NLOGIT 4

Student Version

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1986 - 2007

User's Guide

by

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Preface to the Student Version of NLOGIT 4

NLOGIT is a major suite of programs for the estimation of discrete choice models. It is built on the original **DISCRETE CHOICE** command in *LIMDEP Version 6.0* which provided some of the features that are described with the estimator presented in Chapter 9 of this reference guide. *NLOGIT*, itself, began with the development, in 1996, of the nested logit command, originally an extension of the multinomial logit model. With the additions of the multinomial probit model and the mixed logit model among several others, *NLOGIT* has now grown to a self standing superset of *LIMDEP*. The focus of most of the recent development is the random parameters logit model, or 'mixed logit' model as it is frequently called in the literature. *NLOGIT* is now the only generally available package that contains panel data (repeated measures) versions of this model, in random effects and autoregressive forms. We note, the technology used in the random parameters model, originally proposed by Dan McFadden and Kenneth Train, has proved so versatile and robust, that we have been able to extend it into most of the other modeling platforms that are contained in *LIMDEP*. They, like *NLOGIT*, now contain random parameters versions. Finally, a major feature of *NLOGIT* is the simulation package. With this program, you can use any model that you have estimated to do 'what if' sorts of simulations to examine the effects on predicted behavior of changes in the attributes of choices in your model.

NLOGIT Version 4.0 is the result of an ongoing (since 1985) collaboration of **William Greene** (Econometric Software, Inc.) and **David Hensher** (Econometric Software, Inc., Australia.) Recent developments, especially the random parameters logit in its cross section and panel data variants have also benefited from the suggestions of Kenneth Train of UC Berkeley. Version 4.0 has also been greatly improved by the enthusiastic collaboration of John Rose (Econometric Software, Inc., Australia).

The student version of NLOGIT 4.0 is the entire program. The limitations of the program relate only to the data set size: 1000 observations, 99 variables, and 25 parameters in a model

We note, the recently published work *Applied Choice Analysis: A Primer* (Hensher, D., Rose, J. and Greene, W., Cambridge University Press, 2005) is a wide ranging introduction to discrete choice modeling that contains numerous applications developed with Versions 3.0 and 4.0 of *NLOGIT*. This book should provide a useful companion to the documentation for *NLOGIT*.

Econometric Software, Inc.
January, 2008

Preface to the User's Guide for the Student Version of NLOGIT 4

This user's guide is constructed specifically for the student who is using *NLOGIT* for the first time and is, most likely, taking their first course in econometrics. Since *NLOGIT* is an extension of *LIMDEP*, we assume that you have at hand the manual for the student version of *LIMDEP*. This guide is a distillation of the full manual for *NLOGIT* that will show you how to use the program extensions that comprise *NLOGIT*. Note, however, that *NLOGIT* contains all of *LIMDEP* plus the modeling extensions for analysis of discrete choices.

Having introduced the manual as above, we do emphasize, this user's guide is not an econometrics or statistics text, and does not strive to be one. The material below will present only the essential background needed to illustrate the use of the program. In order to accommodate as many readers as possible, we have attempted to develop the material so that it is accessible to both undergraduates and graduate students. (For the latter, a text that would be useful to accompany this guide is *Econometric Analysis, 6th Edition* (William Greene, Prentice Hall, 2008), which was written by the author of both *NLOGIT* and this manual.)

Student *NLOGIT 4*.0 Table of Contents

Student NLOGIT 4.0 TOC

Chapter 5 Basic Models for Discrete Choice

Chapter 6 The Multinomial Logit Model

Chapter 7 Data Setup for *NLOGIT*

Chapter 8 *NLOGIT* Commands and Results

Chapter 9 The Conditional Logit Model

Student NLOGIT 4.0 TOC

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Chapter 1

Introduction to *LIMDEP* and *NLOGIT*

1.1 The *LIMDEP* Program

LIMDEP is an integrated package for estimating and analyzing econometric models. It is primarily oriented toward cross section and panel data. But, many standard problems in time series analysis can be handled as well. *LIMDEP*'s basic procedures for data analysis include:

- descriptive statistics (means, standard deviations, minima, etc.), with stratification,
- multiple linear regression and stepwise regression,
- time series identification, autocorrelations and partial autocorrelations,
- cross tabulations, histograms, and scatter plots of several types.

You can also model many extensions of the linear regression model such as:

- heteroscedasticity with robust standard errors,
- autocorrelation with robust standard errors,
- multiplicative heteroscedasticity,
- groupwise heteroscedasticity and cross sectional correlation,
- the Box-Cox regression model,
- one and two way random and fixed effects models for balanced or unbalanced panel data
- distributed lag models, ARIMA, and ARMAX models,
- time series models with GARCH effects,
- dynamic linear models for panel data,
- nonlinear single and multiple equation regression models,
- seemingly unrelated linear and nonlinear regression models,
- simultaneous equations models.

LIMDEP is best known for its extensive menu of programs for estimating the parameters of nonlinear models for qualitative and limited dependent variables. (We take our name from *LIM*ited *DEP*endent variables.) No other package supports a greater variety of nonlinear econometric models. Among *LIMDEP*'s more advanced features, each of which is invoked with a single command, are:

- univariate, bivariate and multivariate probit models, probit models with partial observability, selection, heteroscedasticity and random effects,
- Poisson and negative binomial models for count data, with fixed or random effects, sample selection, underreporting, and numerous other models of over and underdispersion,
- tobit and truncation models for censored and truncated data,
- models of sample selection with one or two selection criteria,
- parametric and semiparametric duration models with time varying covariates,
- stochastic frontier regression models,
- ordered probit and logit models, with censoring and sample selection,
- switching regression models,

- nonparametric and kernel density regression,
- fixed effects models, random parameters models and latent class models for over 25 different linear and nonlinear model classes,

and over fifty other model classes. Each of these allows a variety of different specifications. Most of the techniques in wide use are included. Among the aspects of this program which you will notice early on is that regardless of how advanced a technique is, the commands you use to request it are the same as those for the simplest regression.

LIMDEP also provides numerous programming tools, including an extensive matrix algebra package and a function optimization routine, so that you can specify your own likelihood functions and add new specifications to the list of models. All results are kept for later use. You can use the matrix program to compute test statistics for specification tests or to write your own estimation programs. The structure of *LIMDEP*'s matrix program is also especially well suited to the sorts of moment based specification tests suggested, for example, in Pagan and Vella (1989) – all the computations in this paper were done with *LIMDEP*. The programming tools, such as the editor, looping commands, data transformations, and facilities for creating 'procedures' consisting of groups of commands will also allow you to build your own applications for new models or for calculations such as complicated test statistics or covariance matrices.

Most of your work will involve analyzing data sets consisting of externally generated samples of observations on a number of variables. You can read the data, transform them in any way you like, for example, compute logarithms, lagged values, or many other functions, edit the data, and, of course, apply the estimation programs. You may also be interested in generating random (Monte Carlo) samples rather than analyzing 'live' data. *LIMDEP* contains random number generators for 15 discrete and continuous distributions including normal, truncated normal, Poisson, discrete or continuous uniform, binomial, logistic, Weibull, and others. A facility is also provided for random sampling or bootstrap sampling from any data set, whether internal or external, and for any estimation technique you have used, whether one of *LIMDEP*'s routines or your own estimator created with the programming tools. *LIMDEP* also provides a facility for bootstrapping panel data estimators, a feature not available in any other package.

1.2 References for Econometric Methods

This manual will document how to use *LIMDEP* for econometric analysis. There will be a number of examples and applications provided as part of the documentation. However, we will not be able to provide extensive background for the models and methods. A few of the main general textbooks currently in use are:

- Baltagi, B., *Econometric Analysis of Panel Data*, 3rd ed., Wiley, 2005
- Cameron, C. and Trivedi, P., *Microeconometrics: Methods and Applications,* Cambridge University Press, 2005.
- Greene, W., *Econometric Analysis*, 6th Edition, Prentice Hall, 2008.
- Gujarati, D., *Basic Econometrics*, McGraw Hill, 2003.
- Johnston, J. and DiNardo, J., *Econometric Methods,* 4th Edition, McGraw-Hill, 1997.
- Stock, J. and Watson, M., *Introduction to Econometrics*, 2nd. Ed., Addison Wesley, 2007.
- Wooldridge, J., *Econometric Analysis of Cross Section and Panel Data*, MIT Press, 2002.
- Wooldridge, J., *Modern Econometrics*, 2nd ed., Southwestern, 2007.

Chapter 2

Installation and Setup

2.1 Introduction

This chapter will show you how to install *NLOGIT* on your computer. The installation process will take only a few seconds, and does not require you to change any settings or make any decisions about parameters, switches, destination folders, etc.

2.2 Equipment

LIMDEP/NLOGIT is written for use on Windows based microcomputers using Windows 95 or a later version. As of this writing, we do not support operation on any Apple computers or mainframe systems. Emulation software for Apple machines that allows users to run Windows based software may work, but we are unable to offer any assurance, nor any specific advice. We assume that Apple's (spring, 2006) decision to create dual operating system computers with both Windows and OS capabilities will close this gap. Operation of peripheral devices, such as printers, external disk drives, etc. is under the control of the operating system, and does not require any settings within *LIMDEP/NLOGIT*. Use these devices as you do in other programs.

2.3 Installation Procedure

To install *LIMDEP/NLOGIT*, close all applications. Download the software, setup.exe, to your hard drive. Open My Computer or Windows Explorer and make your way to the setup.exe file. Launch the Setup program, by double clicking the file Setup.exe.

The installation will take about 30 seconds.

