

Debt Overhang and Recapitalization in Closed and Open Economies*

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Abstract

I analyze an economy where debt overhang occurs simultaneously in the mortgage market and in the market for bank debt. Overhang in one market reinforces overhang in the other. In a closed economy, it is ex-post Pareto-efficient to tax households and recapitalize the banks. In an open economy, however, some of the gains are transferred abroad, while all the costs are borne by domestic households. Efficient recapitalization programs therefore require global coordination.

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In his classic paper, Myers (1977) shows that debt overhang leads to under-investment. Firms near financial distress find it difficult to raise capital for new investments because the proceeds from these new investments would benefit existing debt holders instead of new investors. In this paper, I use this simple idea in a macroeconomic model where debt overhang appears simultaneously among banks and households.

The debt overhang economy has the following features. Banks own mortgages and can finance new investments. Their willingness to finance new investment is reduced by debt overhang. The extent of debt overhang depends on the expected performance of the mortgages. The performance of mortgages depends on the income of households which itself depends on the new investments financed by the banks. These complementarities amplify the impact of exogenous shocks and can lead to multiple equilibria. Households are less likely to default on their existing mortgages when their income increases. A lower expected default rate improves the balance sheet of the banks. The banks are then more willing to finance new investments, which, through standard general equilibrium effects, increase the income of households.

The debt overhang equilibrium is inefficient because of two macroeconomic externalities in the model. The first externality is that, when deciding on a new investment, a given bank does not take into account its impact on aggregate activity and on the default rate for other banks. The second externality is that, when deciding how much to sacrifice in order to repay its mortgage, an individual household does not take into account its impact the balance sheets of banks, and therefore on their willingness to extend new loans.

Inefficiencies create room for government interventions. In a closed economy, I show that bailing out banks improves efficiency while bailing out indebted households may not. Banks bailout always improve efficiency because they raise investment without taking away resources from households since households are residual claimants of banks' income. Bailing out indebted household can backfire because it involves a transfer of resources from solvent households towards households who make inefficient savings decisions. This can lead to a crowding out of investment.

In the open economy version of the model, I derive a financial equivalent of the fiscal policy dilemma. I show that a domestic bailout can end up benefiting mostly foreign countries, leaving domestic agents worse off. In such cases, efficient interventions require international coordination.

This paper relates to the literature on government bailouts. A large part of this literature focuses on financial institutions. Aghion, Bolton, and Fries (1999) show that bank bailout policies can be designed

so that they do not distort ex-ante lending incentives relative to strict bank closure policies. Diamond (2001) emphasizes that governments should only bail out banks that have specialized knowledge about their borrowers. Gorton and Huang (2004) argue that there is a potential role for the government to bail out banks in distress because the government can provide liquidity more effectively than the private market. Diamond and Rajan (2005) show that bank bailouts can increase excess demand for liquidity, which can cause further insolvency and lead to a meltdown of the financial system.

The concept of debt overhang has received a lot of attention since the start of the financial crisis. Philippon and Schnabl (2009) compare various forms of government bailouts in a partial equilibrium model of debt overhang. They find that equity injections dominate asset buybacks and debt guarantees when banks have more information than the government. Debt overhang also plays a fundamental role in the model of Diamond and Rajan (2009). Kocherlakota (2009) analyzes resolutions to a banking crisis in a setup where insurance provided by the government generates debt overhang. In the macroeconomic literature, debt overhang has typically been analyzed in the context of sovereign debt crisis. Krugman (1988) analyzes the choice between financing and forgiving the debt from the perspective of creditors. Bulow and Rogoff (1991) show that a country cannot gain by openly repurchasing its debt at market prices. Aguiar, Amador, and Gopinath (forthcoming) analyze taxation and debt policy when the government lacks commitment. Importantly, they show that even though investment and government debt are negatively related, debt relief is never Pareto improving. Finally, Lamont (1995) shows that debt overhang can cause multiple equilibria, but he does not analyze government interventions, and he does not consider the case of an open economy.

The remaining of the paper is organized as follows. Section1 presents a simple numerical example and discusses some general theoretical ideas and empirical facts about debt overhang. Section 2 contains the description of the benchmark model. Section 3 characterizes the debt overhang equilibrium. 4 explores government interventions in a closed economy. Section5 explores government interventions in open economies. Section6 concludes.

1 A Simple Example of Debt Overhang

Before presenting the benchmark model, I give a simple overview of the classic debt overhang model of Myers (1977). Like all modern theories of corporate finance, the debt overhang model is based on a well-defined departure from the benchmark of Modigliani and Miller (1958). Here the departure is the assumption that outstanding senior debt contracts cannot be renegotiated.

Consider a simple numerical example. Bank A holds risky securities, which pay off either \$100 or \$0 with equal probability. The bank is financed by equity and debt with a face value of \$90. Assuming investors are risk neutral, debt value is $1/2 \cdot 90 = \$45$ and equity value is $1/2 \cdot (100 - 90) = \$5$. The bank can invest in a safe project, which requires an outlay of \$5 and yields a discounted value of \$6. The NPV of the project is one. The bank needs to raise \$5 from new lenders to pay for the project. If the bank invests, debt value increases to $1/2 \cdot (90 + 6) = \$48$ since debt holders now receive \$6 in the bad state. The new lenders must receive an expected payment of \$5 to break even, therefore they must get \$10 in the good state. Equity value, however, becomes $1/2 \cdot (100 - 90 - 10 + 6) = \3 . Equity value declines by \$2. Investing is not in the interest of shareholders even though the project has a positive NPV. This is the essence of the debt overhang problem.

Debt overhang occurs when it is difficult to renegotiate senior claims. Since investment does not take place, debt value remains \$45 and debt holders would be willing to pay up to \$3 to convince the shareholders to invest. One way to achieve the Pareto efficient outcome is swap debt for equity. We would expect this efficient outcome to prevail when debt is held by a few large institutions. When bonds are dispersed, however, this solution is difficult to implement because it is never in the interest of a nonpivotal bond holder to accept the reduction in face value. By holding on to their original claim, a small bond holder can expect to get 48 cents on the dollar if investment takes place. There is therefore no equilibrium in which investment takes place and where a small bond holder would tender for less than 48 cents on the dollar. At that price, however, the shareholders are better off not investing. This is the well-known free rider problem (see Bulow and Shoven, 1978; Grossman and Hart, 1980; Gertner and Scharfstein, 1991, among others).¹ The same idea explains the results of Bulow and Rogoff (1991). In their model, the government announces that it is buying back some of its debt and creditors sell at market prices after the announcement. The creditors therefore

¹Contract incompleteness can also be an impediment to renegotiation (Bhattacharya and Faure-Grimaud, 2001)

reap all expected efficiency gains and the indebted country cannot gain from buying back its debt.

