Notes on Equilibrium Financial Intermediation

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Abstract

These notes analyze the production of financial services in general equilibrium.

1 Real Economy

1.1 Households

The finance industry provides credit as well as liquidity and payment services to households (and savers more generally). In addition, household debt has an important life-cycle component (i.e., mortgages). The model must therefore incorporate these features. To do so, I consider a setup with two types of households: some households are infinitely lived, the others belong to an overlapping generations structure.[1] Households in the model do not lend directly to one another. Savers lend to intermediaries, and intermediaries lend to firms and households.

Long-Lived Households

Long-lived households (index l) own the capital stock and have no labor endowment. Liquidity services are modeled with money in the utility function (using a cash-in-advance framework gives similar results). The households choose consumption C and holdings of liquid assets M to maximize

$$\mathbb{E} \sum_{t \geq 0} \beta^t u\left(C_t, M_t\right).$$

I specify the utility function as

$$u\left(C_t, M_t\right) = \frac{\left(C_t M_t^{\nu}\right)^{1-\rho}}{1-\rho}.$$

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[1]As argued in Mankiw (2000), neither the pure infinite horizon model nor the pure OLG model is adequate. The pure infinite horizon model misses the importance of life-cycle earnings profile. The pure two-periods OLG model is not appealing because households do not actually borrow: the young ones save, and the old ones eat their savings. In addition bequests are of first order empirical importance. The simplest way to capture all these ideas is the mixed model. The interpretation is that the long-lived household has bequest motives, so it is equivalent to an infinitely lived agent. See also Mehra et al. (2011) for a model where household save for retirement over an uncertain lifetime.

As argued for instance by Lucas (2000), these homothetic preferences are consistent with the absence of trend in the ratio of real balances to income in U.S. data, and the constant relative risk aversion form is consistent with balanced growth.

Let S_{t-1} be the total savings (i.e., their wealth) of savers at the end of time $t-1$. Let r_t be the interest rate received by savers during time t. Households hold liquid assets M_t because these assets provide transaction-services or other ancillary services. These assets, however, yields less than r and the opportunity cost is the price of liquidity services $\psi_{m,t}$.[2] The budget constraint of the savers is therefore

$$S_t + C_t + \psi_{m,t} M_t \leq (1 + r_t) S_{t-1},$$

The Euler equation of long lived households is then

$$u_C(t) = \beta \mathbb{E}_t \left[(1 + r_{t+1}) u_C(t+1) \right]$$

With the functional form above, it can then be written as

$$M_{l,t}^{\nu(1-\rho)} C_{l,t}^{-\rho} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) M_{l,t+1}^{\nu(1-\rho)} C_{l,t+1}^{-\rho} \right].$$

The liquidity demand equation $u_M(t) = \psi_{m,t} u_C(t)$ is simply

$$\psi_{m,t} M_{l,t} = \nu C_{l,t}.$$

Overlapping Generations

The other households live for two periods and are part of on overlapping generation structure. The young (index 1) have a labor endowment η_1 and the old (index 2) have a labor endowment η_2. We normalize the labor supply to one:

$$\eta_1 + \eta_2 = 1.$$

The life-time utility of a young household is

$$u(C_{1,t}, M_{1,t}) + \beta u(C_{2,t+1}, M_{2,t+1}).$$

[2]See Lucas and Stokey (1987) and Sargent and Smith (2009) for a discussion of cash-in-advance models. Lucas (2000) uses the framework of Sidrauski (1967) with a more flexible functional form of the type $\left(C_t \varphi \left(\frac{M_t}{C} \right) \right)^{1-\rho}$. I use a Cobb-Douglass aggregator for simplicity given the complexity of the rest of the model. A more important difference with the classical literature on money demand is that I do not focus on inflation. Households save S at a gross return of $1+r$, while liquid assets yield $(1+r)/(1+\psi_m)$. So this model implies a constant spread between the lending rate and the rate on liquid assets. This is consistent with my interpretation of liquidity as not only money, but also money market funds shares and repurchase agreements.