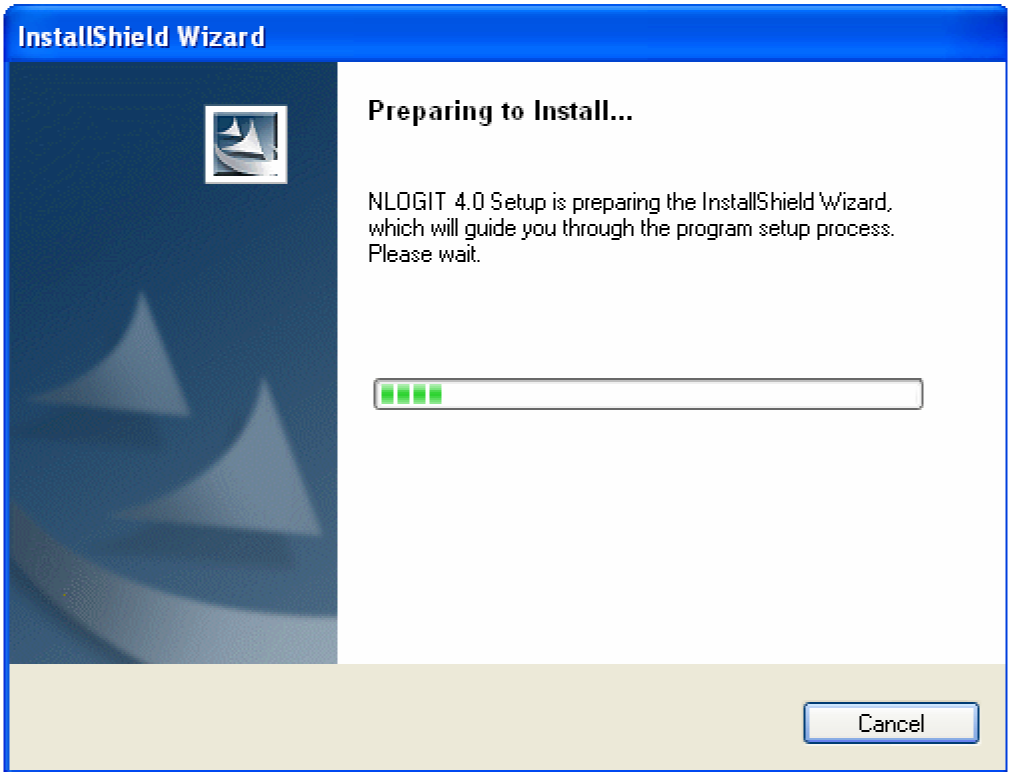

Figure 2.1 Setup Program Initial Screen

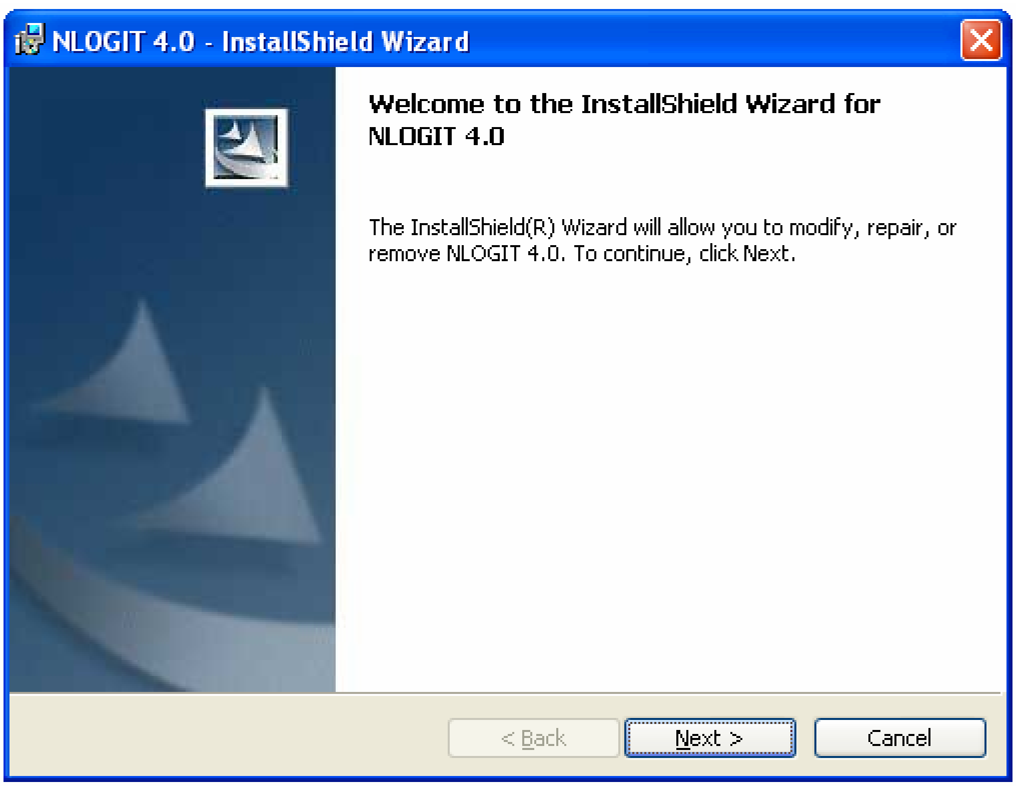

Figure 2.2 Installation Program

If you have a previous version of *NLOGIT* or *LIMDEP* already installed on your computer Setup will request that you uninstall the old version of the program. The Program Maintenance dialog shown in Figure N2.3 is used so that you do not have to use Control Panel/ Add Remove Programs. The first two options in this dialog, Modify and Repair, are not used. Use only the third one if necessary. No changes will be made to your working files; this only removes the old version of the program. After this operation is completed, Setup will close down automatically. You must then restart Setup. This step will now be bypassed, and the installation will be completed. Standard operation of Setup takes only a few seconds. Then, the license agreement and some information about the program are displayed.

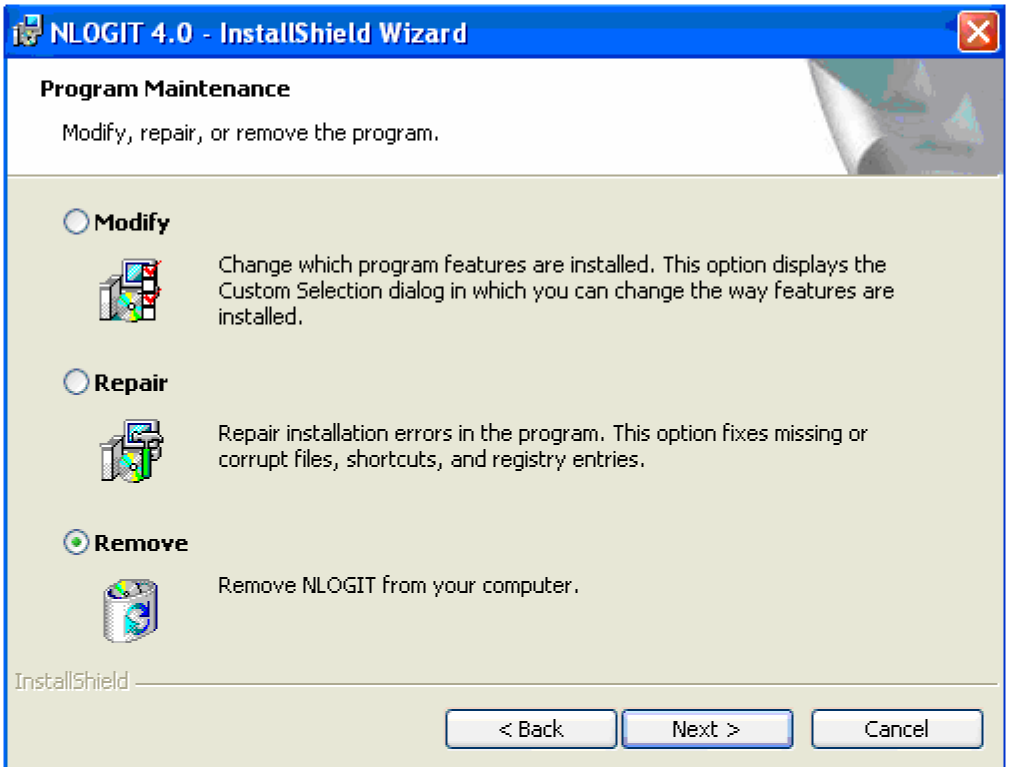

Figure 2.3 Uninstalling a Previous Version of NLOGIT or LIMDEP

The default destination folder for installation of the *LIMDEP/NLOGIT* program is C:\Program Files\Econometric Software. (See Figure 2.4) During the installation, the only information that you need to provide the Setup procedure is the name of the folder where you wish to install the program *if you choose not to use the default*. Unless you have a particular arrangement of your computer's hard drive in mind, we recommend that you use the default choice for where to install the program.