It is also important to understand why debt overhang happens in the first place. An insight of the theoretical literature of the past 20 years is that financial contracts – and debt in particular – are used strategically to create or remedy various types of governance problems. For instance, it is often optimal to make renegotiation difficult for governance reasons. Hart and Moore (1995) model the trade-off between debt overhang and the agency costs of free cash flows. It is precisely because debt is a hard claim that it can be used as a disciplining device on management. Bolton and Scharfstein (1996) argue that borrowing from multiple creditors provides managerial discipline but that it can also reduce efficiency when firms default because of liquidity problems. The optimal capital structure trades off the incentive benefits with the ex-post costs of inefficiencies. Mueller and Panunzi (2004) show that debt can be used to alleviate free riding problems among target shareholders in tender offers.

A large body of empirical research has shown the economic importance of renegotiation costs for firms in financial distress. Gilson, John, and Lang (1990); Asquith, Gertner, and Scharfstein (1994); Hennessy (2004, among others). The social costs of renegotiation may be even larger than the private costs because renegotiation can trigger creditor runs among other firms. In the debt overhang model, the flip side of underinvestment in safe projects is a potential over investment in risky ones. Shareholders could be tempted to invest in negative value projects as long as these projects have enough upside income (Jensen and Meckling, 1976). There is ample evidence that risk shifting is a first order issue. Concerns about risk shifting explain the use of covenants in debt contracts. Of course, to the extent that the covenants perform well, we do not expect to see much risk shifting in equilibrium. The exception is when risk shifting goes undetected, as it did in the run up to the financial crisis of 2007-2009. Banks had sold out-of-the-money liquidity guarantees to conduits (liquidity guarantees were/are charged low capital requirements by Basel regulation). In the end, there was little risk transfer from banks to conduit investors as the primary purpose was not risk transfer but simply to hold greater quantity of assets with as little capital as possible. This is a prime example of risk shifting (Acharya, Schnabl, and Suarez (2010)). Another clear example of risk shifting when banks refuse to sell risky assets even though such sales could significantly lower the expected costs of financial distress (Philippon and Schnabl, 2009; Diamond and Rajan, 2009)

2 Model

2.1 Technology and Preferences

The economy is populated by a continuum of households $i \in [0, 1]$, and a continuum of financial intermediaries (banks) $j \in [0, 1]$. For simplicity I do not introduce a separate non financial corporate sector and I assume that intermediaries own industrial projects. In doing so, I ignore contractual issues between the banks and the industrial sector. The model has two dates, $t = 1, 2$. The utility of household i is:

$$U(i) = E \left[c_1(i) + \frac{c_2(i)}{\delta} \right] \quad (1)$$

Financial intermediaries receive an investment opportunity at time 1. They can spend $x \in \{0, X\}$ units of output at time 1 to create qx units of output at time 2, with $q > \delta$. Investment can only be made by intermediaries. Households receive an endowment which can be used for consumption at date 1, or for investment. For simplicity I assume that all individuals receive the same endowment \bar{y}_1 . Let \bar{x} be aggregate investment, and let \bar{c}_1 be aggregate consumption at time 1. Clearing the goods market requires

$$\bar{y}_1 = \bar{c}_1 + \bar{x}. \quad (2)$$

At time 2, the intermediaries receive exogenous output z from their initial assets, and the proceeds from their new investments qx . Aggregate output at time 2 is therefore

$$\bar{y}_2 = z + qx \quad (3)$$

It is also equal to aggregate consumption: $\bar{c}_2 = \bar{y}_2$.

2.2 Financial Contracts

Households and banks start the first period with outstanding financial claims on each other. The focus of this paper is on the ex-post consequences of debt overhang, not on the ex-ante choice of capital structure. I take the initial claims as given and I do not model where they come from. There is of course an extensive literature

on financial contracting that analyzes the optimal quantity, ownership and maturity of debt instruments (see the discussion in Section 1). In the context of financial firms, Diamond and Rajan (2001) study the optimal capital structure of a bank when bankers have specific skills and limited commitment. They show that the bank will issue claims that cannot be renegotiated (for instance, demand deposits that cannot be renegotiated without triggering a run).² These non-renegotiable contracts are privately optimal as long as the ex-ante probability of an aggregate liquidity crisis is low. In this context, one can imagine that such a low probability event has occurred, and that there is now a significant amount of debt overhang. The important point here is that there are many well-understood reasons for companies to take on debt, and that we should expect ex-post inefficiencies because these can be privately optimal from an ex-ante perspective. Of course, there is no reason to expect these ex-post inefficiencies to be socially optimal. For instance, general equilibrium feedback can lead to excessive amounts of default. This is what this paper is about.

All the claims settle at the end of the second period. The initial financial assets of households are the bonds and stocks issued by the intermediaries. The initial liabilities of households are the mortgages held by the banks. The initial assets of intermediaries are the loans (mortgages) made to households as well as the existing assets that deliver exogenous payoffs z at time 2. The liabilities of intermediaries are the bonds and shares held by households.

For simplicity, I assume that households have the same initial diversified portfolios of assets, and that they differ only by the face value of the loans they must repay at date 2. Let $F(m)$ be the cumulative distribution of the mortgages, with face value $m \in [0, \infty)$. Similarly, I assume that banks hold diversified portfolios of consumer credit, and that they differ only by the face value of the bonds they have issued. Let $G(b)$ be the distribution of the face values of banks' bonds, with face value $b \in [b^{\min}, b^{\max}]$. Note that m and b are given at time 0.

At time 1 the banks must issue new claims in order to finance their investments. Households can consume their endowments or invest them in financial claims issued by banks. Let n be the face value of the new claims issued by banks. These new claims could be common shares, preferred stocks, or junior debt claims. All these are economically equivalent in my model. What is crucial is that the banks cannot issue claims that are senior to the outstanding debt b . I shall assume that the banks issue junior debt since it simplifies

²See Tirole (2006) for a complete overview of this literature.

the accounting.

At time 2 production takes place and financial contracts are settled. Figure 1 presents the balance sheets of households and banks. Note that m, b, n, e_0 are face values, not market values. In case of default, the actual payments are smaller than the face values. The actual payments are discussed in details in the next section.