I consider the case where they want to borrow when they are young (i.e., η_1 is small enough). In the first period, the budget constraint is

$$C_{1,t} + \psi_{m,t} M_{1,t} = \eta_1 W_{1,t} + (1 - \psi_{c,t}) B_t.$$

There is an intermediation cost (for screening and monitoring services) of $\psi_{c,t}$ per unit of borrowing. In the second period, the household consumes $C_{2,t+1} + \psi_{m,t+1} M_{2,t+1} = \eta_2 W_{t+1} - (1 + r_{t+1}) B_t$. The Euler equation for OLG households is

$$(1 - \psi_{c,t}) M_{1,t}^{\nu(1-\rho)} C_{1,t}^{-\rho} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) M_{2,t+1}^{\nu(1-\rho)} C_{2,t+1}^{-\rho} \right].$$

Their liquidity demand is identical to the one of long-lived households.

1.2 Non Financial Businesses

Non financial output is produced with constant returns technology

$$Y_t = F(A_t n_t, K_t).$$

The capital stock K_t is predetermined and depreciates at rate δ. Let $\psi_{k,t}$ be the price of financial intermediation per unit of corporate capital. Non financial firms therefore solve the following program $\max_{n,K} F(A_t n, K) - (r_t + \delta + \psi_{k,t}) K - W_t n$. Capital demand equates the marginal product of capital to its user cost:

$$\frac{\partial F}{\partial K}(A_t n_t, K_t) = r_t + \delta + \psi_{k,t}. \quad (1)$$

Similarly, labor demand equates the marginal product of labor to the real wage:

$$A_t \frac{\partial F}{\partial n}(A_t n_t, K_t) = W_t. \quad (2)$$

2 Financial Intermediation

2.1 Production function

Financial services are produced with capital and labor under constant returns to scale:

Assumption HI (homogenous intermediation). *Intermediation is produced by a constant return technology. With n_t^f units of labor and K_t^f units of capital, the quantity of intermediation is*

$$Q^f = F^f\left(A_t n_t^f, K_t^f\right). \quad (3)$$

Abstracting from physical capital, HI simply says that it takes a given amount of human capital in banking to monitor one unit of entrepreneurial human capital. Let p_t^f be the unit price of financial services. Capital and labor demand are given by

$$p_t^f \frac{\partial F^f}{\partial k} \left(n^f, k^f\right) = r_t + \delta + \psi_{k,t}. \tag{4}$$

Similarly, labor demand equates the marginal product of labor to the real wage:

$$p_t^f A_t \frac{\partial F^f}{\partial \bar{n}} \left(n^f, k^f\right) = W_t. \tag{5}$$

2.2 Market clearing and aggregation

We then need to explain the market clearing condition for financial services. The finance industry produces $F^f\left(A_t n_t^f, K_t^f\right)$ units of financial services. These services are used by households (for B_t and M_t) and firms (for K_t). Market clearing for financial services is then given by

$$Q^f = \mu_c B_t + \mu_m M_t + \mu_k \bar{K}_t, \tag{6}$$

where $\bar{K}_t = K_t + K_t^f$ and μ_i is the intermediation intensity of market i. For instance, if it is more complicated to monitors entrepreneurs than households, we would have $\mu_k > \mu_c$. The parameters μ's are convenient to describe various prediction of the model. For instance, a decrease in μ_k can be interpreted either as an improvement in the creditworthiness of businesses, or as an improvement specific to corporate finance (i.e., that leaves the production function F unchanged, and does not directly affect household finance). It is immediate from (6) that for each market $j = c, m, k$ we have

$$\psi_{jt} = p_t^f \mu_j. \tag{7}$$

The scale of Q^f is arbitrary. It is convenient to define it so that it corresponds to the value of intermediated assets. To to so, I normalize the model so that monitoring one unit of household borrowing requires one unit of financial services:

$$\mu_c \equiv 1.$$

Note that the value added of the finance industry is given by

$$Y_t^f = p_t^f Q_t^f$$

With the normalization above, p_t^f is a equivalent to a spread, i.e., net income over assets. Aggregate GDP in this economy is defined by $\bar{Y}_t \equiv Y_t + Y_t^f$ and the finance share of GDP is

$$\phi_t \equiv \frac{Y_t^f}{Y_t + Y_t^f}.$$

3 Equilibrium with Balanced Growth

Definition 1. An equilibrium is a sequence of the various prices and quantities listed above such that households choose optimal levels of credit and liquidity; financial and non financial firms maximize profits; the labor market clears

$$n_t^f + n_t = 1;$$

the capital market clears (with total capital defined as $\bar{K} \equiv K + K^f$)

$$S_t = B_t + \bar{K}_{t+1};$$

and the market for intermediation services clears using equations (3) and (6).

