Fiscal Policy and the Term Structure of Interest Rates*

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Abstract

Macroeconomists want to understand the effects of fiscal policy on interest rates, while financial economists look for the factors that drive the dynamics of the yield curve. To shed light on both issues, we present an empirical macro-finance model that combines a no-arbitrage affine term structure model with a set of structural restrictions that allow us to identify fiscal policy shocks, and trace the effects of these shocks on the prices of bonds of different maturities. Compared to a standard VAR, this approach has the advantage of incorporating the information embedded in a large cross-section of bond prices. Moreover, the pricing equations provide new ways to assess the model's ability to capture risk preferences and expectations. Our results suggest that government deficits affect long term interest rates, at least temporarily: (i) a one percentage point increase in the deficit to GDP ratio increases the 10-year rate by 35 basis points after 3 years; (ii) this increase is partly due to higher expected spot rates, and partly due to higher risk premia on long term bonds; and (iii) the fiscal policy shocks account for up to 13% of the variance of forecast errors in bond yields.

JEL: E0, G0

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Introduction

Empirical macroeconomic research has not been able to establish if and how government deficits affect interest rates. Yet, this is an important issue for policy making and for academic research. For instance, Evans (1987), Plosser (1987) and Engen and Hubbard (2004) find small or insignificant responses of interest rates, while Laubach (2003) and Gale and Orszag (2003) find significant responses. Evans and Marshall (2002) find different responses of interest rates depending on how they identify fiscal shocks. One reason for this lack of success is that macroeconomists have not fully incorporated long term interest rates into their empirical models, and have mainly relied on simple least-squares estimates.[1] The common feature of the papers in the existing literature is that they do not model the kernel that prices long term bonds.

On the other hand, recent theoretical and empirical research in finance has led to a better understanding of the dynamic properties of the term structure of interest rates: The models are parsimonious, financially coherent, and are able to capture some important stylized facts.[2] Most of these bond pricing models, however, are based on unobserved risk factors which are not easy to interpret. An important task, started by Ang and Piazzesi (2003), is therefore to draw explicit connections between the latent risk factors that drive the dynamics of the term structure and observable macroeconomic variables that characterize the state of the economy. Following Piazzesi (2005), much recent research has focused on monetary policy, and in particular on how the monetary authority responds to inflation and business cycle shocks.[3] This research brings the bond pricing literature closer to traditional monetary economics, and to the work of Taylor (1993) and McCallum (1994) in particular. Our goal is to do the same with the analysis of fiscal policy.

We develop a dynamic term structure model to study the impact of fiscal policy

[1] Elmendorf and Mankiw (1999), Gale and Orszag (2003) and Engen and Hubbard (2004) provide recent reviews of the literature. See Barro (1987) for an earlier discussion. Canzoneri, Cumby, and Diba (2002) and Laubach (2003) use projected deficits, and Evans and Marshall (2002) study the response of the yield curve to a range of macroeconomic shocks

[2] Dai and Singleton (2003) survey the bond pricing literature.

[3] See Gallmeyer, Hollifield, and Zin (2004) and Hördahl, Tristani, and Vestin (2003), among others.

on interest rates. Our affine model combines observable macroeconomic variables with one latent factor. We identify fiscal policy shocks using the macroeconomic restrictions, and we examine the impact of policy shocks on the yield curve.

Our work contributes to the macroeconomic and empirical finance literature in two ways. First, we introduce a fiscal policy variable into a no-arbitrage dynamic term structure model, and we show that the government deficit is a factor for long term interest rates. It has been known at least since Taylor (1993) that there is enough information in inflation and the output gap to account for changes in the short term interest rate. Ang and Piazzesi (2003) show that these same macroeconomic factors do not capture fully the dynamics of long term rates. We show that fiscal policy can account for some of the unexplained long rate dynamics. In other words, we find that in order to price long term bonds correctly, it is important to take into account the fiscal position of the government, above and beyond inflation and real activity. This result is independent of the restrictions that we later introduce to identify fiscal shocks.

Second, we introduce macroeconomic restrictions in a dynamic term structure model. Research in finance has focused on finding a kernel that can price various bonds, but it has not tried to *identify* the economic shocks driving the kernel. To do so, one must impose theoretically motivated restrictions on the covariance matrix of reduced form shocks. We borrow the identification strategy of Blanchard and Perotti (2002) to estimate fiscal *policy* shocks.[4] We find that a fiscal shock that increases the deficit to GDP ratio by 1% leads to 35 basis points increase in the 10-year interest rate after three years. When we decompose this increase into risk premia and expected future short rates, we find that the risk premia explain more than one third of the increase in long term interest rates. Finally, we decompose deficit changes into public spending and taxes, and we find that taxes matter independently from spending.

We also argue that bond pricing equations can provide useful over-identifying restrictions to empirical macroeconomic models. The number of variables one can

[4] The Blanchard and Perotti (2002) identification is the easiest one to introduce in our dynamic pricing model. We discuss the differences with the Ramey and Shapiro (1997) approach in the last section of the paper.

include in a VAR is limited, but how can we be sure that a small state space is actually able to capture technology, preferences and the relevant information sets of economic agents? We show how one can use bond prices to address this key issue. Bond prices are observable and bond returns are predictable.[5] Empirical models should be able to price bonds and predict returns. A failure to do so means that the model does not capture risk aversion, or expectations, or both. These ideas guide our preliminary analysis, and in particular our choice of the variables to be included in the state space. We then conduct a maximum likelihood estimation of the model that incorporate all of the over-identifying restrictions offered by bond prices and returns. We find that a model with four observable macroeconomic variables (federal funds rate, inflation, deficit, real activity) and one latent factor can price bonds, capture return predictability and explain the deviations from the expectations hypothesis.

The rest of the paper is organized as follows. Section 1 introduces the bond pricing model. In, Section 2, we conduct a preliminary analysis of the data using a set of excess returns and yield regressions, and we argue that these regressions can help us choose the state space of the model. Section 3 presents the estimation of the macro-finance model by maximum likelihood. Section 4 discusses identification and presents the impulse responses to fiscal policy shocks. Section 5 contains various robustness checks and a detailed comparison of our results with results from existing macroeconomic studies. In particular, we discuss the extent to which our model can reconcile the conflicting results found in the literature.

1 The Affine Pricing Model

We begin with a description of the main features of the discrete-time dynamic term structure model. Technical details can be found in the appendices. We assume that the state vector y_t follows a Vector Autoregressive process[6] of finite order $L+1$, $y_t = \phi_0 + \sum_{l=1}^{L+1} \phi_l y_{t-l} + u_t$. We defer the discussion of which variables should be included in the state space to section 2. By expanding the state space to the companion form

[5] See Cochrane and Piazzesi (2005) for recent results.

[6] The observation interval is arbitrary at this stage, and is quarterly in the empirical implementation.

$Y_t = [y_t \; .. \; y_{t-L}]$, we can rewrite the state dynamics in the more convenient VAR(1) form (after normalizing the unconditional mean to 0):

$$Y_t = \Phi Y_{t-1} + U_t, \tag{1}$$

where the shocks $U_t = [u_t; 0]$ are jointly normally distributed with constant covariance matrix $\Omega = E\left[U_t U_t'\right]$. To price the government bonds, we assume that the pricing kernel takes the form:

$$\frac{M_{t+1}}{M_t} = \exp\left(-r_t - \frac{\Lambda_t'\Omega\Lambda_t}{2} - \Lambda_t' U_{t+1}\right), \tag{2}$$

where the vector of market prices of risk is given by

$$\Lambda_t = \Omega^{-1}\left(\Lambda_0 + \Lambda Y_t\right), \tag{3}$$

and the short rate (1-quarter) is given by

$$r_t = \delta_0 + \delta' Y_t. \tag{4}$$

We assume that the government will never default on its nominal obligations. Real defaults are possible through high inflation, however. We believe that these are sensible assumptions for the US in the post-war period. By definition of the pricing kernel, the price of a n-period zero-coupon default-free bond at time t must satisfy:

$$P_t^n = E_t\left[\frac{M_{t+1}}{M_t} P_{t+1}^{n-1}\right].$$

In an affine setup, one can easily show that bond prices are given by

$$P_t^n = \exp\left(-A_n - B_n' Y_t\right), \tag{5}$$

where A_n and B_n solve the recursive equations

$$\begin{aligned} A_n &= \delta_0 + A_{n-1} - B_{n-1}'\Lambda_0 - \frac{B_{n-1}'\Omega B_{n-1}}{2}, \\ B_n &= \delta + \left(\Phi - \Lambda\right)' B_{n-1}, \end{aligned} \tag{6}$$

with initial conditions $A_0 = B_0 = 0$. Clearly, $A_1 = \delta_0$, and $B_1 = \delta$.

1.1 Relation to Existing Work on the Term Structure

Equations (1), (2), (3), and (4) constitute a full-fledged term structure model, which belongs to the class of affine term structure models (see, e.g., Duffie and Kan (1996), Dai and Singleton (2000), Dai and Singleton (2002), and Duffee (2002)). In section 2, we will argue that in order to give a reasonable description of both the economic environment and the term structure dynamics, the state vector y_t should include the federal fund rate (f_t), the logarithm of spending over taxes (d_t), the log growth rate of the GDP deflator (π_t), the help wanted index (h_t) [7], and a latent variable. Existing works that are most closely related to our model are Ang and Piazzesi (2003) and Hördahl, Tristani, and Vestin (2003). Ang and Piazzesi (2003) use a no-arbitrage VAR where the maintained assumption is that latent factors (which presumably include monetary and fiscal policies) do not affect output or inflation. Their model is most useful to understand how much of the dynamics of the yield curve can be accounted for by inflation and real activity, but it is not suitable for identifying the effects of monetary and fiscal policies.

In contrast, Hördahl, Tristani, and Vestin (2003) start from a simple textbook model of the macro-economy, with a price setting equation for firms, and a linearized Euler equation for consumption and output. We do not follow this strategy, for two reasons. First, while the price setting equation that governs the inflation process in the textbook model appears to be quite reasonable (see, e.g., Gali and Gertler (1999)), the Euler equation, that supposedly links aggregate dynamics to asset prices suffers from known failures (the most well-known being the equity premium puzzle or the risk-free rate puzzle). Indeed, for the purpose of pricing bonds, Hördahl, Tristani, and Vestin (2003) posit a reduced-form pricing kernel on top of the marginal rate of substitution underlying the Euler equation. Second, to examine the effects of fiscal policy shocks on the term structure of interest rates, it would be necessary to introduce the fiscal variables into either the pricing equation, or the aggregate demand equation, or both. There is hardly any consensus in the macro literature on how this should be achieved.

[7] This is an index of help wanted advertising in newspapers, available on FRED® II (Federal Reserve Economic Data).

2 Choosing the State Space

We now turn to the choice of the variables to be included in the state space. As one can see from equation (5), the affine model predicts linear relations between the components of the state space and bond yields. As it turns out, the same is true for bond returns. One can therefore use simple linear regressions to assess the performance of different candidate variables for the state space. We emphasize that this is *not* our main contribution. We will later perform a full estimation of the pricing model using maximum-likelihood based on the Kalman Filter. The simple regressions presented here are only meant to guide us in our choice of a state space.

We focus on two sets of linear regressions: yield regressions that relate changes in the yield levels to (contemporaneous) changes in the state vector; and excess return regressions that relate the predictable component of bond returns to the current state of the economy. The idea here is simply that, if the state space is correctly specified, it should be able to explain bond prices as well as bond returns through the reduced-form bond pricing equations implied by the model. If the proposed state space fails this test, there is no point going further. If the proposed state space passes the test, then it makes sense to estimate a term structure model that imposes cross-sectional restrictions (which arise from the no-arbitrage assumption) on the regression coefficients. We will discuss the nature of these restrictions after we have reviewed the results from the linear regressions.

Our state space will include quarterly time-series observations of the federal funds rate, the quarterly growth rate of the GDP deflator and the help wanted index. Robustness checks will be made using non durable consumption, GDP and the Stock and Watson index of leading indicators. For spending and taxes, we follow Blanchard and Perotti (2002). Nominal spending G_t is defined as the purchase of goods and services by federal, state and local governments. Nominal net taxes T_t are defined as taxes minus transfers. All the details about the construction of these variables are in appendix A. In our analysis, we always use the log of real variables. Thus, we define

$$g_t = \frac{1}{10} \log \left(\frac{G_t}{P_t} \right) ,$$

for spending and

$$\tau_t = \frac{1}{10} \log \left(\frac{T_t}{P_t} \right) ,$$

for net taxes, where P_t is the GDP deflator. We divide the variables by 10 for convenience when we plot impulse responses. Finally, we define the deficit as the log of government spending over taxes

$$d_t = g_t - \tau_t$$

This measure of the deficit includes interest payments and one of our robustness checks will use the primary deficit, which excludes interest payments. We use two samples of interest rates. Our main sample, from the first quarter of 1970 to the third quarter of 2003, contains zero-coupon bond yields with maturities ranging from 1 to 40 quarters, constructed by extending the Fama-Bliss smoothed data set to the recent quarters. Our long sample, which we use only for reduced form regressions, contains the 10-year rate and starts in the third quarter of 1954. The summary statistics for both samples are reported in **Table 1.**

2.1 Yields Regressions

Consider first the dynamics of bond prices of different maturities. We can rewrite the pricing equation as

$$r_t^n \equiv -\frac{\log \left(P_t^n \right)}{n} = a_n + b_n' Y_t,$$

where r_t^n is the yield at time t on a zero-coupon Treasury with remaining maturity n, and $a_n \equiv A_n/n$ and $b_n \equiv B_n/n$. In the maximum likelihood estimation, we will test and accept the restriction that the loadings of b_n on lagged values of y_t are zero. Thus, we only estimate

$$r_t^n \equiv -\frac{\log \left(P_t^n \right)}{n} = a_n + b_n' y_t, \tag{7}$$

Table 2 presents the yields regressions (7) estimated independently for each maturity n. The coefficients are estimated by ordinary least squares, but the standard errors are corrected for autocorrelations up to 8 quarters using the Newey-West procedure. Columns (i) to (v) are based on the main sample. There are three main findings.