2.3 First Best Equilibrium

Since $q > \delta$, it is optimal to invest as much as possible. The first best saving curve is $S^*(r) = y_1 1_{r \geq \delta}$, while the first best investment curve is $I^*(r) = X 1_{r \leq q}$. If $\bar{y}_1 > X$, there is excess savings, and the equilibrium is such that $\bar{x} = X$, $\bar{c}_1 = \bar{y}_1 - X$ and the interest rate is $r = \delta$. Alternatively, if $\bar{y}_1 < X$, investment is constrained by available savings, so that $\bar{x} = \bar{y}_1$, $\bar{c}_1 = 0$ and the interest rate is q .³ In the remaining of the paper, I assume an interior solution for consumption at time 1:

Assumption A1: Excess Savings. *The aggregate endowment exceeds the investment capacity at time 1:*

$$\bar{y}_1 \geq X.$$

Under A1, the required return in the first best equilibrium is therefore $r^{FB} = \delta$, and the first best quantities are $c_1^{FB} = \bar{y}_1 - X$, and $c_2^{FB} = \bar{y}_2^{FB} = z + qX$.

3 Debt Overhang Equilibrium

The key assumption of the debt overhang model is that outstanding claims (b) are senior to new claims (n).

Assumption A2: Debt Overhang. *The initial banks' bonds b (resp. households' loans m) are senior and cannot be renegotiated.*

³Note that in this case banks are indifferent between investing and not investing. Since projects are indivisible at the micro level, a fraction of banks invest $x = X$, while the remaining banks invest nothing. The fraction is such that the equilibrium condition $\bar{x} = \bar{y}_1$ is satisfied.

3.1 Equilibrium at Time 2

Financial contracts are settled at time 2. I have specified the face values of the contracts: m owed by individuals to banks, b and n owed by banks to individuals. Let ρ_ω be the actual repayment on claim $\omega = \{m, b, n\}$, and let $\bar{\rho}_\omega$ denote aggregate repayments (or averages across banks or households). For equity I use the convention that e is the actual payment since there is no relevant face value.

Banks hold diversified portfolios of consumer credit. They receive $\bar{\rho}_m$ from households and z from their initial assets. Therefore a bank with investment x , initial bonds b , and new liabilities n , defaults at time 2 if and only if:

$$z + qx + \bar{\rho}_m < b + n. \quad (4)$$

The repayments from banks to their claim holders follow strict priority rules. Senior debt holders (holding the initial long term bonds) are paid first:

$$\rho_b = \min(b, z + qx + \bar{\rho}_m).$$

Junior debt holders (holding the notes issued at time 1) are paid second:

$$\rho_n = \min(n, z + qx + \bar{\rho}_m - \rho_b).$$

And shareholders are paid last:

$$e = z + qx + \bar{\rho}_m - \rho_b - \rho_n.$$

Households hold diversified portfolios of equity and debt, so their financial income is only a function of aggregate payments from intermediaries. They receive income from their initial bonds and shares, and from the new savings they made at time 1. Let r be the gross rate of return between dates 1 and 2. The wealth at time 2 of an individual who saved s at time 1 is:

$$w(s) = \bar{\rho}_b + \bar{e} + rs. \quad (5)$$

A household with mortgage loan m defaults if and only if $w(s) < m$. In case of default at date 2, the household consumes nothing and its financial wealth goes to the banks holding the loans, so that $\rho_m = w(s)$. If the household does not default, it repays m to the banks and consumes $w(s) - m$. Let us now examine the consumption and investment decisions at time 1.

3.2 Consumption and saving

At time 1, households choose how much to consume and how much to save in order to maximize their lifetime utility (1). Households have an endowment y_1 and they can sell their stocks and bonds holdings for a value $(\bar{\rho}_b + \bar{e})/r$. The program of the household is therefore

$$\max y_1 - s + E \left[\frac{c_2}{\delta} \right],$$

subject to $s \in [-(\bar{\rho}_b + \bar{e})/r, y_1]$, and $c_2(s, m) = \max(0, w(s) - m)$. The next lemma characterizes the solution.

Lemma 1. *The savings function at date 1 for an individual with mortgage m is*

$$s(m) = y_1 \cdot \mathbf{1}_{m \in [0, \hat{m}]} - \frac{\bar{\rho}_b + \bar{e}}{r} \cdot \mathbf{1}_{m \in (\hat{m}, \infty)}, \quad (6)$$

where the solvency cutoff \hat{m} is defined by:

$$\hat{m} \equiv (r - \delta) \left(y_1 + \frac{\bar{\rho}_b + \bar{e}}{r} \right). \quad (7)$$

Proof. Since the benchmark model is deterministic, the household seeks to maximize $y_1 - s + \frac{1}{\delta} \max(0, w(s) - m)$ subject to $s \in [-(\bar{\rho}_b + \bar{e})/r, y_1]$. If $r < \delta$ it is clear that it is optimal to consume as much as possible and $s = -(\bar{\rho}_b + \bar{e})/r$ for all households. This corresponds to a negative value for the cutoff \hat{m} , and therefore $m > \hat{m}$ for all households. If $r \geq \delta$, we have to consider the incentives of the household given its outstanding mortgage m . If the household decides to consume at time 1 and default at time 2, it gets utility $c_1^{\max} = y_1 + (\bar{\rho}_b + \bar{e})/r$. If, on the other hand, the household is solvent conditional on saving all it

can, then it consumes $c_2^{\max}(m) = ry_1 + \bar{\rho}_b + \bar{e} - m$. The marginal household is therefore characterized by the debt level \hat{m} such that $c_2^{\max}(\hat{m}) = \delta c_1^{\max}$, which delivers equation (7). \square

The intuition for this first Lemma is straightforward. If individuals have too much debt, they have no incentives to save in conventional ways because their savings are given away to their creditors. Households would rather default than save if they expect their savings to be entirely transferred to the bank owning their mortgage. If the household remains insolvent even when it saves the maximum amount, then it is clearly better off defaulting and consuming all it can at date 1. This happens when $m > \bar{\rho}_b + \bar{e} + ry_1$. On the other hand, saving is attractive if the interest rate is high and the household expects to keep most of its savings. This happens when $r > \delta$ and $m < \bar{\rho}_b + \bar{e}$. In between these polar cases lies the marginal household. This household is insolvent conditional on not saving anything, but solvent conditional saving y_1 . The marginal household must then be indifferent between the two strategies, and this defines the solvency cutoff \hat{m} .

Equation (7) is intuitive. The second term in parenthesis is simply the net present value of households' assets. The term $r - \delta$ is the spread of the market expected return over the rate of time preference. The spread is required to induce saving under debt overhang. Since households are risk neutral, their maximization program would be linear without debt overhang. Debt overhang makes it convex which leads households to choose corner solutions: either $s = -(\bar{\rho}_b + \bar{e})/r$ or $s = y_1$. Households with debt above \hat{m} consume as much as possible at date 1 and default at date 2. Households with debt below \hat{m} save all their income (as long as $\delta \geq r$) and repay their mortgages in full at date 2.

