THE TRANSACTIONS DEMAND FOR CASH: AN INVENTORY THEORETIC APPROACH

By William J. Baumol

A stock of cash is its holder's inventory of the medium of exchange, and like an inventory of a commodity, cash is held because it can be given up at the appropriate moment, serving then as its possessor's part of the bargain in an exchange. We might consequently expect that inventory theory and monetary theory can learn from one another. This note attempts to apply one well-known result in inventory control analysis to the theory of money.[1]

I. A Simple Model

We are now interested in analyzing the transactions demand for cash dictated by rational behavior, which for our purposes means the holding of those cash balances that can do the job at minimum cost. To abstract from precautionary and speculative demands let us consider a state in which transactions are perfectly foreseen and occur *in a steady stream.*

Suppose that in the course of a given period an individual will pay out T dollars in a steady stream. He obtains cash either by borrowing it, or by withdrawing it from an investment, and in either case his interest cost (or interest opportunity cost) is i dollars per dollar per period. Suppose finally that he withdraws cash in lots of C dollars spaced evenly throughout the year, and that each time he makes such a withdrawal he must pay a fixed "broker's fee" of b

1. T. M. Whitin informs me that the result in question goes back to the middle of the 1920's when it seems to have been arrived at independently by some half dozen writers. See, e.g., George F. Mellen, "Practical Lot Quantity Formula," *Management and Administration*, Vol. 10, September 1925. Its significant implications for the economic theory of inventory, particularly for business cycle theory, seem to have gone unrecognized until recently when Dr. Whitin analyzed them in his forthcoming *Inventory Control and Economic Theory* (Princeton University Press) which, incidentally, first suggested the subject of this note to me. See also, Dr. Whitin's "Inventory Control in Theory and Practice" (elsewhere in this issue, *supra*, p. 502), and Kenneth J. Arrow, Theodore Harris, and Jacob Marschak, "Optimal Inventory Policy," *Econometrica*, Vol. 19, July 1951, especially pp. 252–255. In addition to Dr. Whitin, I am heavily indebted to Professors Chandler, Coale, Gurley, Lutz, Mr. Turvey, and Professor Viner, and to the members of the graduate seminar at Harvard University, where much of this paper was first presented.

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dollars.[2] Here T, the value of transactions, is predetermined, and i and b are assumed to be constant.

In this situation any value of C less than or equal to T will enable him to meet his payments equally well provided he withdraws the money often enough. For example, if T is $100, he can meet his payments by withdrawing $50 every six months or $25 quarterly, etc.[3] Thus he will make $\frac{T}{C}$ withdrawals over the course of the year, at a total cost in "brokers' fees" given by $\frac{bT}{C}$.

In this case, since each time he withdraws C dollars he spends it in a steady stream and draws out a similar amount the moment it is gone, his average cash holding will be $\frac{C}{2}$ dollars. His annual interest cost of holding cash will then be $\frac{iC}{2}$.

The total amount the individual in question must pay for the use of the cash needed to meet his transaction when he borrows C dollars at intervals evenly spaced throughout the year will then be the sum of interest cost and "brokers' fees" and so will be given by

$$\frac{bT}{C} + \frac{iC}{2}. \tag{1}$$

2. The term "broker's fee" is not meant to be taken literally. It covers all non-interest costs of borrowing or making a cash withdrawal. These include opportunity losses which result from having to dispose of assets just at the moment the cash is needed, losses involved in the poor resale price which results from an asset becoming "secondhand" when purchased by a nonprofessional dealer, administrative costs, and psychic costs (the trouble involved in making a withdrawal) as well as payment to a middleman. So conceived it seems likely that the "broker's fee" will, in fact, vary considerably with the magnitude of the funds involved, contrary to assumption. However, *some* parts of this cost will not vary with the amount involved — e.g., postage cost, bookkeeping expense, and, possibly, the withdrawer's effort. It seems plausible that the "broker's fee" will be better approximated by a function like $b + kC$ (where b and k are constants), which indicates that there is a part of the "broker's fee" increasing in proportion with the amount withdrawn. As shown in a subsequent footnote, however, our formal result is completely unaffected by this amendment.