First, the R^2 are high, and the fiscal variables are highly significant. Second, the fiscal variables become more relevant as maturity increases, while the federal fund rate, help and inflation become relatively less important. Not including the deficit reduces the R^2 by more than 10 percentage points for the 10-year rate. Looking at the fifth column, one cannot reject the hypothesis that taxes and spending enter with opposite coefficients of similar magnitude. Columns (vi) to (ix) in **Table 2** are based on the long sample, and three subsamples. The main finding here is that the loading of the 10-year rate on the deficit is large and significant in all subsamples. There is some evidence of instability of the coefficient, as the point estimate drops in the last subsample, perhaps due to the globalization of the financial markets over the past 15 years. This is a topic for future research. Note, however, that the estimate remains just as statistically significant as in earlier samples. **Figure 1** shows that the results in **Table 2** are not driven by outliers. To construct **Figure 1**, the deficit and the 10-year rate are first regressed on inflation, the help wanted index, and the federal funds rate. We then plot the residuals from these two regressions against each other. Note that in **Figure 1** (and only there), we use annual data to avoid cluttering the picture.

We have also introduced the debt to GDP ratio in our regressions, and we have found it to be insignificant once we control for the current deficit. In other words, when we introduce debt together with its lag, the data chooses to create the deficit by putting coefficients of similar magnitudes (and opposite signs) on the log of debt and its lag. There are two possible theoretical answers for the fact that debt does not appear to matter much for long term rates, while deficits do. The first is that it is really the expected debt that matters, so today's deficit may simply be a better proxy for tomorrow's debt.

The second is that Ricardian equivalence might hold in the long run, but not in the short to medium run. The Ricardian equivalence holds in the Barro-Ramsey model (Barro (1974)) and fails in the Samuelson-Diamond model (Diamond (1965)) when current consumers can shift the tax burden to future generations they do not care about. However, Poterba and Summers (1987) argue that inter-generational linkages are unlikely to be quantitatively important for analyzing the short run effects of government deficits. Rather, they argue that liquidity constraints and myopia might be more important. Similarly, Mankiw (2000) points out that the two standard models – Barro-Ramsey and Samuelson-Diamond – are not fully consistent with the existing micro evidence.[8] Mankiw (2000), like Poterba and Summers (1987), concludes that the Ricardian equivalence might not hold in the short to medium run, but not for the reasons emphasized in the over-lapping generation literature. He then considers a model in which some – but not all – agents are Ricardian. In the long run, the savers pin down the interest rate, therefore, in any steady state the real interest rate equals the rate of time preference of the Ricardian agents. This implies that debt has no effect on interest rate in the long run. Changes in debt, however, have real consequences. The intuition laid out in Mankiw (2000) has been formalized in Gali, Lopez-Salido, and Valles (2005). More generally, any model that is Ricardian in the long run, but not in the short run, would be consistent with our findings.

Finally, note that our analysis ignores the risk that the government could explicitly default on its liabilities. This seems like a safe assumption for the United States in the postwar period, but it might not be such a good assumption for other countries (see Ardagna, Caselli, and Lane (2005) for an analysis of OECD countries).

2.2 Excess Return Regressions

Alternatively, we can examine holding-period returns on bonds of various maturities. By definition, the return on an n-period zero-coupon bond held for τ periods, in excess of the return on a τ-period zero-coupon bond, is given by

$$xr^n_{t\to t+\tau} \equiv \log\left(P^{n-\tau}_{t+\tau}\right) - \log\left(P^n_t\right) - r^\tau_t = A_n + B'_n Y_t - A_{n-\tau} - B'_{n-\tau} Y_{t+\tau} - \tau r^\tau_t, \quad (8)$$

so that the expected excess return is given by

$$E_t\left[xr^n_{t\to t+\tau}\right] = \alpha_n + \beta'_n Y_t,$$

[8] Consumption smoothing is far from perfect, net worth is zero or negative for 18.5% of households, and bequest are an important factor in wealth accumulation. The first two facts contradict both models, and the third contradicts the OLG model.

where $\alpha_n = A_n - A_{n-\tau} - A_\tau$, and $\beta_n = B'_n - B'_\tau - B'_{n-\tau}\Phi^\tau$. Using the recursion for B_n, the slope coefficients can be computed explicitly and are given by

$$\beta'_n = B_{n-\tau}\left[(\Phi - \Lambda)^\tau - \Phi^\tau\right].$$

Clearly, the risk premium is constant for all n and τ if and only if Λ is equal to zero.

As with the yield regressions, we will later test and accept the restriction that the loadings of β_n on lagged values of y_t are zero, and to be consistent, we use y_t in our OLS regressions as well. **Table 3** presents the excess return regressions.[9] The observable state space can predict 24.4% of the excess returns on 5-year bonds, 30% on 2-year bonds, and 21.2% on 10-year bonds. Columns (iv) and (v) show that neither the growth rate of GDP, nor the growth rate of non-durable consumption, are statistically significant. The help wanted index does a better job at forecasting excess returns, presumably because it is a leading indicator. In column (vi), we find that the index of leading indicator of Stock and Watson performs very much like the help index. Note that one qualitative feature appears very robust across specifications: excess returns on long term bonds go down when the state of the economy improves. This is consistent with the view that risk aversion varies over time in a counter-cyclical fashion. The regressions simply indicate that the help wanted index is a better proxy for this time varying risk aversion than current consumption or GDP growth. When we add the 2-year rate to the state space in column (vii), we can account for more than 34% of excess returns. Using the 10-year rate instead of the 2-year rate would increased our R^2 to 37%. As a benchmark, Cochrane and Piazzesi (2005) report predictability of around 40% using all the forward rates (although their study is restricted to maturities of five years or less).

2.3 Taking Stock

We draw two main conclusion from this exercise. First, the observable variables that we propose can potentially account for a large fraction of the dynamics of interest

[9] Again, these are "unrestricted" regressions in the sense that cross-sectional restrictions on α_n and β_n for different n are ignored. The restrictions will be imposed in the maximum likelihood estimation.

rates. Second, we need a latent factor to capture some of the predictable excess returns on long term bonds. The fact that the two-year rate is very useful in predicting excess returns means that there is extra information in the current yield curve about agents' expectations and/or risk aversion. A formal economic interpretation of the latent variable is beyond the scope of this paper. For our purpose, it suffices to say that economic agents presumably form their expectations using a larger information set than by simply looking at past values of inflation, output and the federal funds rate. These expectations are then embedded in the term structure, and, therefore, in our latent variable. Other factors that can affect the supply and demand of long term bonds, but are not adequately captured by the macro-economic variables included here, range from "liquidity preference" to central bank intervention in the currency market.

The yields and returns regressions that we have presented do not enforce the restrictions on the time-series behavior of yields and returns implied by the pricing model. The unrestricted coefficients, however, can be used to construct restricted estimates of the parameters of interest. Specifically, the slope coefficients from the yield regressions must satisfy the recursions (6), and, without latent factor, the mean reversion matrix Φ can be estimated directly by OLS, so the state-dependent portion of the market price of risk can be estimated by choosing $\Lambda = \Lambda^{yield}$ that gives the best cross-sectional fit of the unrestricted yields coefficients. Alternatively, since the excess returns are proportional to the market prices of risk, we can obtain another estimate, Λ^{return}, by regressing the coefficients from the excess return regressions on the coefficients from the yield regressions. Λ^{yield} and Λ^{return} emphasize different aspects of the data.[10] If the model is correctly specified, the two estimates should be similar. Indeed, an important reason why we find it necessary to include a latent variable in the state space is that Λ^{yield} and Λ^{return} are much closer when the latent variable (proxied by the two-year rate in our OLS analysis) is present than when it is absent.[11]

[10] Roughly speaking, Λ^{yield} helps determine how the shape of the yield curve changes over time, whereas Λ^{return} helps determine how the expected returns changes over time.

[11] As a practical matter, when Λ^{yield} and Λ^{return} are sufficiently close, we can use either one of the estimates as starting value for the MLE, and the model has a good chance of explaining the

3 Maximum Likelihood Estimation

Having settled on a state space, we will now estimate the model using maximum likelihood based on the Kalman Filter. Details on the construction of the Kalman filter and the likelihood are presented in the appendices.

3.1 Estimation

At a technical level, the maximum likelihood approach allows us to replace the bond yield in the OLS analysis by a latent variable in order to impose no-arbitrage restrictions in a proper and natural manner. It also allows us to compute asymptotic standard errors for the parameter estimates based on standard inference procedures. At a substantive level, the maximum likelihood approach allows the model to achieve the best trade-off between the time-series properties of the state variables and the cross-sectional behavior of bond yields and returns. This is critically important for our purpose because, as we will elaborate in the next section, fiscal policies affect the term structure through both expectations and risk premia. In order to identify the two effects separately, we do not wish to skew the model toward one channel at the expense of another through arbitrary choices of moment conditions and weighting schemes. It is worth pointing out that the MLE estimation is independent of the structural restrictions that we later impose to identify the policy shocks.

Based on the analysis presented in section 3, we choose the state space $y_t = (f_t, d_t, \pi_t, h_t, q_t)$, where f_t is the federal funds rate, d_t is (one tenth of) log of spending over taxes, π_t is the growth rate of the GDP deflator, h_t is the help wanted index, and q_t is latent. We specify the dynamics of y_t as a $VAR(2)$ using an information criterion to select the number of lags. In addition to the macro variables, we assume that eight bonds (with 1, 2, 4, 8, 16, 20, 30, 40 quarters of maturity) are observed and used in the estimation.[12] The measurement errors on these bonds have a multi-variate

violation of the expectations puzzle. Otherwise, the model tends to be bimodal in the sense that there are (at least) two local optima that emphasize different aspects of the model fit (yield levels versus returns).

[12] We allow f_t to depend on all the macroeconomic variables as well as the latent factor. This specification accommodates backward and forward looking monetary policy rules, and allows the monetary authority to react to the information contained in long term bonds.

normal distribution with zero mean and arbitrary correlation.

The sample period, from 1970:1 to 2003:3, consists of 135 quarters. In each quarter, we observe 12 variables (8 bonds and 4 observable macro variables), for a total of 1620 observations. To reduce the number of free parameters, we restrict the short-rate equation and the market price of risk in such a way that current bond prices depend only upon current values of the state space, i.e., such that A_n and B_n load only on y_t, not y_{t-1}.[13] We test these restrictions using a Lagrange multiplier statistic. The unconstrained model is obtained by freeing up the loadings of δ and λ on the lagged values. The Lagrange multiplier statistic is $LM = 0.9588$ with 10 degrees of freedom. The critical value is essentially zero, and the constrained model cannot be rejected against the unconstrained model; thus, from now on, we will use the constrained model. The parameters of interest characterize the dynamics of the system (Φ, Ω), the short rate (δ_0, δ) and the market price of risk (Λ_0, Λ).[14] Given the model parameters and other necessary normalizations[15], an unbiased estimate of the latent variable q_t can be obtained through the Kalman filter.[16]

Table 4 presents the estimated coefficients and **Table 5** presents the t-statistics.[17] We find that many parameters are well identified. Consider for instance the fourth column of the loadings of the market prices of risk on the current state space. The

[13] Technically, this is achieved by imposing the restriction that (i) the short rate does not load on the lagged state variables, and (ii) the dynamics of y_t is VAR(1) under the risk-neutral measure. This reduces the number of free parameters by 30. As noted earlier, the forecast model for y_t under the physical measure is VAR(2).

[14] The MLE also estimates the covariance matrix of the measurement errors. Intuitively, one of the observed bond yields identifies the latent factor. The resulting state space identifies the VAR parameters. The remaining bond yields identify the pricing kernel and the covariance matrix of the measurement errors through the pricing restrictions and zero-mean restrictions on the measurement errors.

[15] The presence of a latent factor means that the model is invariant to certain affine transformations. We normalize the model by imposing the following restrictions: (i) the loading of the short rate on the latent factor is 1; and (ii) the latent factor is conditionally uncorrelated with the observed factors. See Dai and Singleton (2000) for a more general discussion of these issues.

[16] We impose that the eigenvalues of $I - \sum_{j=1}^{L+1} \phi_j$ lie within the unit circle, so that the state process is stationary under the physical measure. We also rule out complex eigen-values for the mean reversion matrix under the risk-neutral measure to avoid oscillating behavior in the yield loadings.

[17] In computing the t-statistics, we fix some of the parameters to their point estimates if their t-ratios are less than 1 because the information matrix is almost singular when all the parameters are free. This means that some coefficients are hard to identify. It does not mean that they all are. For instance, we can easily reject the null that the loadings on the fiscal deficit are zero.

point estimate for the (4,4) element 0.705 means that a positive shock to h_t increases Λ_t and therefore decreases the expected excess returns on long term bonds. This captures the time varying, counter-cyclical risk aversion of the economy. The t-statistic for this coefficient is 2.233.