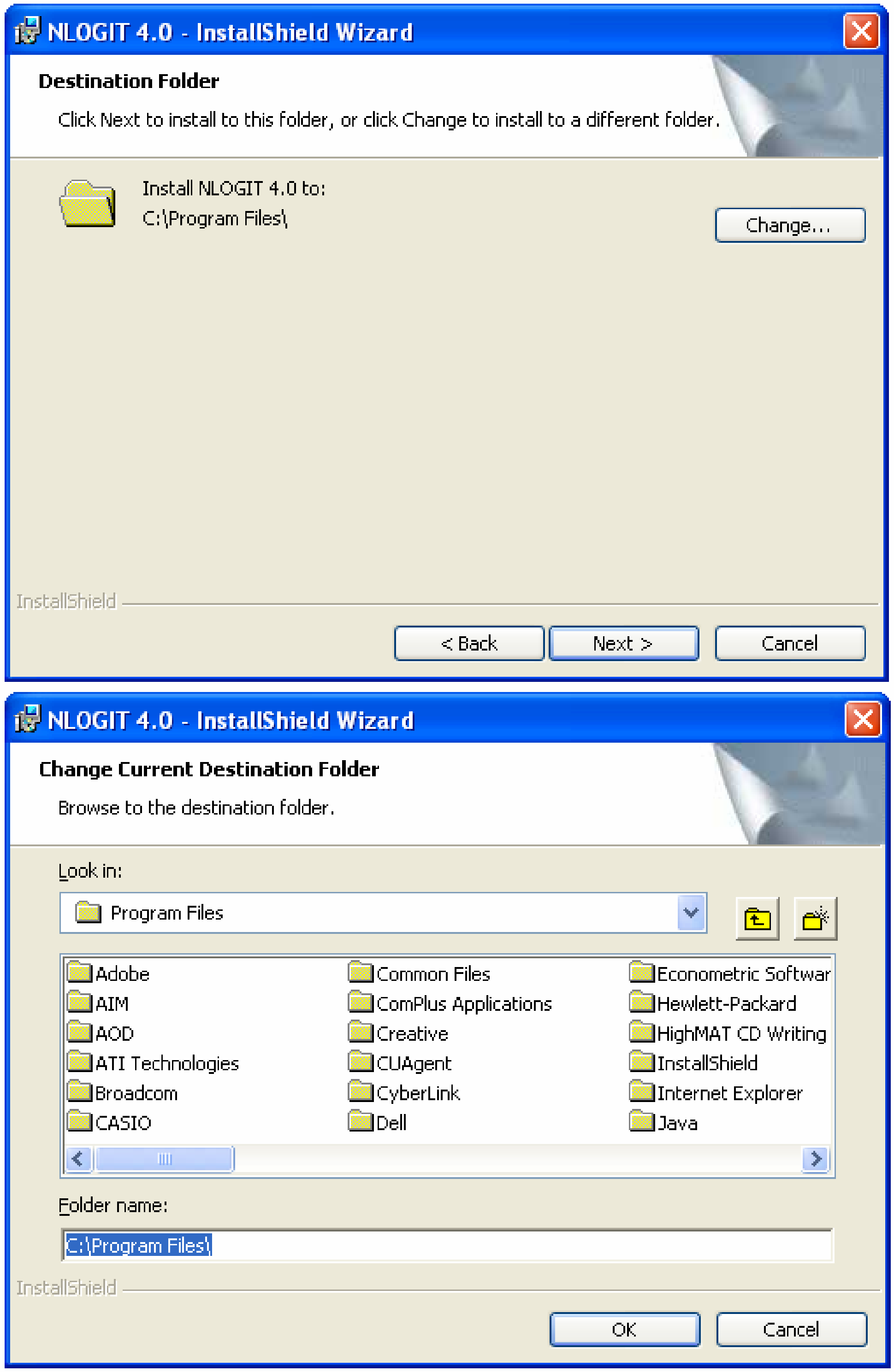

Figure 2.4 Changing the Installation Folder

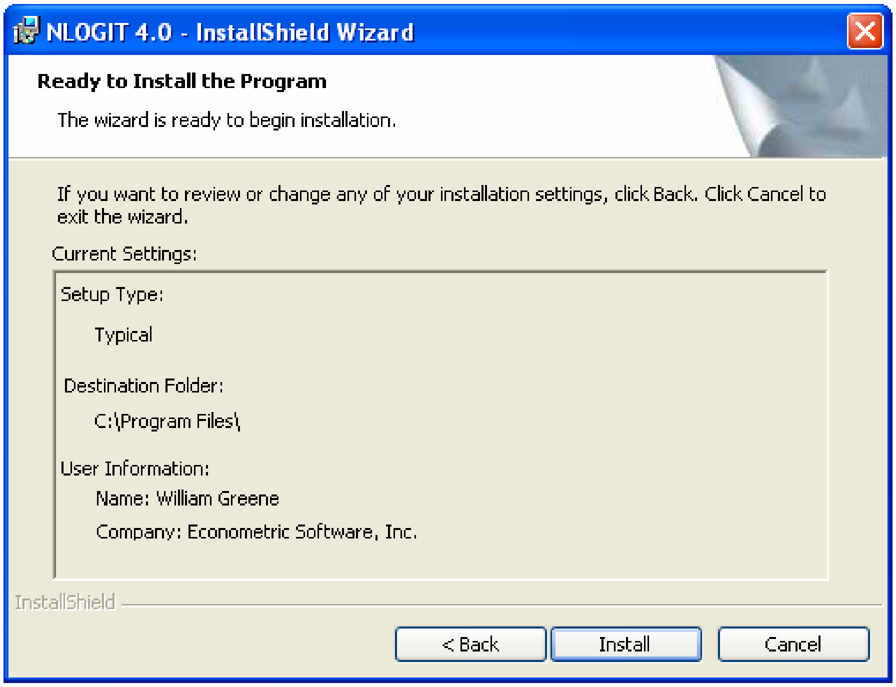

Figure 2.5 Installation Procedure

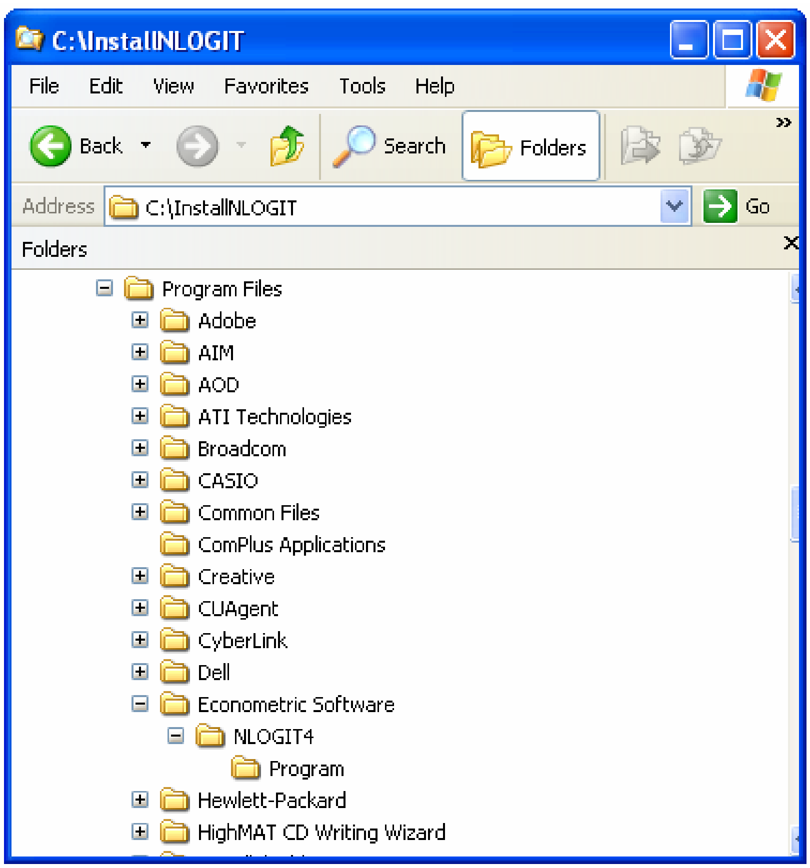

Figure 2.6 Installation Folder

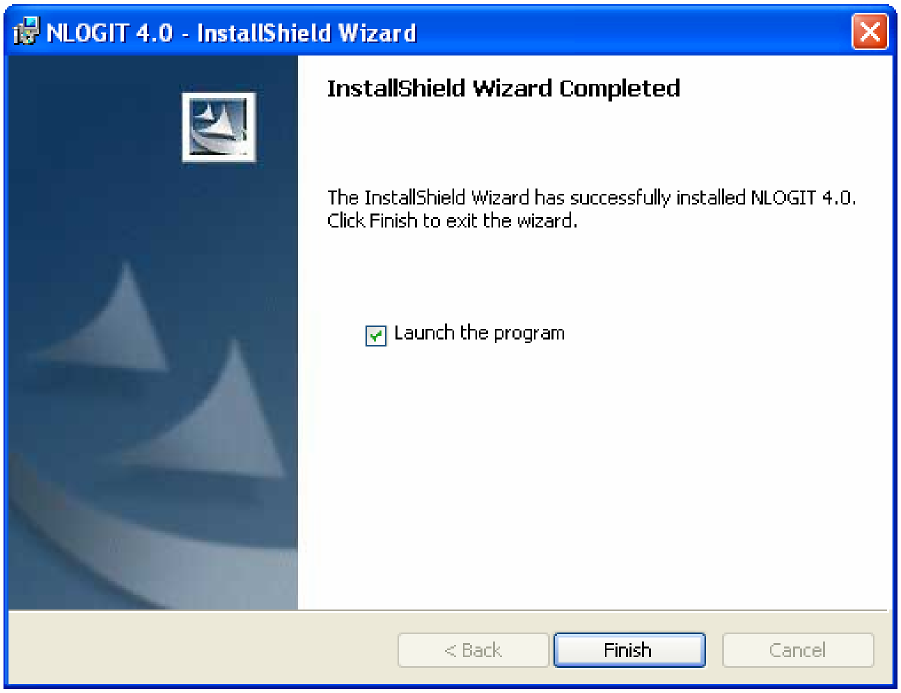

Figure 2.7 Setup Completion

Setup will attempt to place *LIMDEP/NLOGIT* in Programs in your Start menu and will put icons for the software in the Programs menu and on your desktop. If this not possible, you can

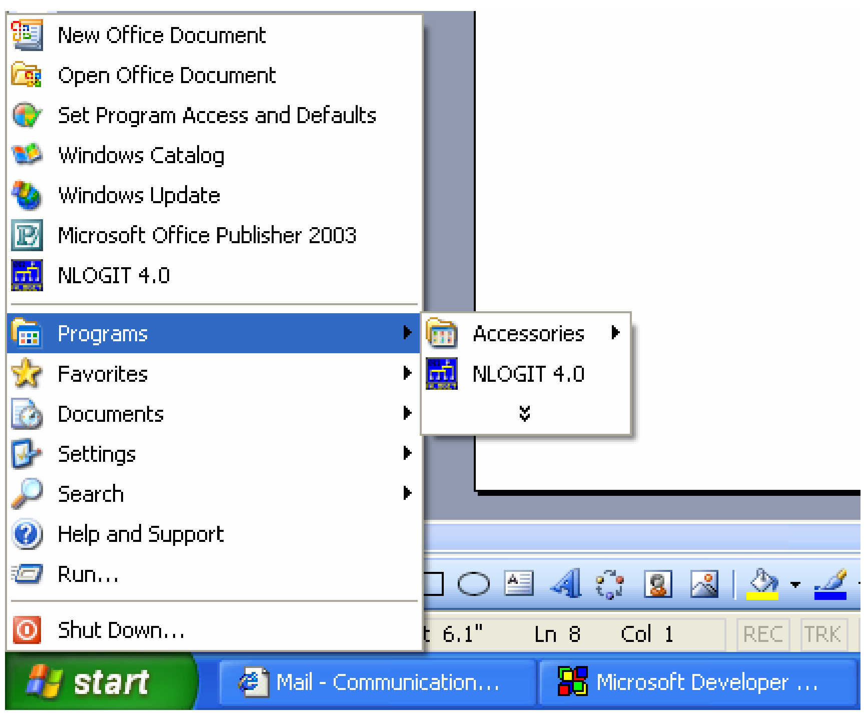

Figure 2.8 NLOGIT Installed in Start Menu

modify the Start menu and create a shortcut on your desktop at the same time using Windows Explorer.

1. Right click Start, then click Explore All Users.
2. Locate the NLOGIT.EXE file (in the Program Files\Econometric Software \NLOGIT4\Program folder).
3. To create a shortcut on your desktop, right click the NLOGIT.EXE file, click Send To, then click Desktop (create shortcut).
4. To put *LIMDEP/NLOGIT* in your Start Menu\Programs folder, scroll to the Start Menu\Programs folder, drag the LIMDEP.EXE file and drop it into the Programs folder of the Start Menu.

2.4 Registration

The first time you use *LIMDEP/NLOGIT* you will be presented with the Welcome and Registration dialog box. There are two steps to register *LIMDEP/NLOGIT.* First, please provide the registration information requested in the dialog box. Carefully input the serial number included with your program. This will place the registration information, including your serial number, in the About box. You must complete this dialog box in order to begin using *LIMDEP/NLOGIT.* See Figure 2.9.

Second, please send your registration information to Econometric Software. You can register with Econometric Software by completing the registration card included with your order and faxing or mailing it to us. You can also send your registration information to Econometric Software online via our website. To submit your registration information on our website, click the Help button, then select NLOGIT Web Site and proceed to the Registration page on our website.

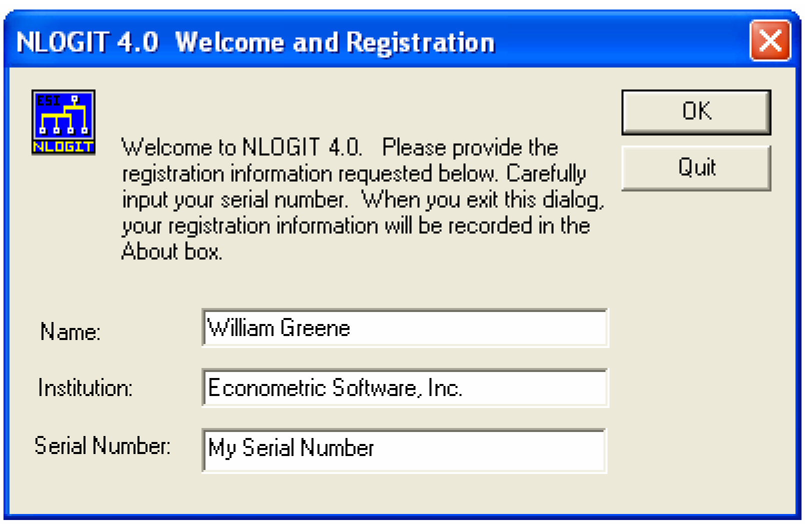

Figure 2.9 Welcome and Registration Dialog Box

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Chapter 3

Discrete Choice Models

3.1 Introduction

This chapter will provide a short, thumbnail sketch of the discrete choice models discussed in this manual. *NLOGIT* supports a large array of models for both discrete and continuous variables, including regression models, survival models, models for counts and, of relevance to this setting, models for discrete outcomes. The group of models described in this manual are those that arise naturally from a random utility framework, that is, those that arise from a consumer choice setting in which the model is of an individual's selection among two or more alternatives. This includes several of the models described in the *LIMDEP* manual, such as the binary logit and probit models, but also excludes some others, including the models for count data and some of the loglinear models such as the geometric regression model.

Two groups of models are considered. The first set are the binary, ordered and multivariate choice models. These form the basic building blocks for the *NLOGIT* extensions that are the main focus of this part of the program. Since they are developed in detail elsewhere, we will only provide the basic forms and only the essential documentation here. The second group of estimators are the multinomial logit models and extensions of them that form the group of tools specific to *NLOGIT*.

3.2 Random Utility Models

The random utility framework starts with a structural model,

$$U(choice\ 1) = f_1\,(attributes\ of\ choice\ 1,\ characteristics\ of\ the\ consumer,\ \varepsilon_1, \mathbf{v}, \mathbf{w}),$$
$$\ldots$$
$$U(choice\ J) = f_J\,(attributes\ of\ choice\ J,\ characteristics\ of\ the\ consumer,\ \varepsilon_J, \mathbf{v}, \mathbf{w}),$$

where $\varepsilon_1,...,\varepsilon_J$ denote the random elements of the random utility functions and in our later treatments, **v** and **w** will represent the unobserved individual heterogeneity built into models such as the error components and random parameters (mixed logit) models. The assumption that the choice made is alternative j such that

$$U(choice\ j) > U(choice\ q)\ \forall\ q \neq j.$$

The observed outcome variable is then

$$y\ =\ \text{the index of the observed choice.}$$

The econometric model that describes the determination of y is then built around the assumptions about the random elements in the utility functions that endow the model with its stochastic characteristics. Thus, where Y is the random variable that will be the observed discrete outcome,

$$\text{Prob}(Y = j)\ =\ \text{Prob}(\text{U(choice } j) > \text{U(choice } q)\ \forall\ q \neq j).$$

The objects of estimation will be the parameters that are built into the utility functions including possibly those of the distributions of the random components and, with estimates of the parameters in hand, useful characteristics of consumer behavior that can be derived from the model, such as partial effects and measures of aggregate behavior.

To consider the simplest example, that will provide the starting point for our development, consider a consumer's random utility derived over a single choice situation, say whether to make a purchase. The two outcomes are 'make the purchase' and 'do not make the purchase.' The random utility model is simply

$$U(\textit{not purchase}) = \boldsymbol{\beta}_0'\mathbf{x}_0 + \varepsilon_0,$$
$$U(\textit{purchase}) = \boldsymbol{\beta}_1'\mathbf{x}_1 + \varepsilon_1.$$

Assuming that ε_0 and ε_1 are random, the probability that the analyst will observe a purchase is

$$\begin{aligned}\text{Prob}(\textit{purchase}) &= \text{Prob}(U(\textit{purchase}) > U(\textit{not purchase}))\\ &= \text{Prob}(\boldsymbol{\beta}_1'\mathbf{x}_1 + \varepsilon_1 > \boldsymbol{\beta}_0'\mathbf{x}_0 + \varepsilon_0)\\ &= \text{Prob}(\varepsilon_1 - \varepsilon_0 < \boldsymbol{\beta}_1'\mathbf{x}_1 - \boldsymbol{\beta}_0'\mathbf{x}_0)\\ &= F(\boldsymbol{\beta}_1'\mathbf{x}_1 - \boldsymbol{\beta}_0'\mathbf{x}_0),\end{aligned}$$

where $F(z)$ is the cdf of the random variable $\varepsilon_1 - \varepsilon_0$. The model is completed and an estimator, generally maximum likelihood, is implied by an assumption about this probability distribution. For example, if ε_0 and ε_1 are assumed to be normally distributed, then the difference is also, and the familiar probit model emerges.

The sections to follow will outline the models described in this manual in the context of this random utility model. The different models derive from different assumptions about the utility functions and the distributions of their random components.