Let us now characterize an equilibrium with constant productivity growth, driven by labor-augmenting technological progress $A_t = (1+\gamma) A_{t-1}$.

Real interest rate. On the balanced growth path, M grows at the same rate as C. The Euler equation for long-lived households becomes

$$1 = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \left(\frac{C_{t+1}}{C_t} \right)^{\nu(1-\rho)-\rho} \right],$$

so the equilibrium interest rate is simply pinned down by

$$\beta (1+r) = (1+\gamma)^{\theta}. \tag{8}$$

where $\theta \equiv \rho - \nu (1-\rho)$. Let lower-case letters denote de-trended variables, i.e. variables scaled by the current level of technology: for capital $k \equiv \frac{K_t}{A_t}$, for consumption of agent i $c_i \equiv \frac{C_{i,t}}{A_t}$, and for the productivity adjusted wage $w \equiv W_t / A_t$.

Marginal production costs. The intermediation production function $F^f \left(A_t n_t^f, K_t^f \right)$ defines a constant marginal cost χ^f which must equal the price of financial services: $p^f = \chi^f (w, r + \delta + \psi_k)$. Using (7) for business capital, we therefore have

$$\psi_k = \mu_k \chi^f (w, r + \delta + \psi_k).$$

This implies a schedule ψ_k that is increasing in the real wage w and decreasing in the efficiency of intermediation. Similarly in the non-financial business sector we have

$$\chi(w, r + \delta + \psi_k) = 1. \tag{9}$$

This implies a schedule ψ_k that is decreasing in the real wage w. These two equations pin down ψ_k and w. Notice that if corporate finance becomes easier (μ_k goes down), then it becomes cheaper (ψ_k goes down) and the real wage goes up (because k/n goes up). The price of financial services is then pinned down $p^f = \chi^f(w, r + \delta + \psi_k)$, so we know the user costs for $j = c, m$:

$$\psi_j = \mu_j \chi^f(w, r + \delta + \psi_k).$$

Consumption and borrowing by OLG households. Let us now consider the OLG households. The Euler equation for short-lived households written with detrended consumption is

$$\mathbb{E}_t \left[\beta(1 + r_{t+1})(1 + \gamma_{t+1})^{-\theta} \left(\frac{c_{2t+1}}{c_{1t}} \right)^{-\theta} \right] = 1 - \psi_c.$$

Since, on the balanced growth path, the interest rate is pinned down by the long horizon savers at $\beta(1 + r) = (1 + \gamma)^{\theta}$, the Euler equation of short lived households becomes simply

$$c_1 = (1 - \psi_c)^{\frac{1}{\theta}} c_2. \tag{10}$$

In addition, we have $\psi_m m = \nu c$ for each cohort. The budget constraints are therefore $(1 + \nu) c_1 = \eta_1 w + (1 - \psi_c) b$ and $(1 + \nu) c_2 = \eta_2 w - \frac{1+r}{1+\gamma} b$. We can then use the Euler equations and budget constraints to compute the borrowing by young households:

$$\frac{b}{w} = \frac{(1 - \psi_c)^{\frac{1}{\theta}} \eta_2 - \eta_1}{1 - \psi_c + (1 - \psi_c)^{\frac{1}{\theta}} \frac{1+r}{1+\gamma}}. \tag{11}$$

If ψ_c is 0, we have $c_1 = c_2$, and consumption growth is the same for all agents and equal to γ, the fundamental growth rate of the economy. From the perspective of current consumption, borrowing costs act as a tax on future labor income. If ψ_c is too high, no borrowing takes place and the consumer credit market collapses. Improvements in corporate finance increase b because they increase w.