Debt overhang induces households to make inefficient decisions. In the simple stylized model presented here, they consume more at date 1. In reality, they could also decrease investment in maintenance of their homes, decrease job searching efforts, increase time spent avoiding repayments, etc. The robust effect is that they decrease investment in assets and activities that can be appropriated by their creditors, and they increase consumptions and activities that cannot be taken away from them in case of personal bankruptcy.

3.3 Borrowing and investment

At time 1, the investment decision is made to maximize shareholder value, taking as given the initial outstanding liabilities b . For any bank, we can use the participation constraint of new investors $E[\rho_n] = rx$ to

write shareholder value at time 1 as:

$$E_1 [e|x] = z + \bar{\rho}_m + (q - r)x - \min(b, z + qx + \bar{\rho}_m). \quad (8)$$

Equation (8) says that the returns to investing are the NPV of the project $(q - r)x$ – as in the first best economy – minus the transfers to existing bond holders. Notice that this equation would be exactly the same if the bank issued equity instead of junior debt at time 1 for a value exactly equal to x . The key point is that shareholders end up paying the full cost of investment but only receive part of the return. We therefore have the following lemma:

Lemma 2. *The investment function of a bank with debt level b is*

$$x(b) = X \cdot 1_{b \leq \hat{b}} \cdot 1_{r \leq q}, \quad (9)$$

where the cutoff for investment is given by

$$\hat{b} \equiv z + \bar{\rho}_m + (q - r)X. \quad (10)$$

Proof. Investment takes place if and only if it is beneficial to shareholders: $E_1 [e|x = X] > E_1 [e|x = 0]$. This condition is simply

$$(q - r)X > \min(b, z + qX + \bar{\rho}_m) - \min(b, z + \bar{\rho}_m)$$

A bank that is always solvent ($b \leq z + \bar{\rho}_m$) will invest if and only if $q > r$. A bank that is not solvent even after investing ($z + \bar{\rho}_m + qX < b$) will never invest. The marginal bank is indifferent between investing and not investing when $(q - r)X = b - z - \bar{\rho}_m$. This defines the debt threshold for investment \hat{b} . \square

We saw earlier how debt overhang led households to make inefficient savings decisions. In the case of banks, we see that debt overhang leads to under-investment in new projects at time 1. Banks with debt above \hat{b} do not invest even though the projects have positive net present value.

3.4 Equilibrium

To understand the macroeconomic equilibrium conditions, we need to bring together the balance sheets of banks and individuals. An important relation comes from the aggregate balance sheets of banks. We know that $\bar{\rho}_b + \bar{e} = z + q\bar{x} + \bar{\rho}_m - \bar{\rho}_n$ and $\bar{\rho}_n = r\bar{x}$, therefore:

$$\bar{\rho}_b + \bar{e} = z + (q - r)\bar{x} + \bar{\rho}_m. \quad (11)$$

Equation (11) says that total payments to claim holders (shareholders and bond holders) equal total revenues of the banks. These revenues are made of income from initial assets, net returns from new projects, and aggregate mortgage payments from households. The important point here is that mortgage payments from households pass through the banks back to households. We can now present the equilibrium conditions of the benchmark model.

Proposition 1. *The equilibrium conditions of the debt overhang economy are the flow of payments from households to banks:*

$$\bar{\rho}_m(\hat{m}) = \int_0^{\hat{m}} m dF(m), \quad (12)$$

the individual solvency threshold:

$$\hat{m} = (r - \delta) \left(y_1 + \frac{z + (q - r)\bar{x} + \bar{\rho}_m(\hat{m})}{r} \right), \quad (13)$$

the aggregate savings condition (for $r > \delta$):

$$\bar{x} = y_1 F(\hat{m}) - \frac{z + (q - r)\bar{x} + \bar{\rho}_m(\hat{m})}{r} (1 - F(\hat{m})), \quad (14)$$

and the aggregate investment condition (for $r < q$):

$$\bar{x} = XG(z + \bar{\rho}_m(\hat{m}) + (q - r)X). \quad (15)$$

Proof. The repayments of a household with debt level m to its bank is m if $m < \hat{m}$ and 0 otherwise. In the

aggregate, we therefore get (12). Using (11) we can rewrite (7) as (13). If we aggregate individual savings (6), we get $S = y_1 F(\hat{m}) - \frac{\bar{\rho}_m + \bar{\epsilon}}{r} (1 - F(\hat{m}))$. Using (11) and $S = \bar{x}$, we then obtain (14). Finally, if we aggregate investment decisions (9), we get (15). \square

The equilibrium conditions involve four equations in four unknowns: mortgage repayments $\bar{\rho}_m$, households' solvency threshold \hat{m} , aggregate investment \bar{x} , and interest rate r . To gain intuition about the properties of the debt overhang economy, it is useful to examine subsets of these equations.

Let us first consider the mortgage repayment system defined by the first two equilibrium conditions, (12) and (13), holding \bar{x} and r constant. Equation (12) says that the aggregate flow of payments from individuals to banks $\bar{\rho}_m$ is increasing in the default threshold \hat{m} . Equation (13) comes from the optimal savings behavior of an individual, given her expectations about future income from intermediaries $q\bar{x} + \bar{\rho}_m$ and given excess returns on savings, $(r - \delta)y_1$. The higher these are the more the individual is willing to save. In this schedule, \hat{m} is therefore increasing in $\bar{\rho}_m$. Formally this means that there are strategic complementarities in savings/repayment decisions since a higher fraction of savers increases the return to savings.

Lemma 3. *Debt overhang creates strategic complementarities in mortgage repayment decisions. The system is unstable if $(1 - \delta/r)\hat{m}f'(\hat{m}) > 1$. When it is stable, we have \hat{m} and $\bar{\rho}_m$, increasing in r and in \bar{x} .*

Proof. From equation (12), we know that repayments $\bar{\rho}_m$ are increasing in \hat{m} : $\frac{\partial \bar{\rho}_m}{\partial \hat{m}} = \hat{m}f'(\hat{m})$. From equation (13) we know that \hat{m} is increasing in $\bar{\rho}_m$: $\frac{\partial \hat{m}}{\partial \bar{\rho}_m} = 1 - \delta/r$. The system is unstable if $\frac{\partial \bar{\rho}_m}{\partial \hat{m}} \frac{\partial \hat{m}}{\partial \bar{\rho}_m} > 1$. \square

The intuition is simple. Individuals eventually receive the income of intermediaries. So if $\bar{\rho}_m$ increases, banks receive more, and so do individuals, because they hold the bonds and stocks of intermediaries. These are the first complementarities in the model. In the remaining of the paper I assume that the mortgage repayment system is stable: $(1 - \delta/r)\hat{m}f'(\hat{m}) < 1$. In this case we get \hat{m} and $\bar{\rho}_m$, increasing in r and in \bar{x} . Figure 2 presents the cycle of mortgage payments.

