagari, 1994). Levine and Zame (2002) consider a single good endowment economy. They show that when agents are perfectly patient and endowment shocks are transitory and idiosyncratic, the equilibrium with trading in a single bond attains the complete market outcome. However, we do not assume that shocks are transitory. Rather, we endogenize transitory income effects and we show that, due to general equilibrium effects, demand shocks do not affect savers' nominal wealth or nominal consumption even when they are permanent. Unlike Levine and Zame (2002), we do not assume that agents are perfectly patient and allow for a discount factor below one.

1 Model

We consider a currency union composed of several countries, each of which is populated by a measure of infinitely lived households. Each country produces a tradable domestic good and households consume both domestic and foreign goods. As in Gali and Monacelli (2008), we assume a continuum of small countries. However, as highlighted in the proofs and the appendix, many of our results extend to the case of a finite number of countries. Following Mankiw (2000) and Eggertsson and Krugman (2012), we assume that within each country, households are heterogeneous in their degree of time preference. Specifically, in each region there is a fraction χ of impatient households, and a fraction $1-\chi$ of patient ones. Patient households (indexed by s for savers) have a higher discount factor than borrowers (indexed by b for borrowers): $\beta \equiv \beta_s > \beta_b$. The economies differ with respect to the menu of traded assets available to savers. For now we leave time subscripts out of the model parameters, although we consider (anticipated or unanticipated) shocks to many of them later.

1.1 Preferences and technology

We introduce equilibrium conditions for the home country, but they are defined analogously for the other countries. Households of each type (borrower or saver) derive utility from consumption and labor through Cole-Obstfeld preferences:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta_i^t \left[\log \mathbf{C}_{i,t} - \nu\left(N_{i,t}\right)\right], \text{ for } i = b, s,$$

where $\mathbf{C}_{i,t}$ is a composite good that aggregates goods produced by the home (C_h) and foreign (C_f) countries

$$\log \mathbf{C}_{i,t} = (1-\alpha)\log\left(C_{h,i,t}\right) + \alpha \log\left(C_{f,i,t}\right),$$

and $\alpha < \frac{1}{2}$ is a measure of the openness to trade of the economy; equivalently, $1-\alpha$ measures home bias in consumption.[1] The home good is a composition of intermediate goods produced and aggregated into the final consumption home good using the following constant elasticity (ϵ) of substitution technologies:

[1] With discount rate shocks the borrowers problem is

$$\mathbb{E}_t \sum_{t=0}^{\infty} \prod_{k=0}^{t} \beta_{b,k} \left[\log C_{b,t} - \nu\left(N_{b,t}\right)\right]$$

.

$$C_{h,i} = \left[\int_0^1 c_i\left(j\right)^{\frac{\epsilon-1}{\epsilon}} \, \mathrm{d}j \right]^{\frac{\epsilon}{\epsilon-1}}.$$

The foreign good is a composition of goods produced in the different countries and aggregated into a final good via the technology

$$\log C_{f,i} = \int_0^1 \log(C_{k,i}) \, \mathrm{d}k.$$

Similarly to the home good, each such foreign good is in turn a composite of intermediate goods:

$$C_{k,i} = \left[\int_0^1 c_{k,i}\left(j\right)^{\frac{\epsilon-1}{\epsilon}} \, \mathrm{d}j \right]^{\frac{\epsilon}{\epsilon-1}}.$$

With these preferences, the home consumption-based price index (CPI) is

$$\mathbf{P} = \left(P_h\right)^{1-\alpha} \left(P_f\right)^{\alpha}.$$

Here the domestic producer price index is

$$P_h = \left[\int_0^1 p\left(j\right)^{1-\epsilon} \, \mathrm{d}j \right]^{\frac{1}{1-\epsilon}},$$

where $p(j)$ are prices of intermediate goods and

$$P_f = exp \int_0^1 log(P_k) \, \mathrm{d}k.$$

Similarly for each foreign country the producer price index is

$$P_k = \left[\int_0^1 p_k\left(j\right)^{1-\epsilon} \, \mathrm{d}j \right]^{\frac{1}{1-\epsilon}}.$$

The production of intermediate goods is linear in labor AN, where A is total factor productivity. In the appendix we introduce capital into the production function.

1.2 Wages and prices

We assume the labor market is rationed uniformly across households. This assumption simplifies the analysis because we do not need to keep track separately of the labor income of patient and impatient households within a country. Not much changes if we relax this assumption, except that we lose some tractability.[2] We assume a general form for the wage

[2]In response to a negative shock, impatient households would try to work more. The prediction that hours increase more for credit constrained households appears to be counter-factual however. One can fix

setting function $W_t = g(z^t)$, where z^t denotes the history of state variables up to time t. We do this to emphasize that the specific form of wage setting is immaterial to our theoretical results. Anticipating the discussion in Sections 2 and 3, this is because our theoretical results will describe the behaviour of nominal variables. The precise form of wage setting is of course critical in determining the behaviour of real variables. In the appendix we study the benefits of money market and capital market unions with a calibrated model. To do so we assume sticky wages, which is important for quantitative results. This assumption is employed also when generating the impulse response figures in the theoretical section.

We assume prices are flexible though many of the results hold also with fixed prices. The monopolistically competitive intermediate goods producers set their prices flexibly every period. It follows that:

$$p_t(j) = P_{h,t} = \mu \frac{W_t}{A}, \ \forall j, t,$$

where $\mu \equiv \epsilon / (\epsilon - 1)$ is a markup over the marginal cost $\frac{W_t}{A}$. Since intermediate goods producers charge a markup over marginal cost, they earn profits

$$\Pi_t = (AP_{h,t} - W_t) N_t = (\mu - 1) W_t N_t.$$

1.3 Borrowers' budget constraint

The budget constraint of impatient households (borrowers) in each country is given by

$$\frac{B_{t+1}}{R_t} + W_t N_t - T_t^b = \mathbf{P}_t \mathbf{C}_{b,t} + B_t.$$

Where B_t is the face value of debt issued in period $t-1$ by borrowers, R_t is the nominal interest rate between t and $t+1$, and T_t are lump sum taxes. Borrowing is denominated in units of the currency of the monetary union and is subject to an exogenous limit $\bar{B}$:

$$B_{t+1} \leq \bar{B}.$$

In the numerical calibrations we assume that the borrowers are impatient enough that they always borrow up to the constraint, so $B_{t+1} = \bar{B}$. However, this assumption is not required for most of the theoretical results.

this by assuming a low elasticity of labor supply, which amounts to assuming that hours worked are rationed uniformly in response to slack in the labor market. Assuming that the elasticity of labor supply is small (near zero) also means that the natural rate does not depend on fiscal policy. In an extension we study the case where the natural rate is defined by the labor supply condition in the pseudo-steady state $\nu'(n_i^\star) = (1-\tau_j)\frac{w_j}{x_{i,j}}$. We can then ration the labor market relative to their natural rate: $n_{i,j,t} = \frac{n_i^\star(\tau)}{\sum_i n_i^\star(\tau)} n_{j,t}$ where $n_i^\star(\tau)$ is the natural rate for household i in country. This ensures consistency and convergence to the correct long run equilibrium. Steady state changes in the natural rate are quantitatively small, however, so the dynamics that we study are virtually unchanged. See Midrigan and Philippon (2010) for a discussion.

1.4 Monetary and fiscal policy

The precise form of the monetary policy rule $\bar{R}_t(z^t)$, given a history of state variables z^t, is not important for the theoretical results. However, we assume that the policy rate does not react to purely domestic shocks facing a small open economy. That is $\bar{R}_t(z^t) = \bar{R}_t(\tilde{z}^t)$, where $\tilde{z}^t$ is a history of aggregate state variables. This is also where the assumption of a fixed exchange rate comes to play. Our key results hold given flexible exchange rates if we assumed that the policy rate does not react to the country specific shocks. However, generally with independent monetary policy the domestic central bank might choose to react to these shocks, which would affect the results.

The government budget constraint is:

$$\frac{B^g_{t+1}}{R_t} = P_{h,t}G_t - T_t + B^g_t. \tag{1}$$

The rate on government debt is R_t and tax receipts are $T_t = \chi T^b_t + (1-\chi) T^s_t$. Here we assume the government consumes only domestic goods, which is important for our results concerning government spending shocks. We assume away from state-contingent fiscal transfers between governments; on fiscal unions see for example Farhi and Werning (2017).[3]

1.5 Savers' budget constraint in each of the economies

Segmented Markets (SMU) and Money Market Union (MMU) Savers save at the rate R_t. The savers' budget constraint is

$$S_t + W_t N_t - T^s_t + \frac{\Pi_t}{1-\chi} = \mathbf{P}_t \mathbf{C}_{s,t} + \frac{S_{t+1}}{R_t},$$

where Π_t are per-capita profits from intermediate good producers. Only savers in each country have claims to these profits, so $\frac{\Pi_t}{1-\chi}$ are profits per saver. Under MMU, the interest rate at home is always equal to the interest rate in the union: $R_t = \bar{R}_t$ for all t. Under SMU, on the other hand, we can have $R_t \neq \bar{R}_t$ and we will need to specify how R_t is determined. SMU is considered primarily in the numerical section in the appendix.

Capital Market Union (CMU) In a capital market union savers can additionally trade a continuum of stocks. Each such stock k represents a claim to the aggregate profit stream in country k. The savers' budget constraint in the home country is

$$S_t + W_t N_t - T^s_t + \int_k \varphi_{t,k}\left(V_{t,k} + \Pi_{t,k}\right) = \int_k \varphi_{t+1,k} V_{t,k} + \mathbf{P}_t \mathbf{C}_{s,t} + \frac{S_{t+1}}{R_t},$$

[3] Note that in our complete markets case the savers, but governments, can write state-contingent contracts.

where $\varphi_{t,k}$ are the home savers' aggregate holdings of the country k stocks and $V_{t,k}$ is the price of country k stock. In an (ideal) CMU this stock trading is frictionless.

Complete Markets In the complete markets economy, savers have access to a full set of state contingent securities. We denote purchases at time t of securities paying off one unit of currency at time $t+1$ contingent on the realization of state z_{t+1} following history z^t by $D_{t+1}(z_{t+1}, z^t)$; this security has a time t price $Q_t(z_{t+1}, z^t)$:

$$S_t + W_t N_t - T_t^s + \frac{\Pi_t}{1-\chi} + \int_{z_{t+1}} Q_t\left(z_{t+1}, z^t\right) D_{t+1}\left(z_{t+1}, z^t\right) dz_{t+1} = D_t\left(z^t\right) + \mathbf{P}_t \mathbf{C}_{s,t} + \frac{S_{t+1}}{R_t}.$$

1.6 Equilibrium conditions

Demand functions for the home and foreign consumption bundles by savers and borrowers are given by

$$P_{h,t} C_{i,t} = (1-\alpha) \mathbf{P}_t \mathbf{C}_{i,t}, \text{ for } i = b, s. \tag{2}$$

Savers are unconstrained and their consumption is determined by their Euler equation and budget constraint (which differs depending on which assets are available, as discussed in Section 1.5):

$$\frac{1}{\mathbf{P}_t \mathbf{C}_{t,s}} = \beta_s R_t \mathbb{E}_t \left[\frac{1}{\mathbf{P}_{t+1} \mathbf{C}_{t+1,s}} \right]. \tag{3}$$

When borrowers are unconstrained their consumption is characterized by a similar Euler equation. Market clearing in goods is given by

$$AN_t = \int_k \left(\chi_k c_{k,h,b,t} + (1-\chi_k) c_{k,h,s,t}\right) + G_t\,, \tag{4}$$

where $c_{k,h,b,t}$ and $c_{i,h,s,t}$ are consumption of home goods by borrowers and savers from country k. Finally, market clearing for borrowing requires

$$\int_k (1-\chi_k) S_{t+1,k} = \int_k \chi_k B_{t+1,k} + \int_k B^g_{t+1,k}\,, \tag{5}$$

and (if available) that for stocks $\int_k (1-\chi_k) \psi_{t+1,k} = 1$ and for Arrow-Debreu securities $\int_k (1-\chi_k) D_{t,k}(z_{t+1}, z^t) = 0$ for all z_{t+1}.

2 Money Market Union

In this section we study demand shocks under MMU: specifically, shocks that come from private borrowing or fiscal policy. Our key theoretical result is that an ideal MMU provides perfect risk sharing with respect to these shocks. Later we argue that this result is robust to several variations of the model structure. Under MMU, the funding cost is the same in all countries. The k-period discount rate from e savers' perspective is $R_{t,k} \equiv R_t \times .. \times R_{t+k-1}$, with the convention $R_{t,0} = 1$. We also define $\tilde{Y}_t \equiv P_{h,t} N_t - T_t$ as nominal private disposable income and F_t as nominal exports.

The first step is to write the current account equilibrium in market values. We then have the following Lemma:

Lemma 1. *The inter-temporal current account condition (for each country) is*

$$\alpha \left((1-\chi) S_t - \chi B_t + \sum_{k=0}^{\infty} \frac{\tilde{Y}_{t+k}}{R_{t,k}} (z^{t+k}) \right) = (1-\chi) S_t - \chi B_t - B_t^g + \sum_{k=0}^{\infty} \frac{F_{t+k}}{R_{t,k}} (z^{t+k}) \quad (6)$$

for each path of history z^{∞}.

Proof. See Appendix. □

The left hand side is the net present value of all future imports, which is a share α of private wealth, which itself equals financial wealth plus the present value of disposable income. On the right we have net foreign assets plus the present value of nominal exports (F_t). The key insight is that the inter-temporal current-account condition pins down the NPV of disposable income as a function of current assets and foreign demand. With unit demand elasticity nominal exports are exogenous to a small country.

The result requires an intratemporal unit demand elasticity over home and foreign goods, but not log intertemporal preferences (which will be necessary for the following results). The result does not depend on form of the production function, the labor supply condition, fiscal policy or whether prices are sticky or flexible. In an open economy model with unit demand elasticities and a fixed α, the NPV of exports and country's net wealth fully determine the NPV of disposable income independently of issues such as productivity, disutility of labor or level of taxation. Still, such features generally have an impact on macroeconomic quantities other than the NPV of disposable nominal income.