Long-lived households own the capital stock and lend to young households. In detrended form this means

$$s = b + (1 + \gamma) \bar{k}.$$

We can then detrend the market clearing condition as

$$q^f = F^f \left(n^f, k^f\right) = \mu_c b + \mu_m m + \mu_k \bar{k} \tag{12}$$

To close the model, we obtain the consumption of the long lived savers (c_0) from the resource constraint. Since $\psi_m m_i = \nu c_i$ for all agents, we have $y = (1+\nu)(c_0 + c_1 + c_2) + \psi_c b + (\gamma + \delta + \psi_k)\bar{k}$. Using the budget constraints of the young and old agents, we get

$$(1+\nu) c_0 = (r - \gamma)\left(\bar{k} + \frac{b}{1+\gamma}\right).$$

Total expenditure of long-lived households is equal to their capital income from loans to corporates and to short-lived households.

Proposition 1. *Under HI, there is a unique balanced growth path where the finance share of GDP ϕ, the unit cost of financial intermediation p^f, and the financial ratios M/Y, B/Y and K/Y are constant. The equilibrium has the following features*

(i) Improvements in corporate finance increase the real wage, the capital stock, household debt, and liquidity;

(ii) Improvements in household finance increase household debt, consumption and liquidity, but do not affect the real wage; household debt increases with the slope of life-cycle earnings profiles (η_2/η_1).

(iii) Improvements in liquidity management increase consumption, but do not affect household debt or the real wage;

(iv) Improvements in finance in general have an ambiguous impact on the GDP share of the finance industry.

Proof. The proposition follows from the discussion above. □

The bigger the ratio η_2/η_1 the larger the borrowing. For instance, increased years schooling generates more borrowing, and a larger financial sector. Improvements in corporate finance increase liquidity demand because they increase the consumption output ratio. When ψ_k goes down, k/y goes up while b/y is unchanged, therefore $\nu c_0/y$ goes up. One important point is that, under HI, the model does not predict an income effect, i.e., just because a country becomes richer (thanks to TFP growth) does not mean that it should spend a higher fraction of its income on financial services.

That homogeneity is required for balanced growth is not surprising. What is more interesting is that it is sufficient even if the production technologies differ between the financial and non-financial sectors. Acemoglu and Guerrieri (2008) discuss the issue of balanced growth with two sectors that differ in their capital intensities. The key difference is that they consider a CES aggregator for the output of the two sectors, which is appropriate for goods and services that are consumed. But credit is not consumed, it is used to facilitate consumption and investment. As a result, it aggregates in a very particular way that ensures balanced growth even if the factor shares differ across industries. For the liquidity demand component, balanced growth comes from the assumed preferences, as discussed in Lucas (2000).

4 Implications of Different Production Functions

This section studies the equilibrium share of finance in GDP, and how different demand for financial services interact. At this point it is useful to restrict the intermediation production function to simplify the notations. The empirical work shows that the labor share is similar in the financial and non-financial sector. To simplify my notations I am therefore going to assume that the production function F and F^f are similar.

Assumption of Similar Technologies (SI). *There exists a constant ζ such that:*

$$F^f\left(n^f, k^f\right) = F\left(n, k\right)/\zeta.$$

The key point is that, under SI, the relative price of financial services is exactly ζ, since $\chi^f = \zeta\chi$ and $\chi = 1$. Therefore we have simply

$$\psi_k = \zeta\mu_k.$$

4.1 Equilibrium size of finance industry, borrowing, and financial crowding out.

As before, the real wage is pinned down by (9). Under SI, the capital labor ratio is the same in both sectors, and pinned down by

$$\frac{\frac{\partial F}{\partial \bar{n}}\left(n, k\right)}{\frac{\partial F}{\partial k}\left(n, k\right)} = \frac{w}{r + \delta + \psi_k}.$$

The capital labor ratio k/n is increasing in w and decreasing in ψ_k. Since $\bar{n} = 1$ in the aggregate, the aggregate stock of capital $\bar{k}$ is an increasing function of $\frac{w}{r+\delta+\psi_k}$. Since $\psi_c = \zeta\mu_c$ and the wage is already pinned down, consumer borrowing is given by (11) as before. Finally, liquidity demand is

$$m = \frac{\nu c}{\psi_m} = \frac{\nu c}{\zeta\mu_m}$$

We already know $\psi_m = \zeta\mu_m$, so we only need to measure aggregate consumption as

$$c = \frac{1}{1+\nu}\left(w - \psi_c b + (r - \gamma)\,\bar{k}\right)$$

Finally the finance share of GDP is

$$\phi = \frac{\zeta q^f}{y + \zeta q^f}$$

where

$$q^f = \mu_c b + \mu_m m + \mu_k \bar{k}.$$

We already know $\bar{k}$ depends on $\zeta\mu_k$. From the money demand equation we have

$$\zeta q^f = \zeta\mu_c b + \nu c + \zeta\mu_k \bar{k}$$

and we know b and c.