Let us now turn to the aggregate savings curve (14). It says that in equilibrium the savings of solvent households must finance investment plus the net dis-savings of insolvent agents. Along the savings curve, \hat{m} must increase in \bar{x} : more capital spending requires more agents to save. A higher interest rate, on the other hand, decreases the NPV of the insolvent agent portfolios of liquid claims, so fewer agents need to save to finance the same \bar{x} , and we would expect \hat{m} to decrease with r .

Consider finally aggregate investment demand represented by equation (15). Since from the savings system we have $\partial \hat{m} / \partial \bar{x} > 0$, we see that both the LHS and the RHS are increasing in \bar{x} . The intuition is the following. When more banks finance new projects, economic value is created. This value eventually trickles down to households, who are thus less likely to default on their loans. This makes banks more solvent, and therefore more willing to finance new investment. Figure 3 illustrates the complementarities in investment decisions.

To emphasize the role of complementarities, we can look at conditions under which there is an equilibrium without any investment. Let us define:

$$\hat{m}_q \equiv (q - \delta) \left(y_1 + \frac{z + \bar{\rho}_m(\hat{m}_q)}{q} \right) \quad (16)$$

We can state a general property about the equilibrium interest rate.

Lemma 4. *All equilibria must have $r > \delta$. If $qy_1F(\hat{m}_q) \leq (z + \bar{\rho}_m(\hat{m}_q))(1 - F(\hat{m}_q))$, then there is an equilibrium with $r \geq q$ and no investment. Otherwise all equilibria have strictly positive investment and $r \in (\delta, q)$.*

Proof. Suppose $r \leq \delta$. Then $\hat{m} \leq 0$, $F(\hat{m}) = 0$, and savings are negative. This is impossible therefore $r > \delta$ in equilibrium. At the other extreme if $r = q$ and $\bar{x} = 0$, the savings equilibrium requires $y_1F(\hat{m}) = \frac{z + \bar{\rho}_m}{r}(1 - F(\hat{m}))$. We can solve for \hat{m}_q defined above. If at $r = q$ we have $y_1F(\hat{m}_q) > \frac{z + \bar{\rho}_m}{q}(1 - F(\hat{m}_q))$ then there is no equilibrium with $r \geq q$. On the other hand, if $y_1F(\hat{m}_q) \leq \frac{z + \bar{\rho}_m}{q}(1 - F(\hat{m}_q))$, then there is a unique equilibrium with $r \geq q$. This equilibrium has no investment. \square

4 Government Interventions in a Closed Economy

If the dead weight losses from debt overhang and bankruptcy is high, there might be room for an intervention by the government. The government can alleviate the debt overhang problem by providing capital to the banks, or by helping insolvent households. It is important to emphasize that I do not consider the ex-ante implications of government bailouts. This is important to the extent that bailouts create moral hazard, as analyzed by Farhi and Tirole (2009) among others. Here I want to emphasize that nothing prevents the

government from firing the managers of the banks. The only assumption I am making is that the government does not repudiate private contracts. Otherwise the government could simply wipe out the shareholders, declare all debt contracts void and replace them with new equity contracts. The focus of this paper is not on the long list of reasons why this policy is not often used in practice, but I will point out that such policies are much more attractive in a small open economy than in a large closed economy.

4.1 Recapitalizing the banks

The government has several ways to bail out the financial system. In an equity injection, the government provides cash to the banks, and asks for equity in exchange. In a debt guarantee program, the government offers to guarantee the new debt issued by banks. The banks can then borrow at the risk free rate. In an asset buyback program, the government offers to buy back the toxic assets from the banks above market price. Philippon and Schnabl (2009) show that, absent private information on the side of banks, all these programs are equivalent. For simplicity, I focus here on a pure cash transfer and I abstract from dead weight losses from taxation.

The government taxes the endowment at time 1 to raise an amount τ . It can then transfer this amount to the banks. After the transfer, each bank must borrow only $X - \tau$ from investors. These investors require expected returns $r(X - \tau)$. If the bank is always solvent with the new cash τ , i.e. if $z + \bar{\rho}_m + r\tau > b$, then the bank invests. The marginal bank must be insolvent if it does not invest but solvent if it does invest. Conditional on investment, shareholders receive $e = z + \bar{\rho}_m + qX - r(X - \tau) - b$. The investment condition therefore becomes

$$b < \hat{b} = z + \bar{\rho}_m + (q - r)X + r\tau. \quad (17)$$

Banks that do not invest can lend the cash τ , and for these banks we have $\rho_b + e = z + \bar{\rho}_m + r\tau$. If we aggregate across all banks, the flow payments equation (11) becomes

$$\bar{\rho}_b + \bar{e} = z + \bar{\rho}_m + r\tau + (q - r)\bar{x}. \quad (18)$$

Let us now consider the response of households to the new taxation. Equation (12) is unchanged while

equation (7) becomes

$$\hat{m} = (r - \delta) \left(y_1 - \tau + \frac{\bar{\rho}_b + \bar{e}}{r} \right). \quad (19)$$

From a partial equilibrium point of view taxing the households lowers \hat{m} and increases the default rate on mortgages. From a macroeconomic perspective, however, things are very different. Using the new flow equation (18), we see that equation (13) does not change

$$\hat{m} = (r - \delta) \left(y_1 - \tau + \frac{z + \bar{\rho}_m + r\tau + (q - r)\bar{x}}{r} \right) = (r - \delta) \left(y_1 + \frac{z + \bar{\rho}_m + (q - r)\bar{x}}{r} \right)$$

The key point here is that taxes taken from households at time 1 are given back in the form of higher dividends and interest payments at time 2. In equilibrium, the two effects exactly cancel out and the equilibrium cutoff function does not depend directly on taxes. The aggregate savings curve (14) is also unchanged because household savings decrease by τ while bank savings increase by τ . Finally, the aggregate investment curve (15) becomes

$$\bar{x} = XG(z + \bar{\rho}_m + (q - r)X + r\tau).$$

To summarize, we have shown that equations (12), (13), and (14) are unchanged, while the schedule of aggregate investment curve (15) moves up. Hence, the direct effect of banks bailouts is to increase investment. In equilibrium, the increase in investment then leads to an increase in aggregate income and a decrease in the default rate on mortgages. We can therefore state the following proposition.