The next step is to consider the program of the savers. With log-preferences, we can reformulate the savers' problem as a choice of nominal consumption:

$$\begin{gathered} \max \mathbb{E}_t \textstyle\sum_{t \geq 0} \beta_s^t \log (\mathbf{P}_t \mathbf{C}_{s,t}) \\ s.t.\ \mathbf{P}_t \mathbf{C}_{s,t} + \frac{S_{t+1}}{R_t} = S_t + \tilde{Y}_t^s. \end{gathered}$$

The inter-temporal budget constraint of savers is

$$\sum_{k=0}^{\infty} \frac{\mathbf{P}_{t+k} \mathbf{C}_{s,t+k}}{R_{t,k}} (z^{t+k}) = S_t + \sum_{k=0}^{\infty} \frac{\tilde{Y}_{t+k}^s}{R_{t,k}} (z^{t+k}), \quad (7)$$

where $\tilde{Y}_t^s = W_t N_t - T_t^s + \frac{\Pi_t}{1-\chi}$ is the disposable income of savers. Savers have a claim on corporate equity and might face different taxes than borrowers who earn $\tilde{Y}_t^b = W_t N_t - T_t^b$. To derive our first result, we need to make a connection between the disposable income of savers $\tilde{Y}_t^s$ that enters Equation (7) and the average disposable income $\tilde{Y}_t = (1-\chi)\,\tilde{Y}_t^s + \chi \tilde{Y}_t^b$ that enters Equation (6). If taxes are arbitrary, there is of course very little that we can say. Therefore, we restrict our attention to a class of fiscal policies where the following condition holds:

Condition 1. For each history z^∞ , the present value of savers' disposable income is a function of the present value of average disposable income and does not depend on any other variable.[4]

$$\sum_{k=0}^{\infty} \frac{\tilde{Y}_{t+k}^s}{R_{t,k}}(z^{t+k}) \sim \sum_{k=0}^{\infty} \frac{\tilde{Y}_{t+k}}{R_{t,k}}(z^{t+k}).$$

Condition 1 ensures that the net present value of aggregate income is a sufficient statistic for the net present value of savers' income. Condition 1 imposes some restrictions on fiscal policy, but it holds in many natural settings and in all the applied models that we have studied.[5] The simplest example is uniform flat taxation of all income at rate τ_t, i.e., $T_t^b = \tau_t W_t N_t + T_t^{b,LS}$ and $T_t^s = \tau_t \left(W_t N_t + \frac{\Pi_t}{1-\chi}\right) + T_t^{s,LS}$ with lump-sum taxes such that $\sum_{k=0}^{\infty} \frac{T_t^{b,LS}}{R_{t,k}}(z^t) \sim \sum_{k=0}^{\infty} \frac{T_t^{s,LS}}{R_{t,k}}(z^t)$. For example when the lump-sum taxes are zero $\tilde{Y}_t^b = (1-\tau_t)\,W_t N_t$ and $\tilde{Y}_t^s = (1-\tau_t)\left(W_t N_t + \frac{\Pi_t}{1-\chi}\right) = (1-\tau_t)\,W_t N_t \left(1 + \frac{\mu-1}{1-\chi}\right)$. Therefore, all taxes, income and profits are proportional to $W_t N_t$. In particular, $\tilde{Y}_t = \mu\,(1-\tau_t)\,W_t N_t$, and therefore $\tilde{Y}_t^s = \frac{1}{\mu}\left(1 + \frac{\mu-1}{1-\chi}\right)\tilde{Y}_t$. With constant markups, all disposable incomes are directly proportional, period-by-period, which is stronger than Condition 1 (we discuss variable markups briefly in the next section). With Lemma 1 and Condition 1, we obtain the following result.

Lemma 2. *Under Condition 1 and log-preferences, nominal spending by savers ($\mathbf{P}_t\mathbf{C}_{s,t}$) does not react to private credit shocks ($\bar{B}_{t+1}$), to borrowers' discount rate shocks ($\beta_{b,t}$) or to fiscal policy (neither G_t nor T_t). Spending only reacts to interest rate and foreign demand shocks.*

Proof. Lemma 1 shows that the net present value of disposable income is a function of exactly four variables:

$$\sum_{k=0}^{\infty} \frac{\tilde{Y}_{t+k}}{R_{t,k}}(z^{t+k}) \equiv \Omega\left(S_t, B_t, B_t^g, \sum_{k=0}^{\infty} \frac{F_{t+k}}{R_{t,k}}(z^{t+k})\right),$$

[4] To rule out ill-defined cases we require that this function be differentiable.

[5] Note that we assume a general form for wages yet flexible prices. In a model with fixed prices but flexible wages this condition may be violated due to fluctuations in markups; we briefly discuss markup fluctuations in Section 3.

where the first three variables (saving, household debt, public debt) are predetermined at time t and the last one (exports in euros) is exogenous given a unit demand elasticity. Therefore under Condition 1, equation (7) is, in fact,

$$\sum_{k=0}^{\infty} \frac{\mathbf{P}_{t+k}\mathbf{C}_{s,t+k}}{R_{t,k}}(z^{t+k}) \sim S_t + \Omega_t(z^{\infty}).$$

In equilibrium with log-preferences, savers' current nominal expenditure ($\mathbf{P}_t\mathbf{C}_{s,t}$) depends only on S_t and Ω_t and the path of nominal interest rates. In particular, for given Ω_t and interest rates, it does not depend on contemporaneous or future private credit, borrowers' discount rate, or fiscal policy. □

Lemma 2 clarifies the behavior of savers. Let us emphasize again that we are focusing on *nominal* variables. The *nominal* spending of savers reacts neither to credit shocks nor to fiscal shocks. Deleveraging shocks affect the savers in two ways. First, if this debt was held by domestic savers, deleveraging results in repayments of debt. However, the savers can substitute these repayments by lending more to foreign countries. The fact that this direct effect does not affect the *net present value* of savers income and therefore their spending is perhaps not surprising. However, deleveraging also lowers the demand of borrowers. Since currency union wide monetary policy does not react to the idiosyncratic deleveraging shock, this causes a fall in employment, lowering labor income and profits received by savers. Intuitively, savers' consumption should therefore fall. But borrowers' demand in future periods increases by virtue of their reduced debt burden, which increases the savers' future income. What is surprising is that for any distribution of deleveraging shocks this future increase in income exactly offsets the initial fall so that the NPV of savers income does not change. As a result, patient agents keep their nominal spending constant.

As regards fiscal policy, Lemma 2 implies that changes in government spending do not affect the nominal consumption of savers; by extension, if there are no borrowers, the result implies that changes in government spending have no effect on nominal household consumption[6]. This result is different from Ricardian equivalence and obtains because Cole-Obstfeld preferences and the SOE assumption imply a nominal fiscal consumption multiplier of zero. This implication can be seen as a version of the Cole and Obstfeld (1991) result and is discussed further in Lemma 5 in Appendix D. In simple economic terms this is because: i) the interest rate does not react due to the small country assumption, ii) nominal exports do not react due to Cole-Obstfeld preferences, i.e. there is (no "leakage"). Appendix G discusses the robustness of this result to deviations from Cole-Obstfeld preferences. Note that irrespective of preferences the *real* fiscal consumption multiplier, a statistic studied for example by Farhi and Werning (2013), is generally not zero.

[6] We discuss this result further and show impulse response function for this case in Appendix F

We now state our first main result:

Proposition 1. *The Money Market Union achieves the Complete Markets allocation subject to (an arbitrary cross-sectional distribution of) country-specific private and public demand shocks ($\bar{B}_{t+1}$, $\beta_{b,t}$, G_t, T_t) using dynamic cross-country borrowing.*

Proof. Under MMU, the interest rate is the same in all countries and is independent of idiosyncratic shocks to the SOE. The complete markets outcome is characterized by the Backus-Smith condition, which, with log preferences, takes the form

$$\frac{\mathbf{C}_{s,t,j}}{\mathbf{C}_{s,t}} \sim \frac{\mathbf{P}_t}{\mathbf{P}_t^j},$$

for arbitrary foreign country j. Since shocks to an SOE do not affect foreign prices or quantities, it follows that the complete markets condition is also that $\mathbf{P}_t\mathbf{C}_{s,t}$ remains constant. Given Lemma 2 in response to deleveraging shocks coming either from a change in the borrowers' credit constraints or the discount rate (or both simultaneously), the MMU replicates the complete markets economy. □

Proposition 1 shows that a money market union is sufficient to deal with any cross-sectional distribution of debt deleveraging and fiscal shocks in a currency union. Martin and Philippon (2017) show that segmented markets, in contrast, can be very inefficient. They find that spreads go up during episodes of private deleveraging, mostly because of stress in the banking sector. This leads savers (or firms under Q-theory) to cut spending precisely when the economy is in recession, exacerbating the downturn. We quantify the welfare gains from MMU in the appendix.

Proposition 1 is consistent with the following heuristic partial equilibrium reasoning about the effects of deleveraging. Assume a fixed interest rate R and that during the first period borrowers reduce debt (and therefore consumption) by 1 euro. This reduces GDP by $(1-\alpha)(1-\chi)$ euros in the first period but increases it by $(1-\alpha)(R-1)(1-\chi)$ euros in all the following periods. The total effect on the NPV of the country's GDP and income is

$$-(1-\alpha)\chi + \frac{(1-\alpha)(R-1)\chi}{R} + \frac{(1-\alpha)(R-1)\chi}{R^2} + \frac{(1-\alpha)(R-1)\chi}{R^3}... = 0.$$

Now a saver can fully smooth this shock, that does not affect her permanent income, by dis-saving in the first period. Proposition 1 shows that this reasoning is *exactly* valid in *general equilibrium* assuming a continuum of small countries, Cole-Obstfeld preferences and that the NPV of savers' income is a function of the NPV of the country's income.

The Proposition is different from previous hedging results in the international macroeconomics literature, such as those found by Coeurdacier and Gourinchas (2016) and Coeurdacier et al. (2010). They consider two country models with trading in two real bonds as well as equity claims and find that countries can share risks using static positions in the real bonds. In contrast, we consider a setting with trading in one nominal bond with a common interest rate and show that countries can share risks through dynamic cross-country borrowing. Our result also differs from the results in Engel and Matsumoto (2009), who show that agents can hedge risks through a static forward position in foreign exchange.

Figure 1 plots the impulse responses to a domestic deleveraging shock (credit shock) given the calibration of the model presented in table 4 in the appendix[7]. Deleveraging affects borrowers' spending and initially creates a recession. Savers smooth this fall in income by borrowing more from foreign countries. After the first period, this deleveraging has a small positive effect on output, wages and profits as borrowers' lower interest expenses boost demand. This additional income offsets the lower interest rate income received by savers who now hold a smaller stock of savings. As implied by Proposition 1, savers' nominal expenditure does not react to these changes. This is because the negative and positive income effects of deleveraging exactly offset each other so that the NPV of savers' income does not change.

Proposition 1: Beyond Cole-Obsteld Similarly to for example Gali and Monacelli (2008), Heathcote and Perri (2013) and Martin and Philippon (2017) our framework assumes Cole-Obstfeld preferences. That is, we assume log-preferences and a unit elasticity of substitution between all goods. However, we next explain that Proposition 1 holds approximately for deleveraging shocks with more general preferences. The appendix provides additional robustness checks for the results, for example explaining that they hold numerically well in the case of a two country model.

Different Demand Elasticities To relax the unit elasticity of demand assumption we now consider the aggregators:

$$\mathbf{C}_{i,t} = \left((1-\alpha)^{\xi_1} \left(C_{h,i,t}\right)^{(\xi_1-1)/\xi_1} + \alpha^{\xi_1} \left(C_{f,i,t}\right)^{(\xi_1-1)/\xi_1}\right)^{\xi_1/(\xi_1-1)}, \text{ for } i = b, s,$$

$$C_{f,i} = (\int_0^1 C_{k,i}^{(\xi_2-1)/\xi_2} \,\mathrm{d}k)^{\xi_2/(\xi_2-1)}.$$

Here ξ_1 is the demand elasticity between the home good and the aggregate foreign good. Moreover, ξ_2 is the demand elasticity between different varieties of foreign goods.

We now consider private deleveraging shocks. Figure 2 shows the response in savers' nominal consumption for four different values of elasticity of substitution between home and foreign goods $\xi_1 = \in \{0.5, 1, 2, 1000\}$. One can see that the results are virtually identical for these different values. When demand elasticity is high, nominal consumption stays

[7]Here the model abstracts away from capital so we do not need the capital share and depreciation rate

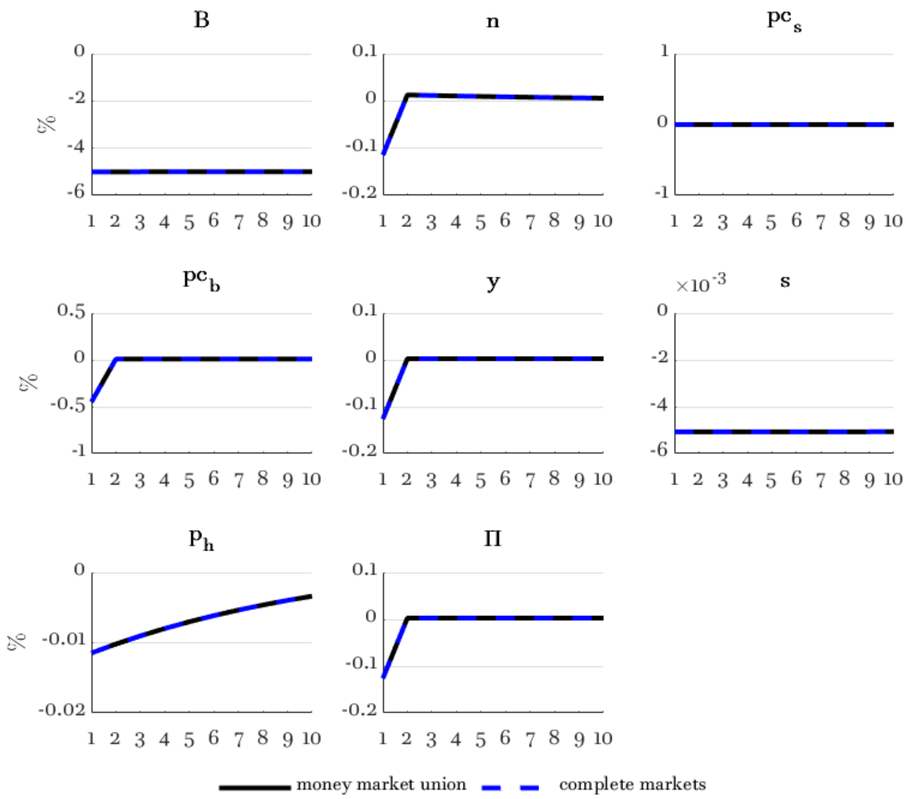


Figure 1: Private Deleveraging

Note: Impulse responses to permanent -5% shock to $\bar{B}_t$. $y_t$is nominal GDP.

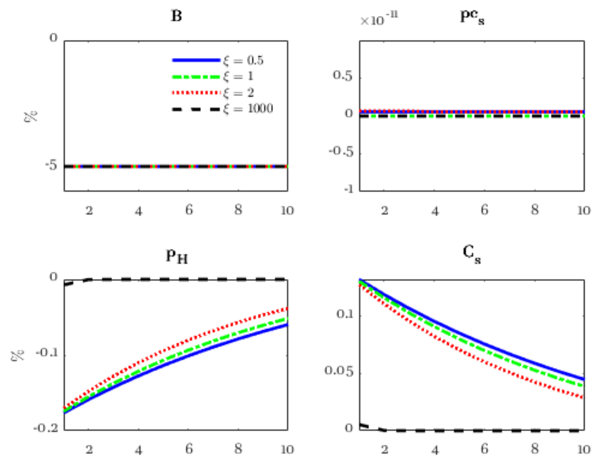


Figure 2: Private deleveraging in a money market union for different values of elasticity of substitution between home and foreign goods
Note: Impulse response to permanent -5% shock to $\bar{B}_t$.

roughly constant because prices and real consumption do not react. When this elasticity is low, the response in nominal consumption is small because increases in real consumption are offset by a lower price of the home good.

We repeat this exercise but now with different values of elasticity of substitution between different varieties of foreign goods $\xi_2 \in \{0.5, 1, 2, 1000\}$ [8] . The results are given in 3 and look similar to those before.