3.3 Binary Choice Models

Continuing the example in the previous section, the choice of alternative 1 (*purchase*) reveals that $U_1 > U_0$, or that

$$\varepsilon_1 - \varepsilon_0 < \boldsymbol{\beta}_1'\mathbf{x}_1 - \boldsymbol{\beta}_0'\mathbf{x}_0.$$

Let $\varepsilon = \varepsilon_1 - \varepsilon_0$ and $\boldsymbol{\beta}'\mathbf{x}$ represent the difference on the right hand side of the inequality - $\mathbf{x}$ is the union of the two sets of covariates, and $\boldsymbol{\beta}$ is constructed from the two parameter vectors with zeros in the appropriate locations if necessary. Then, a binary choice model applies to the probability that $\varepsilon \le \boldsymbol{\beta}'\mathbf{x}$. Two of the parametric model formulations in *NLOGIT* for binary choice models are the probit model based on the normal distribution:

$$F = \int_{-\infty}^{\beta'\mathbf{x}_i} \frac{\exp(-t^2/2)}{\sqrt{2\pi}}dt = \Phi(\boldsymbol{\beta}'\mathbf{x}_i),$$

and the logit model based on the logistic distribution

$$F = \frac{\exp(\boldsymbol{\beta}'\mathbf{x}_i)}{1+\exp(\boldsymbol{\beta}'\mathbf{x}_i)} = \Lambda(\boldsymbol{\beta}'\mathbf{x}_i).$$

Numerous variations on the model can be obtained. A model with multiplicative heteroscedasticity is obtained with the additional assumption

$$\varepsilon_i \sim \text{normal or logistic with variance} \propto [\exp(\boldsymbol{\gamma}'\mathbf{z}_i)]^2,$$

where $\mathbf{z}_i$ is a set of observed characteristics of the individual. A model of sample selection can be extended to the probit and logit binary choice models. In both cases, we depart from

$$\text{Prob}(y_i = 1\,|\mathbf{x}_i) = F(\boldsymbol{\beta}'\mathbf{x}_i),$$

where

$$F(t) = \Phi(t) \text{ for the probit model and } \Lambda(t) \text{ for the logit model,}$$
$$d_i^* = \boldsymbol{\alpha}'\mathbf{z}_i + u_i,\ u_i \sim N[0,1],\ d_i = 1(d_i^* > 0),$$
$$y_i, \mathbf{x}_i \text{ observed only when } d_i = 1.$$

where $\mathbf{z}_i$ is a set of observed charactristics of the individual. In both cases, as stated, there is no obvious way that the selection mechanism impacts the binary choice model of interest. We modify the models as follows: For the probit model,

$$y_i^* = \boldsymbol{\beta}'\mathbf{x}_i + \varepsilon_i,\ \varepsilon_i \sim N[0,1],\ y_i = 1(y_i^* > 0),$$

which is the structure underlying the probit model in any event, and

$$u_i, \varepsilon_i \sim N_2[(0,0),(1,\rho,1)].$$

(We use N_P to denote the P-variate normal distribution, with the mean vector followed by the definition of the covariance matrix in the succeeding brackets.) For the logit model, a similar approach does not produce a convenient bivariate model. The probability is changed to

$$\text{Prob}(y_i = 1 \mid \mathbf{x}_i,\varepsilon_i) = \frac{\exp(\boldsymbol{\beta}'\mathbf{x}_i + \sigma\varepsilon_i)}{1+\exp(\boldsymbol{\beta}'\mathbf{x}_i + \sigma\varepsilon_i)}.$$

With the selection model for z_i as stated above, the bivariate probability for y_i and z_i is a mixture of a logit and a probit model. The log likelihood can be obtained, but it is not in closed form, and must be computed by approximation. We do so with simulation.

There are several formulations for extensions of the binary choice models to panel data setting. These include

- Fixed effects: $\text{Prob}(y_{it} = 1) = F(\boldsymbol{\beta}'\mathbf{x}_{it} + \alpha_i)$, α_i correlated with $\mathbf{x}_{it}$.
- Random effects: $\text{Prob}(y_{it} = 1) = \text{Prob}(\boldsymbol{\beta}'\mathbf{x}_{it} + \varepsilon_{it} + u_i > 0)$, u_i uncorrelated with $\mathbf{x}_{it}$.
- Random parameters: $\text{Prob}(y_{it} = 1) = F(\boldsymbol{\beta}_i'\mathbf{x}_{it})$,
  $\boldsymbol{\beta}_i \mid i \sim h(\boldsymbol{\beta}|i)$ with mean vector $\boldsymbol{\beta}$ and covariance matrix $\boldsymbol{\Sigma}$.
- Latent class: $\text{Prob}(y_{it} = 1|\text{class } j) = F(\boldsymbol{\beta}_j'\mathbf{x}_{it})$,
  $\text{Prob}(\text{class} = j) = G_j(\boldsymbol{\theta},\mathbf{z}_i)$,

where $\mathbf{z}_i$ is a set of observed charactristics of the individual. Other variations include simultaneous equations models and semiparametric formulations.

3.4 Bivariate and Multivariate Binary Choices

The bivariate probit model is a natural extension of the model above in which two decisions are made jointly;

$$y_{i1}^* = \boldsymbol{\beta}_1'\mathbf{x}_{i1} + \varepsilon_{i1},\ y_{i1} = 1 \text{ if } y_{i1}^* > 0,\ y_{i1} = 0 \text{ otherwise,}$$

$$y_{i2}^* = \boldsymbol{\beta}_2'\mathbf{x}_{i2} + \varepsilon_{i2},\ y_{i2} = 1 \text{ if } y_{i2}^* > 0,\ y_{i2} = 0 \text{ otherwise,}$$

$$[\varepsilon_{i1},\varepsilon_{i2}] \sim N_2[0,0,1,1,\rho],\ -1 < \rho < 1,$$

individual observations on y_1 and y_2 are available for all i.

This model extends the binary choice model to two different, but related outcomes. One might, for example, model Y_1 = home ownership (vs. renting) and Y_2 = automobile purchase (vs. leasing). The two decisions are obviously correlated (and possibly even jointly determined).

A special case of the bivariate probit model is useful for formulating the correlation between two binary variables. The tetrachoric correlation coefficient is equivalent to the correlation coefficient in the following bivariate probit model:

$$y_{i1}^* = \mu + \varepsilon_{i1}, \quad y_{i1} = 1(y_{i1}^* > 0),$$

$$y_{i2}^* = \mu + \varepsilon_{i2}, \quad y_{i2} = 1(y_{i2}^* > 0),$$

$$(\varepsilon_{i1},\varepsilon_{i2}) \sim N_2[(0,0),(1,1,\rho)].$$

The bivariate probit model has been extended to the random parameters form of the panel data models. For example, a true random effects model for a bivariate probit outcome can be formulated as follows: Each equation has its own random effect, and the two are correlated. The model structure is

$$y_{it1}^* = \boldsymbol{\beta}_1'\mathbf{x}_{it1} + \varepsilon_{it1} + u_{i1},\ y_{it1} = 1 \text{ if } y_{it1}^* > 0,\ y_{it1} = 0 \text{ otherwise,}$$

$$y_{it2}^* = \boldsymbol{\beta}_2'\mathbf{x}_{it2} + \varepsilon_{it2} + u_{i2},\ y_{it2} = 1 \text{ if } y_{it2}^* > 0,\ y_{it2} = 0 \text{ otherwise,}$$

$$[\varepsilon_{it1},\varepsilon_{it2}] \sim N_2[0,0,1,1,\rho],\ -1 < \rho < 1,$$

$$[u_{i1},u_{i2}] \sim N_2[0,0,1,1,\theta],\ -1 < \theta < 1.$$

Individual observations on y_{i1} and y_{i2} are available for all i. Note, in the structure, the idiosyncratic ε_{itj} creates the bivariate probit model, whereas the time invariant common effects, u_{ij} create the random effects (random constants) model. Thus, there are two sources of correlation across the equations, the correlation between the unique disturbances, ρ, and the correlation between the time invariant disturbances, θ.

The multivariate probit model is the extension to M equations of the bivariate probit model

$$y_{im}^* = \boldsymbol{\beta}_m'\mathbf{x}_{im} + \varepsilon_{im},\ m = 1,\ldots,M$$

$$y_{im} = 1 \text{ if } y_{im}^* > 0, \text{ and } 0 \text{ otherwise,}$$

$$\varepsilon_{im},\ m = 1,\ldots,M \sim N_M[\mathbf{0},\mathbf{R}],$$

where $\mathbf{R}$ is the correlation matrix. Each individual equation is a standard probit model. This generalizes the bivariate probit model for up to $M = 20$ equations.

3.5 Ordered Choice Models

The basic ordered choice model can be cast in an analog to our random utility specification. We suppose that preferences over a given outcome are reflected as earlier, in the random utility function:

$$
\begin{aligned}
y_i^* &= \boldsymbol{\beta}'\mathbf{x}_i + \varepsilon_i,\\
\varepsilon_i &\sim F(\varepsilon_i\,|\boldsymbol{\theta}),\ \boldsymbol{\theta} = \text{a vector of parameters},\\
E[\varepsilon_i|\mathbf{x}_i] &= 0,\\
\text{Var}[\varepsilon_i|\mathbf{x}_i] &= 1.
\end{aligned}
$$

The consumer is asked to reveal the strength of their preferences over the outcome, but is given only a discrete, ordinal scale, $0,1,...,J$. The observed response represents a complete censoring of the latent utility as follows:

$$
\begin{aligned}
y_i &= 0 \text{ if } y_i^* \le \mu_0,\\
&= 1 \text{ if } \mu_0 < y_i^* \le \mu_1,\\
&= 2 \text{ if } \mu_1 < y_i^* \le \mu_2,\\
&\ \ \ldots\\
&= J \text{ if } y_i^* > \mu_{J\text{-}1}.
\end{aligned}
$$

The latent 'preference' variable, y_i^* is not observed. The observed counterpart to y_i^* is y_i. (The model as stated does embody the strong assumption that the threshold values are the same for all individuals. We will relax that assumption below.) The *ordered probit* model based on the normal distribution was developed by Zavoina and McElvey (1975). It applies in applications such as surveys, in which the respondent expresses a preference with the above sort of ordinal ranking. The ordered logit model arises if ε_i is assumed to have a logistic distribution rather than a normal. The variance of ε_i is assumed to be the standard, one for the probit model and $\pi^2/6$ for the logit model, since as long as y_i^*, $\boldsymbol{\beta}$, and ε_i are all unobserved, no scaling of the underlying model can be deduced from the observed data. (The assumption of homoscedasticity is arguably a strong one. We will also relax that assumption.) Since the μs are free parameters, there is no significance to the unit distance between the set of observed values of y_i. They merely provide the coding. Estimates are obtained by maximum likelihood. The probabilities which enter the log likelihood function are

$$\text{Prob}(y_i = j) = \text{Prob}(y_i^* \text{ is in the } j\text{th range}).$$

The model may be estimated either with individual data, with $y_i = 0, 1, 2, ...$ or with grouped data, in which case each observation consists of a full set of J+1 proportions, $p_{i0},...,p_{iJ}$.

There are many variants of the ordered probit model. A model with multiplicative heteroscedasticity of the same form as in the binary choice models is

$$\text{Var}[\varepsilon_i] = [\exp(\boldsymbol{\gamma}'\mathbf{z}_i)]^2.$$

The following describes an ordered probit counterpart to the standard sample selection model. (This is only available for the ordered probit specification.) The structural equations are, first, the main equation, the ordered choice model that was given above and, second, a selection equation, a univariate probit model,

$$d_i^* = \boldsymbol{\alpha}'\mathbf{z}_i + u_i,$$

$d_i \quad = 1$ if $d_i^* > 0$ and 0 otherwise.

The observation mechanism is

$[y_i,\mathbf{x}_i]$ is observed if and only if $d_i = 1$,

$\varepsilon_i, u_i \quad \sim N_2[0,0,1,1,\rho]$; there is 'selectivity' if ρ is not equal to zero.

LIMDEP/NLOGIT's general set of panel data formulations is also available for the ordered probit and logit models.

- Fixed effects: $\text{Prob}(y_{it} = j) = F[\mu_j - (\boldsymbol{\beta}'\mathbf{x}_{it} + \alpha_i)] - F[\mu_{j-1} - (\boldsymbol{\beta}'\mathbf{x}_{it} + \alpha_i)]$,
  α_i correlated with $\mathbf{x}_{it}$.
- Random effects: $\text{Prob}(y_{it} = j) = F[\mu_j - (\boldsymbol{\beta}'\mathbf{x}_{it} + u_i)] - F[\mu_{j-1} - (\boldsymbol{\beta}'\mathbf{x}_{it} + u_i)]$,
  u_i uncorrelated with $\mathbf{x}_{it}$.
- Random parameters: $\text{Prob}(y_{it} = j) = F(\mu_j - \boldsymbol{\beta}_i'\mathbf{x}_{it}) - F(-\mu_{j-1}\boldsymbol{\beta}_i'\mathbf{x}_{it})$,
  $\boldsymbol{\beta}_i \mid i \sim h(\boldsymbol{\beta}|i)$ with mean vector $\boldsymbol{\beta}$ and covariance matrix $\boldsymbol{\Sigma}$.
- Latent class: $\text{Prob}(y_{it} = j|\text{class } c) = F(\mu_{j,c} - \boldsymbol{\beta}_c'\mathbf{x}_{it}) - F(\mu_{j-1,c} - \boldsymbol{\beta}_c'\mathbf{x}_{it})$,
  $\text{Prob}(\text{class} = c) = G_c(\boldsymbol{\theta},\mathbf{z}_i)$.

The hierarchical ordered probit model, or generalized ordered probit model, relaxes the assumption that the threshold parameters are the same for all individuals. Two forms of the model are provided.