Proposition 2. *Changes in ψ_m have no impact on the GDP share of finance, while changes in ψ_c have an inverse U-shape impact. Expansion of consumer credit can crowd out business capital via the labor market for financial intermediaries.*

An expansion of consumer credit increases driven by demand factors (η_2/η_1 for instance) leads to an increase in n^f and therefore lower n and k in the non financial sector.

4.2 Pure Corporate Finance with Homogenous Borrowers

It is useful to study the case where $\psi_c = \psi_m = 0$ to understand the role of the production technology in shaping the demand for financial intermediation. Corporate finance income is

$$y^f = \psi_k \bar{k}.$$

On the other hand, since k/n is the same in both sectors and $n^f + n = 1$, we have GDP

$$\bar{y} = y + \zeta q^f = F\left(1, \bar{k}\right)$$

and therefore

$$\phi = \frac{\psi_k \bar{k}}{F\left(1, \bar{k}\right)}$$

Proposition 3. *In the homogenous-borrowers model, improvements in financial intermediation (lower ψ) lead to an increase in capital, real wages, and consumption. The GDP share of the finance industry decreases as long as*

$$(\sigma - 1)\,\psi < r + \delta, \tag{13}$$

where σ is the elasticity of substitution between capital and labor.

Proof. The fact that k increases when ψ decreases is clear from (1). The impact on w is clear, and $c = w + (r - \gamma)\,k$ increases because both w and k increase. Let us use a production function with a constant elasticity of substitution to understand the behavior of the GDP share of finance. Output is given by

$$y = F(n,k) = \left(\alpha \bar{n}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\, k^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

where α is the labor share and σ is the elasticity of substitution between capital and labor. The optimality condition for factor demands are simply

$$(1-\alpha)\left(\frac{y}{k}\right)^{\frac{1}{\sigma}} = r + \delta + \psi_k,$$

and $\alpha \left(\frac{y}{\bar{n}}\right)^{\frac{1}{\sigma}} = w$. Therefore $\frac{k}{f(k)} = \left(\frac{1-\alpha}{r+\delta+\psi_k}\right)^{\sigma}$ and the finance share of GDP is

$$\phi = \left(\frac{1-\alpha}{r+\delta+\psi_k}\right)^{\sigma} \psi_k.$$

Hence $\log\phi = \sigma \log(1-\alpha) + \log\psi_k - \sigma\log(r+\delta+\psi_k)$, and

$$\frac{d\log\phi}{d\psi_k} = \frac{1}{\psi_k} - \frac{\sigma}{r+\delta+\psi_k}.$$

Therefore $\frac{d\log\phi}{d\psi_k} > 0$ if and only if $r+\delta > (\sigma-1)\,\psi_k$. QED. $\square$

Cobb-Douglass example An important special case is one of Cobb-Douglass technology. In this case, better finance, *in the sense of uniformly better corporate finance,* always leads to smaller finance since $\sigma = 1$. One can easily obtain closed form solutions. Capital is simply $k = \left(\frac{1-\alpha}{r+\delta+\psi}\right)^{\frac{1}{\alpha}}$ and output is $y = k^{1-\alpha}$. The consumption output ratio is $\frac{c}{y} = \alpha + (r-\gamma)\frac{k}{y}$ and we know that $\frac{k}{y} = \frac{r-\gamma}{r+\delta+\psi}$. If ψ goes down, k/y goes up, so c/y goes up (but consumption over capital goes down). With Cobb-Douglass technology, the corporate finance share of income

$$\phi = (1-\alpha)\frac{\psi}{r+\delta+\psi}$$

and it is easy to see that the GDP share decreases when the intermediation cost ψ decreases.