In some cases we can actually show that the key result of 1 holds up to first order for any values of the demand elasticity parameters $\xi_1$and ξ_2. In particular we have the following lemma:

Lemma 3. *Assume the labor supply condition is of the form* $W_t = h(N_t, \mathbf{P}_t^i, \chi \mathbf{P}^i \mathbf{C}_{b,t} + (1-\chi)\,\mathbf{P}_t^i \mathbf{C}_{s,t})$. *Now Proposition 1 holds in a first order approximation for private deleveraging shocks* $(\bar{B}_{t+1},\ \beta_{b,t})$ *for any demand elasticity parameters* ξ_1*and* ξ_2

Proof: See Appendix

This generalizes the results in Lemmas 1 and 2. Now the result of Proposition 1 follows immediately. Effectively changing the demand elasticity parameters alters the response of GDP to a deleveraging shock due to price adjustments. However, in a first order approximation the NPV of these price adjustment effects is still zero. Therefore Proposition 1 still holds up to first order. Alternatively, Cole-Obstfeld preferences imply a type of linearity in the demand effects induced by deleveraging. This linearity is why the effects of deleveraging

[8]Here the demand elasticity changes in all countries simultaneously

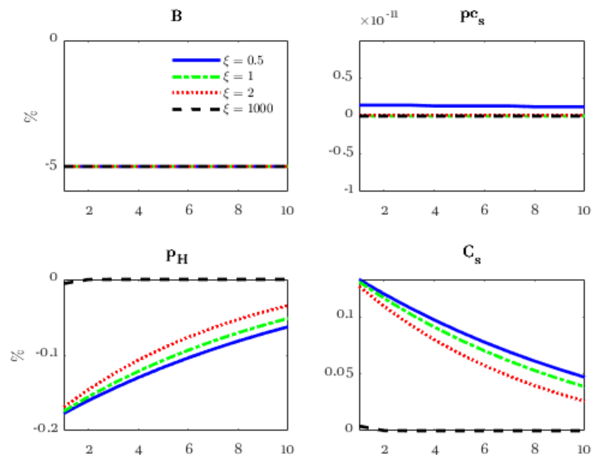


Figure 3: Private deleveraging in a money market union for different values of elasticity of substitution between different varieties of foreign goods
Note: Impulse response to permanent -5% shock to $\bar{B}_t$.

net out so that the NPV of savers' income does not change. Such linearity still holds in a first order approximation for any values of demand elasticities.

The appendix studies the robustness of Proposition 1 with respect to fiscal shocks. Here the results can be more sensitive to changes in demand elasticities depending on the form of household taxes.

CRRA We now consider CRRA preferences over the final good

$$\frac{\mathbf{C}_{i,t}^{1-\gamma}}{1-\gamma} \text{ for } i = b, s,$$

Lemma 1 still holds with CRRA preferences as the proof makes no assumption concerning preferences over the final good. But what about Proposition 1? We next argue that it can also be generalized. Figure4 shows the response of nominal spending to a temporary deleveraging shock given a CRRA parameter of 2 (EIS $=\frac{1}{2}$). One can see that a tightening of the borrowing limit leads to a decrease in nominal spending. This effect is numerically small: a 5% decrease in the borrowing limit leads to a less than 0.07% decrease in savers' nominal spending. However, complete markets predict that marginal utility rather than nominal spending should remain constant, which are generally different given the CRRA assumption. Therefore, the figure also plots the response of nominal spending under complete markets. Interestingly the response under complete markets is very similar to that in MMU.

This observation motivates the following lemma:

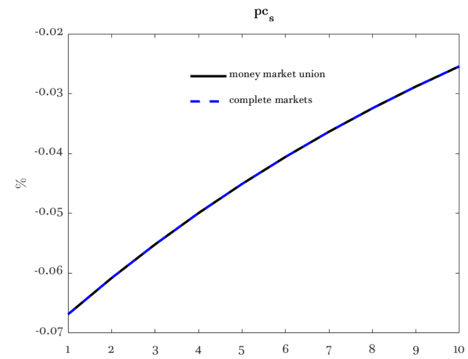


Figure 4: Private deleveraging in a money market union and complete markets with a CRRA parameter of 2 (EIS=0.5).
Note: Impulse response to permanent -5% shock to $\bar{B}_t$.

Lemma 4. *Assume the labor supply condition is of the form* $W_t = h(N_t, \mathbf{P}_t^i, \chi \mathbf{P}^i \mathbf{C}_{b,t} + (1-\chi)\, \mathbf{P}_t^i \mathbf{C}_{s,t})$*. Now Proposition 1 holds in a first order approximation for private demand shocks* ($\bar{B}_{t+1}$, $\beta_{b,t}$) *for any CRRA parameter* γ

Proof: See Appendix

The logic of this proposition is that an CRRA agent prefers to smooth consumption by holding marginal utility constant. In a first order approximation the price effects of demand shocks add up to zero in NPV terms so that keeping marginal utility constant is affordable. These results hold for arbitrary combinations of demand elasticities and CRRA parameters. However, when all demand elasticities equal one, Lemma 4 holds also for fiscal shocks.

We conclude that the key results of the section hold up to first order with general CRRA preferences and arbitrary demand elasticities. That is, while they hold exactly with commonly used log-preferences, they also hold approximately in more general models.

3 Capital Market Union

In this section we focus on the benefits of a capital market union above an ideal money market union. We pay special attention to technology shocks in the form of "quality" shocks to the goods sold by firms. Formally, we model these shocks as changes to quality parameters α_i (possibly correlated across countries). These shocks alter the relative profitability of firms in different countries. The money market union will not be able to share this kind of risk, but the capital market union could, at least in principle. The following proposition characterizes the types of shocks that can be shared efficiently in a CMU.

Proposition 2. *Assume borrowers are impatient enough to borrow up to the borrowing constraint. Using static equity positions and no-cross country borrowing, it is possible to replicate the complete markets allocation in a capital market union subject to (an arbitrary cross-sectional distribution) of quality* (α_t)*, TFP* (A_t)*, monetary policy, and various preference shocks (that can be correlated across countries).*

Proof. To highlight that the result does not depend on the assumption of a continuum of countries we show it in an I country version of the model from which we can see that it holds also when $I \to \infty$.[9] The equilibrium conditions for this version of the model are very similar to those with a continuum of countries. Here the mass of each country is $\frac{1}{I}$. We assume symmetric countries but relax this in the appendix. Given symmetric countries and log preferences the complete markets condition is $\mathbf{P}_t^i \mathbf{C}_{s,t,i} = \mathbf{P}_t^j \mathbf{C}_{s,t,j}$. Imposing symmetric and constant stock positions as well as constant taxes, government spending and borrowing, and borrowing limits, the savers' budget constraints in countries i and j are

$$\mathbf{P}_t^i \mathbf{C}_{s,t,i} = \\ \bar{B}(1 - \frac{1}{R_t}) + W_{t,i}N_{t,i} + \varphi \frac{(\mu - 1)W_{t,i}N_{t,i}}{1-\chi} + \sum_{j \neq i} \frac{(1-\varphi)}{I-1} \frac{(\mu-1)W_{t,j}N_{t,j}}{1-\chi},$$

where we used the assumption for the production function and the fact that taxes and transfers cancel assuming no new borrowing by government. Moreover, to simplify expressions in this case of symmetric countries, but without loss of generality, we here choose a different normalization of stock supply. Namely, each unit of the home stock entitles a saver to a dividend of $\frac{\Pi_t}{1-\chi}$. Deducting the conditions for two countries i and $j \neq i$ we obtain

$$\mathbf{P}_t^i \mathbf{C}_{s,t,i} - \mathbf{P}_t^j \mathbf{C}_{s,t,j} = \\ W_{t,i}N_{t,i} - W_{t,j}N_{t,j} + \varphi \frac{(\mu-1)W_{t,i}N_{t,i} - (\mu-1)W_{t,j}N_{t,j}}{1-\chi} \\ -(\mu-1)\frac{W_{t,i}N_{t,i} - W_{t,j}N_{t,j}}{1-\chi}\frac{1-\varphi}{I-1} = (W_{t,i}N_{t,i} - W_{t,j}N_{t,j})\left(1 + \varphi\frac{\mu-1}{1-\chi} - \frac{1-\varphi}{I-1}\frac{\mu-1}{1-\chi}\right).$$

Imposing the complete markets condition and ignoring the indeterminacy case (discussed in the appendix), we need

$$1 + \varphi\frac{\mu-1}{1-\chi} - \frac{1-\varphi}{I-1}\frac{\mu-1}{1-\chi} = 0. \tag{8}$$

From this one can solve

$$\varphi = \frac{1}{I} - \frac{I-1}{I}\frac{1-\chi}{\mu-1}. \tag{9}$$

[9] In the limit there is a countable infinity of countries instead of a continuum of countries. However, the limiting model is effectively equivalent to a continuum economy, see Sihvonen (2019) for a discussion. Moreover, we could prove all the results by imposing a continuum of countries a priori.

With these stock positions the complete markets condition holds for arbitrary labor income realizations. The complete markets condition also ensures that the Euler equations for stocks and borrowing hold. Therefore, the above stock positions and no-cross country borrowing constitute an equilibrium that replicates the complete markets outcome. In the small open economy limit $I \to \infty$, a saver should hold $-\frac{1-\chi}{\mu-1}$ home stocks and $1+\frac{1-\chi}{\mu-1}$ foreign stocks split equally. $\square$

To efficiently share quality shocks, savers should underweight home stocks. As explained in the appendix, various frictions often lead savers to do the opposite and overweight home stocks. Hence we could define a CMU as the removal of such equity market frictions. However, if these frictions cannot be removed perfectly, a full CMU might be unattainable. Here a capital market union with partially segmented equity markets is able to share some but not all of the risks associated with the shocks.

Note that the proposition holds for various different types of shocks, including quality shocks, TFP shocks and monetary policy shocks. It also holds for all types of preference shocks that do not alter the complete markets condition. This includes shocks to the disutility of labor that typically affects the relationship between labor supply and wages. Moreover, the number of shocks can be higher than the number of assets; this is in contrast to the usual finding that obtaining the complete markets outcome requires at least as many assets as shocks (see e.g. Coeurdacier and Gourinchas (2016)). The exact theoretical result hinges on log-preferences as well as the assumed form of the production function. However, it does not require a unit elasticity of substitution or a continuum of countries. [10]

The assumption that the borrowers borrow up to the constraint rules out cases in which a supply shock would indirectly induce leveraging or deleveraging. We relax this assumption in Proposition 3. Note that as explained by Lemma 5, due to Cole-Obstfeld preferences, TFP shocks do not affect nominal consumption assuming fixed quality parameters α_i. However with stochastic quality parameters, sharing TFP shocks can require diversification in stock positions.

Figure 5 shows the outcomes of a home quality shock in a money market union, a partial capital market union (with equal weights on home and foreign stocks), and complete markets (equivalently, a CMU with optimal weights). With complete markets savers' spending reacts neither in the home country nor in the foreign countries. Proposition 2 shows that if stock positions are chosen correctly, the capital market outcome coincides with the complete markets case. With equal weights on home and foreign stocks, savers' spending in the home country increases. This increase, however, is smaller than in a money market union without cross-border equity claims.

[10] The production function implies a perfect correlation between dividends and labor income. The result would also hold in a model with a fixed capital stock but not in a model with investment. However, it holds approximately in a model with investment with realistic investment adjustment costs.

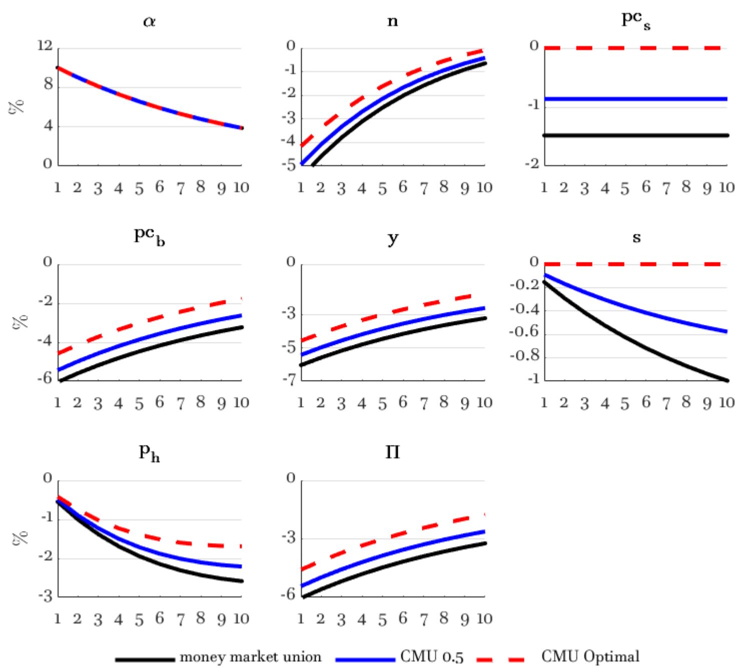


Figure 5: Quality Shocks in MMU and CMU
Notes: Impulse responses to 10% shock to α. CMU 0.5 has an exogenous weight of 0.5 on the home stock and a weight of 0.5 on foreign stocks split equally. Complete markets is equivalent to a CMU with optimal weights, as explained in Proposition 2. MMU is CMU with zero weight on foreign stocks.

Note in an optimal CMU cross-border equity holdings provide full insurance against supply shocks and savers have no incentives for cross-country borrowing. However, in a partial CMU savers also borrow more from foreign countries to gain additional smoothing.

Note that our definition of a MMU implies perfect home bias in equity, whereas we define a CMU as featuring optimal cross-border holdings of equity. We have in mind a situation in which some friction prevents savers from optimally diversifying their equity holdings, and a capital market union can be thought of as the removal of this friction. We do not explicitly model such frictions in this paper; for more elaborate micro-foundations of equity home bias and related discussions see, for example, Coeurdacier and Rey (2012) and Sihvonen (2018).

Simultaneous Supply and Demand Shocks Proposition 1 shows that by using dynamic borrowing a MMU is able to share demand shocks. Proposition 2 argues that by using static equity positions a CMU can share quality shocks. In a first-order approximation these results add up in a fairly straightforward way. In our framework we also obtain the following exact result:

Proposition 3. *Using static equity positions and dynamic cross-country borrowing it is possible to replicate the complete markets allocation in a capital market union subject to (an arbitrary cross-sectional distribution of) (country specific) private deleveraging as well as arbitrary foreign quality, productivity, monetary policy, and various preference shocks.*

Proof: See Appendix.

Shocks that Can Be Shared Neither in MMU or CMU We have provided results for the types of shocks that can be shared perfectly either in MMU or CMU. We have covered a broad array of shocks including credit, discount rate, taxation, government spending, quality, productivity, monetary policy and disutility of labor shocks. Are there shocks, then, for which the CMU does not attain replicate the complete markets outcome? Yes: a salient example is a redistributive shock such as a mark-up shock that alters the relative shares of labor and dividend income. In case of such shocks neither a MMU nor a CMU exactly obtains the complete markets outcome.

What could be done to attain the complete markets outcome in the case of mark-up shocks? The issue with such shocks is that they tend to redistribute income between borrowers and savers in way the savers cannot hedge using bond or equity positions. However, this effect could be offset using redistributive fiscal transfers[11] A detailed analysis of such fiscal policies is beyond the scope of this paper.