Form 1: $\mu_{ij} = \exp(\theta_j + \boldsymbol{\delta}'\mathbf{z}_i)$,
Form 2: $\mu_{ij} = \exp(\theta_j + \boldsymbol{\delta}_j'\mathbf{z}_i)$.

Note that in form 1, each μ_j has a different constant term, but the same coefficient vector, while in form 2, each threshold parameter has its own parameter vector.

Harris and Zhao (2004, 2005) have developed a zero inflated ordered probit (ZIOP) counterpart to the zero inflated Poisson model. The ZIOP formulation would appear

$$
\begin{aligned}
d_i^* &= \boldsymbol{\alpha}'\mathbf{z}_i + u_i, & d_i &= 1\ (d_i^* > 0),\\
y_i^* &= \boldsymbol{\beta}'\mathbf{x}_i + \varepsilon_i, & y_i &= 0 \text{ if } y_i^* \le 0 \text{ or } d_i = 0,\\
& & & 1 \text{ if } 0 < y_i^* \le \mu_1 \text{ and } d_i = 1,\\
& & & 2 \text{ if } \mu_1 < y_i^* \le \mu_2 \text{ and } d_i = 1,\\
& & & \text{and so on.}
\end{aligned}
$$

The first equation is assumed to be a probit model (based on the normal distribution) – this estimator does not support a logit formulation. The correlation between u_i and ε_i is ρ, which by default equals zero, but may be estimated instead. The latent class nature of the formulation has the effect of inflating the number of observed zeros, even if u and ε are uncorrelated. The model with correlation between u_i and ε_i is an optional specification that analysts might want to test. The zero inflation model may also be combined with the hierarchical (generalized) model given above.

The bivariate ordered probit model is analogous to the seemingly unrelated regressions model for the ordered probit case:

$$
\begin{aligned}
y_{ij}^* &= \boldsymbol{\beta}_j'\mathbf{x}_{ji} + \varepsilon_{ij}, \\
y_{ij} &= 0 \text{ if } y_{ij}^* \leq 0, \\
& \quad 1 \text{ if } 0 < y_{ij}^* < \mu_1, \\
& \quad 2, \ldots \text{ and so on, } j = 1,2,
\end{aligned}
$$

for a pair of ordered probit models that are linked by Cor(ε_{i1},ε_{i2}) = ρ. The model can be estimated one equation at a time using the results described earlier. Full efficiency in estimation and an estimate of ρ are achieved by full information maximum likelihood estimation. Either variable (but not both) may be binary. (If both are binary, the bivariate probit model should be used.)

The polychoric correlation coefficient is used to quantify the correlation between discrete variables that are qualitative measures. The standard interpretation is that the discrete variables are discretized counterparts to underlying quantitative measures. We typically use ordered probit models to analyze such data. The polychoric correlation measures the correlation between $y_1 = 0,1,\ldots,J_1$ and $y_2 = 0,1,\ldots,J_2$. (Note, J_1 need not equal J_2.) One of the two variables may be binary as well. (If both variables are binary, we use the tetrachoric correlation coefficient described in Section E21.3.) For the case noted, the polychoric correlation is the correlation in the bivariate ordered probit model, so it can be estimated just by specifying a bivariate ordered choice model in which both right hand sides contain only a constant term.

3.6 Multinomial Logit Model

The canonical random utility model is as follows:

$$
\begin{aligned}
&U(\text{ alternative } 0\ ) = \boldsymbol{\beta}_0'\mathbf{x}_{i0} + \varepsilon_{i0}, \\
&U(\text{ alternative } 1\ ) = \boldsymbol{\beta}_1'\mathbf{x}_{i1} + \varepsilon_{i1}, \\
&\ldots \\
&U(\text{ alternative } J\ ) = \boldsymbol{\beta}_J'\mathbf{x}_{iJ} + \varepsilon_{iJ}, \\
&\text{Observed } y_i = \text{choice } j \text{ if } U_i(\text{ alternative } j\ ) > U_i(\text{ alternative } q\ )\ \forall\ q \neq j.
\end{aligned}
$$

The 'disturbances' in this framework (individual heterogeneity terms) are assumed to be independently and identically distributed with identical type 1extreme value distribution; the CDF is

$$F(\varepsilon_j) = \exp(-\exp(-\varepsilon_j)).$$

Based on this specification, the choice probabilities are

$$
\begin{aligned}
\text{Prob(choice } j) &= \text{Prob}(U_j > U_q),\ \forall\ q \neq j \\
&= \frac{\exp(\boldsymbol{\beta}_j'\mathbf{x}_{ij})}{\sum_{q=0}^{J}\exp(\boldsymbol{\beta}_q'\mathbf{x}_{iq})},\ j = 0,\ldots,J.
\end{aligned}
$$

At this point we make a purely semantic distinction between two cases of the model. When the observed data consist of individual choices and (only) data on the characteristics of the individual, identification of the model parameters will require that the parameter vectors differ across the utility functions, as they do above. The study on labor market decisions by Schmidt and Strauss (1975) is a classic example. For the moment, we will call this the *multinomial logit model*. When the data also include attributes of the choices that differ across the alternatives, then the forms of the utility functions can change slightly – and the coefficients can be generic, that is the same across alternatives. Again, only for the present, we will call this the *conditional logit model*. (It will emerge that the multinomial logit is a special case of the conditional logit model, though the reverse is not true.)

The general form of the *multinomial logit* model is

$$\text{Prob(choice } j) = \frac{\exp(\boldsymbol{\beta}_j'\mathbf{x}_i)}{\sum_{q=0}^{J}\exp(\boldsymbol{\beta}_q'\mathbf{x}_i)}, \; j = 0,...,J.$$

A possible J+1 *unordered* outcomes can occur. In order to identify the parameters of the model, we impose the normalization $\boldsymbol{\beta}_0 = \mathbf{0}$. This model is typically employed for individual or grouped data in which the '**x**' variables are characteristics of the observed individual(s), not the choices. The data will appear as follows:

- Individual data: y_i coded 0, 1, ..., J,
- Grouped data: $y_{i0}, y_{i1},...,y_{iJ}$ give proportions or shares.

3.6.1 Random Effects and Common (True) Random Effects

The structural equations of the multinomial logit model are

$$U_{ijt} = \boldsymbol{\beta}_j'\mathbf{x}_{it} + \varepsilon_{ijt},\; t = 1,...,T_i,\; j = 0,1,...,J, i=1,...,N,$$

where U_{ijt} gives the utility of choice j by person i in period t – we assume a panel data application with $t = 1,...,T_i$. The model about to be described can be applied to cross sections, where $T_i = 1$. Note also that as usual, we assume that panels may be unbalanced. We also assume that ε_{ijt} has a type 1 extreme value distribution and that the J random terms are independent. Finally, we assume that the individual makes the choice with maximum utility. Under these (IIA inducing) assumptions, the probability that individual i makes choice j in period t is

$$P_{ijt} = \frac{\exp(\boldsymbol{\beta}_j'\mathbf{x}_{it})}{\sum_{q=0}^{J}\exp(\boldsymbol{\beta}_q'\mathbf{x}_{it})}.$$

We now suppose that individual i has latent, unobserved, time invariant heterogeneity that enters the utility functions in the form of a random effect, so that

`

$$U_{ijt} = \boldsymbol{\beta}_j'\mathbf{x}_{it} + \alpha_{ij} + \varepsilon_{ijt},\; t = 1,...,T_i,\; j = 0,1,...,J, i=1,...,N.$$

The resulting choice probabilities, conditioned on the random effects, are

$$P_{ijt} \mid \alpha_{i1},...,\alpha_{iJ} = \frac{\exp(\boldsymbol{\beta}_j'\mathbf{x}_{it} + \alpha_{ij})}{\sum_{q=0}^{J}\exp(\boldsymbol{\beta}_q'\mathbf{x}_{it} + \alpha_{iq})}.$$

To complete the model, we assume that the heterogeneity is normally distributed with zero means and $(J+1)\times(J+1)$ covariance matrix, $\boldsymbol{\Sigma}$. For identification purposes, one of the coefficient vectors, $\boldsymbol{\beta}_q$, must be normalized to zero and one of the α_{iq}s is set to zero. We normalize the first element – subscript 0 – to zero. For convenience, this normalization is left implicit in what follows. It is automatically imposed by the software. To allow the remaining random effects to be freely correlated, we write the $J\times1$ vector of nonzero αs as

$$\boldsymbol{\alpha}_i = \boldsymbol{\Gamma}\,\mathbf{v}_i$$

where $\boldsymbol{\Gamma}$ is a lower triangular matrix to be estimated and $\mathbf{v}_i$ is a standard normally distributed (mean vector $\mathbf{0}$, covariance matrix, $\mathbf{I}$) vector.

3.6.2 A Dynamic Multinomial Logit Model

The preceding random effects model can be modified to produce the dynamic multinomial logit model proposed in Gong, van Soest and Villagomez (2000). The choice probabilities are

$$P_{ijt} \mid \alpha_{i1},...,\alpha_{iJ} = \frac{\exp(\boldsymbol{\beta}_j'\mathbf{x}_{it} + \boldsymbol{\gamma}_j'\mathbf{z}_{it} + \alpha_{ij})}{\sum_{q=1}^{J}\exp(\boldsymbol{\beta}_q'\mathbf{x}_{it} + \boldsymbol{\gamma}_q'\mathbf{z}_{it} + \alpha_{iq})}\quad t = 1,...,T_i,\ j = 0,1,...,J, i=1,...,N,$$

where $\mathbf{z}_{it}$ contains lagged values of the dependent variables (these are binary choice indicators for the choice made in period t) and possibly interactions with other variables. The $\mathbf{z}_{it}$ variables are now endogenous, and conventional maximum likelihood estimation is inconsistent. The authors argue that Heckman's treatment of initial conditions is sufficient to produce a consistent estimator. The core of the treatment is to treat the first period as an equilibrium, with no lagged effects,

$$P_{ij0} \mid \theta_{i1},...,\theta_{iJ} = \frac{\exp(\boldsymbol{\delta}_j'\mathbf{x}_{i0} + \theta_{ij})}{\sum_{q=1}^{J}\exp(\boldsymbol{\delta}_q'\mathbf{x}_{i0} + \theta_{iq})},\ t = 0,\ j = 0,1,...,J, i=1,...,N,$$

where the vector of effects, $\boldsymbol{\theta}$, is built from the same primitives as $\boldsymbol{\alpha}$ in the later choice probabilities. Thus, $\boldsymbol{\alpha}_i = \boldsymbol{\Gamma}\mathbf{v}_i$ and $\boldsymbol{\theta}_i = \boldsymbol{\Phi}\,\mathbf{v}_i$, for the same $\mathbf{v}_i$, but different lower triangular scaling matrices. (This treatment slightly less than doubles the size of the model – it amounts to a separate treatment for the first period.) Full information maximum likelihood estimates of the model parameters, $(\boldsymbol{\beta}_1,...,\boldsymbol{\beta}_J,\boldsymbol{\gamma}_1,...,\boldsymbol{\gamma}_J,\boldsymbol{\delta}_1,...,\boldsymbol{\delta}_J,\boldsymbol{\Gamma},\boldsymbol{\Phi})$ are obtained by maximum simulated likelihood, by modifying the random effects model. The likelihood function for individual i consists of the period 0 probability as shown above times the product of the period $1,2,...,T_i$ probabilities defined earlier.

3.7 Conditional Logit Models

If the utility functions are conditioned on observed individual, choice invariant characteristics, $\mathbf{z}_i$, as well as the attributes of the choices, $\mathbf{x}_{ij}$, then we write

$$U(\textit{ choice } j \textit{ for individual } i\ ) = U_{ij} = \boldsymbol{\beta}'\mathbf{x}_{ij} + \boldsymbol{\gamma}_j'\mathbf{z}_i + \varepsilon_{ij},\ j = 1,...,J_i.$$

(For this model, which uses a different part of *NLOGIT*, we number the alternatives $1,...,J_i$ rather than $0,...,J_i$. There is no substantive significance to this – it is purely for convenience in the context of the model development for the program commands.) The random, individual specific terms, $(\varepsilon_{i1},\varepsilon_{i2},...,\varepsilon_{iJ})$ are once again assumed to be independently distributed across the utilities, each with the same type 1 extreme value distribution

$$F(\varepsilon_{ij}) = \exp(-\exp(-\varepsilon_{ij})).$$

Under these assumptions, the probability that individual t chooses alternative j is

$$\text{Prob}(U_{ij} > U_{iq}) \text{ for all } q \neq j.$$

It has been shown that for independent type 1 extreme value distributions, as above, this probability is

$$\text{Prob}(y_i = j) = \frac{\exp\left(\boldsymbol{\beta}'\mathbf{x}_{ij} + \boldsymbol{\gamma}_j'\mathbf{z}_i\right)}{\sum_{q=1}^{J_i}\exp\left(\boldsymbol{\beta}'\mathbf{x}_{iq} + \boldsymbol{\gamma}_q'\mathbf{z}_i\right)}$$

where y_i is the index of the choice made. We note at the outset that the IID assumptions made about ε_j are quite stringent, and induce the 'Independence from Irrelevant Alternatives' or IIA features that characterize the model. This is functionally identical to the multinomial logit model. Indeed, the earlier model emerges by the simple restriction $\boldsymbol{\gamma}_j = \mathbf{0}$. We have distinguished it in this fashion because the nature of the data suggests a different arrangement than for the multinomial logit model and, second, the models in the section to follow are formulated as extensions of this one.