Proposition 2. *The recapitalization of banks improves welfare in the debt overhang economy. Their direct effect is to increase investment. In equilibrium, the increase in investment leads to a lower default rate on mortgages.*

As before, much intuition is gained by looking at the small open economy case. In this case it is easy to see that the cutoff changes according to

$$\left(\frac{r}{r - \delta} - (1 + \hat{\gamma}) \hat{m} f(\hat{m}) \right) d\hat{m} = \hat{\gamma} r d\tau,$$

with $\hat{\gamma} = (q - r) Xg(\hat{b})$. This equation shows that the fraction of insolvent households decrease with the

financial bailout. The effect is stronger when the density of banks around the investment cutoff and the density of households around the default cutoff are large. Similarly, aggregate investment increases by

$$d\bar{x} = Xg\left(\hat{b}\right) (\hat{m}f(\hat{m})d\hat{m} + rd\tau).$$

So investment goes up for two reasons: the direct impact on banks' liquidity, and the indirect impact on the performance of their mortgages. Without dead weight losses from taxation, it is optimal to increase taxes until the debt overhang problem is entirely alleviated. In the more realistic case where taxes create distortions the bailout should stop short of achieving the first best. We will see in Section 5 how financial globalization changes this outcome even without dead weight losses from taxes.

4.2 Households bailouts

Consider the simple model with two groups of households. A fraction $1 - \pi$ of households have no mortgages and a fraction π have mortgage $m = M$. Households without mortgages never default, and we want to characterize equilibria where the default rate among indebted households is η . In terms of our earlier notations, this simply means that $F(\hat{m}) = 1 - \pi + (1 - \eta)\pi = 1 - \eta\pi$. The aggregate mortgage payments from households to banks are therefore $\bar{\rho}_m = (1 - \pi) \times 0 + \pi(1 - \eta) \times M = \pi(1 - \eta)M$. If the default rate η is between 0 and 1, indebted households must be indifferent between defaulting and paying. In any such equilibrium, we must have $\hat{m} = M$. On the investment side we have the same equation as before.

Let us now analyze the possible equilibria in this economy. The key endogenous variable is η . Consider first a good equilibrium where there are no defaults: $\eta = 0$, $\bar{\rho}_m = \pi M$. The investment curve is $\bar{x} = XG(z + \pi M + (q - r)X)$. This can be an equilibrium if and only if it is indeed the case that $M \leq \hat{m}$:

$$M \leq (r - \delta) \left(y_1 + \frac{z + (q - r)XG(z + \pi M + (q - r)X) + \pi M}{r} \right)$$

Note that this equilibrium can only happen in the open economy because it implies excess savings since $y > X$. This equilibrium requires that the economy runs a surplus. Consider now the polar opposite equilibrium, where all indebted agents default: $\eta = 1$ and $\bar{\rho}_m = 0$. The investment curve is $\bar{x} = XG(z + (q - r)X)$. This

is indeed an equilibrium with $\eta = 1$ if and only if $M > \hat{m}$:

$$M > (r - \delta) \left(y_1 + \frac{z + (q - r) X G(z + (q - r) X)}{r} \right)$$

Combining the previous two conditions, we see that multiple equilibria are possible if the shift in the fraction of banks that decide to invest $G(z + \pi M + (q - r) X) - G(z + (q - r) X)$ is large enough.

In the context of a closed economy, imagine that the government redistributed resources from solvent to insolvent households in an attempt to lower the default rate on mortgages. The government levies a per-capita tax τ on the households with no debt and transfers τ' it to the households with debt M . The budget constraint requires $(1 - \pi)\tau = \pi\tau'$. The default threshold becomes

$$\hat{m} = (r - \delta) \left(y_1 + \tau' + \frac{z + (q - r) \bar{x} + \bar{\rho}_m}{r} \right) \quad (20)$$

The savings curve becomes

$$\bar{x} = (1 - \eta\pi) y_1 - \eta\pi \left(\frac{z + (q - r) \bar{x} + \bar{\rho}_m}{r} + \tau' \right)$$

The investment curve remain the same. We have seen in the previous section that taxing households to redistribute money to banks always led to an improvement in welfare. One could imagine that a similar proposition would hold for household bailouts. It turns out that this is not true in general.

Proposition 3. *In a closed economy, households' bailouts can backfire in the sense that they can lead to an equilibrium with lower investment and a higher default rate.*

Proof. Consider an equilibrium with $\eta = 0$ and $r = q$. All indebted households default if and only if $\hat{m}_q < M$ where $\hat{m}_q = (q - \delta) \left(y_1 + \tau' + \frac{z}{q} \right)$. Without investment, the savings of solvent households is less than the dis-saving of insolvent ones if $(1 - \pi) y_1 \leq \pi \left(\frac{z}{q} + \tau' \right)$. The condition for $\eta = 0$ and $r = q$ to be an equilibrium is therefore

$$\frac{y_1}{\pi} \leq y_1 + \tau' + \frac{z}{q} \leq \frac{M}{q - \delta}$$

This condition might be violated when $\tau' = 0$ but satisfied when $\tau' > 0$, for instance when $(1 - \pi) y_1 >$

$\pi z/q$.

□

The idea of the proof is simply to check the conditions under which conditions an inefficient equilibrium can happen. We therefore consider a candidate bad equilibrium with high interest rate, maximum default, and no investment. There are two conditions to check. One is that all indebted households indeed default. Transfers to indebted households improve the solvency threshold in equation (20). This make the condition for a bad equilibrium less likely to hold. This is the intended consequence of the intervention.

There is however a second condition, involving the savings curve. In the bad equilibrium there is no investment because the rate is $r = q$. This is inconsistent with the aggregate supply of savings when $(1 - \pi) y_1 > \pi z/q$ because solvent households would then generate strictly positive net savings. This rules out a total collapse of investment in the decentralized equilibrium. With the intervention, however, the savings curve goes down because the government transfers money from savers to insolvent households with excessive consumption. If the transfer is such that $\pi z/q + \pi \tau' > (1 - \pi) y_1$ then the bad outcome becomes an equilibrium because of the intervention.

Bailing out insolvent households therefore has two opposite effects. On the one hand, it improves the performance of mortgages that were not excessively under water to start with. On the other hand, it crowds out savings by transferring resources away from solvent households. Whether such an intervention helps or hurts the economy therefore depends on which effect is stronger.

5 Recapitalization in Financially Integrated Economies

5.1 Decentralized Equilibrium

In this section I study bailout in an economy where financial markets are integrated. Consider a world made of identical small open economies. Households own domestic and foreign stocks and bonds. Let α be the share of foreign assets in domestic households' portfolios. Equation (12) is unchanged but the solvency threshold now depends on foreign income (as usual I use * to denote foreign variables)

$$\hat{m} = (r - \delta) \left(y_1 + (1 - \alpha) \frac{\bar{p}_b + \bar{e}}{r} + \alpha \frac{\bar{p}_b^* + \bar{e}^*}{r} \right).$$

Banks own domestic and foreign mortgages. Let β be the share of foreign mortgages in domestic banks' portfolios. The aggregate flow payments to by banks to their shareholders and bondholders (both foreign and domestic) is therefore

$$\bar{\rho}_b + \bar{e} = z + (1 - \beta) \bar{\rho}_m + \beta \bar{\rho}_m^* + (q - r) \bar{x}.$$

Finally, the investment demand curve becomes

$$\bar{x} = XG(z + (1 - \beta) \bar{\rho}_m + \beta \bar{\rho}_m^* + (q - r) X).$$

If we aggregate the economies and we consider symmetric equilibria, it is clear that the equilibrium conditions for the global economy are identical to the equilibrium condition for the closed economy of analyzed in Section 3. Figure 4 illustrates the open economy equilibrium.