On Empirical Tests of Model Predictions Testing the empirical validity of our theoretical results about the types of shocks that can be shared efficiently either in MMU or CMU is challenging because our results describe counterfactuals. For example, according to Proposition 1 an idealized MMU, in which risk-free rates are fully equalized, could efficiently share deleveraging shocks. However, actual deleveraging episodes such as those observed during the Eurozone crisis tend to be associated with segmentation in risk-free rates. Perhaps the best way to test this proposition would be to consider a region such as US that is closer to a money market and banking union type arrangement with smaller regional differences in state level funding costs. If the Eurozone is also able to implement a well-functioning banking union, future deleveraging periods could also be used for such tests. However, note that this would require carefully identifying a deleveraging shock. Similarly, Proposition 2 could be tested using a region with a high level of capital market integration such as the US. Again, this would require identifying supply shocks.

Giroud and Mueller (2016) show that the pattern of investment and employment across US locations during the great recession is consistent with what we call a money market union. Following the terminology of Holmström and Tirole (1997), they show that there is

[11]Introducing additional financial instruments can of course help in attaining the complete market case with respect to such shocks.

no local credit crunch but there is some collateral squeeze. Using census data, Giroud and Mueller (2016) find that the employment of manufacturing establishment does not respond to local house price shocks. This is what our model predicts for traded goods and assuming that costs of funds are not affected by local shocks. Aggregating at the firm level they find results consistent with money market union (no local credit crunch) together with balance sheet/cash flow channels. When firms with low leverage are hit by local demand shocks they do not decrease investment. Instead, they increase short and long term debt to smooth the shocks. This shows that funding costs are equalized in the cross section, or, in the the terminology of Holmström and Tirole (1997), there are no local credit crunches. This does not mean, however, that there are no credit constraints: in fact, Giroud and Mueller (2016) find that firms with high leverage do not smooth these shocks. This is exactly what we assume in our model, except that we focus on household credit constraints (the model works in the same way with credit constrained small firms, as explained in Gourinchas et al. (2016)).

4 Conclusion

Failures of risk sharing lie at the heart of many economic crises. Such crises are particularly acute in the context of a currency union in which constituent countries are hit by large, asymmetric shocks; the Eurozone crisis of 2009-12 stands as a particularly striking example.

This paper presents two main theoretical findings. The first is that in the case of demand shocks - for example, private or public deleveraging - an idealized money market union in which risk-free rates are equalized across constituent members of the currency union provides the same level of insurance as complete markets. The second finding illustrates the limitations of this ideal money market union: in the case of supply shocks, the money market union does not provide full insurance, but an idealized capital market union, in which savers frictionlessly choose optimal portfolios, does.

References

Afonso, A. and D. Furceri (2008). EMU enlargement, stabilization costs and insurance mechanisms. *Journal of International Money and Finance 27*(2), 169–187.

Aiyagari, S. (1994). Uninsured idiosyncratic risk and aggregate saving. *Quarterly Journal of Economics 109*(3), 659–684.

Asdrubali, P., B. Sorensen, and O. Yosha (1996). Channels of interstate risk sharing: United States 1963–1990. *The Quarterly Journal of Economics 111*(4), 1081–1110.

Auray, S. and A. Eyquem (2014). Welfare reversals in a monetary union. *AEJ: Macroeconomics 6*(4), 246–90.

Bayoumi, T. and P. R. Masson (1995). Fiscal flows in the United States and Canada: Lessons for monetary union in europe. *European Economic Review 39*(2), 253 – 274. Symposium of Industrial Oganizational and Finance.

Benigno, P. and F. Romei (2014). Debt deleveraging and the exchange rate. *Journal of International Economics 93*(1), 1 – 16.

Bilbiie, F. (2008). Limited asset market participation, monetary policy and (inverted) aggregate demand logic. *Journal of Economic Theory 140*(1), 162–196.

Blanchard, O., C. J. Erceg, and J. Linde (2014). Jump-starting the euro area recovery: Would a rise in core fiscal spending help the periphery? mimeo.

Bolton, P. and O. Jeanne (2011). Sovereign default risk and bank fragility in financially integrated economies. *IMF Economic Review 59*(2), 162–194.

Brunnermeier, M. and Y. Sannikov (2015). International credit flows and pecuniary externalities. *AEJ: Macroeconomics 71*(1), 297–338.

Chari, V., P. Kehoe, and E. McGrattan (2005). Sudden stops and output drops. *American Economic Review Papers and Proceedings 95*(2), 381–387.

Coeurdacier, N. and P. Gourinchas (2016). When bonds matter: Home bias in goods and assets. *Journal of Monetary Economics 82*, 119–137.

Coeurdacier, N., R. Kollmann, and P. Martin (2010). International portfolios, capital accumulation and foreign assets dynamics. *Journal of International Economics 80*(1).

Coeurdacier, N. and H. Rey (2012). Home bias in open economy financial macroeconomics. *Journal of Economic Literature 51*(1), 63–115.

Cole, H. and M. Obstfeld (1991). Commodity trade and international risk sharing: How much do financial markets matter. *Journal of Monetary Economics 28*, 3–24.

Coval, J. and T. Moskowitz (1999). Home bias at home: local equity preference in domestic portfolios. *Journal of Finance 54*(6), 2045–2074.

Debortoli, D. and J. Gali (2017). Monetary policy with heterogeneous agents: Insights from tank models.

Devereux, M. and A. Sutherland (2011a). International credit flows and pecuniary externalities. *European Economic Review 55*(3), 427–442.

Devereux, M. B. and A. Sutherland (2011b). Country portfolios in open economy macro-models. *Journal of the European Economic Association 9*(2), 337–369.

Eggertsson, G. and P. Krugman (2012). Debt, deleveraging, and the liquidity trap:a fisher-minsky-koo approach. *Quarterly Journal of Economics* (127(3)), 1469–1513.

Engel, C. and A. Matsumoto (2009). The international diversification puzzle when goods prices are sticky: It's really about exchange-rate hedging, not equity portfolios. *American Economic Journal: Macroeconomics 1*, 155–188.

Epstein, L. G. and S. E. Zin (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica 57*, 937–969.

Farhi, E. and I. Werning (2013). Fiscal multipliers: Liquidity traps and currency unions. MIT Working Paper.

Farhi, E. and I. Werning (2017). Fiscal unions. *American Economic Review 107*(12), 3788–3834.

Fornaro, L. (2018). International debt deleveraging. *Journal of the European Economic Association 16*(5).

Gali, J. and T. Monacelli (2008). Optimal monetary and fiscal policy in a currency union. *Journal of International Economics 76*, 116–132.

Ghironi, F. (2006). Macroeconomic interdependence under incomplete markets. *Journal of International Economics 70*(2), 428 – 450.

Giroud, X. and H. Mueller (2016). Firm leverage, consumer demand, and employment losses during the great recession. Quarterly Journal of Economics, forthcoming.

Gourinchas, P.-O. and M. Obstfeld (2012). Stories of the twentieth century for the twenty-first. *American Economic Journal Macro 4*(1), 226–265.

Gourinchas, P.-O., T. Philippon, and D. Vayanos (2016). The analytics of the greek crisis. In *NBER Macroeconomics Annual 2016, Volume 31*, NBER Chapters. National Bureau of Economic Research, Inc.

Heathcote, J. and F. Perri (2002). Financial autarky and international business cycles. *Journal of Monetary Economics 49*(3), 601 – 627.

Heathcote, J. and F. Perri (2013). The international diversification puzzle is not as bad as you think. *Journal of Political Economy 121*(6), 1108–1159.

Hepp, R. and J. von Hagen (2013). Interstate risk sharing in germany: 1970–2006. *Oxford Economic Papers 65*(1), 1–24.

Hoffmann, M., E. Maslov, B. Sorensen, and I. Stewen (2018). Are banking and capital markets union complements?evidence from channels of risk sharing in the eurozone. CEPR DP13254.

Holmström, B. and J. Tirole (1997). Financial intermediation, loanable funds and the real sector. *Quarterly Journal of Economics 112*, 663–691.

Kehoe, P. J. and F. Perri (2002). International business cycles with endogenous incomplete markets. *Econometrica 70*(3), 907–928.

Kenen, P. (1969). The theory of optimum currency areas: An eclectic view. In R. Mundell and A. Swoboda (Eds.), *Monetary Problems of the International Economy*. Chicago University Press.

Lane, P. (2012). The european sovereign debt crisis. *Journal of Economic Perspectives 26*(3), 49–68.

Lane, P. (2013). Capital flows in the euro area. *European Economy, Economic Papers 497*.

Levine, D. and W. Zame (2002). Does market incompleteness matter? *Econometrica 70*(5), 1805–1839.

Mankiw, N. G. (2000). The savers-spenders theory of fiscal policy. *American Economic Review Papers and Proceedings*, 120–125.

Martin, P. and T. Philippon (2017, July). Inspecting the mechanism: Leverage and the great recession in the eurozone. *The American Economic Review*.

Mendoza, E. G. (2010). Sudden stops, financial crises, and leverage. *American Economic Review 100*, 1941–1966.

Mendoza, E. G. and K. Smith (2006). Quantitative implications of a debt-deflation theory of sudden stops and asset prices. *The Journal of International Economics Volume 70, Issue 1*, 82–114.

Midrigan, V. and T. Philippon (2010). Household leverage and the recession. NYU Working Paper.

Modigliani, F. and M. H. Miller (1958). The cost of capital, corporation finance and the theory of investment. *American Economic Review 48*, 261–297.

Mundell, R. (1973). The economics of common currencies. In H. Johnson and A. Swoboda (Eds.), *Uncommon Arguments for Common Currencies*. Allen and Unwin.

Mundell, R. A. (1961). Theory of optimum currency areas. *The American Economic Review 51*(4), 657–665.

Obstfeld, M. and K. Rogoff (1995). Exchange rate dynamics redux. *Journal of Political Economy 103*(3), 624.

Schmitt-Grohe, S. and M. Uribe (2016). Downward nominal wage rigidity, currency pegs, and involuntary unemployment. *Journal of Political Economy 124*, 1466–1514.

Sihvonen, M. (2018). Equity home bias and efficiency. Working Paper.

Sihvonen, M. (2019). Market selection with idiosyncratic uncertainty. *Journal of Economic Theory 182*, 143–160.

Tille, C. and E. van Wincoop (2010). International capital flows. *Journal of International Economics 80*(2), 157–175.

Yaari, M. (1976). A law of large numbers in the theory of consumer's choice under uncertainty. *Journal of Economic Theory 12*(2), 202–217.

Appendix

A Proofs

A.1 Proof of Lemma 1

Define the k-period ahead discount rate for $k \geq 1$ from the savers' perspective:

$$R_{t,k} \equiv \prod_{i=1}^{k} (1 + r_{t+i-1}).$$

and the convention $R_{t,0} = 1$.

Let us start from market clearing for the home good (productivity is normalized to 1):

$$Y_t = (1-\alpha)(\chi \mathbf{P}_t \mathbf{C}_{b,t} + (1-\chi) \mathbf{P}_t \mathbf{C}_{s,t}) + F_t + P_{h,t} G_t,$$

where Y_t is nominal GDP. Using the budget constraints of the agents and of the government we get

$$\alpha \tilde{Y}_t = (1-\alpha)\chi \left(\frac{B^h_{t+1}}{1+r_t} - B^h_t \right) - (1-\alpha)(1-\chi) \left(\frac{S_{t+1}}{1+r_t} - S_t \right) + F_t + \frac{B^g_{t+1}}{1+r_t} - B^g_t,$$

where$\tilde{Y}_t$ is total disposable income. Summing and rearranging the terms, we get

$$\begin{aligned} \alpha \left(\tilde{Y}_t + \frac{\tilde{Y}_{t+1}}{R_{t,1}} \right) &= (1-\alpha)\chi \left(\frac{1}{R_{t,1}} \frac{B^h_{t+2}}{1+r_{t+1}} - B^h_t \right) \\ &\quad - (1-\alpha)(1-\chi) \left(-S_t + \frac{1}{R_{t,1}} \frac{S_{t+2}}{1+r_{t+1}} \right) + F_t + \frac{F_{t+1}}{R_{t,1}} \\ &\quad + \frac{1}{R_{t,1}} \frac{B^g_{t+2}}{1+r_{t+1}} - B^g_t. \end{aligned}$$

to write:

$$\begin{aligned} \alpha \left(\tilde{Y}_t + \frac{\tilde{Y}_{t+1}}{R_{t,1}} + \frac{\tilde{Y}_{t+2}}{R_{t,2}} \right) &= -(1-\alpha)\chi \left(B^h_t - \frac{1}{R_{t,2}} \frac{B^h_{t+3}}{1+r_{t+2}} \right) \\ &\quad + (1-\alpha)(1-\chi) \left(S_t - \frac{S_{t+3}}{R_{t,3}} \right) + F_t + \frac{F_{t+1}}{R_{j,t,1}} + \frac{F_{t+2}}{R_{t,2}} \\ &\quad - B^g_t + \frac{1}{R_{t,2}} \frac{B^g_{t+3}}{1+r_{t+2}}. \end{aligned}$$

Therefore for a generic horizon K

$$\begin{aligned}\sum_{k=0}^{K}\frac{\alpha\tilde{Y}_{t+k}}{R_{t,k}} &= (1-\alpha)\left((1-\chi)\,S_t-\chi B_t^h\right)-B_t^g+\sum_{k=0}^{K}\frac{F_{t+k}}{R_{t,k}}\\ &\quad -(1-\chi)\,(1-\alpha)\,\frac{S_{t+K+1}}{R_{t,K+1}}+\frac{1}{R_{t,K}}\left(\frac{(1-\alpha)\,\chi B_{t+K+1}^h}{1+r_{t+K}}+\frac{B_{t+K+1}^g}{1+r_{t+K}}\right).\end{aligned}$$

We take the limit and we impose the No-Ponzi conditions

$$\begin{aligned}\lim_{K\to\infty}\frac{S_{t+K+1}}{R_{t,K+1}}(z^{t+K}) &= 0\\ \lim_{K\to\infty}\frac{1}{R_{t,K}}\frac{B_{t+K+1}^h}{1+r_{t+K}}(z^{t+K}) &= 0\\ \lim_{K\to\infty}\frac{1}{R_{t,K}}\frac{B_{t+K+1}^g}{1+r_{t+K}}(z^{t+K}) &= 0.\end{aligned}$$

The inter-temporal current account condition is

$$\alpha\sum_{k=0}^{\infty}\frac{\tilde{Y}_{t+k}}{R_{t,k}}(z^{t+k}) = \sum_{k=0}^{\infty}\frac{F_{t+k}}{R_{t,k}}(z^{t+k})-(1-\alpha)\left(\chi B_t^h-(1-\chi)\,S_t\right)-B_t^g.$$

A.2 Proof of Lemma 3

Now the market clearing condition is

$$Y_t=(1-\alpha)\,(\mathbf{P}_t^i/P_{t,i})^{\xi_1-1}\left(\chi\mathbf{P}_t^i\mathbf{C}_{b,t}+(1-\chi)\,\mathbf{P}_t^i\mathbf{C}_{s,t}\right)+(P_{t,i}^{1-\xi_2}/\mu)F.$$

Log-linearizing

$$\hat{y}_t=(1-\alpha)\left((1-\chi)(\hat{\mathbf{P}}_t+\hat{\mathbf{C}}_{s,t})+\chi(\hat{\mathbf{P}}_t+\hat{\mathbf{C}}_{b,t})\right)+a_1\hat{p}_t.$$