3.8 Error Components Logit Model

When the sample consists of a 'panel' of data, that is, when individuals are observed in more than one choice situation, the conditional logit model can be augmented with individual effects, similar to the use of common effects models in regression and other single equation cases. A 'panel data' form of this model that is a counterpart to the random effects model is what we label the 'error components model.' (This has been called the 'kernel logit model' in some treatments in the literature.) The model arises by introducing M up to $\max_i J_i$ alternative and individual specific random terms in the utility functions as in

$$\begin{aligned} &U(\textit{ choice } j \textit{ for individual } i \textit{ in choice setting } t\ ) \\ &\quad = U_{ijt} \\ &\quad = \boldsymbol{\beta}'\mathbf{x}_{ij} + \boldsymbol{\gamma}_j'\mathbf{z}_i + \varepsilon_{ij} + \Sigma_{m=1}^{M} d_{jm}\sigma_m u_{im},\ j = 1,...,J_i,\ t = 1,...,T_i. \end{aligned}$$

where d_{jm} = 1 if effect m appears in utility function j, 0 if not,

σ_m = the standard deviation of effect m (to be estimated)

v_{im} = effect m for individual i.

The M random individual specifics are ($\sigma_m u_{im}$). The are distributed as normal with zero means and variances σ_m^2. The constants d_{jm} equal one if random effect m appears in the utility function for alternative j, and zero otherwise. The error components account for unobserved, alternative specific variation. With this device, the sets of random effects in different utility functions can overlap, so as to accommodate correlation in the unobservables across choices. The random effects may also be heteroscedastic, with

$$\sigma_{m,i}^2 = \sigma_m^2 \exp(\theta_m'\mathbf{z}_i).$$

The probabilities attached to the choices are now

$$\text{Prob}(y_i = j) = \frac{\exp\left(\beta'\mathbf{x}_{ij} + \gamma_j'\mathbf{z}_i + \Sigma_{m=1}^{M} d_{jm}\sigma_m u_{im}\right)}{\sum_{q=1}^{J_i}\exp\left(\beta'\mathbf{x}_{iq} + \gamma_q'\mathbf{z}_i \Sigma_{m=1}^{M} d_{qm}\sigma_m u_{im}\right)}.$$

This is precisely an analog to the random effects model for single equation models. Given the patterns of d_{jm}, this can provide a nesting structure as well.

3.9 Heteroscedastic Extreme Value

In the conditional logit model,

$$U(\textit{ choice } j \textit{ for individual } i\ ) = U_{ij} = \beta'\mathbf{x}_{ij} + \gamma_j'\mathbf{z}_i + \varepsilon_{ij}, j = 1,...,J_i,$$

$$\text{Prob}(y_i = j) = \frac{\exp\left(\beta'\mathbf{x}_{ij} + \gamma_j'\mathbf{z}_i\right)}{\sum_{m=1}^{J_i}\exp\left(\beta'\mathbf{x}_{im} + \gamma_m'\mathbf{z}_i\right)},$$

an implicit assumption is that the variances of ε_{ji} are the same. With the type 1extreme value distribution assumption, this common value is $\pi^2/6$. This assumption is a strong one, and it is not necessary for identification or estimation. The heteroscedastic extreme value model relaxes this assumption. We assume, instead, that

$$F(\varepsilon_{ij}) = \exp(-\exp(-\theta_j\varepsilon_{ij})],$$

$$\text{Var}[\varepsilon_{ij}] = \sigma_j^2\,(\pi^2/6) \text{ where } \sigma_j^2 = 1/\theta_j^2,$$

with one of the variance parameters normalized to one for identification. A further extension of this model allows the variance parameters to be heterogeneous, in the standard fashion

$$\sigma_{ij}^2 = \sigma_j^2 \exp(\gamma'\mathbf{z}_i).$$

3.10 Nested and Generalized Nested Logit

The nested logit model is an extension of the conditional logit model. The models supported by *NLOGIT* are based on variations of a four level tree structure such as the following:

```
ROOT                                  root
                                       |
                     ------------------------------------
                     |                                  |
TRUNKS             trunk1                             trunk2
                     |                                  |
              ---------------                    ---------------
              |             |                    |             |
LIMBS       limb1         limb2                limb3         limb4
              |             |                    |             |
          ---------     ---------            ---------     ---------
          |       |     |       |            |       |     |       |
BRANCHES branch1 branch2 branch3 branch4  branch5 branch6 branch7 branch8
          |       |       |       |        |       |       |       |
        -----   -----   -----   -----    -----   -----   -----   -----
        |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |
ALTS    a1  a2  a3  a4  a5  a6  a7  a8   a9 a10 a11 a12 a13 a14 a15 a16
```

The choice probability under the assumption of the nested logit model is defined to be the conditional probability of alternative *j* in branch *b*, limb *l*, and trunk *r*, $j|b,l,r$:

$$P(j|b,l,r) = \frac{\exp(\boldsymbol{\beta}'\mathbf{x}_{j|b,l,r})}{\sum_{q|b,l,r}\exp(\boldsymbol{\beta}'\mathbf{x}_{q|b,l,r})} = \frac{\exp(\boldsymbol{\beta}'\mathbf{x}_{j|b,l,r})}{\exp(J_{b|l,r})},$$

where $J_{b|l,r}$ is the inclusive value for branch *b* in limb *l*, trunk *r*, $J_{b|l,r} = \log \Sigma_{q|b,l,r}\exp(\boldsymbol{\beta}'\mathbf{x}_{q|b,l,r})$. At the next level up the tree, we define the conditional probability of choosing a particular branch in limb *l*, trunk *r*,

$$P(b|l,r) = \frac{\exp(\boldsymbol{\alpha}'\mathbf{y}_{b|l,r} + \tau_{b|l,r}J_{b|l,r})}{\sum_{s|l,r}\exp(\boldsymbol{\alpha}'\mathbf{y}_{s|l,r} + \tau_{s|l,r}J_{s|l,r})} = \frac{\exp(\boldsymbol{\alpha}'\mathbf{y}_{b|l,r} + \tau_{b|l,r}J_{b|l,r})}{\exp(I_{l|r})},$$

where $I_{l|r}$ is the inclusive value for limb *l* in trunk *r*, $I_{l|r} = \log \Sigma_{s|l,r}\exp(\boldsymbol{\alpha}'\mathbf{y}_{s|l,r} + \tau_{s|l,r}J_{s|l,r})$. The probability of choosing limb *l* in trunk *r* is

$$P(l|r) = \frac{\exp(\boldsymbol{\delta}'\mathbf{z}_{l|r} + \sigma_{l|r}I_{l|r})}{\sum_{s|r}\exp(\boldsymbol{\delta}'\mathbf{z}_{q|r} + \sigma_{s|r}I_{s|r})} = \frac{\exp(\boldsymbol{\delta}'\mathbf{z}_{l|r} + \sigma_{l|r}I_{l|r})}{\exp(H_r)},$$

where H_r is the inclusive value for trunk *r*, $H_r = \log \Sigma_{s|l}\exp(\boldsymbol{\delta}'\mathbf{z}_{s|r} + \sigma_{s|r}I_{s|r})$. Finally, the probability of choosing a particular limb is

$$P(r) = \frac{\exp(\boldsymbol{\theta}'\mathbf{h}_r + \phi_r H_r)}{\sum_s \exp(\boldsymbol{\theta}'\mathbf{h}_s + \phi_s H_s)}.$$

By the laws of probability, the unconditional probability of the observed choice made by an individual is

$$P(j,b,l,r) = P(j|b,l,r) \times P(b|l,r) \times P(l|r) \times P(r).$$

This is the contribution of an individual observation to the likelihood function for the sample.

The 'nested logit' aspect of the model arises when any of the $\tau_{b|l,r}$ or $\sigma_{l|r}$ or ϕ_r differ from 1.0. If all of these deep parameters are set equal to 1.0, the unconditional probability reduces to

$$P(j,b,l,r) = \frac{\exp(\boldsymbol{\beta}'\mathbf{x}_{j|b,l,r} + \boldsymbol{\alpha}'\mathbf{y}_{b|l,r} + \boldsymbol{\delta}'\mathbf{z}_{l|r} + \boldsymbol{\theta}'\mathbf{h}_r)}{\sum_r \sum_l \sum_b \sum_j \exp(\boldsymbol{\beta}'\mathbf{x}_{j,b,l,r} + \boldsymbol{\alpha}'\mathbf{y}_{b,l,r} + \boldsymbol{\delta}'\mathbf{z}_{l,r} + \boldsymbol{\theta}'\mathbf{h}_r)},$$

which is the probability for a one level conditional (multinomial) logit model.

The generalized nested logit model is an extension of the nested logit model in which alternatives may appear in more than one branch. Alternatives that appear in more than one branch are allocated across branches probabilistically. The model estimated includes the usual nested logit framework (only two levels are supported in this framework), as well as the matrix of allocation parameters. The only difference between this and the more basic nested logit model is the specification of the tree. For the allocations of choices to branches, a multinomial logit form is used,

$$\pi_{j,b} = \text{Prob}(\textit{alternative j is in branch b}) = \exp(\theta_{j,b}) / \Sigma_s \exp(\theta_{j,s}),$$

where the parameters θ are estimated by the program. Note the denominator summation is over branches that the alternative appears in. The probabilities sum to one. The identification rule that one of the θs for each alternative modeled equals one is imposed. These allocations may depend on an individual characteristic (not a choice attribute), such as income. In this instance, the multinomial logit probabilities become functions of this variable,

$$\pi_{j,b} = \text{Prob}(\textit{alternative j is in branch b}) = \exp(\theta_{j,b} + \gamma_{j,b}z_i) / \Sigma_s \exp(\theta_{j,s} + \gamma_{j,s}z_i).$$

Now, to achieve identification, one of the θs and one of the γs is set equal to zero. It is convenient to form the matrix $\Pi = [\pi_{j,b}]$. This is a $J \times B$ matrix of allocation parameters. The rows sum to one, and note that some values in the matrix are zero. But, no rows have all zeros – every alternative appears in at least one branch, and no columns have all zeros – every branch contains at least one alternative. The probabilities for the observed choices are formed as

$$\begin{aligned} \text{Prob}(\textit{alternative}, \textit{branch}) &= P(j,b) \\ &= P(j|b) \times P(b) \end{aligned}$$

where

$$P(j\,|\,b) = \frac{[\pi_{j,b}U_j]^{\sigma_b}}{\sum_{s=1}^{B} [\pi_{j,s}U_s]^{\sigma_s}}$$

(the denominator summation is over the alternatives in that branch) and

$$P(b) = \frac{\left[\sum_{j|b} [\pi_{j,b} U_j]^{\sigma_b}\right]^{1/\sigma_b}}{\sum_{b=1}^{B}\left[\sum_{j|b} [\pi_{j,b} U_j]^{\sigma_b}\right]^{1/\sigma_b}}$$

3.11 Random Parameters Logit

In its most general form, we write the multinomial logit probability as

$$P(j \mid \mathbf{v}_i) = \frac{\exp(\alpha_{ji} + \boldsymbol{\theta}_j'\mathbf{z}_i + \boldsymbol{\phi}_j'\mathbf{f}_{ji} + \boldsymbol{\beta}_{ji}'\mathbf{x}_{ji})}{\sum_{q=1}^{J}\exp(\alpha_{qi} + \boldsymbol{\theta}_q'\mathbf{z}_i + \boldsymbol{\phi}_q'\mathbf{f}_{qi} + \boldsymbol{\beta}_{qi}'\mathbf{x}_{qi})},$$

where

$U(j,i) = \alpha_{ji} + \boldsymbol{\theta}_j'\mathbf{z}_i + \boldsymbol{\phi}_j'\mathbf{f}_{ji} + \boldsymbol{\beta}_{ji}'\mathbf{x}_{ji}$, $j = 1,...,J_i$ alternatives in individual i's choice set,

α_{ji} is an alternative specific constant which may be fixed or random, $\alpha_{Ji} = 0$,

$\boldsymbol{\theta}_j$ is a vector of nonrandom (fixed) coefficients, $\boldsymbol{\theta}_{Ji} = \mathbf{0}$,

$\boldsymbol{\phi}_j$ is a vector of nonrandom (fixed) coefficients,

$\boldsymbol{\beta}_{ji}$ is a coefficient vector that is randomly distributed across individuals; $\mathbf{v}_i$ enters $\boldsymbol{\beta}_{ji}$,

$\mathbf{z}_i$ is a set of choice invariant individual characteristics such as age or income,

$\mathbf{f}_{ji}$ is a vector of M individual and choice varying attributes of choices, multiplied by $\boldsymbol{\phi}_j$,

$\mathbf{x}_{ji}$ is a vector of L individual and choice varying attributes of choices, multiplied by $\boldsymbol{\beta}_{ji}$.