Figure 2 presents the yield loadings b_n implied by the MLE estimates of Φ and Λ. The loadings do not have a structural interpretation because the variables in the system are jointly endogenous, and also because we can rotate the model by adding any linear combination of the observed variables to the latent variable. The shapes of the loading curves, however, are still informative. As expected from the OLS regressions, we find that the loadings on the deficit increase with maturity. This is an interesting property since all the observed factors we are aware of tend to display the opposite pattern when they are embedded in an affine model.

3.2 Tests of the Pricing Model

We now check whether the model fits the data correctly, by considering four questions. Does the model match the mean and volatility of yields? How large are the pricing errors? Are the no-arbitrage restrictions accepted? Can the model explain the observed deviations from the expectation hypothesis?

Panels (a) and (b) of **Figure 3** shows that the model does a good job at matching the mean and volatility of the yield curve. In each subplot, we include the sample moments (circles), the moments computed from the model-implied yields evaluated (equation 5) at the sample values of Y_t (crosses), and the population moments evaluated at the MLE estimates (solid line) together with one standard-error bands (dashed lines). For the most part, the sample moments for both observed and model-implied yields are within one standard error of the population moments. **Figure 4** shows that the observed factors account for 80% to 95% of the variance of interest rates, that the latent factor explains most of the remaining variance, and that the pricing errors are small.[18]

[18] By definition, our model implies that $r_t^n = a_n + b_n' y_t + \text{error}_t^n$. The orthogonal part of the latent factor is the residual from projecting the latent factor onto the observed variables. This ordering gives as much explanatory power as possible to the observed factors.

As explained in section 2, the no-arbitrage restrictions mean that the full model is overidentified. The loadings of **Figure 3** are not free parameters, they are functions of the underlying Φ and Λ. We perform a Lagrange multiplier test, and we find that we cannot reject these pricing restrictions. The test is described in the appendix.

Another important test is to check whether the model can explain the violation of the expectations hypothesis. Under the expectations hypothesis, the slope coefficients in the following regressions,

$$r_{t+1}^{n-1} - r_t^n = \text{constant} + c^n \times \frac{r_t^n - r_t^1}{n-1} + \text{errors}, \tag{9}$$

should be equal to 1. Campbell and Shiller (1991) show, however, that the c^n coefficients are negative for all maturities, suggesting that the expectations hypothesis is violated. For our sample, the slope coefficients c^n range from -0.5 to -3, as indicated by circles in **Figure 5**. The solid line in the same graphs represents the population values of the slope coefficients from our model (solid line), which is computed as 1 minus the linear projection coefficient from the expected excess return on the slope of the yield curve.[19] The fact that the Campbell-Shiller coefficients lie within the predicted standard-error bands (dotted lines) of the population coefficients means that our model explains the expectations puzzle. For comparison, we also plot the coefficients (stars) implied by the filtered yields from MLE. The latter are even closer to their sample counterparts, indicating that the remaining difference between the sample coefficients and the population coefficients may be explained by small sample biases.

We conclude that the deficit is a factor for long term rates, above and beyond inflation and real activity. This is a new result. It is also clear-cut: the p-value for the test that the deficit is not a factor (that its loadings on interest rates are all zero)

[19] See Dai and Singleton (2002), who show that, by definition, the following equation must hold:

$$r_{t+\tau}^{n-\tau} - r_t^n + \frac{E_t[xr_{t\to t+\tau}^n]}{n-\tau} = \frac{r_t^n - r_t^\tau}{n-\tau}.$$

It follows that the downward bias from 1 (representing the expectations hypothesis) is equal to the linear projection coefficient from the expected excess return, $E_t[xr_{t\to t+\tau}^n]$, on the slope of the yield curve, $r_t^n - r_t^\tau$.

is essentially 0.[20] On the other hand, this result does not address directly the issue of causality, and it certainly tells us little about the dynamic effects of a fiscal shock. This is why we also need to identify exogenous fiscal shocks. The fact that our model explains the expectations puzzle should then allow us to separate the effects of fiscal policy shocks on risk premia from their effects on expectations of future short rates. This does not mean that the structural restrictions that we are going to impose in the next section are warranted. We are simply going to use an off-the-shelf procedure. One can agree or disagree with the identifying restrictions independently from the bond pricing model. However, if one accepts the identifying restrictions, then, given that our bond pricing model is reasonably successful, one should take seriously the impulse responses and variance decomposition derived from the model.

4 Fiscal Policy Shocks and Interest Rates

Our goal in this paper is to study the effects of fiscal policy on the term structure of interest rates. In the previous sections, we have presented an affine model that seems to capture expectations and risk premia reasonably well. We now turn to the issue of identifying the policy shocks. Identification is the central issue in empirical macroeconomics, but it has not received the same attention in empirical finance.[21] We borrow the identification strategy of Blanchard and Perotti (2002). This methodology is not uncontroversial, and we discuss various robustness checks in section 5.

4.1 Identification of Structural Shocks

In Blanchard and Perotti (2002) fiscal shocks are defined as fiscal innovations not resulting from automatic responses to real output. The main difference between our setup and the original Blanchard-Perotti setup is that we include inflation in the state space while they did not. Inflation is crucial for us, since we are pricing nominal bonds. The general strategy, however, is still applicable, provided that we know the

[20] Bikbov and Chernov (2004) have recently confirmed and extended this result, also using an affine model, but following a different estimation strategy that shows that the latent residual which is orthogonal to the *entire history* of past GDP growth and inflation, is strongly correlated with the growth rate of government debt.

[21] This is one difference between our work and Bikbov and Chernov (2004).

elasticities of government purchases and net taxes to inflation shocks. Fortunately, Perotti (2004) provides estimates of these elasticities. Without loss of generality, we write the reduced form shocks to real spending u_t^g as the sum of the reduced form shocks to the other variables in the state space – federal funds rate u_t^f, inflation u_t^π, real activity u_t^h, and latent factor u_t^q – and to the fiscal policy shocks ε_t^g:

$$u_t^g = \eta_t^{g,f} u_t^f + \eta_t^{g,\pi} u_t^\pi + \eta_t^{g,h} u_t^h + \eta_t^{g,q} u_t^q + \varepsilon_t^g. \tag{10}$$

Similarly, for real net taxes, we write:

$$u_t^\tau = \eta_t^{\tau,f} u_t^f + \eta_t^{\tau,\pi} u_t^\pi + \eta_t^{\tau,h} u_t^h + \eta_t^{\tau,q} u_t^q + \varepsilon_t^\tau. \tag{11}$$

Equation (11) emphasizes the fact that not all changes in tax revenues are due to changes in fiscal policy. Tax revenues respond automatically to economic activity because the tax base depends on output. Real tax revenues also respond automatically to inflation because tax brackets are not indexed to inflation within the quarter. The problem is that one cannot use macroeconomic data to identify the elasticities $\eta_t^{i,j}$ for $i = g, \tau$ and $j = f, \pi, h, q$. So we need to bring in new information. Note that we only need to restrict the within-quarter elasticities, and that we impose no restrictions on the effects of lagged variables.

Like in Perotti (2004), we assume that the elasticities of net taxes and of expenditures to interest rates are zero. This allows us to set $\eta^{g,f} = \eta^{g,q} = 0$ for expenditures, and $\eta^{\tau,f} = \eta^{\tau,q} = 0$ for net taxes. This seems like a safe assumption for expenditures. The situation is potentially more complex for revenues, because of taxes on dividend, capital gains and interest income. These elasticities are probably not exactly equal to zero, but they are much smaller than $\eta_t^{\tau,h}$ and $\eta_t^{\tau,\pi}$. The impact of inflation and real activity on tax revenues is certainly a first order concern, however. Fortunately, using detailed knowledge of the tax system of the United States, Blanchard and Perotti (2002) and Perotti (2004) have already *calibrated* the automatic responses of spending and taxes to shocks to inflation and output. Perotti's estimates are based on annual elasticities of different types of taxes, as computed by the OECD. The elasticities $\eta_t^{\tau,h}$ and $\eta_t^{\tau,\pi}$ vary over time because of changes in the tax base and in the tax system. Over the period 1970:1-2003:3, $\eta_t^{\tau,h}$ has a mean of 1.92 and a standard deviation of 0.12, while $\eta_t^{\tau,\pi}$ has a mean of 1.35 and a standard deviation of 0.42. For spending, $\eta_t^{g,h} = 0$ because spending does not react to news about real activity within the quarter, and $\eta_t^{g,\pi} = -0.5$ because roughly half of spending is not directly indexed to inflation. We will later conduct robustness checks around these values.

The elasticity $\eta_t^{\tau,h}$ in equation (10) allows us to estimate how much tax revenue increase when the economy expands. The elasticity $\eta_t^{\tau,\pi}$ does the same thing for inflation shocks. Using both elasticities, we can construct the residual ε_t^{τ}. This residual is the unexpected change in net taxes which is not caused by changes in real activity or by shocks to the price level. We therefore interpret it as a policy shock to real net taxes. Similarly, we interpret ε_t^{g} as a policy shock to real government spending. Finally, since the deficit is simply $d = g - \tau$, we have $u^d = u^g - u^\tau$, and we can use the elasticities

$$\eta_t^{d,h} = \eta_t^{g,h} - \eta_t^{\tau,h} \ ; \ \eta_t^{d,\pi} = \eta_t^{g,\pi} - \eta_t^{\tau,\pi}$$

to identify structural shocks to the deficit ε_t^d.[22]

4.2 Responses to Deficit Shocks

Now that we have identified the structural shocks ε_t^d, we can compute the impulse responses of the various variables to fiscal policy shocks. We present the impulse responses of the state space, and then the implied responses of the short rate and of the 10-year rate. In the next subsection, we will discuss the extension to a state space with 6 variables, where we consider spending and taxes separately.

Figure 6 presents the responses to the deficit shock. The initial shock is $\varepsilon_0^d = 1\%$. This can be compared with either the standard deviation of ε_t^d of 0.35%, or with the standard deviation of d_t of 1.2% reported in **Table 1**. Because of the automatic stabilizers, the initial increase in deficit is only 0.8%. Inflation and real activity increase, while the federal fund rate does not react immediately. Eventually, the federal fund rate increases by substantially more than inflation. To get a sense of the

[22] Remember that, for aesthetic reasons, we have scaled the fiscal variables by a factor of 10, $g = \frac{1}{10}\log(\frac{G}{p})$, so the elasticities must also be divided by 10. Another issue is that the elasticities of Perotti apply to GDP, not directly to the help index, so another adjustment is needed. See the appendix for details.

magnitudes involved, remember that spending is roughly 20% of GDP, and that we have normalized $d = \frac{1}{10} \log(G/T)$. We are therefore looking at a shock that would increase spending by 10%, or the deficit to GDP ratio by 2 percentage points before macroeconomic feedbacks. Most existing papers report changes in interest rates for a 1 percentage point increase in the deficit ratio, so a rough comparison can be made by dividing our values by 2. **Figure 7** presents the impulse response of the 10-year rate together with its asymptotic standard error bands. The peak response occurs after three years and is approximately 70 basis points. This suggests an elasticity of long rates to the deficit to GDP ratio of around 35 basis points. By comparison, the estimates in Gale and Orszag (2003) range from 25 to 35 basis points.

Figure 8 decomposes the response of the 10-year rate into expected future short rates and risk premia. Under the expectations hypothesis, the 10-year rate at any point in time is the average of future short rates over the following 10 years. The difference between the actual 10-year rate and the 10-year rate under EH reflects the risk premia on long-term bonds. The risk premium is initially negative before becoming positive. After 4 years, the risk premium explains more that a third of the increase in long term rates. The initial drop and subsequent increase in the risk premium come from the dynamics of the state space and of the market price of risk. For instance, deficit spending is expansionary, so h_t goes up. As discussed above, this reduces risk aversion and lowers the premium on long term bonds. Similarly, we see that the real rate is below its long run mean for at least one year, which contributes to the low risk premia. After 3 years, the expansionary effect dampens out, while the real rate is above its long run mean, and the risk premium increases. **Figures 9** shows that fiscal policy shocks matter more at longer horizons, and that they explain up to 13% of the variance of interest rates.

To summarize, we have found that fiscal policy matters for long term interest rates, and that fiscal shocks lead to departures from the expectation hypothesis. This last fact can explain why some researchers estimate large coefficients when regressing long rates on current deficits, but small (and sometimes insignificant) coefficients when regressing current short rates on current deficits or current debt to GDP ratios. Our results suggest that part of the reason is that high long rates do not necessarily turn into high future short rates.

4.3 Responses to Spending and Taxes

While separating government purchases of goods and services from tax revenues and transfers is important for macroeconomics, it is not the main focus of our paper. Moreover, existing papers have already investigated the issue (see Blanchard and Perotti (2002)). Here, we simply wish to present some evidence that we hope will be informative for future research. Introducing taxes and spending separately is not straightforward because the two series are non-stationary and need to be either detrended, or introduced as growth rates. Growth rates do not have much explanatory power for yields: the data want the deficit (the difference between log spending and taxes), not the change in the deficit, and using additional lags does not solve the problem. In other words, the model in growth rates is misspecified. Detrending is also problematic, however, since it assumes that economic agents know the actual trends. This does not seem like an ideal assumption,[23] especially for the purpose of pricing bonds. For lack of a better alternative, we will nonetheless proceed with linearly detrended series. Another issue is that the number of free parameters increases substantially when we move to a state space with six variables, and the MLE becomes hard to implement.