5.2 Bank Recapitalization

Consider now the financial bailouts of Section 4 in the context of financially integrated economies. For the world economy it is clear that Proposition 2 applies. Global bailouts improve efficiency by increasing global investment and lowering global default rates. Global bailouts are not feasible however, because their require government to levy taxes on foreign individuals and redistribute the money to domestic banks.

The relevant question is therefore whether an individual country would find it in its own interest to bail out its financial system by taxing its own citizens. This is the issue that I now analyze. The country is small and does not take into account the impact of its decision on the foreign payouts $\bar{\rho}_b^* + \bar{e}^*$. The government taxes the endowment at time 1 and raises an amount τ . It can then transfer this amount to the domestic banks. With the transfer, the solvency threshold becomes $\hat{m} = (r - \delta) \left(y_1 - \tau + (1 - \alpha) \frac{\bar{\rho}_b + \bar{e}}{r} + \alpha \frac{\bar{\rho}_b^* + \bar{e}^*}{r} \right)$, the investment cutoff becomes $\hat{b} = z + (1 - \beta) \bar{\rho}_m + \beta \bar{\rho}_m^* + (q - r) X + r\tau$ and the flow payments equation (11) becomes $\bar{\rho}_b + \bar{e} = z + (1 - \beta) \bar{\rho}_m + \beta \bar{\rho}_m^* + r\tau + (q - r) \bar{x}$. These three equations are similar to the ones obtained in the closed economy.

The crucial difference between the closed-economy bailout and the open-economy bailout come entirely

from the macroeconomic feed backs.. Putting the pieces together, we get the solvency threshold equation

$$\frac{r}{r-\delta}\hat{m} = ry_1 - \alpha r\tau + (1-\alpha)(z + (1-\beta)\bar{\rho}_m + \beta\bar{\rho}_m^* + (q-r)\bar{x}) + \alpha(\bar{\rho}_b^* + \bar{e}^*).$$

Note that as long as α is positive, the transfer τ appears in the solvency condition. The taxes are levied on domestic households and given to domestic banks, but some of the shareholders of these banks are foreign. The domestic households therefore do not recover the full value of their taxes. Similarly, the investment condition becomes

$$\bar{x} = XG(z + (1-\beta)\bar{\rho}_m + \beta\bar{\rho}_m^* + (q-r)X + r\tau).$$

Any improvement in domestic mortgage payments is scaled down by $1-\beta$ because a fraction β of all mortgages are held by foreign banks. If we differentiate the system,⁴ we get

$$\left(\frac{r}{r-\delta} - (1-\alpha)(1-\beta)(1+\hat{\gamma})\hat{m}f(\hat{m})\right)d\hat{m} = ((1-\alpha)\hat{\gamma} - \alpha)r d\tau.$$

with $\hat{\gamma} = (q-r)Xg(\hat{b})$. This equation makes it clear that bailouts are less efficient in an open economy. The multiplier is reduced by both α and β . When α is large enough that $(1-\alpha)\hat{\gamma} - \alpha < 0$ the intervention backfires and mortgage defaults increase. This also has a negative impact on investment.

Proposition 4. *Banks recapitalizations are less efficient when banks operate more internationally and when households hold more foreign assets. A domestic recapitalization will backfire and lead to more defaults on mortgages when households hold diversified portfolios.*

This last proposition highlights the problems created by financial integration. Since the global economy is equivalent to the closed economy analyzed in Section 4, we know that a sequence of coordinated, identical domestic bailouts would always improve efficiency. It is clear however that it might not be in any individual country's interest to intervene alone. This problem is the financial equivalent of the fiscal policy dilemma in open economies. Government spending in an open economy generates a trade deficit and has a smaller effect on output than in a closed economy. The deficit and the smaller multiplier are due to the fact that

⁴ $(r/(r-\delta) - (1-\alpha)(1-\beta)\hat{m}f(\hat{m}))d\hat{m} = (1-\alpha)(q-r)d\bar{x} - \alpha rd\tau$ and $d\bar{x} = Xg(\hat{b})((1-\beta)\hat{m}f(\hat{m})d\hat{m} + r d\tau)$

the increase in demand now falls not only on domestic goods but also on foreign goods. Here I obtain a similar effect with bailouts. It is clear that countries have an incentive to free-ride on foreign recapitalization programs, so a corollary of Proposition is that efficient financial stabilization is likely to require international coordination.

Corollary. *Financial globalization creates the need for coordination in recapitalization programs.*

6 Conclusion

I have presented a simple model where debt overhang occurs simultaneously in two markets. Households' mortgages can be under-water, and this can lead them to take inefficient actions to avoid transferring wealth to their creditors. Banks' bonds can be under-water, and this can lead them to pass on positive NPV projects. The two problems interact in equilibrium in a way that amplifies shocks and can lead to multiple equilibria.

When the economy is in a debt overhang equilibrium, bailing out banks improves economic efficiency, while bailing out indebted households can backfire. In open economies, I find that bailouts might require coordination among countries, since it is possible that no country would chose to bail out its financial system, even though coordinated bailouts would be globally efficient.

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Fig 2: Mortgage Performance Cycle