We must also extend the meaning of the interest rate to include the value of protection against loss by fire, theft, etc., which we obtain when someone borrows our cash. On the other hand, a premium for the risk of default on repayment must be deducted. This protection obtained by lending seems to be mentioned less frequently by theorists than the risk, yet how can we explain the existence of interest free demand deposits without the former?

3. In particular, if cash were perfectly divisible and no elapse of time were required from withdrawal through payment he could make his withdrawals in a steady stream. In this case he would never require any cash balances to meet his payments and C would be zero. However, as may be surmised, this would be prohibitive with any b greater than zero.

Since the manner in which he meets his payments is indifferent to him, his purpose only being to pay for his transactions, rationality requires that he do so at minimum cost, i.e., that he choose the most economical value of C. Setting the derivative of (1) with respect to C equal to zero we obtain[4]

$$-\frac{bT}{C^2}+\frac{i}{2}=0,$$

i.e.,

$$C=\sqrt{\frac{2bT}{i}}. \tag{2}$$

Thus, in the simple situation here considered, the rational individual will, given the price level,[5] demand cash in proportion to the square root of the value of his transactions.

Before examining the implications of this crude model we may note that, as it stands, it applies to two sorts of cases: that of the individual (or firm) obtaining cash from his invested capital and that of the individual (or firm) spending out of borrowing in anticipation of future receipts. Since our problem depends on non-coincidence of cash receipts and disbursements, and we have assumed that cash disbursements occur in a steady stream, one other case seems possible, that where receipts precede expenditures. This differs from the first case just mentioned (living off one's capital) in that the individual now has the option of withholding some or all of his receipts from investment and simply keeping the cash until it is needed. Once this withheld cash is used up the third case merges into the first: the individual must obtain cash from his invested capital until his next cash receipt occurs.

We can deal with this third case as follows. First, note that any receipts exceeding anticipated disbursements will be invested, since, eventually, interest earnings must exceed ("brokerage") cost of investment. Hence we need only deal with that part of the cash influx which is to be used in making payments during the period

4. This result is unchanged if there is a part of the "broker's fee" which varies in proportion with the quantity of cash handled. For in this case the "broker's fee" for each loan is given by $b + kC$. Total cost in "broker's fees" will then be

$$\frac{T}{C}(b+kC)=\frac{T}{C}b+kT.$$

Thus (1) will have the constant term, kT, added to it, which drops out in differentiation.

5. A doubling of *all* prices (including the "broker's fee") is like a change in the monetary unit, and may be expected to double the demand for cash balances.

between receipts. Let this amount, as before, be T dollars. Of this let I dollars be invested, and the remainder, R dollars, be withheld, where either of these sums may be zero. Again let i be the interest rate, and let the "broker's fee" for withdrawing cash be given by the linear expression $b_w + k_w C$, where C is the amount withdrawn. Finally, let there be a "broker's fee" for investing (depositing) cash given by $b_d + k_d I$ where the b's and the k's are constants.

Since the disbursements are continuous, the $R = T - I$ dollars withheld from investment will serve to meet payments for a fraction of the period between consecutive receipts given by $\frac{T-I}{T}$. Moreover, since the average cash holding for that time will be $\frac{T-I}{2}$, the interest cost of withholding that money will be $\frac{T-I}{T} i \frac{T-I}{2}$. Thus the total cost of withholding the R dollars and investing the I dollars will be

$$\frac{T-I}{2} i \frac{T-I}{T} + b_d + k_d I.$$

Analogously, the total cost of obtaining cash for the remainder of the period will be

$$\frac{C}{2} i \frac{I}{T} + (b_w + k_w C) \frac{I}{C}.$$

Thus the total cost of cash operations for the period will be given by the sum of the last two expressions, which when differentiated partially with respect to C and set equal to zero once again yields our square root formula, (2), with $b = b_w$.