The term 'mixed logit' is often used in the literature (e.g., Revelt and Train (1998)) for this model. The choice specific constants, α_{ji} and the elements of $\boldsymbol{\beta}_{ji}$ are distributed randomly across individuals such that for each random coefficient, ρ_{ki} = any (not necessarily all of) α_{ji} or β_{jki}, the coefficient on attribute x_{jik}, k=1,...,K,

$$\rho_{jki} = \alpha_{ji} \text{ or } \beta_{jki} = \rho_{jk} + \boldsymbol{\delta}_k'\mathbf{w}_i + \sigma_k v_{ki},$$

or

$$\rho_{jki} = \alpha_{ji} \text{ or } \beta_{jki} = \exp(\rho_{jk} + \boldsymbol{\delta}_{jk}'\mathbf{w}_i + \sigma_{jk} v_{jki}).$$

The vector $\mathbf{w}_i$ (which does not include *one*) is a set of choice invariant characteristics that produce individual heterogeneity in the means of the randomly distributed coefficients; ρ_{jk} is the constant term and $\boldsymbol{\delta}_{jk}$ is a vector of 'deep' coefficients which produce an individual specific mean. The random term, v_{jki} is normally distributed (or distributed with some other distribution) with mean 0 and standard deviation 1, so σ_{jk} is the standard deviation of the marginal distribution of ρ_{jki}. The v_{jki}s are individual and choice specific, unobserved random disturbances - the source of the heterogeneity. Thus, as stated above, in the population

$$\alpha_{ji} \text{ or } \beta_{jki} \sim \text{Normal or Lognormal } [\rho_{jk} + \boldsymbol{\delta}_{jk}'\mathbf{w}_i, \sigma_{jk}^2].$$

(Other distributions may be specified.) For the full vector of K random coefficients in the model, we may write

$$\boldsymbol{\rho}_i = \boldsymbol{\rho} + \boldsymbol{\Delta}\mathbf{w}_i + \boldsymbol{\Gamma}\mathbf{v}_i.$$

where Γ is a diagonal matrix which contains σ_k on its diagonal. A nondiagonal Γ allows the random parameters to be correlated. Then, the full covariance matrix of the random coefficients is $\Sigma = \Gamma\Gamma'$. The standard case of uncorrelated coefficients has $\Gamma = \text{diag}(\sigma_1,\sigma_2 ,\ldots,\sigma_k)$. If the coefficients are freely correlated, Γ is a full, unrestricted, *lower triangular* matrix and Σ will have nonzero off diagonal elements. An additional level of flexibility is obtained by allowing the distributions of the random parameters to be heteroscedastic,

$$\sigma_{ijk}^2 = \sigma_{jk}^2 \times \exp(\gamma_{jk}'\mathbf{h}_i).$$

This is now built into the model by specifying

$$\rho_i = \rho + \Delta\mathbf{w}_i + \Gamma\,\Omega_i\,\mathbf{v}_i$$

where $$\Omega_i = \text{diag}[\sigma_{ijk}^2]$$

and now, Γ is a lower triangular matrix of constants with ones on the diagonal. Finally, autocorrelation can also be incorporated by allowing the random components of the random parameters to obey an autoregressive process,

$$v_{ki,t} = \tau_{ki}\, v_{ki,t-1} + c_{ki,t}$$

where $c_{ki,t}$ is now the random element driving the random parameter.

This produces, then, the full random parameters logit model

$$P(j\,|\,\mathbf{v}_i) = \frac{\exp(\alpha_{ji} + \beta_i'\mathbf{x}_{ji})}{\sum_{m=1}^{J}\exp(\alpha_{mi} + \beta_i'\mathbf{x}_{mi})},$$

$$\beta_i = \beta + \Delta\mathbf{z}_i + \Gamma\,\Omega_i\,\mathbf{v}_i$$
$\mathbf{v}_i$ ~ with mean vector $\mathbf{0}$ and covariance matrix $\mathbf{I}$.

The specific distributions may vary from one parameter to the next. We also allow the parameters to be lognormally distributed so that the preceding specification applies to the logarithm of the specific parameter.

3.12 Multinomial Probit

In this model, the individual's choice among J alternatives is the one with maximum utility, where the utility functions are

$$U_{ji} = \beta'\mathbf{x}_{ji} + \varepsilon_{ji}$$

where U_{ji} = utility of alternative j to individual i

$\mathbf{x}_{jit}$ = union of all attributes that appear in all utility functions. For some alternatives, $x_{jit,k}$ may be zero by construction for some attribute k which does not enter their utility function for alternative j.

The multinomial logit model specifies that ε_{ji} are draws from independent extreme value distributions (which induces the IIA condition). In the multinomial probit model, we assume that ε_{ji} are normally distributed with standard deviations Sdv[ε_{ji}] = σ_j and correlations Cor[ε_{ji}, ε_{qi}] = ρ_{jq} (the same for all individuals). Observations are independent, so Cor[ε_{ji},ε_{qs}] = 0 if i is not equal to s, for all j and q. A variation of the model allows the standard deviations and covariances to be scaled by a function of the data, which allows some heteroscedasticity across individuals.

The correlations ρ_{jq} are restricted to $-1 < \rho_{jq} < 1$, but they are otherwise unrestricted save for a necessarily normalization. The correlations in the last row of the correlation matrix must be fixed at zero. The standard deviations are unrestricted with the exception of a normalization - two standard deviations are fixed at 1.0 - *NLOGIT* fixes the last two.

This model may also be fit with panel data. In this case, the utility function is modified as follows:

$$U_{ji,t} = \boldsymbol{\beta}'\mathbf{x}_{ji,t} + \varepsilon_{ji,t} + v_{ji,t}$$

where 't' indexes the periods or replications. There are two formulations for $v_{ji,t}$,

Random effects $\quad v_{ji,t} = v_{ji,t}$ (the same in all periods)

First order autoregressive $\quad v_{ji,t} = \alpha_j v_{ji,t-1} + a_{ji,t}$.

It is assumed that you have a total of T_i observations (choice situations) for person i. Two situations might lend themselves to this treatment. If the individual is faced with a set of choice situations that are similar and occur close together in time, then the random effects formulation is likely to be appropriate. However, if the choice situations are fairly far apart in time, or if habits or knowledge accumulation are likely to influence the latter choices, then the autoregressive model might be the better one.

You can also add a form of individual heterogeneity to the disturbance covariance matrix. The model extension is

$$\text{Var}[\boldsymbol{\varepsilon}_i] = \exp[\boldsymbol{\gamma}'\mathbf{h}_i] \times \boldsymbol{\Sigma}$$

where $\boldsymbol{\Sigma}$ is the matrix defined earlier (the same for all individuals), and $\mathbf{h}_i$ is an individual (not alternative) specific set of variables not including a constant.

Chapter 4

Model and Command Summary for Discrete Choice Models

4.1 Introduction

The chapters to follow will provide details on the various discrete choice models you can estimate with *NLOGIT* and on the model commands you will use to request the estimates. This chapter will provide a brief summary listing of the models and model commands. The variety of logit models now use a set of specific names, rather than qualifiers to more general model classes as in earlier versions of *NLOGIT* and *LIMDEP*. For example, the model name **OLOGIT** can be used instead of **ORDERD;Logit**. The earlier formats remain available, but the newer ones may prove more convenient. The full listing of these commands is also given below. The commands below specify the essential parts needed to fit the model. The numerous options and different forms are discussed in the chapters to follow (and, were noted, in the *LIMDEP Econometric Modeling Guide* as well).

4.2 Model Summary

The descriptions below present the different discrete choice models that are the main feature of *NLOGIT*. Note, once again, *NLOGIT* contains all of *LIMDEP*, so all of the models documented in the *Econometric Modeling Guide*, including the regression models, limited dependent vriable models, generalized linear models, sample selection models, and so on are supported in *NLOGIT*., as well as the ancillary tools including **MATRIX**, etc. The models described below include several listed in Section N4.3 that are part of the general *LIMDEP/NLOGIT* econometric modeling package, two listed in Section N4.4 that provide a bridge between the discrete choice models in *LIMDEP* and *NLOGIT*, then the set listed in Section N4.5 that are supported only by *NLOGIT*.

4.3 Basic Discrete Choice Models

The binomial probit and logit and the ordered probit and logit models are *LIMDEP*'s primary model frameworks for single equation, single decision, discrete choice models. The ordered choice and the bivariate and multivariate probit models are multivariate extensions of the simple probit model.

4.3.1 Binary Choice Models

There are numerous binary choice models. The ones that interest us here are the binary probit and logit models. The probit model is requested with

```
PROBIT      ; Lhs = dependent variable
            ; Rhs = independent variables $
```

The binary logit model may be invoked with

BLOGIT ; Lhs = dependent variable
; Rhs = independent variables $

In earlier versions, you would use the **LOGIT** comand, which is still useable. **LOGIT** is the same as **BLOGIT** when the data on the dependent variable are either binary (zeros and ones) or proportions (strictly between zero and one).

4.3.2 Bivariate Binary Choices

The command for the bivariate probit model is

BVPROBIT ; Lhs = variable 1, variable 2
; Rh1 = independent variables for equation 1
; Rh2 = independent variables for equation 2 $

In this form, the Lhs specifies two binary dependent variables. You may use proportions data instead, in which case, you will provide four proportions variables, in order, *p*00, *p*01, *p*10, *p*11. This command is the same as **BIVARIATE PROBIT** in earlier versions. (You may also use **BIVARIATE PROBIT**.)

4.3.3 Multivariate Binary Choice Models

The multivariate probit model is specified with

MVPROBIT ; Lhs = y1, y2, ..., yM
; Eq1 = Rhs variables for equation 1
; Eq2 = Rhs variables for equation 2
...
; EqM = Rhs variables for equation M $

Data for this model must be individual. The Lhs specifies a set of binary dependent variables. This command is the same as **MPROBIT** (which may still be used) in earlier versions of *NLOGIT*.

4.3.4 Ordered Choice Models

Chapter E22 of the *LIMDEP Econometric Modeling Guide* describes four forms for the ordered choice model, probit, logit, complementary log log and Gompertz. The first two interest us here. The ordered probit model is requested with

OPROBIT ; Lhs = dependent variable
; Rhs = independent variables $

This is the same as the **ORDERED PROBIT** command, which may still be used. In this model, the dependent variable is integer valued, taking the values 0, 1, ..., *J*. All *J*+1 values must appear in the data set, including zero. You may supply a set of *J*+1 proportions variables instead. Proportions will sum to 1.0 for every observation.

The ordered logit model is requested with

OLOGIT ; Lhs = dependent variable
; Rhs = independent variables $

The same arrangement for the dependent variables as for the ordered probit model is assumed. This command is the same as **ODRERED ; Logit** in earlier versions of *NLOGIT* and *LIMDEP*.

4.4 Multinomial Logit Models

The 'multinomial logit model' is an early, restrictive version of the conditional logit model, which, itself, is the gateway model to the main model extensions described in Section N4.5.

4.4.1 Multinomial Logit

The multinomial logit model is invoked with

MLOGIT ; Lhs = dependent variable
; Rhs = independent variables $

Data for the **MLOGIT** command/model consist of an integer valued variable taking the values 0, 1, ..., *J*. This model may also be fit with proportions data. In that case, you will provide the names of *J*+1 Lhs variables that will be strictly between zero and one, and will sum to one at every observation. The **MLOGIT** command is the same as **LOGIT**. The program inspects the command (Lhs) and the data, and determines internally whether **BLOGIT** or **MLOGIT** is appropriate. Note, on proportions data, if you want to fit a binary logit model with proportions data, you will supply a single proportions variable, not two. (What would be the second one is just one minus the first.) If you want to fit a multinomial logit model with proportions data with three or more outcomes, you must provide the full set of proportions. Thus, you would never supply two Lhs variables in a **LOGIT**, **BLOGIT** or **MLOGIT** command.

4.4.2 Conditional Logit

The command for the conditional model, and the commands in the sections to follow, are variants of the **NLOGIT** command. This is a full class of estimators based on the conditional logit form. The commands that follow this one are also specific to *NLOGIT*, and are not available in *LIMDEP*.)

There are several forms of the essential command for fitting the conditional logit model with *NLOGIT*. The simpler one is

CLOGIT ; Lhs = dependent variable
; Choices = the names of the J alternatives
; Rhs = list of choice specific attributes
; Rh2 = list of choice invariant individual characteristics $

The data for this estimator consist of a set of *J* observations, one for each alternative. (The observation resembles a group in a panel data set.) The command just given assumes that every individual in the sample chooses from the same size choice set, *J*. The choice sets may have different numbers of choices, in which case, the command is changed to

; Lhs = dependent variable, choice set size variable

The second Lhs variable is structured exactly the same as a **;Pds** variable for a panel data estimator. In the second form of the model command, the utility functions are specified directly, symbolically.