Here hats denote log-deviations and

$$a_1=(1-\alpha)\,(\xi_1-1)-(\xi_1-1)\,(1-\alpha)^2+\alpha(1-\xi_2).$$

Using the borrowers' and savers' budget constraints and rearranging

$$\alpha\hat{y}_t=(1-\alpha)\left(\chi\left(\frac{\hat{b}_{t+1}^h}{R}-\hat{b}_t^h\right)-(1-\chi)\left(\frac{\hat{s}_{t+1}^h}{R}-\hat{s}_t^h\right)\right)+a_1\hat{p}_t.$$

The interest rate is constant. Now we have $W_t=h(N_t,\mathbf{P}_t^i,\chi\mathbf{P}_t^i\mathbf{C}_{b,t}+(1-\chi)\,\mathbf{P}_t^i\mathbf{C}_{s,t})$. Using the price setting condition, this implies $P_{t,i}/\mu=h(Y_t/P_{t,i},\mathbf{P}_t^i,\chi\mathbf{P}^i\mathbf{C}_{b,t}+(1-\chi)\,\mathbf{P}_t^i\mathbf{C}_{s,t})$. Using the savers' and borrowers' budget constraints, $P_{t,i}/\mu=h(Y_t/P_{t,i},\mathbf{P}_t^i,\chi\left(\frac{B_{t+1}}{1+r_t}-B_t\right)-$

$(1-\chi)\left(\frac{S_{t+1}}{1+r_t}-S_t\right)$). Linearizing we obtain $\hat{y}_t = a_2\hat{p}_t + a_3\left(\chi\left(\frac{\hat{b}^h_{t+1}}{R}-\hat{b}^h_t\right)-(1-\chi)\left(\frac{\hat{s}^h_{t+1}}{R}-\hat{s}^h_t\right)\right)$ for some a_2 and a_3. Plugging this back to the market clearing condition implies

$$\hat{p_t} = \left(\chi\left(\frac{\hat{b}^h_{t+1}}{R}-\hat{b}^h_t\right)-(1-\chi)\left(\frac{\hat{s}^h_{t+1}}{R}-\hat{s}^h_t\right)\right)a_4,$$

where $a_4 = \frac{1-\alpha-\alpha a_3}{\alpha a_2 - a_1}$. But therefore

$$\alpha\hat{y}_t = \left(\chi\left(\frac{\hat{b}^h_{t+1}}{R}-\hat{b}^h_t\right)-(1-\chi)\left(\frac{\hat{s}^h_{t+1}}{R}-\hat{s}^h_t\right)\right)\left((1-\alpha)+a_1a_4\right).$$

Now iterating similarly to before and imposing transversality conditions

$$\alpha\sum_{k=0}^{\infty}\frac{\hat{y}_{t+k}}{R_k} = \left((1-\chi)\hat{s}^h_t - \chi\hat{b}^h_t\right)\left((1-\alpha)+a_1a_4\right).$$

A.3 Proof of Lemma 4

The Euler equation is

$$\beta\mathbb{E}_t\left(\frac{\mathbf{C_{t+1}}}{\mathbf{C_t}}\right)^{-\gamma}\frac{P_t}{P_{t+1}}R = 1.$$

$$\frac{\mathbf{C_t}^{-\gamma}}{P_t} = \beta R\mathbb{E}_t\frac{\mathbf{C_{t+1}}^{-\gamma}}{P_{t+1}}.$$

In a first order approximation this becomes

$$mu_t = \mathbb{E}_t mu_{t+1}$$

Here mu_t is log-marginal utility. We can solve

$$mu_t = \underset{T\to\infty}{lim}\mathbb{E}_t mu_{t+T}.$$

Hence the agent maintains constant marginal utility following the shock. I next argue that she keeps it at the pre-shock level. Up to first order the present value of consumption is

$$\sum_{k=0}^{\infty}\frac{\hat{\mathbf{C}}_{t+k}+\hat{\mathbf{P}}^i_t}{R_k}.$$

Note that as argued in the previous proof

$$\hat{p_t} = \left(\chi\left(\frac{\hat{b}^h_{t+1}}{R}-\hat{b}^h_t\right)-(1-\chi)\left(\frac{\hat{s}^h_{t+1}}{R}-\hat{s}^h_t\right)\right)a_4.$$

Hence

$$\sum_{k=0}^{\infty} \frac{\hat{p}_t}{R_k} = -\left(\chi \hat{b}_t^h + (1-\chi)\hat{s}_t^h\right) a_4.$$

is predetermined as is $\sum_{k=0}^{\infty} \frac{\hat{\mathbf{P}}_t^i}{R_k}$. The condition for constant marginal utility can be linearized as

$$-\gamma \hat{\mathbf{C}}_t - \hat{\mathbf{P}}_t^i = 0$$

which implies

$$\hat{\mathbf{C}}_t + \hat{\mathbf{P}}_t^i = (1 - \frac{1}{\gamma})\hat{\mathbf{P}}_t^i$$

and

$$\sum_{k=0}^{\infty} \frac{\hat{\mathbf{C}}_{t+k} + \hat{\mathbf{P}}_t^i}{R_k} = (1 - \frac{1}{\gamma}) \sum_{k=0}^{\infty} \frac{\hat{\mathbf{P}}_t^i}{R_k}.$$

which is predetermined and hence does not respond to the shock. Hence choosing the old marginal utility is feasible. Now if some other marginal utility than the previous one were optimal, it would have been so already so before the shock. Therefore following the shock marginal utility remains constant which is also the complete markets condition.

A.4 Proof of Proposition 3

We need to first extend the argument in Proposition 1 to include static equity positions. Using manipulations similar to those in the proof of Lemma 1, we can write

$$W_{t,i}N_{t,i}(\mu - (1-\alpha)(1+\varphi(\mu-1))) = F_{t,i} + (1-\chi)(1-\alpha)\left(\frac{B_{t+1,i}}{R_t} - B_{t,i}\right)$$
$$-\chi(1-\alpha)\left(\frac{S_{t+1,i}}{R_t} - S_{t,i}\right) + (1-\alpha)(1-\chi)\Gamma_{t,i}.$$

Here Γ_t is the savers' income from foreign stocks. We also assumed away from public deleveraging and spending shocks. From this we can solve

$$W_{t,i}N_{t,i} = a_1 F_{t,i} + a_2\left(\frac{B_{t+1,i}}{R_t} - B_{t,i}\right) - a_3\left(\frac{S_{t+1,i}}{R_t} - S_{t,i}\right) + a_4\Gamma_{t,i},$$

where

$$a_1 = \frac{1}{\mu-(1-\alpha)(1+\varphi(\mu-1))}, a_2 = \frac{(1-\chi)(1-\alpha)}{\mu-(1-\alpha)(1+\varphi(\mu-1))},$$

$$a_3 = \frac{\chi(1-\alpha)}{\mu-(1-\alpha)(1+\varphi(\mu-1))}, a_4 = \frac{(1-\alpha)(1-\chi)}{\mu-(1-\alpha)(1+\varphi(\mu-1))}.$$

The borrowers' budget constraint is

$$S_t + W_t N_t + \varphi(\mu-1) W N_t + \Gamma_t = \mathbf{P}_t \mathbf{C}_{s,t} + \frac{S_{t+1}}{R_t}.$$

Plugging in the previous result and rearranging we obtain:

$$\mathbf{P}_t \mathbf{C}_{s,t} = (1+\varphi(\mu-1)) a_1 F_{t,i} + (1+\varphi(\mu-1)) a_2 \left(\frac{B_{t+1,i}}{R_t} - B_{t,i}\right)$$
$$-(1+\varphi(\mu-1)) a_3 + 1) \left(\frac{S_{t+1,i}}{R_t} - S_{t,i}\right) + (1+\varphi(\mu-1)) a_4 + 1) \Gamma_{t,i}.$$

Similarly to the proof of Lemma 1, it now follows that $\sum_{k=0}^{\infty} \frac{\mathbf{P}_{t+k}\mathbf{C}_{s,t+k}}{R_{t,k}}(z^t)$ is only a function of S_t, B_t, $\sum \frac{F_{t+k}}{R_{t,k}}$ and $\sum_{k=0}^{\infty} \frac{\Gamma_{t+k}}{R_{t,k}}$ that do not react to domestic deleveraging shocks:

$$\sum_{k=0}^{\infty} \frac{\mathbf{P}_{t+k}\mathbf{C}_{s,t+k}}{R_{t,k}}(z^{t+k}) = \widehat{\Omega}\left(S_t, B_t, B_t^g, \sum_{k=0}^{\infty} \frac{F_{t+k}}{R_{t,k}}(z^{t+k}), \sum_{k=0}^{\infty} \frac{\Gamma_{t+k}}{R_{t,k}}(z^{t+k})\right)$$

This generalizes the argument of Proposition 1 to static equity positions.

The Main Argument Given symmetric borrowing patterns the correct stock positions perfectly share shocks affecting labor income such as quality shocks by the argument in Proposition 2. These shocks need not be idiosyncratic. Idiosyncratic deleveraging shocks do not distort symmetry and the savers' consumption expenditure stays constant by the argument in Proposition 1. While the proof assumes that the home quality stays constant it also goes through with unanticipated home quality shocks. Moreover, it works for preference shocks that do not alter the complete markets condition such as shocks to the disutility of labor. While this proof assumes that home quality stays constant, the Proposition also holds for unanticipated home quality shocks. Under certain further restrictions on fiscal policy, the proof can be generalized to public deleveraging.

B Numerical Welfare Gains

In this section, we quantitatively assess the welfare benefits of a money market and capital market union. To do so, we extend the model to include physical capital, an important feature in assessing the benefits of capital market integration because investment lowers the correlation between dividends and labor income, which reduces the hedging benefits of foreign equity.[12] We also specify a monetary policy rule and the relationship between wages and labor supply.

B.1 Model Structure

Final goods producers As before, competitive final goods producers produce the consumption good using a CES technology that aggregates intermediate goods:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Intermediate goods producers Intermediate goods are produced by monopolistically competitive firms using a Cobb-Douglas technology with labor and capital as inputs:

$$Y_{j,t} = A_t N_{j,t}^{1-\theta} K_{j,t}^{\theta}.$$

Where A_t is an aggregate, country-specific productivity shock. Intermediate goods producers are owned by shareholders in the home and foreign country and maximize dividend payoffs to shareholders ($d_{j,t}$), discounted using the average discount factor ($\bar{m}_{0,,t}$) of savers in the two countries

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \bar{m}_{t,t+s} d_{j,t+s}$$

The weights for the discount factors are given by the stock positions. For example if home savers hold most of the equity of home firms, home firms put more weight on the discount factor of home savers. The firms can transfer the aggregate consumption good into capital through investment. Dividends are:

$$d_{j,t} = P_{j,t} Y_{j,t} - W_t N_{j,t} - \mathbf{P}_t I_{j,t} - \mathbf{P}_t f(I_{j,t}).$$

Where $I_{j,t}$, $P_{j,t}$, $N_{j,t}$ and $Y_{j,t}$ are intermediate producer j's investment, price, employment and output at time t and W_t is the wage rate in the country. Moreover, $f(I_{j,t})$ is the investment adjustment cost. Here we set

[12] This is because firms invest in good times, which therefore lowers dividends in booms. This logic is explained e.g. in Heathcote and Perri (2013).

$$f(I_{j,t}) = \frac{\zeta}{2}\left(\frac{I_{t,j}}{I_{t-1,j}} - 1\right)^2.$$

Firm j's capital evolves according to:

$$K_{j,t+1} = (1-\delta)K_{j,t} + I_{j,t}.$$

And it faces a downward sloping demand curve from producers of the final good:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_{h,t}}\right)^{-\epsilon} Y_t.$$

Intermediate goods producers set prices flexibly. It follows that they all set the same price, labor demand and investment level.

$$N_t = N_{j,t},\ I_t = I_{j,t},\ P_{h,t} = P_{j,t},\ K_t = K_{j,t}.$$

Optimal investment is determined by the following equation:

$$\mathbf{P}_t + \mathbf{P}_t\zeta\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)\frac{1}{I_{j,t-1}} = \mathbb{E}_t\bar{m}_{t,t+1}\left[P_{j,t+1}\zeta\frac{Y_{j,t+1}}{K_{j,t+1}} + \mathbf{P}_{t+1}\zeta\left(\frac{I_{j,t+1}}{I_{j,t}} - 1\right)\frac{I_{j,t+1}}{I_{j,t}^2} + \Psi_{t+1}\right]$$

Here

$$\Psi_{t+1} = (1-\delta)\mathbb{E}_{t+1}\bar{m}_{t+1,t+2}P_{j,t+2}\zeta\frac{Y_{j,t+2}}{K_{j,t+2}} + ...$$

This can be written in recursive form as

$$\mathbf{P}_t + \mathbf{P}_t\zeta\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)\frac{1}{I_{j,t-1}} = \mathbb{E}_t\bar{m}_{t,t+1}\left[P_{j,t+1}\eta\frac{Y_{j,t+1}}{K_{j,t+1}} + \mathbf{P}_{t+1}\zeta\left(\frac{I_{j,t+1}}{I_{j,t}} - 1\right)\frac{I_{j,t+1}}{I_{j,t}^2} + \mathcal{A}_{t+1}\right].$$

Here

$$\mathcal{A}_{t+1} = (1-\delta)\left[\mathbf{P}_{t+1} + \mathbf{P}_{t+1}\zeta\left(\frac{I_{j,t+1}}{I_{j,t}} - 1\right)\frac{1}{I_{j,t}} - \mathbb{E}_{t+1}\bar{m}_{t+1,t+2}\mathbf{P}_{t+2}\zeta\left(\frac{I_{j,t+2}}{I_{j,t+1}} - 1\right)\frac{I_{j,t+2}}{I_{j,t+1}^2}\right].$$

The price is a constant markup over marginal cost

$$P_{h,t} = \mu MC_t.$$

Where the markup over marginal cost MC_t is given by $\mu \equiv \frac{\epsilon}{\epsilon-1}$ and $MC_t = \frac{W_t}{(1-\theta)Y_t/N_t}$.

Monetary policy rule For the small open economy model we assume a constant policy rate from the perspective of the home country. For the two country version considered later we assume the central bank sets the interest rate according to

$$\bar{R}_t = R_{ss}\left(\left(\frac{Y_t}{Y_{ss}}\right)\left(\frac{Y_t^*}{Y_{ss}^*}\right)\right)^{\phi_Y}\left(\left(\frac{\pi_t}{\pi_{ss}}\right)\left(\frac{\pi_t^*}{\pi_{ss}^*}\right)\right)^{\phi_\pi},$$

where R_{ss}, Y_{ss} and π_{ss} are the steady state interest rate, output and inflation.

Wages and labor supply Following Martin and Philippon (2017), wage dynamics are determined by a Phillips curve with slope κ

$$W_t = W_{t-1}\left(1 + \kappa\left(N_t - N_{ss}\right)\right),$$

where N_{ss} is steady-state employment.