Thus, in this case, the optimum cash balance after the initial cash holding is used up will again vary with the square root of the volume of transactions, as is to be expected by analogy with the "living off one's capital" case.

There remains the task of investigating $R/2$, the (optimum) average cash balance before drawing on invested receipts begins. We again differentiate our total cost of holding cash, this time partially with respect to I, and set it equal to zero, obtaining

$$-\frac{T-I}{T} i + k_d + \frac{Ci}{2T} + \frac{b_w}{C} + k_w = 0,$$

i.e.,

$$R = T - I = \frac{C}{2} + \frac{b_w T}{Ci} + \frac{T(k_d + k_w)}{i},$$

or since from the preceding result, $C^2 = 2Tb_w/i$, so that the second term on the right hand side equals $C^2/2C$,

$$R = C + T\left(\frac{k_w + k_d}{i}\right).$$

The first term in this result is to be expected, since if *everything* were deposited at once, C dollars would have to be withdrawn at that same moment to meet current expenses. On this amount two sets of "broker's fees" would have to be paid and no interest would be earned — a most unprofitable operation.[6]

Since C varies as the square root of T and the other term varies in proportion with T, R will increase less than in proportion with T, though more nearly in proportion than does C. The general nature of our results is thus unaffected.[7]

Note finally that the entire analysis applies at once to the case of continuous receipts and discontinuous payments, taking the period to be that between two payments, where the relevant decision is the frequency of investment rather than the frequency of withdrawal. Similarly, it applies to continuous receipts and payments where the two are not equal.

II. Some Consequences of the Analysis

I shall not labor the obvious implications for financial budgeting by the firm. Rather I shall discuss several arguments which have been presented by monetary theorists, to which our result is relevant.

The first is the view put forth by several economists,[8] that in a

6. Here the assumption of constant "brokerage fees" with $k_d = k_w = 0$ gets us into trouble. The amount withheld from investment then is never greater than C dollars only because a strictly constant "broker's fee" with no provision for a discontinuity at zero implies the payment of the fee even if nothing is withdrawn or deposited. In this case it becomes an overhead and it pays to invest for any interest earning greater than zero.

For a firm, *part* of the "broker's fee" may, in fact, be an overhead in this way. For example, failure to make an anticipated deposit will sometimes involve little or no reduction in the bookkeeping costs incurred in keeping track of such operations.

7. If we replace the linear functions representing the "broker's fees" with more general functions $f_w(C)$ and $f_d(I)$ which are only required to be differentiable, the expression obtained for R is changed merely by replacement of k_w, and k_d by the corresponding derivatives $f_w'(C)$ and $f_d'(I)$.

8. See, e.g., Frank H. Knight, *Risk, Uncertainty and Profit* (Preface to the Re-issue), No. 16 in the series of Reprints of Scarce Tracts in Economic and Political Science (London: The London School of Economics and Political Science, 1933), p. xxii; F. Divisia, *Économique Rationelle* (Paris: G. Doin, 1927), chap. XIX and the Appendix; and Don Patinkin, "Relative Prices, Say's Law and the Demand for Money," *Econometrica*, Vol. 16, April 1948, pp. 140–145. See also, P. N. Rosenstein-Rodan, "The Coordination of the General Theories of Money and Price," *Economica*, N. S., Vol. III, August 1936, Part II.

stationary state there will be no demand for cash balances since it will then be profitable to invest all earnings in assets with a positive yield in such a way that the required amount will be realized at the moment any payment is to be made. According to this view no one will want any cash in such a stationary world, and the value of money must fall to zero so that there can really be no such thing as a truly static monetary economy. Clearly this argument neglects the transactions costs involved in making and collecting such loans (the "broker's fee").[9] Our model is clearly compatible with a static world and (2) shows that it will generally pay to keep some cash. The analysis of a stationary monetary economy in which there is a meaningful (finite) price level does make sense.