For all these reasons, the results in this section are not based on MLE. Rather, we use the simpler, two-steps, "matching moments" approach described at the end of section 2. We use a state space with 6 variables: f_t, π_t, h_t plus detrended spending g_t and taxes τ_t, and the 2-year interest rate r_t^8. The state space is fully observable since there is no latent factor, and we can estimate $\hat{\Phi}$ and $\hat{\Omega}$ as in a standard VAR. In the first step, we also run 40 yields regressions (7), and 36 one-year excess returns regressions (8). In the second step, we choose the parameters $(\delta_0, \delta, \lambda_0, \lambda)$ to minimize the distance between the coefficients predicted by the recursive equations (6), using $\hat{\Phi}$ and $\hat{\Omega}$ from the VAR, and the estimates from the the 76 yields and returns regressions.

[23] The deficit is stationary, has been and is expected to be. But whether the government will choose to satisfy his budget constraint by adjusting taxes or spending is far from obvious, and certainly hard to forecast.

This method is clearly less efficient than MLE, since $\left[\hat{\Phi}, \hat{\Omega}\right]$ and $[\delta_0, \delta, \lambda_0, \lambda]$ are not jointly estimated, and since the choice of the 2-year interest rate is arbitrary. To build some confidence, we have checked that it delivers broadly similar results for the five-variables model of the previous section. Finally, the policy shocks are constructed using the elasticities $\eta_t^{\tau,h}$, $\eta_t^{\tau,\pi}$, $\eta_t^{g,h}$, $\eta_t^{g,\pi}$ as described above.

Figure 10 reports the response of the 10-year rate to spending and tax shocks, as well as the hypothetical response under EH. **Figures 10** shows that the 10-year rate responds to spending and tax shocks in qualitatively similar ways. The initial shocks are always 1% and the initial response of the other fiscal variable (of spending to taxes or taxes to spending) is set to 0.[24] Note that the standard deviation of ε_t^g is only 0.1%, while the standard deviation of ε_t^τ is 0.33%, so the spending shock is more unusual than the tax shock. To save space, we do not report the responses of the state space itself. These are similar to the ones in **Figure 6**. After a spending shock, inflation increases immediately, while the fed funds rate increases only over time. The responses to tax shocks are just the opposite. One feature is worth emphasizing: Spending does not seem react to the tax shock. Changes in tax policies are not systematically followed by changes in government spending. We did not impose or expect this result, but we note that it allows us to talk about the reactions of the economy to changes in the timing of tax revenues, while keeping the path of spending roughly constant. The response of the economy to tax shocks, in terms of both prices and quantities, does not appear consistent with Ricardian equivalence in the short and medium run. Fiscal policy is neutral in the long run by assumption, since our model is stationary in the level of interest rates and detrended taxes and spending.

[24] Changing the ordering changes nothing to our results since the two reduced form shocks are almost uncorrelated. See Blanchard and Perotti (2002) for similar results and a more detailled discussion.

5 Robustness and Comparison with Existing Work

5.1 Robustness

5.1.1 Elasticities

The main area of uncertainty concerns the elasticity of real spending to inflation, but it turns out that the precise value of this elasticity is not crucial for our results. Our baseline calibration uses $\eta_t^{g,\pi} = -0.5$. In this case, the maximum response of the 10-year rate in **Figure 7** is 70 basis points. If we use $\eta_t^{g,\pi} = -0.25$, this maximum response becomes 65 basis points. If we use $\eta_t^{g,\pi} = -0.75$, it becomes 74 basis points. This is because changing $\eta_t^{g,\pi}$ directly affects the estimated impulse response of inflation to the deficit shock, and the response of inflation affects the response of the long rate. We have also performed similar robustness checks with respect to all the other elasticities. For instance, if we multiply the elasticity of deficit to output by 1.25, the maximum response becomes 76 basis points. If we multiply it by 0.75, the maximum response becomes 64.5 basis points. At the end of the day, our main results appear robust to reasonable changes in the calibrated elasticities.

5.1.2 Alternative Macroeconomic Variables

Our state space includes the help wanted index, the log growth rate of the GDP deflator, and the log of government purchases of goods and services over net taxes. We now discuss the pros and cons of each of these variables.

As argued in section 2, the help wanted index seems to predict bond returns better than GDP growth does. The elasticities $\eta_t^{g,h}$ and $\eta_t^{\tau,h}$, however, apply to GDP, and innovations to h and to GDP need not be the same. In practice, fortunately, the cyclical fluctuations in h explain around 70% of the cyclical fluctuations in GDP (see appendix C). We have conducted extensive checks, and found that in all cases, we identify essentially the same fiscal shocks with h or with GDP. The second issue with the help wanted index is that it might not be stationary because of the increase in advertising on the internet. To deal with both issues, we have estimated a model with GDP growth instead of the help index. In this model, the performance of the bond pricing model deteriorates a bit, but the main qualitative and quantitative features of the impulse responses to identified fiscal shocks remain the same. We have also experimented with the leading indicator of Stock and Watson, and the results are unchanged.

Our model is robust to using the CPI or the core-CPI as our measure of inflation, instead of the GDP deflator. Finally, we have estimated a model with the primary deficit instead of the unified deficit. The primary deficit excludes interest payments. In this case, the estimated effects become larger. For instance, the regression coefficients in the first three columns of **Table 2** become 0.503, 0.771 and 0.890 for the 2-year, 5-year and 10-year yields. The t-statistics are almost the same. The impulse responses are also correspondingly larger. This is consistent with Gale and Orszag (2003) who also find larger elasticities when they use the primary deficit.

5.2 Comparison with Previous Work

We now discuss how our results compare with the large existing literature on deficits and interest rates (see Barro (1987) and Elmendorf and Mankiw (1999) for reviews). There is no consensus in the literature: Evans (1987), Plosser (1987) and Engen and Hubbard (2004) find small or nonexistent responses of interest rates; Laubach (2003) and Gale and Orszag (2003) find significant responses; Evans and Marshall (2002) find different responses of interest rates depending on whether they identify fiscal shocks using the Blanchard and Perotti (2002) approach or using the Ramey and Shapiro (1997) approach.

There are two ways to estimate fiscal shocks. The first is to follow the narrative approach of Ramey and Shapiro (1997). The second is to use the elasticities computed by Blanchard and Perotti (2002) and Perotti (2004).[25] The narrative approach is cleaner, in the sense that it provides a less noisy measure of the true shock, but there are only four episodes to work with over the post-war period, and they only capture spending shocks. The elasticity approach relies on stronger assumptions, but it is more flexible and it can identify tax shocks as well as spending shocks. The two

[25] See Ramey (2006) for a recent discussion.

approaches are different, and it is not surprising that they sometimes give different results. It would be troublesome, however, if they suggested very different answers to the same question. In fact, a broad overview of the literature suggests that there are two main areas of disagreement between the two approaches. The first one concerns the responses of consumption and investment to fiscal shocks.[26] We focus on the second inconsistency, which concerns interest rates. Evans and Marshall (2002) find an effect of fiscal shocks on interest rates only with the Ramey-Shapiro methodology. They find no response using the Blanchard-Perotti approach.[27] One payoff from using our model is that we can solve this puzzle.

The first reason that our results differ from the ones in Evans and Marshall (2002) is straightforward, and it applies to any empirical study of the term structure. In a VAR, one must have as many yields included in the VAR as there are yields that one is interested in. By contrast, we use only one latent factor in addition to the macro factors in order to price all the bonds. Our approach therefore improves the efficiency of the estimation, especially if the sample is short, and if there are idiosyncratic liquidity shocks to yields of different maturities. Given that macro data sets are small, and that macro variables are likely to be measured with error, we feel that this could explain part of the difference between our model and a VAR. Indeed, when we use a VAR similar to the one in Evans and Marshall (2002), we often fail to reject the null hypothesis of no response of interest rates.[28]

The second reason is that VARs are designed to forecast, while our model is designed to price bonds. To forecast the future 10-year rate, it is best to use the current 10-year rate. But what is the economic interpretation of the lagged 10-year rate on the right hand side of the VAR? Yields are market prices, they are not subject to adjustment costs the way investment is, and they actually change every day unlike goods prices. In an asset pricing model, we do not put the lagged 10-year rate on the right hand side. Rather, we attempt to price it. This is an important difference

[26] Burnside, Eichenbaum, and Fisher (2004) and Gali, Lopez-Salido, and Valles (2005) focus on this issue.

[27] The lack of response of interest rates to the Blanchard and Perotti (2002) shocks is puzzling, since it is hard to understand how a shock could affect GDP and leave interest rates unchanged.

[28] In our VAR estimations, the point estimates are smaller than with our model, but the main problem is that standard errors become very large.

because interest rates are persistent, and because our measures of fiscal policy shocks are noisy.[29] Thus, it is perfectly possible for short run movements in the deficit to contain little information about short run movement in interest rates, even when fiscal policy does affect long term interest rates. In this case, the lagged 10-year rate will improve the forecasting power of the model, but might prevent it from identifying the true underlying relationship.

In the appendix, we describe a stylized Monte-Carlo experiment to illustrate this point. The simulations show how including lagged interest rates in the estimation equation might bias the results. **Table 6** shows the relevance of our concern. The four columns of **Table 6** are directly comparable to columns (vi) to (ix) in **Table 2**. The only difference is that **Table 6** includes the lagged 10-year rate as a right-hand-side variable. When we use the lagged 10-year rate, there is an increase in the R^2 (and in the one-quarter ahead forecasting ability), and a sharp drop in the significance of the fiscal variable d_t. In fact, while d_t is still significant in the whole sample, it is not significant in the last two subsamples, once we 'control' for the lagged 10-year rate. One can also get a sense of the issue from **Figure 1**: while the overall picture is quite striking, it is not hard to find two consecutive years where the slope would be negative, 83-84 or 98-99 for instance.

The bias in estimating the effects of the deficit on interest rates is stronger at high frequency, because short run movement in the deficit are more likely to be either pure noise, or hard-to-measure consequences of the business cycle. This might explain why our results differ from the ones of Evans (1987) and Plosser (1987), who regress innovations to interest rates on innovations to spending and taxes.[30]

Our model seems also able to reconcile the results obtained with the 'narrative' approach of Ramey and Shapiro (1997) and the 'elasticity' approach of Blanchard and Perotti (2002). In the VAR of Evans and Marshall (2002), only the 'narrative' approach gives significant responses of interest rates. This is consistent with our

[29] In fact, there is not even agreement on a conceptually correct measure of the government's fiscal position, because implicit liabilities are difficult to estimate, and government assets even more difficult to value. See Elmendorf and Mankiw (1999) for a discussion.

[30] There are other differences between our approach and the ones of these earlier papers, such as the fact that we take into account changes in risk premia and expected excess returns.

Monte Carlo simulations. Because the narrative approach provides clean estimates of policy changes, the impulse responses computed with the Ramey-Shapiro dummies might not be biased, even in a standard VAR setup. On the other hand, there is always going to be some amount of noise left with the elasticity approach, and the VAR-based impulse responses can be severely biased.

Our model, however, is less sensitive to high frequencies errors and delivers significant responses even with the Blanchard and Perotti (2002) approach. Indeed, our impulse responses are strikingly similar to the ones obtained by Evans and Marshall (2002) when they use the Ramey and Shapiro (1997) approach. More precisely, in response to a positive deficit shock, the nominal short rate does not change much and then increases, inflation jumps up and then comes down, the real rate jumps down and then goes back up, the nominal long rate increases over time, and the term premium goes down and then up.

Overall, we feel that our estimation strategy strikes a balance between flexibility, parsimony and robustness to measurement errors. While we acknowledge that limitations remain, especially in the identification of the fiscal shocks, our approach, which is more structural than a simple VAR, seems able to solve at least some of the puzzles found in the existing empirical literature.

6 Conclusion

We have presented and estimated an empirical macro-finance model of the term structure. Based on bond pricing equations, we have chosen a state space that includes the federal funds rate, the government deficit, inflation, real activity and one latent factor. The model successfully explains the dynamics of the term structure of interest rates, and deviations from the expectation hypothesis. The model shows that risk-premia are counter-cyclical and increasing with the level of real rates.

We have found that government deficits increase interest rates, and that the fiscal shocks affect long rates through expectations of future spot rates as well as risk premia. Following an expansionary fiscal shock, the response of the risk premium is initially small or negative before turning positive after a few years, where it accounts for more than one third of the increase in the 10-year rate. Our results emphasize that the usual macroeconomic approach of equating long rates with average future short rates is rejected by the data, and that not recognizing this fact can lead to inconsistent estimates of the effects of fiscal policy. We have also provided some evidence that taxes temporarily affect interest rates for a given path of government spending, which suggests that the Ricardian equivalence might not hold in the medium run. Finally, we have argued that the asset pricing approach provides an efficient and robust alternative to the traditional VAR approach, while retaining its flexibility and identification techniques.