B.2 Numerical Welfare Benefits of a Money Market Union

We now use the model with capital to estimate the welfare benefits of a money market union. Under segmented markets, the private costs of funds are not equalized across regions. It is important to understand that we do not start from any segmented market model. We start from a model that actually describes the behavior of the Eurozone. Martin and Philippon (2017) and Gourinchas et al. (2016) quantify the extent of the dispersion in funding costs during the Eurozone crisis. The simplest interpretation is that domestic banks intermediate savings and investment, and, thus, the private cost of funds is pinned down by the banking system. Formally, in log-deviations from steady state, we have

$$r_t = r_t^b$$

where r_t^b is the banks' funding cost. We can then consider a small country subject to a spread shock r_t^b and a private leverage shock $\bar{B}_t$. We estimate these shocks using data from the Eurozone as in Martin and Philippon (2017) but otherwise consider the baseline calibration discussed in the next section. The idea is to model the joint dynamics of spreads and private debt. Debt is well described by an AR(2) process and spreads by an AR(1) process. The processes are correlated because negative shocks cause spread to rise and banks to cut lending. Our calibration uses data from a volatile period, the Eurozone crisis, so our welfare calculations capture the value of a money market union during periods of heightened financial risks.[13]

Table 2 summarizes our quantitative results. Spread differences between countries increase consumption volatility and lower welfare. The volatilities in the segmented markets case are fairly high since the model is calibrated to a volatile period. The money market union reduces consumption volatility by equalizing interest rates between countries. Table 2 shows the volatilities of (annualized quarterly log changes) real consumption for savers and borrowers as well as for aggregate consumption. The money market union eliminates almost all of the consumption volatility of savers. This is consistent with Proposition 1, according to which the MMU attains the complete markets outcome subject to deleveraging shocks. It also suggests that the Proposition holds well in the extended model with capital. The MMU also leads to a substantial reduction in the consumption volatility of borrowers and a clear decline in the volatility of total consumption. [14]

Consumption Volatility	Segmented Markets	Money Market Union
Savers	6.7%	0.1%
Borrowers	5.1%	1.9%
Total	6.1%	0.5%

Table 2: Consumption volatilities under segmented markets and a money market union, no supply shocks

Table 3 describes the volatilities when adding supply shocks modeled as quality and productivity shocks. The estimation of these shocks is described in section. Now the money market union does not lead to zero volatility for savers but still implies a clear reduction in all consumption volatilities.

Note that the point that eliminating market segmentation improves welfare is not entirely obvious. For example Devereux and Sutherland (2011a) and Brunnermeier and Sannikov (2015) find that free bond trading can reduce welfare relative to financial autarky. But these papers do not consider the kind of segmentation witnessed during the Eurozone crisis. The key feature is that spreads tend to *increase* during deleveraging episodes when it would be

[13] The borrowing limit follows the process

$$\log \bar{B}_{i,t} - \log \bar{B}_{i,t-1} = -0.01 \times (\log \bar{B}_{i,t-1} - \log \bar{B}) + 0.85 \times \left(\log \bar{B}_{i,t-1} - \log \bar{B}_{i,t-2}\right) + 0.04\epsilon^b_{i,t}$$

and the spread the process

$$r^b_{i,t} = 0.9 r^b_{i,t-1} + 0.003\epsilon^r_{i,t}$$

and the correlation between the two shocks is

$$corr\left(\epsilon^b_{i,t}, \epsilon^r_{i,t}\right) = -0.3.$$

The investment adjustment cost is estimated in the next section.

[14] With log-preferences the welfare benefits of these changes are still relatively small. However, we could increase this welfare gain by raising savers' risk aversion, for example through the use of recursive preferences ((Epstein and Zin, 1989)). An approximate consumption equivalent gain for log preferences is given by the Lucas formula $0.5Var(\Delta c_{SM}) - 0.5Var(\Delta c_{BU})$. However, a convex disutility of labor function raises this estimate somewhat.

efficient for countries to smooth shocks by borrowing. The comparison here is between a model with a counter-cyclical spread (segmented markets) and a model with no spread in riskless borrowing rates (a MMU) not between free bond trading and financial autarky.[15] The finding that the money market union can clearly lower consumption volatility is consistent with the message of Martin and Philippon (2017) who find that segmentation in funding costs was a major contributor to the Eurozone crisis.

Consumption Volatility	Segmented Markets	Money Market Union
Savers	7.5%	2.7%
Borrowers	6.3%	3.7%
Total	7.0%	2.9%

Table 3: Consumption volatilities under segmented markets and a money market union, including supply shocks

Figure 6 plots the response of savers' consumption and firm investment to a spread shock. Higher spreads lead to lower nominal consumption. Spreads affect business cycles partly through an investment channel. When funding costs increase, firms cut investment. This effect is numerically large.

B.3 Numerical Welfare Benefits of a Capital Market Union

In this section we argue that the welfare gains of moving from a money market union to a capital market union can be also be significant. As before we employ the model with capital but now with two countries. We assume three different kinds of shocks: deleveraging, quality and productivity shocks.

The benefits of CMU depend on the relative importance of these shocks. First, in line with Proposition 1, deleveraging shocks can be shared well through borrowing and saving at a constant rate and, therefore, require little equity market diversification. Second, due to Cole-Obstfeld preferences, TFP shocks do not create large changes in the total value of output or dividends in each country, consistent with Lemma 5. Sharing such shocks, therefore, requires fairly little equity market diversification, and consumption volatilities in each country are generally insensitive to the level of diversification. On the other hand, using such shocks only tends to lead to a counterfactually low correlation between dividends and labor income. Moreover, these shocks imply high correlations between the consumption levels in the two countries, in contrast to the low levels of international risk sharing seen in the data.

[15] In any case, financial autarky implying a zero trade balance and net wealth is not a realistic policy option.

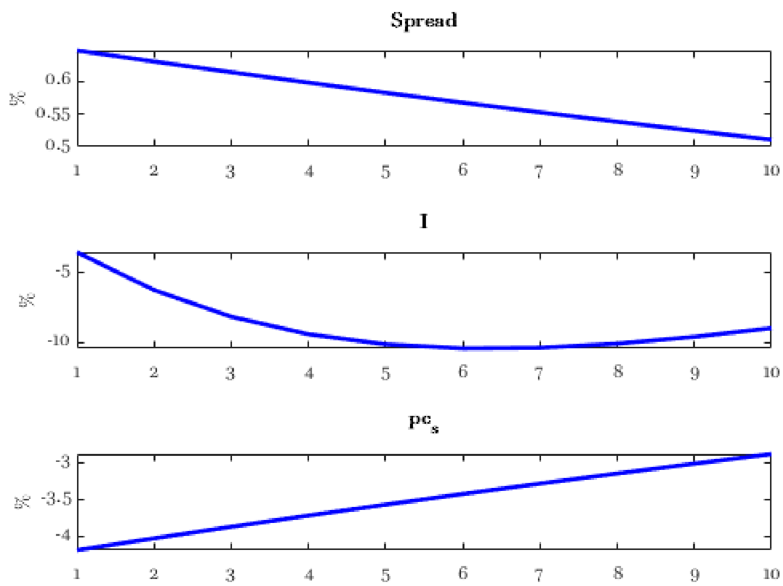


Figure 6: Response of investment to a spread shock in the quantitative model

Calibration Our baseline model for the CMU assumes quality, productivity and deleveraging shocks. Most of the parameters take standard values (see table 4). However, we calibrate the quality and deleveraging shock volatilities and persistences to match consumption and export data from France obtained from Eurostat. We also match the correlation between relative dividends and labor income (Home - Foreign values, $Corr(W_tN_t - W_t^*N_t^*, d_t - d_t^*)$). We take the persistence of the productivity shocks from Heathcote and Perri (2013) but estimate their volatility. Following e.g. Auray and Eyquem (2014) and Heathcote and Perri (2013) we assume home and foreign shocks are uncorrelated, largely because uncorrelated shocks can be used to match the data. These parameter values are given in Table 5. Moreover, Table 6 compares the key model simulated moments to those seen in the data.

We calibrate the shock processes using a stock position of $\varphi = 0.8$. That is we start from a reasonable empirical benchmark with low levels of within union cross-border equity holdings. After that we numerically solve for the optimal home stock position from an individual saver's perspective using the method described by Devereux and Sutherland (2011b). The optimal home stock position is constant up to second order and given by $\varphi = \varphi^* = 0.08$. We do not model the friction that leads agents to choose a larger-than-optimal home stock position. As in Tille and van Wincoop (2010), for example, we can think of this friction as a second-order term that affects macroeconomic conditions through its impact on stock positions. The implied correlation between relative dividends and labor income is roughly

Parameter	Description	Value
χ	Fraction of impatient	0.5
β_s	Discount factor of savers	0.995
α	Openness to trade	0.25
κ	Slope wage Phillips curve	0.1
ϵ	Elasticity domestic intermediates	4
θ	Capital share	0.36
δ	Depreciation rate	0.015
ϕ_Y	Taylor rule - output gap	0.5
ϕ_π	Taylor rule - inflation	1.5

Table 4: Calibration of baseline parameters

Parameter	Value
Quality shock (α_t) volatility	3.64%
Quality shock (α_t) persistence	0.995
TFP shock (A_t) persistence (Heathcote and Perri (2013))	0.91
TFP shock (A_t) volatility	0.75%
Deleveraging shock ($\bar{B}_{t+1}$) volatility	0.7%
Deleveraging shock ($\bar{B}_{t+1}$) persistence	0.90
Investment adjustment cost (ζ)	1.95

Table 5: Rest of the parameters

Statistic	Model	Data
Volatility of consumption growth	2.0%	2.1%
Volatility of export growth	5.2%	5.0%
Dividend-labor income correlation	0.80	0.77 (Coeurdacier et al. (2010))

Table 6: Key simulated and empirical moments

0.8, which is close to that for France as well as close to the average number for EU countries calculated by Coeurdacier et al. (2010). If we match a smaller correlation value, the welfare benefit of a CMU is somewhat lower but still significant. The model is solved using perturbation methods.[16]

Results We now compare the volatility of (log first differences in) consumption under the two different levels of equity market diversification. The results are given in Table 7. Note that we have slightly modified the definition of a money market union to match the empirical extent of equity home bias instead of assuming perfect home bias. Further, the numbers are not directly comparable with the previous tables because we use the two country version of the model to produce Table 7.

Consumption Volatility	Money Market Union $\varphi = 0.8$	Capital Market Union $\varphi = \varphi^* = 0.08$
Savers	1.52%	0.88%
Borrowers	3.46%	2.96%
Total	2.04%	0.85%

Table 7: Consumption volatilities under a money market union and a capital market union

The first order effect of increasing equity market diversification is a 62% reduction in savers' consumption volatility. Interestingly, through general equilibrium effects, increased risk sharing by savers also leads to a reduction in the consumption volatility of borrowers, and therefore a greater reduction in aggregate consumption volatility than would be implied by a reduction in savers' volatility alone. Table 8 illustrates the positive externalities of a CMU. Savers do not internalize the gains that accrue to borrowers, so the reduction in borrowers' consumption volatility amounts to a positive externality. However, there are also positive externalities for savers. If a single saver lowers her stock position to $\varphi = 0.08$, she would face a consumption volatility of 0.94%. That is, roughly 10% of the volatility reduction gains accruing to savers are not internalized. This is due both to pecuniary and aggregate demand externalities.

	Uninternalized Volatility Reduction	Share of Total Volatility Reduction
Savers	0.06%	10%
Borrowers	0.5%	100%

Table 8: Positive Externalities of a CMU

[16]To give a well-defined portfolio choice problem, it is important to approximate the Euler equations at least up to 2nd order. Note that all of our theoretical results, except Lemma 3, are exact.

Sensitivity Analysis The results depend on the types of shocks that we assume. Table 9 shows the results if we estimate the model with deleveraging and productivity shocks only. Because home equity provides a good hedge to shocks to labor income, stock positions are mildly biased towards home stocks even absent frictions. More specifically, the frictionless equilibrium stock position is $\varphi = 0.6$. Overall, consumption volatilities are less sensitive to equity market diversification in line with Lemma 5. We can see from the table that now the CMU brings essentially zero benefits. The deleveraging shocks can be shared through borrowing and saving. Moreover, the productivity shocks do not create large differences in the value of output in the two countries. Similar results have been found in the literature on equity home bias, where it has been shown that equilibrium stock positions can be biased towards home stocks even absent frictions (e.g. Coeurdacier and Gourinchas (2016), Heathcote and Perri (2013)). Moreover, here better diversification in stock positions can even result in a small increase in savers' consumption volatility, reminiscent of the welfare reversal result of Auray and Eyquem (2014).[17] However, as mentioned before, the calibration with quality shocks matches important features of the data that cannot be matched with productivity shocks alone. Furthermore, as explained below this calibration is better in line with reduced-form evidence from the US.

Consumption Volatility	Money Market Union $\varphi = \varphi^* = 0.8$	Capital Market Union $\varphi = \varphi^* = 0.6$
Savers	1.92%	1.94%
Borrowers	3.2%	3.2%
Aggregate	2.2%	2.2%

Table 9: Consumption volatilities under a money market union and a capital market union, no quality shocks

In unreported simulations we also study how the degree of wage rigidity affects these numerical results. More rigid wages seem to lead to higher consumption volatilities given any degree of capital market integration. However, changing the stickiness of wages does not lead to large differences in the relative change in consumption volatilities when moving from a MMU to a CMU. On the other hand, with stickier wages a larger share of this reduction is due to aggregate demand externalities.

Pareto Efficient Solution Our results highlight the cases in which a MMU or a CMU can replicate the complete markets outcome for savers. This equilibrium might still not be

[17] Similar to this alternative calibration, theirs does not match the dividend-labor income correlation. Unlike ours, their baseline model assumes sticky prices but flexible wages and higher price dispersion results in lower welfare, and they do not assume Cole-Obstfeld preferences. Their baseline model considers financial autarky, which as mentioned before is not the most natural benchmark when thinking about a currency union. They also do not consider segmentation in funding costs, which we think is critical for understanding the Eurozone crisis.

Pareto efficient, however, for two reasons. First, it does not attain the complete markets allocation between borrowers in different countries or between borrowers and savers. The allocation can therefore feature pecuniary externalities as the marginal rates of substitutions between all agents are generally not equalized.

The second reason is that we assumed that wages are sticky. This is not important for the main results of the paper. However, as explained by Farhi and Werning (2017) such rigidities can give rise to aggregate demand externalities. This can imply that even the full complete markets allocation is not Pareto efficient.

Providing an analytical solution for the Pareto efficient allocation in our setup seems infeasible. However, using a somewhat simpler model Sihvonen (2018) shows that absent frictions the equilibrium stock positions tend to be socially optimal even despite aggregate demand externalities. Numerically this property seems to hold well in our model. In the baseline model, the frictionless equilibrium stock position is 0.08. Aggregate consumption volatility is minimized with a stock position of -0.18. However, this volatility is fairly flat in the region of the socially optimal stock position so that the equilibrium stock position attains 94% of the total volatility reduction gains. This suggests that the complete markets/equilibrium stock positions are close to the socially optimal ones in a setting where all stock market frictions have been removed (a CMU).

We also show numerically that a MMU and a CMU leads to lower consumption volatilities. Moreover, we numerically evaluate the positive externalities of a CMU. Here we find that these externalities are fairly large, at least in terms of consumption volatilities. That is a substantial part of the gains from moving from an equilibrium given frictions to a frictionless equilibrium are uninternalized.

B.4 On the Empirical Plausibility of the Estimate of the Benefits of CMU

According to table 7 aggregate consumption volatility in a perfect CMU would be less than half that in a money market union. As discussed this estimate is sensitive to model assumptions; for instance, calibrating the model with productivity shocks only lowers the benefit of CMU. However, we argue that our relatively large estimated volatility reduction is consistent with reduced form evidence from the US.