Another view which can be reëxamined in light of our analysis is that the transactions demand for cash will vary approximately in proportion with the money value of transactions.[1] This may perhaps even be considered the tenor of quantity theory though there is no necessary connection, as Fisher's position indicates. If such a demand for cash balances is considered to result from rational behavior, then (2) suggests that the conclusion cannot have general validity. On the contrary, the square root formula implies that

9. It also neglects the fact that the transfer of cash takes time so that in reality we would have to hold cash at least for the short period between receiving it and passing it on again.

It is conceivable, it is true, that with perfect foresight the difference between money and securities might disappear since a perfectly safe loan could become universally acceptable. There would, however, remain the distinction between "real assets" and the "money-securities." Moreover, there would be a finite price for, and non-zero yield on the former, the yield arising because they (as opposed to certificates of their ownership) are not generally acceptable, and hence not perfectly liquid, since there is trouble and expense involved in carrying them.

1. Marshall's rather vague statements may perhaps be interpreted to support this view. See, e.g., Book I, chap. IV in *Money, Credit and Commerce* (London, 1923). Keynes clearly accepts this position. See *The General Theory of Employment, Interest and Money* (New York, 1936), p. 201. It is also accepted by Pigou: "As real income becomes larger, there is, prima facie, reason for thinking that, just as, up to a point, people like to invest a larger proportion of their real income, so also they like to hold real balances in the form of money equivalent to a larger proportion of it. On the other hand, as Professor Robertson has pointed out to me, the richer people are, the cleverer they are likely to become in finding a way to *economize* in real balances. On the whole then we may, I think, safely disregard this consideration . . . for a close approximation. . . . ." *Employment and Equilibrium*, 1st ed. (London, 1941), pp. 59–60. Fisher, however, argues: "It seems to be a fact that, at a given price level, the greater a man's expenditures the more rapid his turnover; that is, the rich have a higher rate of turnover than the poor. They spend money faster, not only absolutely but relatively to the money they keep on hand. . . . We may therefore infer that, if a nation grows richer per capita, the velocity of circulation of money will increase. This proposition, of course, has no reference to *nominal* increase of expenditure." *The Purchasing Power of Money* (New York, 1922), p. 167.

demand for cash rises less than in proportion with the volume of transactions, so that there are, in effect, economies of large scale in the use of cash.

The magnitude of this difference should not be exaggerated, however. The phrase "varying as the square" may suggest larger effects than are actually involved. Equation (2) requires that the average transactions velocity of circulation vary exactly in proportion with the quantity of cash, so that, for example, a doubling of the stock of cash will *ceteris paribus*, just double velocity.[2]

A third consequence of the square root formula is closely connected with the second. The effect on real income of an injection of cash into the system may have been underestimated. For suppose that (2) is a valid expression for the general demand for cash, that there is widespread unemployment, and that for this or other reasons prices do not rise with an injection of cash. Suppose, moreover, that the rate of interest is unaffected, i.e., that none of the new cash is used to buy securities. Then so long as transactions do not rise so as to maintain the same proportion with the square of the quantity of money, people will want to get rid of cash. They will use it to demand more goods and services, thereby forcing the volume of transactions to rise still further. For let ΔC be the quantity of cash injected. If a proportionality (constant velocity) assumption involves transactions rising by $k \Delta C$, it is easily shown that (2) involves transactions rising by more than twice as much, the magnitude of the excess increasing with the ratio of the injection to the initial stock of cash. More precisely, the rise in transactions would then be given by[3]

$$2\,k\,\Delta C + \frac{k}{C}\Delta C^2.$$

Of course, the rate of interest would really tend to fall in such circumstances, and this would to some extent offset the effect of the influx of cash, as is readily seen when we rewrite (2) as