What are the quantitative implications of the model? Most empirical macroeconomic studies use the Cobb-Douglass technology. In this case the model predicts that improvement in financial intermediation should lead to a lower share of finance in GDP. We can go further and ask how high σ would need to be to overturn this prediction. In the empirical section, I find ψ in the range of 1 to 2%. Let us take 2% to be conservative. Standard estimates of the real interest rate are around 1-3%, and of depreciation around 5-10%. Let us take $r+\delta$ of 10%. To violate the condition, we would need $\sigma > 1 + 10/2 = 6$. This is outside the range of plausible elasticities. Most macroeconomic estimates suggest that σ is close to 1. Chirinko et al. (2011) estimate less than 1 for the U.S., while Raurich et al. (2011) estimate ranges of 0.6-0.9 for the U.S. and 1.2-1.6 for Spain. It is therefore safe to conclude that condition

(13) is satisfied.[3]

5 Consequences of Borrower Heterogeneity

The homogenous borrower model described above is a useful benchmark, but it fails to capture some important features of corporate finance. To give an extreme example, corporate finance involves commercial paper for blue chip companies as well as equity for high-technology start-ups. It is plainly obvious that the monitoring requirements per unit of intermediated funding are vastly different. The homogenous borrower model also fails to capture the idea that financial development gives access to credit to borrowers who would otherwise be shut out from the markets. As we will see, modeling this feature is important when thinking about the GDP share of finance, technological progress, and shocks to credit demand.

Let us therefore consider a model with heterogeneity and decreasing returns at the firm level.[4] The previous section has discussed in details the consequences of different production functions, in particular the elasticity of substitution between capital and labor. Here I want to focus on heterogeneity among borrowers. For simplicity, we specify the production function of financial services as using the final good as intermediate input, with constant marginal cost ψ.

Let k be the (endogenous) number of firms, and let $n = \bar{n}/k$ be employment per-firm. Each firm operates A_t units of capital and hires n workers to produce $A_t f(n)$ units of output, where f is increasing and concave. Decreasing returns come from the fact that capital is fixed at the firm level. Firms choose employment to maximize (detrended) net income

$$\pi(w) \equiv \max_n f(n) - w_t n$$

Each firm hires n employees such that $f'(n) = w$. The labor demand schedule is the decreasing function $n(w)$.

Macroeconomic adjustment to the stock of capital takes place at the extensive margin, i.e., by firms' entry (and exit) decisions. Firms differ in their need for intermediation services, characterized by the monitoring requirement μ per unit of capital:

Assumption: *Monitoring an entrepreneur of type μ with capital A requires μA units of intermediation*

The mass of potential firms with monitoring requirements below μ is $G(\mu)$. Let us define $G_0 \equiv G(0)$ as the mass of (potential) firms that do not require intermediation services.[5] It is convenient to define the density $g(\mu)$

[3] What is a plausible interpretation of the homogenous borrower model? The key assumption is that all financial flows are intermediated at the same cost. This model therefore applies either to an economy where firms are fairly homogenous, or, more realistically, it applies to the part of intermediation services that are required by all borrowers. A good example would be passively managed mutual funds. They provide a cheap way for households to hold diversified portfolios of stocks and bonds. Progress in information technology has lowered the cost per unit of asset held. And they are used by (almost) all households and (almost) all firms. In this case the model predicts that mutual funds should increase the stock of corporate assets (measured at market value) but at the same time decrease the GDP share of intermediation. Funds, trusts and other financial vehicles account for only approximately 0.3% of GDP, which is small relative to the more than 8% share of GDP for finance and insurance as a whole.

[4] Decreasing returns in production are required to make room for heterogeneity since with constant returns borrowers that have even a slight financial disadvantage would not be able to enter.

[5] A straightforward extension is to connect the distribution of monitoring needs G to the correlation between cash flows and investment opportunities. When the correlation is high, firms with investment opportunities also have high cash flows and there is no need for

for $\mu > 0$. We can then write $G(\mu) = G_0 + \int_0^\mu g(x)\,dx$. I assume that there are enough potential entrants to have an interior solution. Recall that we have defined k^* as the neoclassical level of detrended capital. I assume that

$$G_0 \leq k^* < G(\infty).$$

Finally, and as in the homogenous borrower model, I let ψ be the unit cost of monitoring services.