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A Description of the Data

Our data come from different sources:

- The federal fund rate and the help wanted index come from FRED, the web site of Federal Reserve Bank of St. Louis. We divided the original help index by 1,000. The leading indicator, originally developed by Stock and Watson, is the Chicago Fed National Activity Index (CFNAI). We divide it by 100.
- Consumption, GDP and the GDP deflator come from the Bureau of Economic Analysis. We annualize by multiplying the quarterly changes by 4.
- For the main sample, 1970:1 to 2003:3, the yields data are constructed by extending the Fama-Bliss smoothed data set to the recent quarters. For the long sample, we use the 10-year nominal interest rate from FRED.

For the fiscal data, we follow Blanchard and Perotti (2002) and use data from the National Income and Product Accounts. We consider nominal purchases of goods and services G on one hand, and nominal transfers minus taxes T on the other. In what follows, we use the Citibase mnemonics.

- Government spending is defined as the purchase of goods and services by federal and state and local governments.

$$G = ggfe + ggse$$

- Net taxes are defined as taxes minus transfers. Note that they include interest payments by the government.

$$T = \underbrace{ggfr + ggsr}_{\text{receipts}} - \underbrace{ggaid - ggftp - ggst}_{\text{aid and transfers}} - \underbrace{ggfint - ggsint}_{\text{net interest paid}} + \underbrace{ggsdiv}_{\text{dividend}}$$

B Model Specification and Parameterization

In this section of the appendix, we collect all of the assumptions and analytical results needed for a complete description of the dynamic term structure model. The model is based on $L+1$ lags of a $N \times 1$ state vector. Let y_t be the state vector. We assume that

$$y_t = \phi_0 + \sum_{l=1}^{L+1} \phi_l y_{t-l} + u_t,$$

where u_t is multi-variate normal with zero mean and covariance matrix ω. It is convenient to re-write the dynamics in terms of the expanded vector: $Y_t = (y_t, y_{t-1}, \ldots, y_{t-L})'$, which is VAR(1):

$$Y_t = \Phi_0 + \Phi Y_{t-1} + U_t,$$

where

$$\Phi_0 = \begin{pmatrix} \phi_0 \\ 0_{NL\times 1} \end{pmatrix},\ \Phi = \begin{bmatrix} \phi_1 ... \phi_{L+1} \\ I_{NL\times NL} \quad 0_{NL\times N} \end{bmatrix},$$
$$U_t = \begin{pmatrix} u_t \\ 0_{NL\times 1} \end{pmatrix},\ \Omega = \begin{bmatrix} \omega & 0_{N\times NL} \\ 0_{NL\times N} & 0_{NL\times NL} \end{bmatrix} \equiv cov(U_t).$$

B.1 Pricing Kernel and Risk-Neutral Dynamics

Let δ be a $N(L+1)\times 1$ vector, λ_0 be a $N\times 1$ vector, and λ be a $N\times N(L+1)$ matrix. We assume that the pricing kernel takes the following form:[31]

$$\frac{M_{t+1}}{M_t} = e^{-r_t - \frac{1}{2}\Lambda_t'\Omega\Lambda_t - \Lambda_t' u_{t+1}},$$
$$r_t = \delta_0 + \delta' Y_t, \quad \Lambda_t = \Lambda_0 + \Lambda Y_t,$$

where

$$\Lambda_0 = \begin{pmatrix} \omega^{-1}\lambda_0 \\ 0_{NL\times 1} \end{pmatrix},\ \Lambda = \begin{bmatrix} \omega^{-1}\lambda \\ 0_{NL\times N(L+1)} \end{bmatrix}.$$

It follows that under the risk-neutral measure Q, the state-dynamics follows:

$$Y_t = \Phi_0^Q + \Phi^Q Y_{t-1} + U_t^Q,$$
$$\Phi_0^Q = \Phi_0 - \Omega\Lambda_0 = \begin{pmatrix} \phi_0 - \lambda_0 \\ 0_{NL\times 1} \end{pmatrix},$$
$$\Phi^Q = \Phi - \Omega\Lambda = \begin{bmatrix} (\phi_1 ... \phi_{L+1}) - \lambda \\ I_{NL\times NL} \quad 0_{NL\times N} \end{bmatrix}.$$

and U_t^Q is multi-variate normal with zero mean and covariance matrix Ω under Q.

B.2 Bond Pricing

Under the above assumptions, the price of a zero-coupon bond with maturity n periods is given by $P_t^n = e^{-A_n - B_n' Y_t}$, where $A_0 = 0$, $B_0 = 0_{N(L+1)\times 1}$, and for $n \geq 0$,

$$A_{n+1} = \delta_0 + A_n + (\Phi_0^Q)' B_n,$$
$$B_{n+1} = \delta_1 + (\Phi^Q)' B_n.$$

[31] The zero-restrictions in Λ_t implies that the pricing kernel can be alternatively written as

$$\frac{M_{t+1}}{M_t} = e^{-r_t - \frac{1}{2}\lambda_t'\omega\lambda_t - \lambda_t'\sigma\epsilon_{t+1}},$$

where $\lambda_t = \omega^{-1}(\lambda_0 + \lambda Y_t)$. This captures the idea that only the shocks at $t+1$ is priced. Dependence of lagged shocks can be normalized away even if allowed.

It follows that the zero-coupon bond yields are given by

$$r_t^n = a_n + b_n' Y_t,$$

where $a_n \equiv A_n/n$ and $b_n \equiv B_n/n$.

B.3 Kalman Filter and Likelihood Function

In this section, we collect all of the assumptions and analytical results for constructing the Kalman Filter and the likelihood function. Suppose that we include K bonds in the estimation, with maturities n_k, $k = 1, 2, \ldots, K$, then the observed time-series variables can be collected in the vector:

$$X_t \equiv \begin{pmatrix} r_t^{n_1} & r_t^{n_2} & \ldots & r_t^{n_K} & z_t' \end{pmatrix}',$$

where z_t is equal to y_t excluding any latent variables. Let's assume that, out of N state variables, M are observed. Without loss of generality, we assume that y_t is ordered in such a way that all latent variables follow the observed variables. Then the observation equation can be written as[32]

$$X_t = G' + H'Y_t + v_t,$$

where,

$$\begin{aligned} G &= \begin{pmatrix} a_{n_1} & a_{n_2} & \ldots & a_{n_K} & 0_{1\times M} \end{pmatrix}, \\ H &= \begin{bmatrix} b_{n_1} & b_{n_2} & \ldots & b_{n_K} & \begin{bmatrix} I_{M\times M} & 0_{M\times(N(L+1)-M)} \end{bmatrix}' \end{bmatrix}. \end{aligned}$$

As part of the econometric specification, we assume that the "measurement errors" v_t are *i.i.d.*, multi-variate normal, with zero mean and covariance matrix R. In addition, we assume that the observed state variables do not contain measurement errors, so that the last M elements of the $K + M$ vector v_t are identically zero, and R is identically zero except the upper-left $K \times K$ sub-matrix, which represents the covariance matrix of the measurement errors in the observed yields.[33]

Let $\mathcal{I}_t = (X_s : s \leq t)$ be the current information set, and let

$$\hat{Y}_{t+1|t} \equiv E\left(Y_{t+1}|\mathcal{I}_t\right), \; P_{t+1|t} \equiv E\left[(Y_{t+1} - \hat{Y}_{t+1|t})^2|\mathcal{I}_t\right],$$

[32] For the Kalman Filter, we will follow closely the notation and algorithms developed in *Time Series Analysis* by James D. Hamilton. Accordingly, we set, without loss of generality, $\phi_0 = 0$ and therefore $\Phi_0 = 0$ by taking out the unconditional (or sample) means of the state variables througout the paper.

[33] In principle, we can allow the observed state variables z_t to contain measurement errors, in which case the matrix R has full rank.

be the optimal forecast of the state vector and the associated mean square forecast errors (MSE). The Kalman-Filter algorithm allows us to compute the forecasts and the associated MSE recursively as follows:

$$\begin{aligned}\hat{Y}_{t+1|t} &= \Phi\hat{Y}_{t|t-1} + \Phi P_{t|t-1}H(H'P_{t|t-1}H+R)^{-1}\left(X_t - G' - H'\hat{Y}_{t|t-1}\right), \\ P_{t+1|t} &= \Phi\left[P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H+R)^{-1}H'P_{t|t-1}\right]\Phi' + \Omega,\end{aligned}$$

starting with the unconditional mean and covariance matrix $\hat{Y}_{1|0} = E(Y_t)$ and $P_{1|0} = cov(Y_t)$. Under our VAR specification, the unconditional covariance matrix is given by $\text{vec}(P_{1|0}) = [I - \Phi \otimes \Phi]^{-1} \times \text{vec}(\Omega)$. The likelihood function can be constructed by noting that, given the information set $\mathcal{I}_t$, the conditional distribution of the observed vector X_{t+1} is multi-variate normal.[34] That is,

$$X_{t+1}|\mathcal{I}_t \sim N\left(G' + H'\hat{Y}_{t+1|t}, H'P_{t+1|t}H + R\right),\ t \geq 0.$$

All of the parameters (Φ, Ω, G, H, R) that determine the behavior of the Kalman Filter are completely determined by the primitive parameters $\beta \equiv (\phi_j, j = 0, 1, \ldots, L + 1, \omega, \delta_0, \delta, \lambda_0, \lambda)$ through deterministic transformations and the no-arbitrage pricing restrictions. In particular, the no-arbitrage pricing restrictions are encapsulated in the vector G and matrix H, which are completely determined by the yield loadings.

B.4 Test of Arbitrage Restrictions

The model implies non linear restrictions on (a_n, b_n). In our setup, these restrictions enter the observation equation: $X_t = G' + H'Y_t + v_t$. According to our model specification, H and G are determined by Φ according to the no-arbitrage restrictions (6). The log likelihood function of the model can be written as $\log \mathcal{L}(\theta, H, G)$ where θ is the vector of parameters, excluding Φ. In the unconstrained model, H and G are free, whereas in the constrained model, the likelihood must be written as $\log \mathcal{L}(\theta, H(\Phi), G(\Phi))$. Φ contains 10 parameters, while H contains 40 parameters and G 18. The Lagrange multiplier statistic, LM, is equal to $\frac{d\log\mathcal{L}}{dx'} \times [Information\_Matrix]^{-1} \times \frac{d\log\mathcal{L}}{dx}$, where x is the vector of unconstrained parameters, and the information matrix is computed under the unconstrained model. Ideally, we would like to test the restrictions on H and G simultaneously, but the information matrix becomes almost singular when we free up all the parameters at once. We therefore test the restrictions on H and on G separately. For H, we find that $LM = 0.9923$, which, for a χ^2 with 30 degrees of freedom gives a p-value (tail probability) of more than 99.9%. For G, we find that $LM = 0.9090$, which implies a p-value of 99.9%.

[34] By convention, $\mathcal{I}_0$ means no information and therefore X_1 is drawn from the unconditional distribution.

C Estimation of the Elasticities

The elasticities are computed separately for $g = \frac{1}{10}\log\left(\frac{G}{P}\right)$ and for $\tau = \frac{1}{10}\log\left(\frac{T}{P}\right)$ where P is the GDP deflator.

C.1 Government purchases of goods and services

The main assumption in Blanchard and Perotti (2002), maintained in Perotti (2004), is that government purchases are predetermined within the quarter. The authors could not find an item on the purchase side that could adjust to news about real activity within a quarter. This is why we set $\eta^{g,h} = 0$. The same is not true, however, for inflation shocks since some of the predetermined expenses are in nominal dollars, while others are in goods, and therefore indexed. Perotti estimates the fraction of spending indexed to roughly $\frac{1}{2}$, and we therefore use $\eta^{g,\pi} = -0.5$ as our benchmark. We also conduct robustness checks with $\eta^{g,\pi} = -0.25$ and $\eta^{g,\pi} = -0.75$.

C.2 Net taxes

Perotti (2004) breaks down total revenues into five components, each with a different elasticity: individual income taxes, corporate income taxes, indirect taxes, social security taxes, and a residual including all other current components and capital transfers. Here we describe the case of individual labor income taxes, which is typically the largest component of tax revenues.

$$T_t^i = \theta_t^i \left(W_t P_t\right) W_t \left(E_t\right) E_t \left(V_t\right)$$

where $\theta^i(.)$ is the tax rate for individual income, W_t the real wage, P_t the price level, E_t total employment and V_t is value added (i.e., GDP). Taking logs and differentiating, we get (lower case letters are logs)

$$d\log T_t^i = \left(\left(\frac{\partial \log \theta_t^i}{\partial w} + 1\right)\frac{\partial w}{\partial e} + 1\right)\frac{\partial e}{\partial v}dv + \frac{\partial \log \theta_t^i}{\partial p}dp$$

In this case, we would define

$$\eta_t^{i\tau,v} = \left(\left(\frac{\partial \log \theta_t^i}{\partial w} + 1\right)\frac{\partial w}{\partial e} + 1\right)\frac{\partial e}{\partial v}$$

and

$$\eta_t^{i\tau,\pi} = \frac{\partial \log \theta_t^i}{\partial p}$$

For most of its member countries, the OECD computes periodically the elasticity of tax revenues per person to average real earnings – i.e. the term $\frac{\partial \log \theta_t^i}{\partial w} + 1$ – using information on the tax code and the distribution of individuals over the different tax brackets. Perotti estimates the terms $\frac{\partial w}{\partial e}$ and $\frac{\partial e}{\partial v}$ for each country. A similar exercise is performed for each tax (corporate income tax, etc..).