$$T = C^2\, i/2b. \tag{3}$$

Moreover, prices will rise to some extent,[4] and, of course, (3) at best

2. Since velocity equals $\frac{T}{C} = \frac{i}{2b} C$ by (2).

3. This is obtained by setting $k = C\, i/2b$ in (3), below, and computing ΔT by substituting $C + \Delta C$ for C.

4. Even if (2) holds, the demand for cash may rise only in proportion with the money value of transactions when all prices rise exactly in proportion, the rate of interest and transactions remaining unchanged. For then a doubling of all prices and cash balances leaves the situation unchanged, and the received argument holds. The point is that b is then one of the prices which has risen.

is only an approximation. Nevertheless, it remains true that the effect of an injection of cash on, say, the level of employment, may often have been underestimated.[5] For whatever may be working to counteract it, the force making for increased employment is greater than if transactions tend, *ceteris paribus*, toward their original proportion to the quantity of cash.

Finally the square root formula lends support to the argument that wage cuts can help increase employment, since it follows that the Pigou effect and the related effects are stronger than they would be with a constant transactions velocity. Briefly the phenomenon which has come to be called the Pigou effect[6] may be summarized thus: General unemployment will result in reduction in the price level which must increase the purchasing power of the stock of cash provided the latter does not itself fall more than in proportion with prices.[7] This increased purchasing power will augment demand for commodities[8] or investment goods (either directly, or because it is used to buy securities and so forces down the rate of interest). In any case, this works for a reduction in unemployment.

Now the increase in the purchasing power of the stock of cash which results from fallen prices is equivalent to an injection of cash with constant prices. There is therefore exactly the same reason for suspecting the magnitude of the effect of the former on the volume of transactions has been underestimated, as in the case of the latter. Perhaps this can be of some little help in explaining why there has not been more chronic unemployment or runaway inflation in our economy.

III. The Simple Model and Reality

It is appropriate to comment on the validity of the jump from equation (2) to conclusions about the operation of the economy. At

5. But see the discussions of Potter and Law as summarized by Jacob Viner, *Studies in the Theory of International Trade* (New York, 1937), pp. 37–39.

6. See A. C. Pigou, "The Classical Stationary State," *Economic Journal*, Vol. LIII, December 1943.

7. Presumably the "broker's fee" will be one of the prices which falls, driven down by the existence of unemployed brokers. There is no analogous reason for the rate of interest to fall, though it will tend to respond thus to the increase in the "real stock of cash."

8. The term "Pigou effect" is usually confined to the effects on consumption demand while the effect on investment demand, and (in particular) on the rate of interest is ordinarily ascribed to Keynes. However, the entire argument appears to antedate Pigou's discussion (which, after all, was meant to be a reformulation of the classical position) and is closely related to what Mr. Becker and I have called the Say's Equation form of the Say's Law argument. See our article "The Classical Monetary Theory; the Outcome of the Discussion," *Economica*, November 1952.

best, (2) is only a suggestive oversimplification, if for no other reason, because of the rationality assumption employed in its derivation. In addition the model is static. It takes the distribution of the firm's disbursements over time to be fixed, though it is to a large extent in the hands of the entrepreneur how he will time his expenditures. It assumes that there is one constant relevant rate of interest and that the "broker's fee" is constant or varies linearly with the magnitude of the sum involved. It posits a steady stream of payments and the absence of cash receipts during the relevant period. It deals only with the cash demand of a single economic unit and neglects interactions of the various demands for cash in the economy.[9] It neglects the precautionary and speculative demands for cash.

These are serious lacunae, and without a thorough investigation we have no assurance that our results amount to much more than an analytical curiosum. Nevertheless I offer only a few comments in lieu of analysis, and hope that others will find the subject worth further examination.