This model is rich enough to capture shifts in demand and supply in the market for financial intermediation. I will use G_0 as an inverse measure of demand shocks and ψ as an inverse measure of supply shocks. To solve the equilibrium, we need to consider firms' entry decisions, and then clear the labor market. Consider entry decisions first. The profits of a firm with monitoring requirements μ are $A_t \pi(w) - (r + \delta + \psi\mu) A_t$. The marginal firm $\hat{\mu}$ is defined by:

$$\pi(w) = r + \delta + \psi\hat{\mu}. \tag{14}$$

Firms above the cutoff $\hat{\mu}$ do not enter. The number of firms, which is also the detrended capital stock, is

$$k = G(\hat{\mu}).$$

Households' preferences are unchanged but their budget constraint becomes $K_{t+1} + C_t \leq (1 + r_t) K_t + W_t + \Pi_t$, where Π_t are aggregate corporate profits, and the aggregate capital is simply $K_t = A_t k$. The Euler equation (8) is unchanged, therefore r is still given by preferences (on the balanced growth path). Since there are k firms, and each employs $n(w)$ workers, clearing the labor market requires $\bar{n} = kn(w) = 1$. From (14), we then get the second equilibrium condition

$$G_0 + \int_0^{\hat{\mu}} g(x)\,dx = \frac{1}{n(w)}. \tag{15}$$

The general equilibrium is the solution $(w, \hat{\mu})$ to the system of equations (14) and (15). The equilibrium is depicted on Figure 1.

This model nests the homogenous borrower case with a vertical demand curve at $\hat{\mu} = 1$ and the neoclassical growth model with $\hat{\mu} = 0$. The finance share of GDP is now equal to

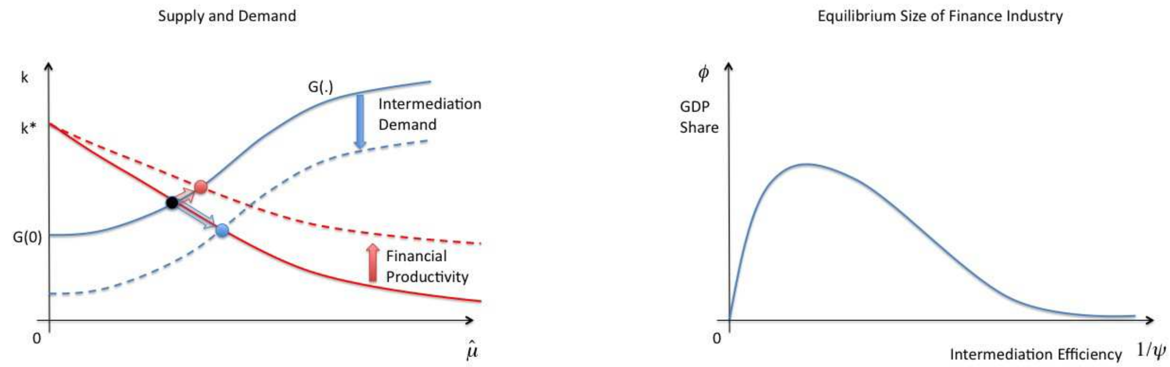
$$\phi = \frac{\psi}{y} \int_0^{\hat{\mu}} \mu g(\mu)\,d\mu. \tag{16}$$

We have the following proposition:

Proposition 4. *In the heterogenous borrowers model, improvements in financial intermediation (a decrease in ψ)*

intermediation (this corresponds to a high value of G_0). The demand for corporate finance services increases when the correlation decreases. From an equilibrium perspective, there are two advantages of this type of micro-foundation. First, the model with moral hazard introduces a new state variable in the system, namely the correlation between cash flows and investment opportunities. Since cash flows are observable, it is then possible (in theory at least) to perform a model-based decomposition of supply and demand shifts. Second, in a model with adverse selection, it is possible to discuss equilibria where the economy is not constrained-efficient and there is room for regulation. These issues are not the main focus of this paper, however.