Transfers react to output mainly because of unemployment insurance. Perotti estimates an elasticity of transfers to output of around -0.2. Most transfers are indexed to the CPI, but indexation typically occurs with a lag, substantially longer than a quarter. Perotti reviewed the indexation clauses of all OECD countries and could not find any government spending program that is (or was) indexed on inflation within one quarter. So the elasticity of real transfers to inflation is -1.

The output-elasticity ($\eta_t^{\tau,v}$) and price-elasticity ($\eta_t^{\tau,\pi}$) of net taxes are the weighted averages of the elasticities of revenues and transfers. Note that all these elasticities are allowed to vary over time. Finally, since we use the help wanted index instead of GDP, we define

$$\eta_t^{i\tau,h} = \eta_t^{i\tau,v} \times \frac{\partial v}{\partial h} \ .$$

To estimate $\frac{\partial v}{\partial h}$ we first extract the cyclical component of GDP as the residual from a quadratic trend. We then regress this cyclical component on the (similarly detrended) help index. This gives a coefficient of 1.28 with a standard error of 0.072 and a R^2 of 0.7046. So in our identification of the fiscal shock with the help index, we scale up the Perotti's elasticities by 1.28.

D Monte-Carlo Experiment

We construct a stylized example to emphasize the difference between short run forecasting and long run pricing in the presence of measurement errors. Suppose that the (demeaned) 10-year rate is given by

$$r_t^{10} = \delta_t + u_t \ , \tag{12}$$

where δ_t follows

$$\delta_t = \rho^d \delta_{t-1} + \sigma^d \varepsilon_t \ , \tag{13}$$

Assume, however, that δ_t is not observable. Rather, we observe

$$d_t = \delta_t - u_t \ . \tag{14}$$

In our interpretation, δ_t is the correct theoretical measure of fiscal policy, d_t is the observed deficit and u_t is a business cycle shock. Positive shocks lead to lower deficits and higher interest rates. We assume that u_t and ε_t are independent and normally distributed with mean 0 and standard deviation 1, and we simulate this model with $\rho^d = 0.95$ and $\sigma^d = 0.75$ for 130 periods (quarters). With these parameters the persistence of the observed deficit and the share of residual interest variance explained by the deficit are in line with what we find in the data. We then run two sets of linear regressions in the simulated data

$$r_t^{10} = \alpha + \beta d_t \ , \tag{15}$$

and

$$r_t^{10} = \alpha + \beta' d_t + \gamma r_{t-1}^{10} \ . \tag{16}$$

We repeat this exercise $10,000$ times and we study the distribution of β and β'. Obviously, both are biased estimators. The more interesting question is: which one is more biased toward 0? The distribution of β has a mean of 0.55 and a $t - stat$ of 3.28. The distribution of $\beta'/(1-\gamma)$ has a mean of 0.35 and a $t - stat$ of 1.47. The fifth percentile of the distribution of β is $+0.25$, while the fifth percentile of the distribution of $\beta'/(1-\gamma)$ is $-.065$. In this example, the tension between short run forecasting and long run pricing is clear. The mean squared error of the one period forecast is at least 20% smaller when we use equation (16) than when we use equation (15). Conceptually, a VAR is like equation (16), while our model is like equation (15). Thus, the VAR may well be better for short run forecasts and yet unable to identify the true relation between δ and r.

Table 1: Summary Statistics

Main Sample: 1970:1 to 2003:3					
	Obs	Mean	St. Dev.	Min	Max
3-month Yield	135	0.063	0.029	0.007	0.154
2-year Yield	135	0.071	0.027	0.013	0.158
5-year Yield	135	0.075	0.025	0.026	0.152
10-year Yield	135	0.078	0.023	0.038	0.150
Federal Fund Rate	135	0.067	0.031	0.010	0.175
Log (Spending / Taxes)/10	135	0.028	0.012	0.001	0.055
Inflation	135	0.041	0.026	0.008	0.122
Help Wanted Index	135	0.076	0.017	0.037	0.105
Detrended Log(Real Spending) /10	135	0.000	0.008	-0.014	0.016
Detrended Log(Real Net Taxes) /10	135	-0.001	0.012	-0.025	0.022
Growth Rate of Real GDP	135	0.030	0.034	-0.082	0.151
Growth Rate of Real Non Durable Consumption	135	0.010	0.010	-0.018	0.045
Index of Leading Indicators	135	0.000	0.011	-0.053	0.026
Long Sample: 1954:3 to 2003:3					
10-year Yield	197	0.065	0.025	0.024	0.143
Federal Fund Rate	197	0.057	0.031	0.009	0.175
Log (Spending / Taxes)	197	0.024	0.012	0.001	0.055
Inflation	197	0.036	0.024	-0.002	0.122
Help Wanted Index	197	0.067	0.022	0.025	0.105

Table 2: Yield Regressions

	Dependent variable is yield at time t								
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)
Sample starts	1970:1	1970:1	1970:1	1970:1	1970:1	1954:3	1954:3	1971:1	1987:1
Sample ends	2003:3	2003-3	2003:3	2003:3	2003:3	2003:3	1970:4	1986:4	2003:3
Maturity	2-year	5-year	10-year	10-year	10-year	10-year	10-year	10-year	10-year
Federal Fund Rate (t)	0.769	0.682	0.62	0.675	0.614	0.662	0.64	0.641	0.389
	0.062	*0.065*	*0.061*	*0.086*	*0.055*	*0.066*	*0.064*	*0.06*	*0.088*
Log(Spending / Taxes) /10 (t)	0.376	0.597	0.708			0.803	0.604	0.846	0.515
	0.092	*0.097*	*0.094*			*0.083*	*0.139*	*0.216*	*0.067*
Inflation (t-1 to t)	-0.044	-0.062	-0.075	-0.178	-0.119	-0.032	0.035	-0.145	0.222
	0.065	*0.072*	*0.073*	*0.11*	*0.069*	*0.072*	*0.078*	*0.075*	*0.102*
Help Wanted Index (t)	0.098	0.051	0.022	-0.178	0.036	0.087	-0.045	0.003	0.25
	0.076	*0.073*	*0.066*	*0.108*	*0.062*	*0.054*	*0.095*	*0.153*	*0.092*
Detrended Log(Real Spending) /10 (t)					0.509				
					0.133				
Detrended Log(Real Net Taxes) /10 (t)					-0.731				
					0.109				
Number of Observations	135	135	135	135	135	197	66	64	67
R^2 of OLS regression	0.91	0.87	0.854	0.73	0.862	0.914	0.924	0.847	0.865

Notes: Newey-West standard errors are displayed in italics under the regression coefficients. The error structure is assumed to be heteroskedastic and possibly autocorrelated up to a lag of 8 quarters.

Table 3: One Year Excess Return Regressions

	The dependent variable is the excess return between t and t+1						
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
Maturity	2-year	5-year	10-year	10-year	10-year	10-year	10-year
Federal Fund Rate (t)	0.306	0.683	1.11	0.551	0.581	0.449	-2.023
	0.069	*0.224*	*0.444*	*0.399*	*0.372*	*0.373*	*0.962*
Log(Spending / Taxes) /10 (t)	0.146	0.511	1.076	1.851	1.385	1.702	-0.518
	0.124	*0.391*	*0.719*	*0.71*	*0.753*	*0.707*	*0.737*
Inflation (t-1 to t)	-0.46	-1.336	-2.457	-1.695	-1.5	-2.033	-2.254
	0.082	*0.24*	*0.449*	*0.543*	*0.501*	*0.399*	*0.483*
Help Wanted Index (t)	-0.535	-1.318	-1.96				-2.165
	0.099	*0.336*	*0.678*				*0.651*
Growth Rate of Real GDP (t-1 to t)				-0.326			
				0.307			
Growth Rate of Real Non Durable Consumption (t-1 to t)					-0.612		
					0.325		
Index of Leading Indicators (t)						-2.518	
						1.133	
2-year Rate (t)							4.133
							1.302
Number of Observations	131	131	131	131	131	131	131
R^2	0.332	0.279	0.253	0.206	0.221	0.234	0.332

Notes: Robust standard errors are displayed in italics under the regression coefficients. Sample period is 1970:1 to 2003:3.

Table 4: MLE Estimates

$$r_t = 1.5812\% + \begin{pmatrix} 0.153 \\ -0.069 \\ 0.008 \\ 0.113 \\ 0.250 \end{pmatrix}' y_t + \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}' y_{t-1}, \text{ (per quarter)}$$

$$\Lambda_t = \begin{pmatrix} -0.014 & -0.006 & 0.038 & -0.017 & 0.011 \end{pmatrix}'$$

$$+ \begin{pmatrix} -0.449 & 0.048 & 0.200 & 1.441 & 0.092 \\ -0.077 & -0.202 & 0.039 & -0.392 & 0.169 \\ 0.495 & -0.841 & -0.869 & -0.396 & 0.184 \\ -0.222 & 0.141 & 0.270 & 0.705 & -0.414 \\ 0.118 & -0.146 & -0.200 & -0.339 & -0.351 \end{pmatrix} y_t + \begin{pmatrix} 0.163 & -0.147 & 0.148 & -0.874 & -0.027 \\ 0.032 & 0.281 & 0.009 & 0.433 & -0.154 \\ -0.030 & 0.017 & 0.234 & -0.488 & 0.001 \\ 0.034 & 0.167 & 0.028 & -0.288 & 0.228 \\ 0.060 & 0.035 & 0.086 & 0.015 & 0.179 \end{pmatrix} y_{t-1}$$

$$y_{t+1} = \begin{pmatrix} 0.530 & 0.005 & 0.058 & 1.296 & 0.352 \\ -0.022 & 0.686 & -0.003 & -0.477 & 0.180 \\ 0.170 & -0.071 & 0.631 & 0.384 & -0.303 \\ -0.126 & -0.141 & 0.015 & 1.355 & -0.090 \\ 0.055 & 0.028 & -0.051 & -0.131 & 0.463 \end{pmatrix} y_t + \begin{pmatrix} 0.163 & -0.147 & 0.148 & -0.874 & -0.027 \\ 0.032 & 0.281 & 0.009 & 0.433 & -0.154 \\ -0.030 & 0.017 & 0.234 & -0.488 & 0.001 \\ 0.034 & 0.167 & 0.028 & -0.288 & 0.228 \\ 0.060 & 0.035 & 0.086 & 0.015 & 0.179 \end{pmatrix} y_{t-1}$$

$$+ \begin{pmatrix} 2.230 & \mathbf{-0.195} & \mathbf{0.139} & \mathbf{0.175} & \cdot \\ -0.131 & 0.657 & \mathbf{-0.184} & \mathbf{-0.209} & \cdot \\ 0.306 & -0.353 & 2.155 & \mathbf{-0.031} & \cdot \\ 0.121 & -0.124 & -0.059 & 0.668 & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0.842 \end{pmatrix} \epsilon_{t+1} \times 10^{-2}$$

$R =$ (bp)

Q	1	2	4	8	16	20	30	40
1	47.165	**0.849**	**0.304**	**−0.411**	**−0.838**	**−0.766**	**−0.550**	**−0.406**
2	27.933	17.394	**0.701**	**−0.121**	**−0.933**	**−0.944**	**−0.771**	**−0.626**
4	7.263	19.998	10.854	**0.567**	**−0.582**	**−0.806**	**−0.880**	**−0.825**
8	−7.317	7.687	12.962	6.021	**0.298**	**−0.080**	**−0.481**	**−0.600**
16	−13.421	−6.733	0.867	4.554	3.122	**0.917**	**0.622**	**0.443**
20	−13.823	−10.028	−4.300	1.009	3.597	1.282	**0.878**	**0.753**
30	−13.640	−14.261	−12.588	−6.497	2.679	3.304	2.581	**0.971**
40	−12.902	−16.932	−17.893	−11.975	2.695	7.245	5.379	2.248

Parameters that are fixed to 0 are represented by a "·". The lower triangle of the volatility matrix for y_t contains the Cholesky decomposition of its conditional covariance matrix and the upper triangle contains the correlation matrix (which is not scaled by $10^{-}2$). Similarly, the lower triangle of R represents the Cholesky decomposition of the covariance matrix of the measurement errors, and the upper triangle represents the correlation matrix.