1. It is no doubt true that a majority of the public will find it impractical and perhaps pointless to effect every possible economy in the use of cash. Indeed the possibility may never occur to most people. Nevertheless, we may employ the standard argument that the largest cash users may more plausibly be expected to learn when it is profitable to reduce cash balances relative to transactions. The demand for cash by the community as a whole may then be affected similarly and by a significant amount. Moreover, it is possible that even small cash holders will sometimes institute some cash economies instinctively or by a process of trial and error not explicitly planned or analyzed.

2. With variable b and i the validity of our two basic results — the non-zero rational transactions demand for cash, and the less than proportionate rise in the rational demand for cash with the real volume of transactions, clearly depends on the nature of the responsiveness of the "brokerage fee" and the interest rate to the quantity of cash involved. The first conclusion will hold generally provided the "broker's fee" never falls below some preassigned level, e.g., it never falls below one mill per transaction, and provided the interest rate, its rate of change with C and the rate of change of the "broker's fee" all (similarly) have some upper bound, however large, at least when C is small.

9. I refer here particularly to considerations analogous to those emphasized by Duesenberry in his discussion of the relation between the consumption functions of the individual and the economy as a whole in his *Income, Saving and the Theory of Consumer Behavior* (Cambridge, Mass., 1950).

The second conclusion will not be violated persistently unless the "brokerage fee" tends to vary almost exactly in proportion with C (and it pays to hold zero cash balances) except for what may roughly be described as a limited range of values of C. Of course, it is always possible that this "exceptional range" will be the one relevant in practice. Variations in the interest rate will tend to strengthen our conclusion provided the interest rate never decreases with the quantity of cash borrowed or invested.[1]

It would perhaps not be surprising if these sufficient conditions for the more general validity of our results were usually satisfied in practice.

3. If payments are lumpy but foreseen, cash may perhaps be employed even more economically. For then it may well pay to obtain cash just before large payments fall due with little or no added cost in "brokers' fees" and considerable savings in interest payments. The extreme case would be that of a single payment during the year

1. For people to want to hold a positive amount of cash, the cost of cash holding must be decreasing after $C = 0$. Let b in (1) he a differentiable function of C for $C > 0$ (it will generally be discontinuous and equal to zero at $C = 0$). Then we require that the limit of the derivative of (1) be negative as C approaches zero from above, where this derivative is given by

$$\text{(i)} \quad -b\frac{T}{C^2} + \frac{T}{C}b' + \frac{i + i'C}{2}.$$

Clearly this will become negative as C approaches zero provided b is bounded from below and b', i, and i' are all bounded from above.

The second conclusion, the less than proportionate rise in minimum cost cash holdings with the volume of transactions, can be shown, with only b not constant, to hold if and only if $b - b'C + b''C^2$ is positive. This result is obtained by solving the first order minimum condition (obtained by setting (i), with the i' term omitted, equal to zero) for $\frac{T}{C}$ and noting that our conclusion is equivalent to the derivative of this ratio with respect to C being positive.

Now successive differentiation of (i) with the i' term omitted yields as our second order minimum condition $2(b - b'C) + b''C^2 > 0$ (note the resemblance to the preceding condition). Thus if our result is to be violated we must have

$$\text{(ii)} \quad b - Cb' \leqq -b''C^2 < 2(b - Cb'),$$

which at once yields $b'' \leq 0$. Thus if b' is not to become negative (a decreasing *total* payment as the size of the withdrawal increases!) b'' must usually lie within a small neighborhood of zero, i.e., b must be approximately linear. However we know that in this case the square root formula will be (approximately) valid except in the case $b = kC$ when it will always (by (i)) pay to hold zero cash balances. Note incidentally that (ii) also yields $b - Cb' \geq 0$ which means that our result must hold if ever the "brokerage fee" increases more than in proportion with C.