Asdrubali et al. (1996) provides a method for estimating risk sharing gains from regional capital market integration. They measure this using the regression slope coefficient

$$\beta_K = \frac{Cov(\Delta log(grp) - \Delta log(ri), \Delta log(grp))}{Var(\Delta log(grp))},$$

where grp is gross regional product and ri is regional income. Here the difference between the two measures reflects dividend, interest and rental payments. A coefficient of one implies that such capital market transfers perfectly offset variation in regional income; when it is zero there is no such counteracting effect. Asdrubali et al. (1996) estimate that between US states $\beta_K^{US} \approx 39\%$ with a number close to 50% in the later part of the sample. On the other hand, Afonso and Furceri (2008) estimate that between Eurozone countries $\beta_K^{EUR} \approx 8\%$. While these measures are based on somewhat different sample periods, capital market integration in the US seems to increase the measure by 30-40 percentage points. On the other hand, using our calibrated model we find simulated coefficients of $\beta_K^{BU} \approx 16\%$ and $\beta_K^{CMU} \approx 67\%$. This implies that moving from a low degree of capital market integration to perfectly integrated capital markets increases the risk sharing measure by roughly 50 percentage points. This estimate is not implausibly large given that capital markets between US states are probably not perfectly integrated. For example Coval and Moskowitz (1999) find evidence of state level home bias in US equity markets.

C A Simple Banking Model

Generally, a natural definition of an ideal banking union is that borrowing (and lending) rates depend only on the risk characteristics of the borrower. In particular the borrowing rate should not depend on the location of the borrower, after controlling for risk attributes. Effectively there is just a single union wide bank market for borrowing and saving and banks have no incentive to discriminate between customers based on location. Local banking conditions and the local health of banks do not affect the borrowing rates. This implies in particular that the risk-free rates are equalized across countries. As explained before we call this feature of an ideal banking union a money market union.

In practice these money market flows would plausibly be intermediated by banks. However, we have abstracted away from explicitly modeling banks as this would simply complicate the model without bringing new insights. For illustrative purposes, we now sketch a simple banking model consistent with our interpretation. Note that this is not the only model consistent with our interpretation.

Segmented Markets Household saving and borrowing is intermediated by banks. In each country there is a competitive representative bank. Domestic households can only transact with this local bank but the bank can also borrow from foreign banks. As in the main text, we assume all contracts are one period and there is no default[18] . For simplicity, we abstract away from bank equity[19] so that

$$B^l_{t+1,i} = B^d_{t+1,i} + B^f_{t+1,i},$$

[18] We considered default in a previous version of the paper but removed it because it brought additional complications yet few additional insights..

[19] We could think of this as a limit of a model with no bank equity.

where $B^l_{t+1,i}$is loans to domestic households, $B^d_{t+1,i}$is deposits of domestic savers and $B^f_{t+1,i}$is total borrowing from foreign banks (can be negative). Here we could add lending to the government to the model without altering the key results. The bank's profit is given by

$$\frac{B^l_{t+1,i}}{R^l_{t,i}} - \frac{B^d_{t+1,i}}{R^d_{t,i}} - \frac{B^f_{t+1,i}}{R^f_{t,i}}.$$

Here $R^l_{t,i}$, $R^d_{t,i}$and $R^f_{t,i}$are the interest rates for loans, deposits and borrowing from foreign banks. The bank profits are distributed to households according to some rule. The local bank faces a (generally multi-dimensional) constraint of the form

$$\Xi(B^l_{t+1,i}, B^d_{t+1,i}, \mathbf{B}^f_{t+1,i}, Z_{t,i}) \geq 0,$$

for some function Ξ, where $Z_{t,i}$ is a set of state variables. The $\mathbf{B}^f_{t+1,i}$ includes the bank's borrowing from foreign banks but also its portfolio of loans to banks in other countries. This constraint represents country specific lending and borrowing frictions. This constraints plausibly becomes tighter during a crisis period as captured by the state variable $Z_{t,i}$. Moreover, the form of constraint implies that a bank can be particularly constrained to lending banks in a specific country, such as a country with bad economic conditions. This might capture, in a reduced form way, effects similar to bank default risk. The high bank funding costs would then generally be transmitted to the rates faced by households in that country.

The problem of the bank is to choose $B^l_{t,i}$, $B^d_{t,i}$and $\mathbf{B}^f_{t+1,i}$ to maximize profits subject to this constraint. This problem then defines the rates faced by households in each country as well as the rates in the bank funding market. However, fully specifying and solving a model with a continuum of local banks is beyond the scope of this paper. Therefore in the numerical part we follow Martin and Philippon (2017) and take spreads as exogenous, matching them to data from the eurozone.

Money Market Union In a MMU there is just one competitive representative bank and households in each country transact with this bank. The constraint takes the form

$$\Xi(B^l_{t+1}, B^d_{t+1}, Z_t) \geq 0,$$

where $B^l_{t+1}$is aggregate loans to domestic households and $B^d_{t+1}$is aggregate deposits and Z_t is a set of aggregate state variables. Because there is no default and country specific variables affecting the constraint, all households face the same interest rates and there are no country specific spreads. This would still hold if we assume a continuum of nonidentical banks. In a MMU the banks are not constrained to lend to households in a particular country and have no incentive to discriminate between households in different countries.

This equalization of interest rates in different countries is the key condition to facilitate risk sharing within the currency union. Because of the constraint, the rates faced by households might still differ from the policy rate.[20] For example borrowers in all countries might face a slightly higher rate than savers. The effect of this would be immaterial to our results. For example consider the exercise in Figure 1 but now assuming the borrowers face 1% higher rate than the savers. A deleveraging shock of 5% increases the savers' nominal spending by roughly 0.002%. The effect is not generally zero because deleveraging is not anymore a zero NPV transaction when calculated using the savers' rate. However, this effect is numerically extremely small. Note that our numerical exercise for segmented markets instead shows that volatile and countercyclical country specific spreads can instead be highly detrimental to welfare.

Hence under a money market union, we can consider a bank problem with no constraint. By bank profit maximization the lending and borrowing rates in each country are now equal and the bank makes zero profits. Such a banking model is homeomorphic to our model of a MMU. For the purposes of this paper, in an ideal MMU, banks are largely a veil.

D Productivity and government spending shocks only

Due to Cole-Obstfeld preferences, price adjustments give a natural hedge against productivity and government spending shocks. This can be formalized in the following lemma that generalizes the famous Cole and Obstfeld (1991)result to a borrower-saver agent economy with rigidities. Note also the limitations of the lemma: it considers a setting with only productivity shocks and government spending shocks. That it does not hold for example in an environment with both productivity and quality shocks in which case the CMU still attains the complete markets outcome.

Lemma 5. ***Cole-Obstfeld 91 Result with Borrowers*** *Consider the baseline model of the paper but subject to productivity shocks only. The optimal stock positions are indeterminate and the equilibrium always attains the complete markets allocation for both borrowers and savers (absent any cross-country borrowing). The result holds also when we add idiosyncratic government spending shocks financed through (potentially distortionary) taxes absent government borrowing. This effectively implies a nominal fiscal consumption multiplier of zero.*

Proof. Similarly to before we perform the proof in an Icountry version of the model. For any country i

$A_{t,i}N_{t,i}P_{t,i} = \mu W_{t,i}N_{t,i}.$

[20] We can justify the effect of monetary policy in the standard way of assuming the households can also hold money but considering the cashless limit of this economy.

Here $P_{t,i}$ is the price of the good produced by country i. Conjecture that the model attains the complete markets outcome for both savers and borrowers. That is for any countries i and j:

$\mathbf{C}_{s,t,i}\mathbf{P}_t^i = \mathbf{C}_{s,t,j}\mathbf{P}_t^j$,

where P_t^i is the consumer price index in country i and

$\mathbf{C}_{b,t,i}\mathbf{P}_t^i = \mathbf{C}_{b,t,j}\mathbf{P}_t^j$.

Now we have,

$\frac{A_{t,i}N_{t,i}}{A_{t,j}N_{t,j}} = \frac{(1-\alpha)\chi\mathbf{C}_{s,t,i}\mathbf{P}_t^i/P_{t,i}+(1-\alpha)(1-\chi)\mathbf{C}_{b,t,i}\mathbf{P}_t^i/P_{t,i}+\frac{\alpha}{I-1}\sum_{k\neq i}(\chi\mathbf{C}_{s,t,k}\mathbf{P}_t^k/P_{t,i}+(1-\chi)\mathbf{C}_{b,t,k}\mathbf{P}_t^k/P_{t,i})}{(1-\alpha)\chi\mathbf{C}_{s,t,j}\mathbf{P}_t^j/P_{t,j}+(1-\alpha)(1-\chi)\mathbf{C}_{b,t,j}\mathbf{P}_t^j/P_{t,j}+\frac{\alpha}{I-1}\sum_{k\neq j}(\chi\mathbf{C}_{s,t,k}\mathbf{P}_t^k/P_{t,j}+(1-\chi)\mathbf{C}_{b,t,k}\mathbf{P}_t^k/P_{t,j})}$.

Then applying the complete markets conditions, we obtain

$\frac{A_{t,i}N_{t,i}}{A_{t,j}N_{t,j}} = \frac{\chi\mathbf{C}_{s,t,i}\mathbf{P}_t^i+(1-\chi)\mathbf{C}_{b,t,i}\mathbf{P}_t^i}{\chi\mathbf{C}_{s,t,i}\mathbf{P}_t^i+(1-\chi)\mathbf{C}_{b,t,i}\mathbf{P}_t^i}\frac{P_{t,j}}{P_{t,i}} = \frac{P_{t,j}}{P_{t,i}}$.

Prices and output levels move inverse one-to-one. But this implies

$W_{t,i}N_{t,i} - W_{t,j}N_{t,j} = 0$.

Now one can see that the budget constraints support the complete markets conditions for both savers and borrowers for any symmetric stock positions. Note that α can be arbitrary so the result also holds with respect to symmetric quality shocks. However, it does not hold with respect to arbitrary quality shocks such as shocks that only affect some countries.

What is the intuition behind the result? Assume that markets are complete. Now due to Cole-Obstfeld preferences relative output levels and prices must move one-to-one. This means that the value of output in each country must be the same. Higher production implies lower prices. But the assumption for production technology implies that labor income is a constant fraction of the total value of output in each country. This means that total labor income in each country must be the same. Finally, this implies that the budget constraints support the complete markets allocation. The result holds also in the SOE limit $I \to \infty$.

To see that the result holds when adding idiosyncratic government spending shocks financed through current taxes (in the SOE limit) note that in the proof of lemma 1, we have the line

$$\alpha\tilde{Y}_t = (1-\alpha)\chi\left(\frac{B_{t+1}^h}{1+r_t} - B_t^h\right) - (1-\alpha)(1-\chi)\left(\frac{S_{t+1}}{1+r_t} - S_t\right) + F_t + \frac{B_{t+1}^g}{1+r_t} - B_t^g.$$

Now absent any borrowing this becomes

$$\alpha\tilde{Y}_t = F_t.$$

In SOE government spending shock does not affect F_t and therefore $\tilde{Y}_t$does not react. Foreign demand solely determines income. By Condition 1 neither the borrowers' nor the savers' income reacts. By the budget constraints the nominal consumption levels do not react either. Because private consumption does not react in any country, the total value of production in the home country must rise by the value of nominal government spending.

Therefore the fiscal multiplier is one. A similar simplified argument could be used for productivity shocks, but the former proof highlights that this first result holds also in the finite country case.

E Asymmetries

We now generalize the results concerning equity to asymmetric initial stock positions, markups and shares of savers. The complete markets condition is $\mathbf{P}_t^i\mathbf{C}_{s,t,i} = \lambda_{i,j}\mathbf{P}_t^j\mathbf{C}_{s,t,j}$, where $\lambda_{i,j}$ is the relative Pareto weight. We first show the result in a two country version of the model and then tackle the I country case. The budget constraints are

$$\bar{B} + N_tW_t - T + \varphi(\mu-1)N_tW_t + (\frac{1}{1-\chi} - \frac{1-\chi^*}{1-\chi}\varphi^*)(\mu^*-1)N_t^*W_t^* = \mathbf{P}_t\mathbf{C}_{s,t} + \frac{\bar{B}}{R_t}$$

$$\bar{B} + N_t^*W_t^* - T + (\frac{1}{1-\chi^*} - \frac{1-\chi}{1-\chi^*}\varphi)(\mu-1)N_tW_t + \varphi^*(\mu^*-1)N_t^*W_t^* = \mathbf{P}_t^*\mathbf{C}_{s,t}^* + \frac{\bar{B}}{R_t},$$

where starred values refer to the foreign country. Deducting the budget constraints and imposing the complete markets condition yield

$$N_tW_t\left(1+(\mu-1)(\varphi - \tfrac{1}{1-\chi^*} + \tfrac{1-\chi}{1-\chi^*}\varphi)\right) \\ -N_t^*W_t^*\left(1+(\mu^*-1)(\varphi^* - \tfrac{1}{1-\chi} + \tfrac{1-\chi^*}{1-\chi}\varphi^*)\right) = (\lambda-1)\mathbf{P}_t^*\mathbf{C}_{s,t}^*.$$

or

$$N_tW_t\left(1+(\mu-1)(\varphi - \tfrac{1}{1-\chi^*} + \tfrac{1-\chi}{1-\chi^*}\varphi) - \tfrac{\lambda-1}{1+\lambda}\mu\right) \\ -N_t^*W_t^*\left(1+(\mu^*-1)(\varphi^* - \tfrac{1}{1-\chi} + \tfrac{1-\chi^*}{1-\chi}\varphi^*) + \tfrac{\lambda-1}{1+\lambda}\mu^*\right) = 0.$$

From this we solve

$$\varphi = \frac{1}{2-\chi-\chi^*} + \frac{\lambda-1}{1+\lambda}\frac{\mu}{\mu-1}\frac{1-\chi^*}{2-\chi-\chi^*} - \frac{1}{\mu-1}\frac{1-\chi^*}{2-\chi-\chi^*}$$

and

$$\varphi^* = \frac{1}{2-\chi-\chi^*} + \frac{\lambda-1}{1+\lambda}\frac{\mu^*}{\mu^*-1}\frac{1-\chi^*}{2-\chi-\chi^*} - \frac{1}{\mu^*-1}\frac{1-\chi^*}{2-\chi-\chi^*}.$$

The relative Pareto weight λ depends on initial conditions and can be solved numerically. φ is increasing in λ and φ^* decreasing. The result can be generalized to different tax rates.

The above derivations generalize Proposition 2. Proposition 3 can be generalized similarly.