Table 5: t-Ratios

$$
\begin{aligned}
r_t &= \cdot + \begin{pmatrix} 10.755 \\ -1.848 \\ \cdot \\ 6.039 \\ \cdot \end{pmatrix}' y_t + \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}' y_{t-1} \\
\Lambda_t &= \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}' \\
&+ \begin{pmatrix} -3.596 & 0.306 & 1.801 & 3.416 & 0.789 \\ -2.233 & -2.141 & 1.037 & -3.356 & 1.544 \\ 1.108 & -1.294 & -2.401 & -0.491 & 0.481 \\ -1.182 & 0.434 & 1.453 & 2.233 & -1.526 \\ 0.942 & -0.760 & -1.652 & -1.817 & -2.455 \end{pmatrix} y_t + \begin{pmatrix} \cdot & \cdot & \cdot & -1.990 & \cdot \\ \cdot & 5.873 & \cdot & 3.903 & -1.434 \\ \cdot & \cdot & 1.732 & -1.226 & \cdot \\ \cdot & \cdot & \cdot & -2.867 & 1.631 \\ \cdot & \cdot & 1.794 & \cdot & \cdot \end{pmatrix} y_{t-1} \\
y_{t+1} &= \begin{pmatrix} 5.445 & \cdot & 0.603 & 2.994 & \cdot \\ \cdot & 14.367 & \cdot & -4.298 & 1.671 \\ 2.138 & \cdot & 5.556 & 0.965 & -1.768 \\ -4.887 & \cdot & 0.691 & 13.505 & -0.673 \\ 1.784 & \cdot & -1.063 & -2.098 & 5.883 \end{pmatrix} y_t + \begin{pmatrix} \cdot & \cdot & \cdot & -1.990 & \cdot \\ \cdot & 5.873 & \cdot & 3.903 & -1.434 \\ \cdot & \cdot & 1.732 & -1.226 & \cdot \\ \cdot & \cdot & \cdot & -2.867 & 1.631 \\ \cdot & \cdot & 1.794 & \cdot & \cdot \end{pmatrix} y_{t-1} \\
&+ \begin{pmatrix} 10.263 & \mathbf{-12.817} & \mathbf{11.920} & \mathbf{13.055} & \cdot \\ \cdot & 12.330 & \mathbf{-1.345} & \mathbf{-1.610} & \cdot \\ \cdot & -1.142 & 11.339 & \mathbf{-0.903} & \cdot \\ \cdot & -1.294 & \cdot & 12.963 & \cdot \\ \cdot & \cdot & \cdot & \cdot & 6.984 \end{pmatrix} \epsilon_{t+1} \\
R &= \left(\begin{array}{c|rrrrrrrr} \mathbf{Q} & \mathbf{1} & \mathbf{2} & \mathbf{4} & \mathbf{8} & \mathbf{16} & \mathbf{20} & \mathbf{30} & \mathbf{40} \\ \hline \mathbf{1} & 9.589 & \mathbf{23.713} & \mathbf{6.263} & \mathbf{-7.314} & \mathbf{-43.835} & \mathbf{-12.416} & \mathbf{-8.174} & \mathbf{-6.640} \\ \mathbf{2} & 13.389 & 7.535 & \mathbf{16.265} & \mathbf{-0.730} & \mathbf{-71.397} & \mathbf{-30.189} & \mathbf{-6.955} & \mathbf{-4.255} \\ \mathbf{4} & \cdot & 4.512 & 5.298 & \mathbf{3.707} & \mathbf{-17.814} & \mathbf{-20.232} & \mathbf{-13.409} & \mathbf{-7.640} \\ \mathbf{8} & \cdot & 1.575 & 4.487 & 5.227 & \mathbf{2.196} & \mathbf{-0.477} & \mathbf{-3.847} & \mathbf{-5.506} \\ \mathbf{16} & -14.424 & \cdot & \cdot & 5.852 & 19.126 & \mathbf{34.377} & \mathbf{6.480} & \mathbf{3.543} \\ \mathbf{20} & -13.097 & -3.471 & -1.914 & \cdot & 14.654 & 15.679 & \mathbf{24.335} & \mathbf{11.257} \\ \mathbf{30} & -14.685 & -2.096 & -2.125 & -3.699 & \cdot & \cdot & \cdot & \mathbf{116.015} \\ \mathbf{40} & \cdot & -1.582 & -1.935 & -3.619 & \cdot & 12.465 & 7.338 & \cdot \end{array} \right) \\
\log L &= -59.28344
\end{aligned}
$$

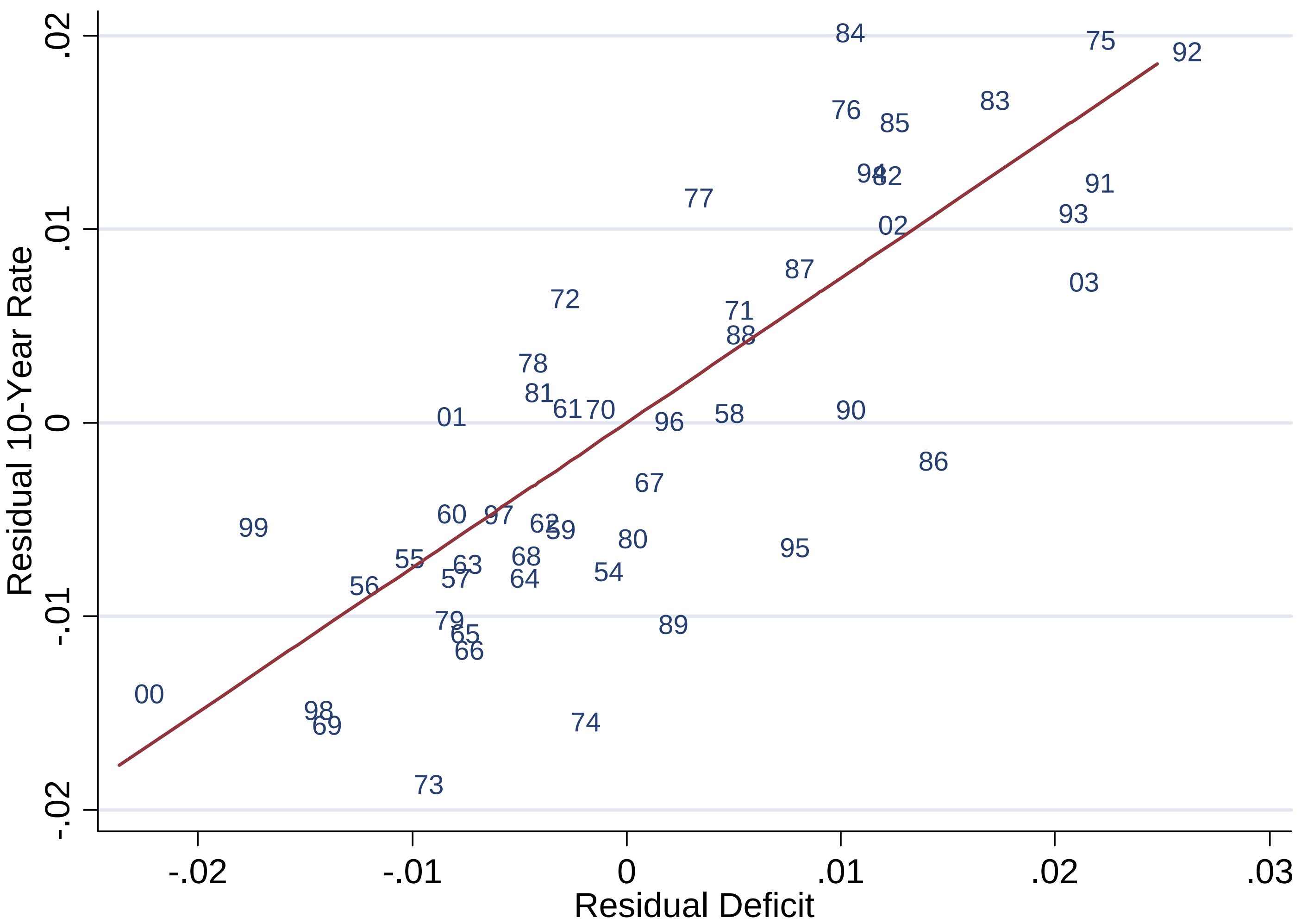
Fixed parameters are represetned by a “$\cdot$”. Some of these parameters are fixed as a normalization, and some are fixed to their MLE point estimates because their t-ratios are relatively small even if they are free.

Table 6: Yield Regressions with Lagged Rate

	Dependent variable is yield at time t			
	(i)	(ii)	(iii)	(iv)
Sample starts	1954:3	1954:3	1971:1	1987:1
Sample ends	2003:3	1970:4	1986:4	2003:3
Maturity	10-year	10-year	10-year	10-year
Federal Fund Rate (t)	0.159	0.147	0.185	-0.019
	0.03	*0.062*	*0.042*	*0.077*
Log(Spending / Taxes) /10 (t)	0.117	0.103	0.098	0.107
	0.04	*0.096*	*0.103*	*0.069*
Inflation (t-1 to t)	0.019	-0.005	-0.001	0.161
	0.018	*0.035*	*0.038*	*0.064*
Help Wanted Index (t)	0.027	0.045	0.021	0.176
	0.021	*0.053*	*0.046*	*0.048*
10-year Rate (t-1)	0.764	0.737	0.753	0.763
	0.044	*0.08*	*0.068*	*0.101*
Number of Observations	196	65	64	67
R^2	0.976	0.972	0.953	0.934

Notes: Robust standard errors are displayed in italics under the regression coefficients.

Figure 1: Deficit and Long Term Rate, 1954-2003



Notes: 10-year rate is the yield on a 10-year treasury bond. Deficit is the log of Government Purchases of Goods and Services over Net Taxes (Taxes minus Transfers). The residuals are obtained by first regressing each variable on the federal fund rate, the help wanted index and inflation (measured by GDP deflator). Each point represents one year.