Figure 1: Equilibrium Corporate Finance



increase the wage and the number of firms but have an ambiguous impact on the GDP share of finance. A higher demand for intermediation (a decrease in G_0) decreases the wage and the number of firms and increases the GDP share of finance.

Proof. The RHS of (15), $\frac{1}{n(w)}$, is increasing in w. Since $\pi(w)$ is decreasing, we see from (16) that w is decreasing in $\hat{\mu}$. The RHS is therefore a downward schedule in $\hat{\mu}$ that is steeper when the marginal cost ψ is higher. The LHS is obviously an increasing schedule. The equilibrium is unique and depicted in Figure 1. A decrease in ψ increases $\hat{\mu}$, therefore $k = G(\hat{\mu})$ and therefore w from $kn(w) = 1$. The fact that ϕ decreases with G_0 is also clear. Suppose G_0 goes up, then $\hat{\mu}$ goes down while k, and therefore y, go up. From (16), ϕ goes down. □

In general, we see that ϕ is non monotonic in ψ. This property is intuitive. When finance is very inefficient (ψ is very high), all would-be users are priced out: $\hat{\mu}_t$ is 0 and so is ϕ, and only G_0 firms enter. Starting from this level, an improvement in financial intermediation must increase the GDP share of finance. The GDP share of finance reaches its maximum for intermediate levels of financial development. When finance is fully efficient (ψ goes to zero), we get the Walrasian benchmark with equality of lending and borrowing rates and ϕ tends to zero.

With Cobb-Douglass technology, we get net income $\pi(w) = (1-\alpha)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}$, labor demand $n = \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}}$. Therefore $k_t = \left(\frac{\alpha}{w_t}\right)^{\frac{-1}{1-\alpha}}$ and we can substitute in the equilibrium condition (15) to get the simple condition

$$G_0 + \int_0^{\hat{\mu}} g(x)\,dx = \left(\frac{1-\alpha}{r+\delta+\psi\hat{\mu}}\right)^{\frac{1}{\alpha}}.$$

6 Accounting

This section lays out the accounting in the economy with three sectors: households, non financial firms, financial intermediaries. Long-lived households own the capital stock

$$S_t = K_{t+1} + K^f_{t+1} + B_t$$

Adding up the budget constraints we have

$$W_t + (1+r)\,S_{t-1} + (1-\psi_c)\,B_t - (1+r)\,B_{t-1} - \psi_m M_t = C_{0t} + C_{1t} + C_{2t} + S_t$$

The two sides of GDP are

$$\begin{aligned} Y_t &= W_t + (r+\delta+\psi_k)\,\bar{K}_t \\ Y_t &= \bar{K}_{t+1} + C_{0t} + C_{1t} + C_{2t} - (1-\delta-\psi_k)\,\bar{K}_t + \psi_m M_t + \psi_c B_t \end{aligned}$$

Combining them we get

$$\bar{K}_{t+1} + C_{0t} + C_{1t} + C_{2t} = W_t + (1+r)\,\bar{K}_t - \psi_c B_t - \psi_m M_t$$

Combining with the budget constraint and capital market equilibrium we get

$$(1-\psi_c)\,B_t = -\psi_c B_t + B_t$$

which is simply the zero profit condition for consumer credit intermediaries.

On the balanced growth path we have

$$\begin{aligned} y &= (1+\gamma)\,\bar{k} + c_0 + c_1 + c_2 - (1-\delta-\psi_k)\,\bar{k} + \psi_m m + \psi_c b \\ y &= (\gamma+\delta+\psi_k)\,\bar{k} + c_0 + c_1 + c_2 + \psi_m m + \psi_c b \end{aligned}$$

We can write

$$(1+\nu)\,(c_0 + c_1 + c_2) = w + (r-\gamma)\,\bar{k} - \psi_c b$$

The budget constraint of short lived households is

$$(1+\nu)\,(c_1 + c_2) = w + (1-\psi_c)\,b - \frac{1+r}{1+\gamma}b = w - \psi_c b - \frac{r-\gamma}{1+\gamma}b$$

so

$$(1+\nu)\,c_0 = (r-\gamma)\left(\bar{k} + \frac{b}{1+\gamma}\right).$$

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