With I countries the budget constraints are:

$$\bar{B} + N_{t,i}W_{t,i} + \sum \varphi_{i,k}(\mu_k - 1)N_{t,k}W_{t,k} = \mathbf{P}_t^i \mathbf{C}_{s,t} + \frac{\bar{B}}{R_t}, i = 1, .., I.$$

The complete market condition is$\mathbf{P}_t^i \mathbf{C}_{s,t,i} = \lambda_{i,j} \mathbf{P}_t^j \mathbf{C}_{s,t,j}$. Deducting the budget constraints and using this condition we obtain:

$$\begin{gathered} N_{t,i}W_{t,i}(1 + \varphi_{i,i}(\mu_i - 1)) - N_{t,j}W_{t,j}(1 + \varphi_{i,j}(\mu_j - 1)) \\ + \sum_{k \neq i,j} (\varphi_{i,k}\text{-}\varphi_{j,k})(\mu_k - 1)N_{t,k}W_{t,k} = (\lambda_{ij} - 1)\mathbf{P}_t^i \mathbf{C}_{s,t,i}, j \neq i. \end{gathered}$$

Using the fact that value of total consumption equals value of total output as well as the complete market conditions:

$$\begin{gathered} N_{t,i}W_{t,i}(1 + (\varphi_{i,i} - \varphi_{j,i})(\mu_i - 1)) - N_{t,j}W_{t,j}(1 + (\varphi_{i,j}\text{-}\varphi_{j,j})(\mu_j - 1)) \\ + \sum_{k \neq i,j} (\varphi_{i,k}\text{-}\varphi_{j,k})(\mu_k - 1)N_{t,k}W_{t,k} = (\lambda_{ij} - 1)\frac{\sum_k \mu_k W_{t,k} N_{t,k}}{1 + \sum_{k \neq j} \lambda_{jk}}, j \neq i. \end{gathered}$$

We need to set the multiplier on each $N_{t,k}W_{t,k}$to zero, which gives a well-defined problem. For each j we get I restrictions in total. There are $I - 1$ such equations. Together with the stock market clearing conditions we have $I \times I$ equations and unknowns and can now solve for the static equity positions replicating the complete market outcome. The result holds also in the small country limit $I \to \infty$.

F Government Spending Shocks: an Example

Note that if there are no borrowers, Proposition 1 implies that the nominal fiscal consumption multiplier is zero in a money market union. This holds irrespective of the financing method of the spending increase. To understand that this holds even with distortionary taxes, consider the following simple example. Now abstract away from borrowers. Assume a disutility of labor function $v(N) = \frac{N^{1+\sigma}}{1+\sigma}$ and that spending increases are financed using a contemporaneous labor tax.

Now figure 7 shows the impulse responses subject to a government spending increase.[21] Nominal spending by households stays constant. Higher government demand for the home good pushes its price up. Before tax wages increase. The reason why nominal consumption does not react is roughly the following. An increase in government spending implies higher taxes, which reduces the disposable income of households. However, at the same time higher government spending increases production improving nominal profits and labor income. With Cole-Obstfeld preferences these two effects exactly offset each other so that the disposable income of savers does not react. This implies that nominal consumption also

[21] We set $\sigma = 3$.

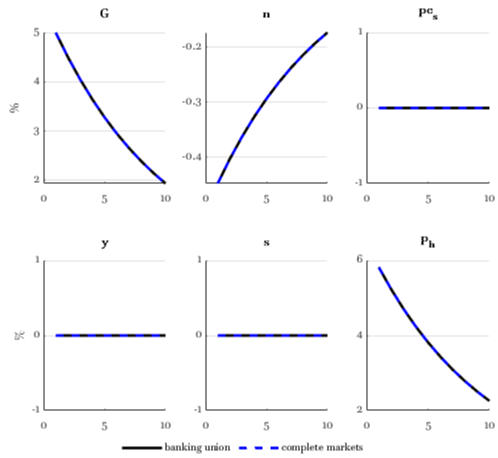


Figure 7: Government Spending Shock, No Borrowers
Note: Impulse responses to a 5% shock to G_t. $y_t$is disposable income.

stays constant. Because disposable income stays constant each period, savings do not react either.

G Proposition 1: Beyond Small Countries

Consider now the case of deleveraging shocks hitting a large economy. Proposition 1 is exactly correct in a small open economy or with a continuum of countries; with two economies, foreign demand depends (partly) on domestic demand and, therefore, on domestic deleveraging. In addition, the central bank reacts by changing the risk free rate.

In spite of these differences we find that the result of Proposition 1 remains essentially correct. The intuition is as follows. First, we know that savers do not react in a SOE. With two countries, foreign demand is endogenous, but this effect is small because it depends on two consecutive cross-border spillovers: the pass-through of domestic demand onto foreign income and then from foreign income back to foreign demand for home goods. The spillover is quantitatively small. Proposition 1 is also approximately correct for reasonable values of the elasticity of substitution other than one.

The second important difference is the Taylor rule. Of course, the reaction of the monetary authority has a direct impact on the dynamics of the currency union. But the key point is that this impact is the same under MMU and under complete markets. Why? Because savers face the same interest rate in both countries.

Figure 8 depicts the impulse responses to a domestic deleveraging shock (credit shock) in each of the two regions of the currency union. The responses of all variables except S_t

are virtually the same under MMU and under complete markets. Domestic savings S_t need to adjust more in the MMU than in the complete markets economy because of the lack of explicit state contingent contracts.

The aggregate (currency union-wide) response to a deleveraging shock obviously depends on how monetary policy reacts. Our results show that, irrespective of the central bank's reaction, the MMU and complete markets economies behave in virtually identical ways after the deleveraging shock. One might wonder, however, if this result could be over-turned if the central bank was constrained by the zero lower bound. We find that this is not the case: our result also holds when the ZLB binds. Figure 9 depicts impulse responses to a deleveraging shock large enough to make the ZLB bind. Naturally, when the ZLB binds the central bank is unable to lower the interest rate enough to stabilize aggregate employment in the currency union.

We conclude that an ideal money market union – a union that guarantees that risk-free rates are equalized across regions – is enough to deal with all domestic demand shocks, both private and public.

H Proposition 1: Beyond Cole-Obsteld for Fiscal Shocks

Pure government *deleveraging*, that does not affect purchases G_t, works similarly to private deleveraging and is not sensitive to the demand elasticity parameters. However, government *spending* shocks can have a large effect on the overall level of taxation not just the timing of taxes. When government spending increases are financed through distortionary taxes, demand elasticities can have a larger effect on how nominal consumption reacts. Figure 10 repeats the exercise of figure 7 but now with different values of the elasticity of substitution between home and foreign goods $\xi_1 \in \{0.5, 1, 2, 1000\}$. Note that a demand elasticity of one (for home and foreign countries) is close to empirically reasonable values (see e.g. Heathcote and Perri 2013 for a discussion).

Figure 11 performs the exercise in figure 10 but now with different values of elasticity of substitution between different varieties of foreign goods $\xi_2 \in \{0.5, 1, 2, 1000\}$ [22] . The results are similar to those before.

The result that government spending shocks do not affect nominal consumption is also sensitive to the assumption that the government purchases only home goods. However, this sensitivity vanishes as the demand elasticity approaches infinity.

[22] Here the demand elasticity changes in all countries simultaneously

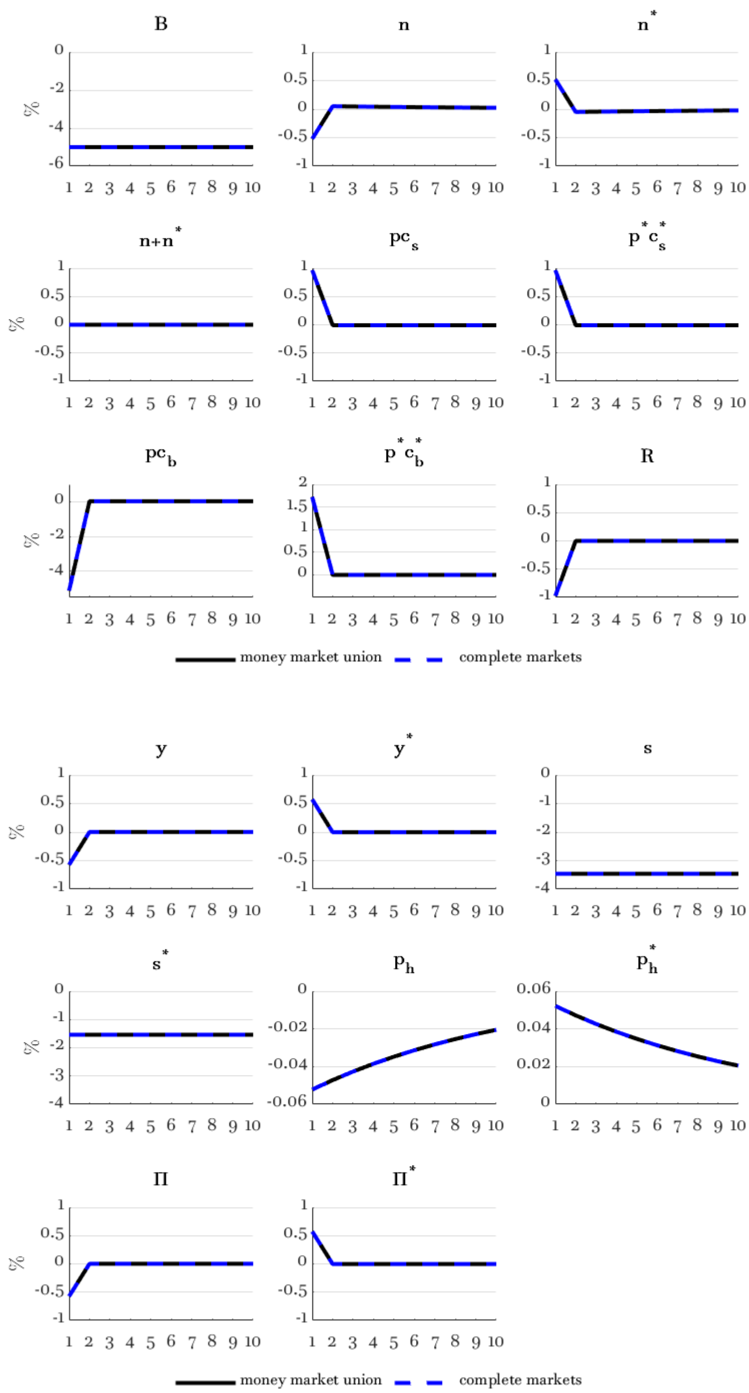
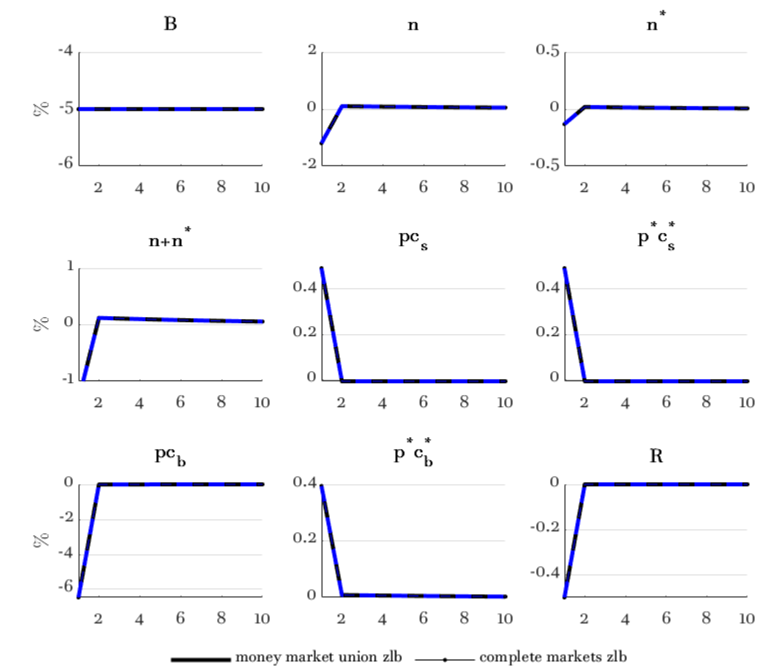


Figure 8: Private Deleveraging in 2-Country Model

Note: Impulse responses to permanent -5% shock to $\bar{B}_t$.



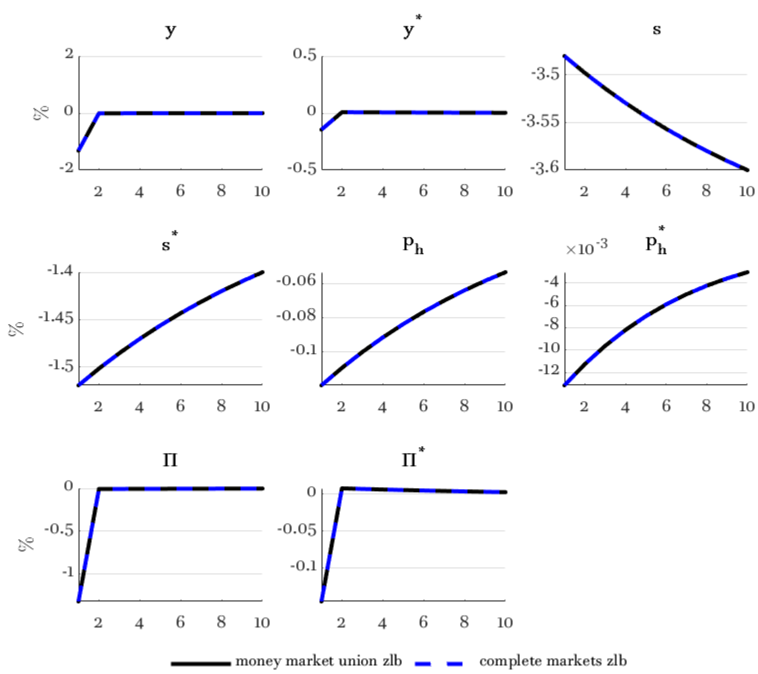


Figure 9: Private Deleveraging in 2-Country Model with ZLB

Note: Impulse responses to permanent -5% shock to $\bar{B}_t$.

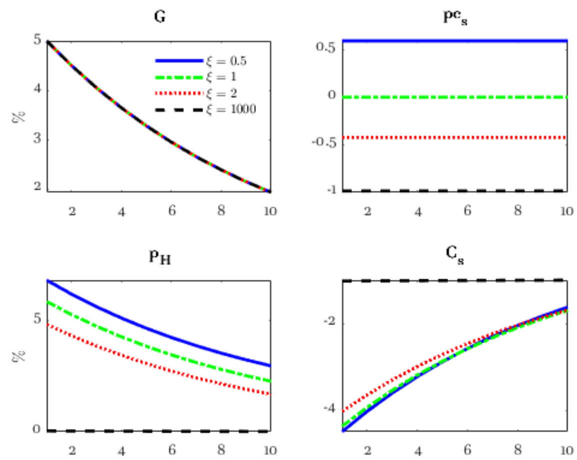


Figure 10: Government spending shock in a money market union for different values of elasticity of substitution between home and foreign goods
Note: Impulse response to a 5% shock to G_t.

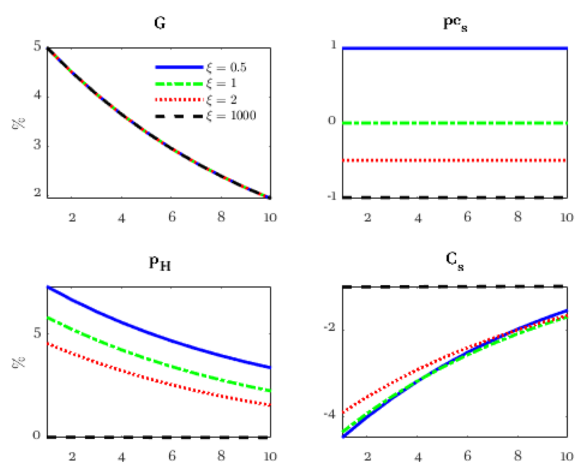


Figure 11: Government spending shock in a money market union for different values of elasticity of substitution between different varieties of foreign goods
Note: Impulse response to a 5% shock to G_t.