Figure 2: Yield Loadings

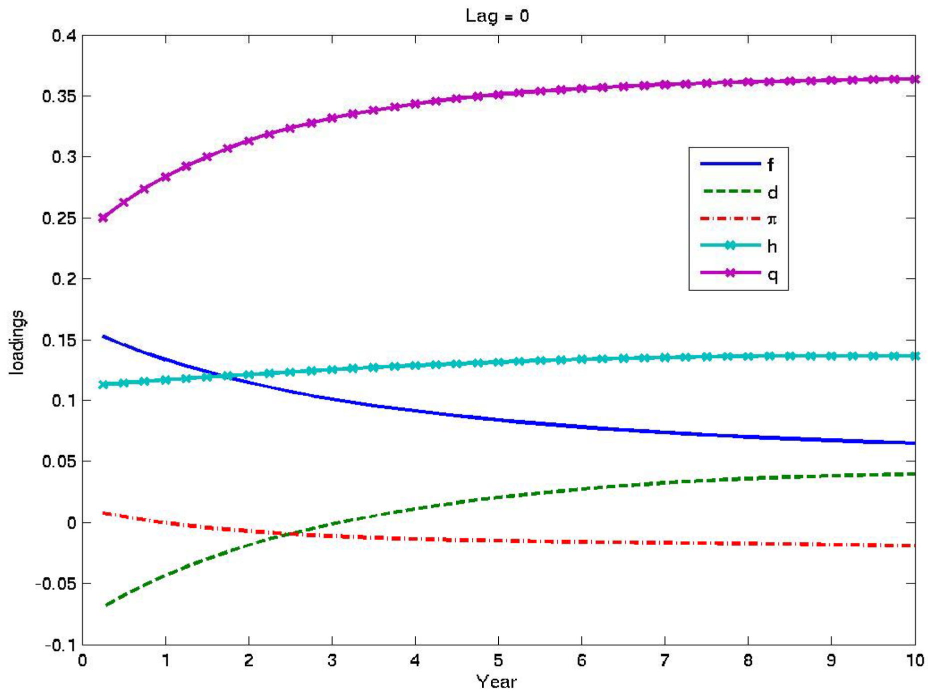


Figure 3: Moments of Yields

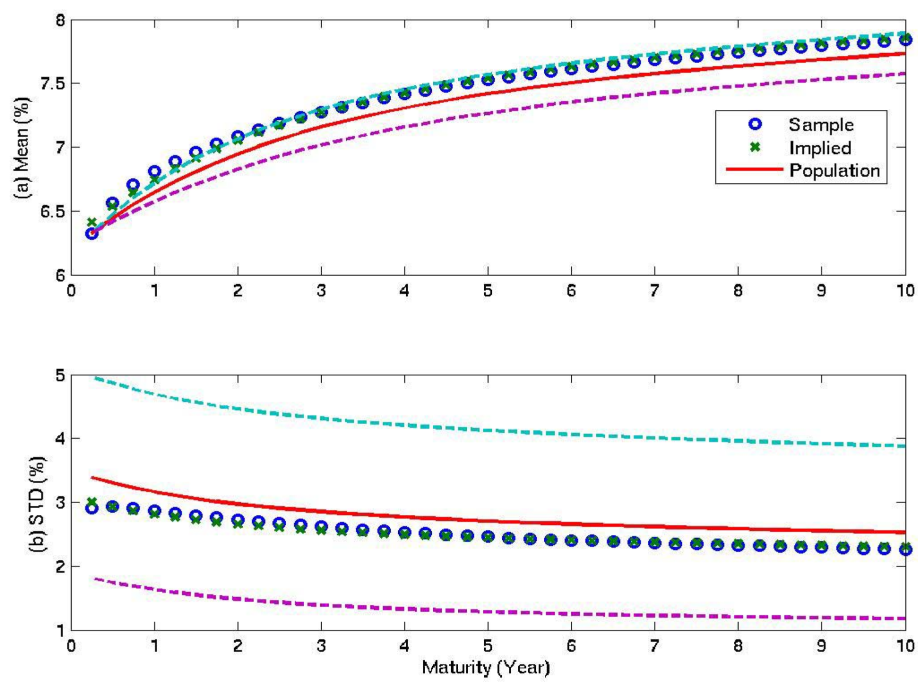


Figure 4: Variance Decomposition of Yields

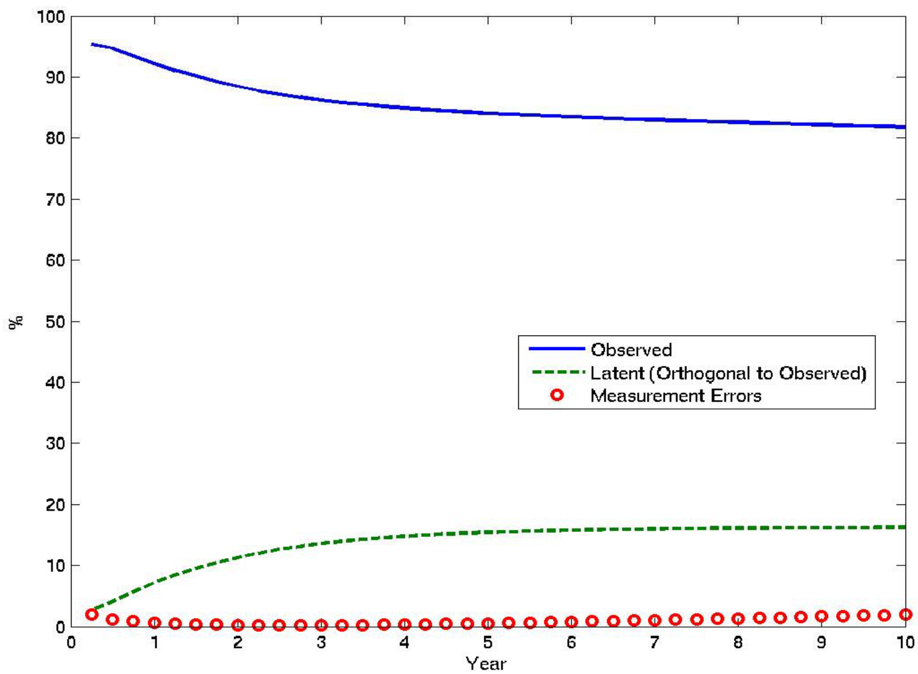


Figure 5: Campbell-Shiller

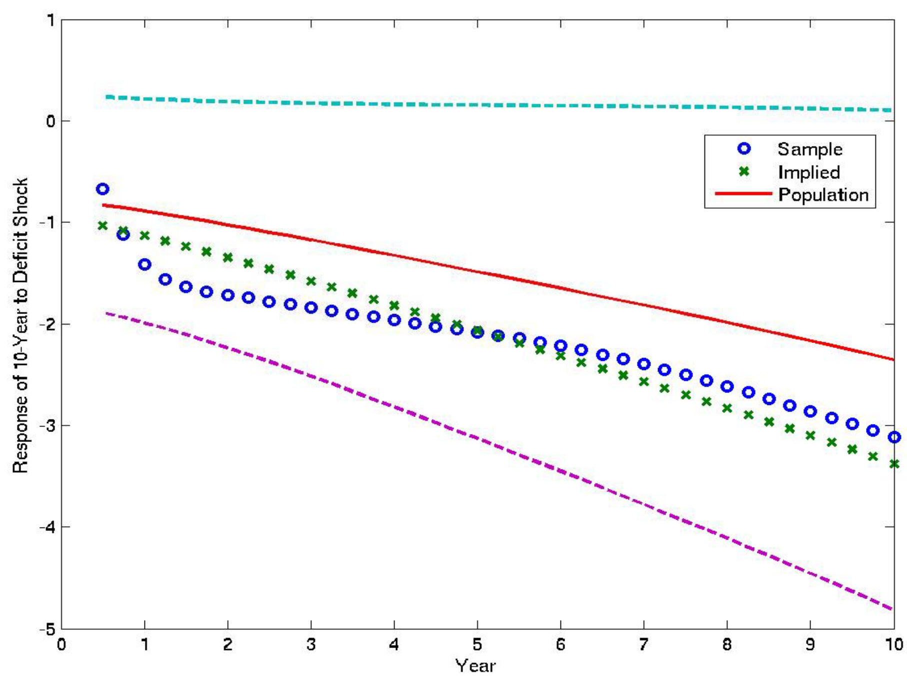


Figure 6: Response of State Variables to Deficit Shock

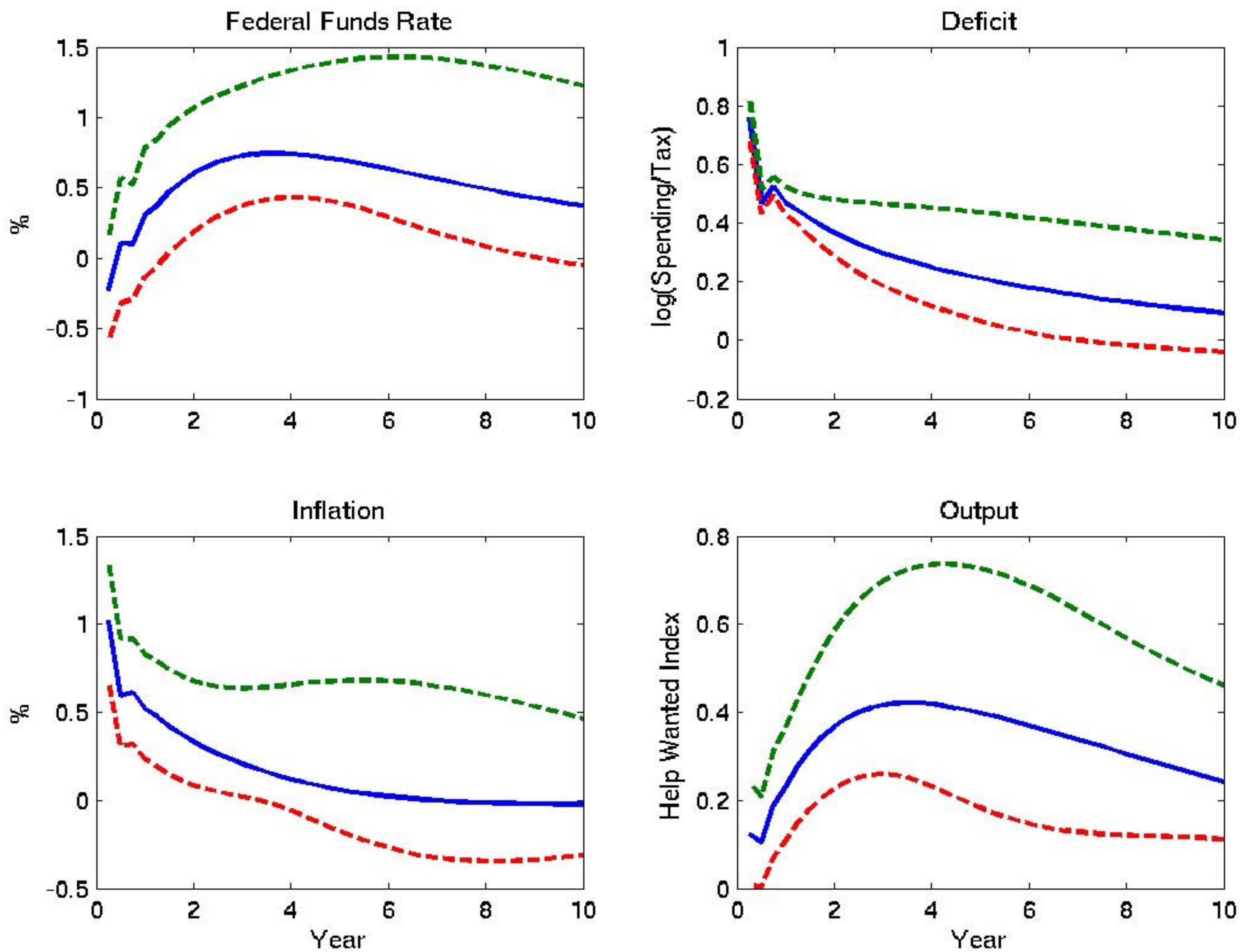


Figure 7: Response of 10-Year Rate to Deficit Shock

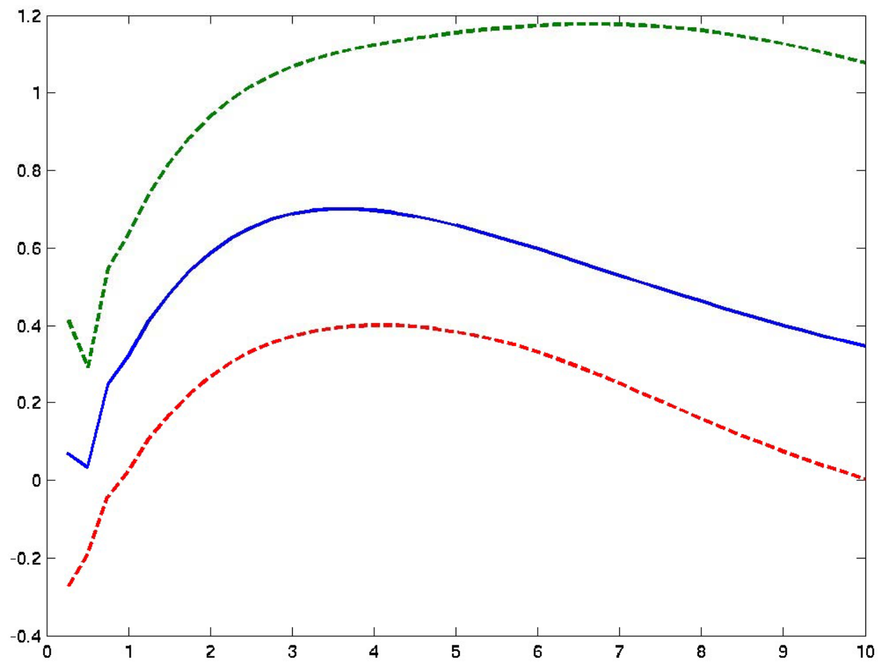


Figure 8: Decomposition of the 10-Year Rate Response

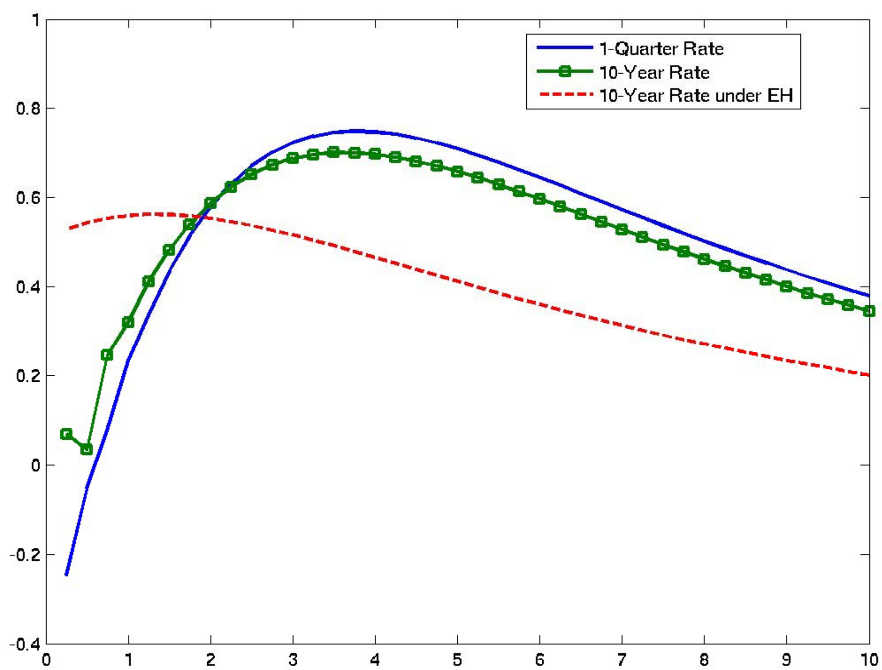


Figure 9 : Variance of Yields due to Identified Fiscal Shocks

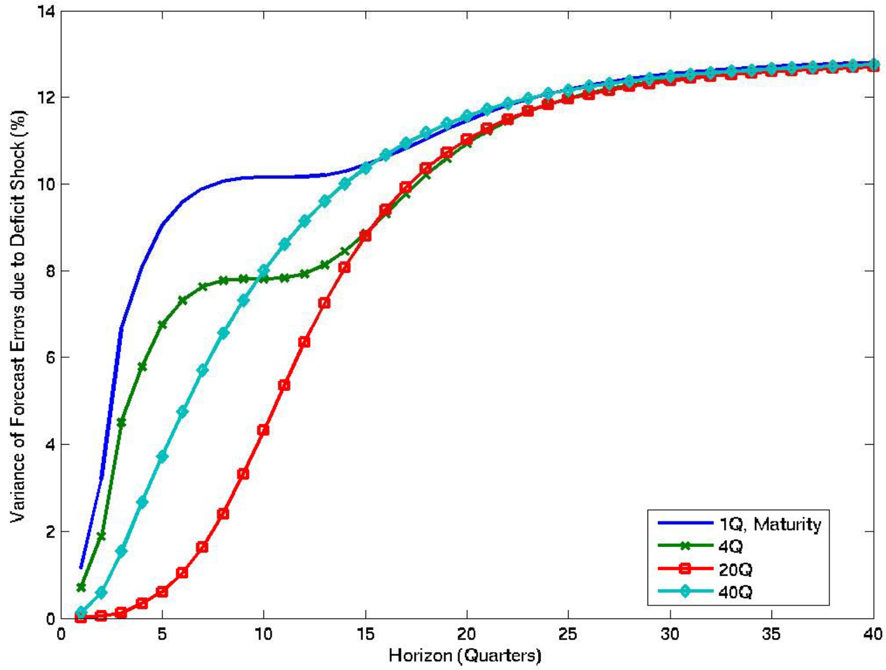


Figure 10: Response of 10-Year Rate to Spending and Taxes