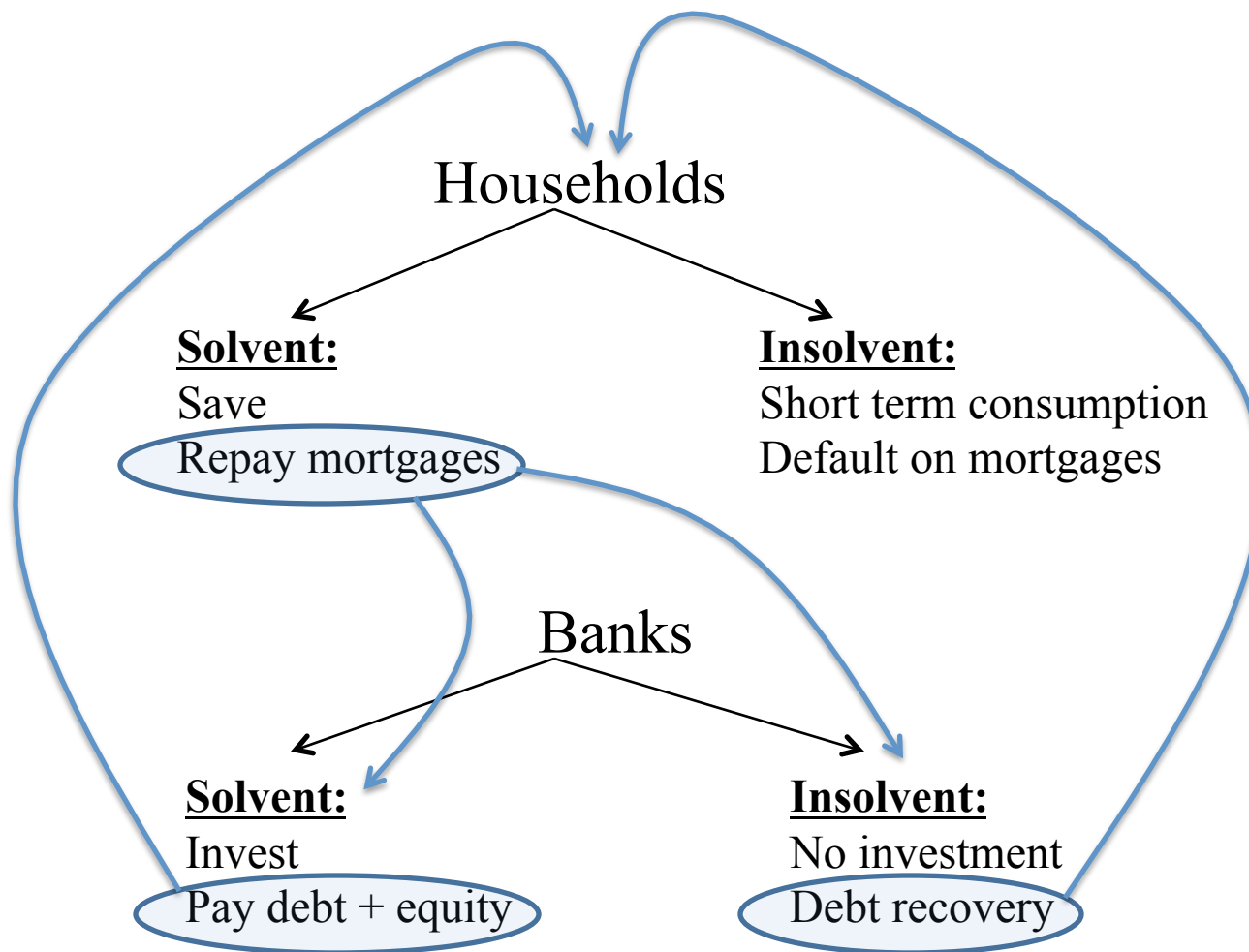


Fig 3: Complementarities in Investment

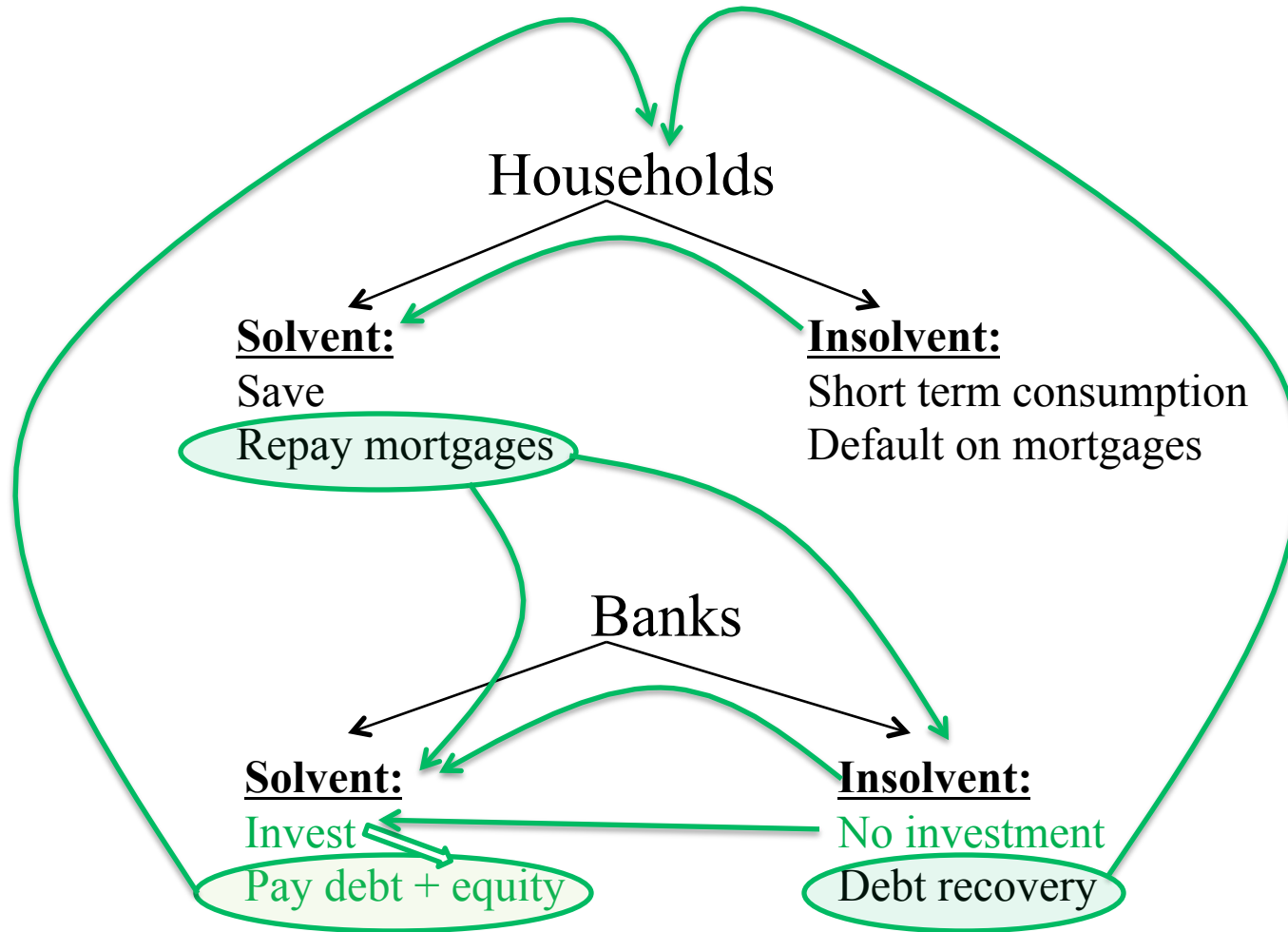


Fig 4: Open Economy