Note, finally, that if i varies with C the first order condition becomes a cubic and, provided $\infty > i' > 0$, our conclusion is strengthened, since T now tends to increase as C^3.

which would call for a zero cash balance provided the cash could be loaned out profitably at all. Cash receipts during the relevant period may have similar effects, since they can be used to make payments which happen to be due at the moment the receipts arrive. Here the extreme case involves receipts and payments always coinciding in time and amounts in which case, again, zero cash balances would be called for. Thus lumpy payments and receipts of cash, with sufficient foresight, can make for economies in the use of cash, i.e., higher velocity. This may not affect the rate of increase in transactions velocity with the level of transactions, but may nevertheless serve further to increase the effect of an injection of cash and of a cut in wages and prices. With imperfect foresight, however, the expectation that payments may be lumpy may increase the precautionary demand for cash. Moreover, the existence of a "broker's fee" which must be paid on lending or investing cash received during the period is an added inducement to keep receipts until payments fall due rather than investing, and so may further increase the demand for cash.

4. The economy in a single person's use of cash resulting from an increase in the volume of his transactions may or may not have its analogue for the economy as a whole. "External economies" may well be present if one businessman learns cash-economizing techniques from the experiences of another when both increase their transactions. On the diseconomies side it is barely conceivable that an infectious liquidity fetishism will permit a few individuals reluctant to take advantage of cash saving opportunities to block these savings for the bulk of the community. Nevertheless, at least two such possible offsets come to mind: (a) The rise in the demand for brokerage services resulting from a general increase in transactions may bring about a rise in the "brokerage fee" and thus work for an increase in average cash balances (a decreased number of visits to brokers). If cash supplies are sticky this will tend to be offset by rises in the rate of interest resulting from a rising total demand for cash, which serve to make cash more expensive to hold. (b) Widespread cash economizing might require an increase in precautionary cash holdings because in an emergency one could rely less on the ability of friends to help or creditors to be patient. This could weaken but not offset the relative reduction in cash holdings entirely, since the increase in precautionary demand is contingent on there being some relative decrease in cash holdings.

5. A priori analysis of the precautionary and the speculative demands for cash is more difficult. In particular, there seems to be

little we can say about the latter, important though it may be, except that it seems unlikely that it will work consistently in any special direction. In dealing with the precautionary demand, assumptions about probability distributions and expectations must be made.[2] It seems plausible offhand, that an increase in the volume of transactions will make for economies in the use of cash for precautionary as well as transactions purposes by permitting increased recourse to insurance principles.

Indeed, here we have a rather old argument in banking theory which does not seem to be widely known. Edgeworth,[3] and Wicksell[4] following him, suggested that a bank's precautionary cash requirements might also grow as the square root of the volume of its transactions (!). They maintained that cash demands on a bank tend to be normally distributed.[5] In this event, if it is desired to maintain a fixed probability of not running out of funds, precautionary cash requirements will be met by keeping on hand a constant multiple of the standard deviation (above the mean). But then the precautionary cash requirement of ten identical banks (with independent demands) together will be the same as that for any one of them multiplied by the square root of ten. For it is a well-known result that the standard deviation of a random sample from an infinite population increases as the square root of the size of the sample.

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2. See Arrow, Harris and Marschak, *op. cit.* for a good example of what has been done along these lines in inventory control analysis.

3. F. Y. Edgeworth, "The Mathematical Theory of Banking," *Journal of the Royal Statistical Society*, Vol. LI (1888), especially pp. 123–127. Fisher (*loc. cit.*) points out the relevance of this result for the analysis of the cash needs of the public as a whole. The result was independently rediscovered by Dr. Whitin (*op. cit.*) who seems to have been the first to combine it and (2) in inventory analysis.

4. K. Wicksell, *Interest and Prices* (London, 1936), p. 67.

5. The distribution would generally be approximately normal if its depositors were large in number, their cash demands independent and not very dissimilarly distributed. The independence assumption, of course, rules out runs on banks.