The **;Rhs** and **;Rh2** specifications can be replaced with

; Model: ... specification of the utility functions.

The **CLOGIT** command is the same as **DISCRETE CHOICE** in *LIMDEP*. It is also the same as **NLOGIT** when the only information given in the command is that specified above, that is when none of the specifications that invoke the model extensions that are described in the sections to follow are provided.

4.4.3 Random Parameters Logit

The random parameters logit model (mixed logit model) is requested by specifying a conditional logit model, and adding the specification of the random parameters. The model command is

```
RPLOGIT     ; Lhs = dependent variable
            ; Choices = the names of the J alternatives
            ; Rhs = list of choice specific attributes
            ; Rh2 = list of choice invariant individual characteristics
            ; Fcn = the specifications of the random parameters
            ; ... other specifications for the random parameters model $
```

Once again, variable choice set sizes and utility function specifications are specified as in the **CLOGIT** command. This command is the same as

```
NLOGIT      ; RPL
            ; ... the rest of the command $
```

There is one modification that might be necessary. If you are providing variables that affect the means of the random parameters, you would generally use

```
NLOGIT      ; RPL = the list of variables
            ; ... the rest of the command $
```

The RPL specification may still be used this way. The command can be NLOGIT as above, or

```
RPLOGIT     ; RPL = the list of variables
            ; ... the rest of the command $
```

These are identical.

The random parameters model may also include an error components specification defined in the next section. The command will be

RPLOGIT ; Lhs = dependent variable
; Choices = the names of the J alternatives
; Rhs = list of choice specific attributes
; Rh2 = list of choice invariant individual characteristics
; Fcn = the specifications of the random parameters
; ... other specifications for the random parameters model
; ECM = specification $

4.4.4 Latent Class Logit

The essential form of the command for the latent class model is

LCLOGIT ; Lhs = dependent variable
; Choices = the names of the J alternatives
; Rhs = list of choice specific attributes
; Rh2 = list of choice invariant individual characteristics
; Pts = the number of classes $

Like the **RPLOGIT** command, you need to modify this command if you are providing variables that affect the class probabilities. You would generally use

NLOGIT ; LCM = the list of variables
; ... the rest of the command $

The LCM specification may still be used this way. The command can be **NLOGIT** as above, or identically,

LCLOGIT ; LCM = the list of variables
; ... the rest of the command $

4.4.5 Multinomial Probit

The essential command for the multinomial probit model is

MNPROBIT ; Lhs = dependent variable
; Choices = the names of the J alternatives
; Rhs = list of choice specific attributes
; Rh2 = list of choice invariant individual characteristics $

Variable choice set sizes and utility function specifications are specified as in the **CLOGIT** command. This command is the same as

NLOGIT ; MNP
; ... the rest of the command $

4.6 Command Summary

The following lists the current and where applicable, alternative forms of the discrete choice model commands. The two sets of commands are identical, and for each model, in *NLOGIT 4.0*, either command may be used for that model.

Models	Command	Alternative Command Form
Binary Choice Models in *NLOGIT* and *LIMDEP*		
Binary Probit	**PROBIT**	**PROBIT**
Binary Logit	**BLOGIT**	**LOGIT**
Bivariate Probit	**BVPROBIT**	**BIVARIATE PROBIT**
Multivariate Probit	**MVPROBIT**	**MPROBIT**
Ordered Choice Models in *NLOGIT* and *LIMDEP*		
Ordered Probit	**OPROBIT**	**ORDERED PROBIT**
Ordered Logit	**OLOGIT**	**ORDERED;Logit**
Multinomial Logit Mode in *NLOGIT* and *LIMDEP*		
Multinomial Logit	**MLOGIT**	**LOGIT**
Conditional Logit	**CLOGIT**	**DISCRETE CHOICE**
Conditional Logit Extensions in *NLOGIT*		
Conditional Logit	**CLOGIT**	**CLOGIT**
Multinomial Logit	**NLOGIT**	**NLOGIT** (Same as **CLOGIT**)
Error Components Logit	**ECLOGIT**	**NLOGIT;ECM=...**
Heteroscedastic Extreme Value	**HLOGIT**	**NLOGIT;HET**
Nested Logit	**NLOGIT;Tree=...**	**NLOGIT;Tree=...**
Generalized Nested Logit	**GNLOGIT;Tree=...**	**NLOGIT;GNL;Tree=...**
Random Parameters Logit	**RPLOGIT**	**NLOGIT;RPL**
Latent Class Logit	**LCLOGIT**	**NLOGIT;LCM**
Multinomial Probit	**MNPROBIT**	**NLOGIT;MNP**

Chapter 5

Basic Models for Discrete Choice

5.1 Introduction

We define models in which the response variable being described is inherently discrete as qualitative response (QR) models. This chapter will describe two of *LIMDEP*'s many estimators for qualitative dependent variable model estimators. The simplest of these is the binomial choice models. The ordered choice model is an extension of the binary choice model in which there are more than two ordered, nonquantitative outcomes, such as scores on a preference scale.

5.2 Modeling Binary Choice

A binomial response may be the outcome of a decision or the response to a question in a survey. Consider, for example, survey data which indicate political party choice, mode of transportation, occupation, or choice of location. We model these in terms of probability distributions defined over the set of outcomes. There are a number of interpretations of an underlying data generating process that produce the binary choice models we consider here. All of them are consistent with the models that *LIMDEP* estimates, but the exact interpretation is a function of the modeling framework.

> The essential model command for the parametric binary choice models is
>
> $$\left.\begin{array}{c}\textbf{PROBIT}\\ \textbf{or}\\ \textbf{LOGIT}\end{array}\right\}\ \textbf{; Lhs = dependent variable ; Rhs = regressors \$}$$

A latent regression is specified as

$$y^* = \boldsymbol{\beta}'\mathbf{x} + \varepsilon.$$

The observed counterpart to y^* is

$$y = 1 \text{ if and only if } y^* > 0.$$

This is the basis for most of the binary choice models in econometrics, and is described in further detail below. It is the same model as the reduced form in the previous paragraph. Threshold models, such as labor supply and reservation wages lend themselves to this approach.

The probabilities and density functions for the most common binary choice specifications are as follows:

Probit

$$F = \int_{-\infty}^{\beta'\mathbf{x}_i} \frac{\exp(-t^2/2)}{\sqrt{2\pi}} dt \;=\; \Phi(\beta'\mathbf{x}_i), \qquad f = \phi(\beta'\mathbf{x}_i)$$

Logit

$$F = \frac{\exp(\beta'\mathbf{x}_i)}{1+\exp(\beta'\mathbf{x}_i)} \;=\; \Lambda(\beta'\mathbf{x}_i), \qquad f = \Lambda(\beta'\mathbf{x}_i)[1 - \Lambda(\beta'\mathbf{x}_i)]$$

5.2.1 Model Commands

The model commands for the five binary choice models listed above are largely the same:

$$\left.\begin{array}{l}\textbf{PROBIT}\\ \textbf{or}\\ \textbf{LOGIT}\end{array}\right\} \textbf{; Lhs = dependent variable ; Rhs = regressors \$}$$

Data on the dependent variable may be either individual or proportions. You need not make any special note of which. *LIMDEP* will inspect the data to determine which type of data you are using. In either case, you provide only a single dependent variable. As usual, you should include a constant term in the model unless your application specifically dictates otherwise.

5.2.2 Output

The binary choice models generate a very large amount of output. Computation begins with least squares estimation in order to obtain starting values.

NOTE: The OLS results will not normally be displayed in the output. To request the display, use **; OLS** in any of the model commands.

Reported Estimates

Final estimates include:

- $\log L$ = the log likelihood function at the maximum,
- $\log L_0$ = the log likelihood function assuming all slopes are zero. If your Rhs variables do not include *one*, this statistic will be meaningless. It is computed as

$$\log L_0 = n[P\log P + (1-P)\log(1-P)]$$

where P is the sample proportion of ones.

- The chi squared statistic for testing H_0: $\boldsymbol{\beta} = \mathbf{0}$ (not including the constant) and the significance level = probability that χ^2 exceeds test value. The statistic is

$$\chi^2 = 2(\log L - \log L_0).$$

Numerous other results, listed in detail, will appear with these in the output. The standard statistical results, including coefficient estimates, standard errors, t ratios, and descriptive statistics for the Rhs variables appear next. A complete listing is given below with an example. After the coefficient estimates are given, two additional sets of results appear, an analysis of the model fit and an analysis of the model predictions.

We will illustrate with binary logit and probit estimates of a model for visits to the doctor using the German health care data described in Chapter E2. The first model command is

```
LOGIT     ; Lhs = doctor
          ; Rhs = one,age,hhninc,hhkids,educ,married
          ; OLS $
```

Note that the command requests the optional listing of the OLS starting values. The results for this command are as follows. With the exception of the table noted below, the same results (with different values, of course) will appear for all five parametric models. Some additional optional computations and results will be discussed later.

The initial OLS estimates are generally not reported unless requested with **; OLS**.

```
+-------------------------------------------------+
| Binomial logit model for binary choice          |
| These are the OLS values based on the           |
| binary variables for each outcome Y(i) = j.     |
+-------------------------------------------------+
+--------+--------------+----------------+--------+--------+----------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|
+--------+--------------+----------------+--------+--------+----------+
---------+Characteristics in numerator of Prob[Y = 1]
 Constant|     .56661068       .02118790    26.742   .0000
 AGE     |     .00468710       .00029114    16.099   .0000   43.5256898
 HHNINC  |    -.03976003       .01726656    -2.303   .0213    .35208362
 HHKIDS  |    -.05217181       .00680260    -7.669   .0000    .40273000
 EDUC    |    -.01071245       .00131378    -8.154   .0000   11.3206310
 MARRIED |     .01946888       .00757540     2.570   .0102    .75861817
```

Standard results for maximum likelihood estimation appear next (or first if OLS is not presented). These are the results generated for all models fit by maximum likelihood. The Hosmer-Lemeshow chi squared statistic is specific to the binary choice models. The information criteria are computed from the log likelihood, $\log L$, and the number of parameters estimated, K, as follows:

AIC = Akaike Information Criterion	$= -2(\log L - K)/n$
BIC = Bayesian Information Criterion	$= -2(\log L - K\log K)/n$
Finite Sample AIC	$= -2(\log L - K - K(K+1)/(n-K-1))/n$
HQIC	$= -2(\log L - K\log(\log n))/n$

```
Normal exit from iterations. Exit status=0.
+---------------------------------------------+
| Binomial Logit Model for Binary Choice      |
| Maximum Likelihood Estimates                |
| Dependent variable               DOCTOR     |
| Weighting variable                 None     |
| Number of observations            27326     |
| Iterations completed                  4     |
| Log likelihood function       -17673.10     |
| Number of parameters                  6     |
| Info. Criterion: AIC =          1.29394     |
|   Finite Sample: AIC =          1.29394     |
| Info. Criterion: BIC =          1.29574     |
| Info. Criterion:HQIC =          1.29452     |
| Restricted log likelihood     -18019.55     |
| McFadden Pseudo R-squared      .0192266     |
| Chi squared                    692.9077     |
| Degrees of freedom                    5     |
| Prob[ChiSqd > value] =         .0000000     |
| Hosmer-Lemeshow chi-squared = 110.37153     |
| P-value=  .00000 with deg.fr. =       8     |
+---------------------------------------------+
+--------+--------------+----------------+--------+--------+----------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|
+--------+--------------+----------------+--------+--------+----------+
---------+Characteristics in numerator of Prob[Y = 1]
 Constant|     .25111543       .09113537     2.755   .0059
 AGE     |     .02070863       .00128517    16.114   .0000   43.5256898
 HHNINC  |    -.18592232       .07506403    -2.477   .0133    .35208362
 HHKIDS  |    -.22947000       .02953694    -7.769   .0000    .40273000
 EDUC    |    -.04558783       .00564646    -8.074   .0000   11.3206310
 MARRIED |     .08529305       .03328573     2.562   .0104    .75861817
```

The next set of results computes various fit measures for the model. This table of information statistics is produced only for the logit model. It is generally used for analysis of the generalized maximum entropy (GME) estimator of the multinomial logit model, but it also provides some useful information for the binomial model even when fit by ML instead of GME. The entropy statistics are computed as follows:

$$Entropy = -\Sigma_i P_i \log P_i$$

where P_i is the probability predicted by the model. The three 'models' are '*M*,' the model fit by maximum likelihood, '*MC*,' the model in which all predicted probabilities are the sample proportion of ones (here 0.6291), and '*M*0,' (no model) in which all predicted probabilities are 0.5. The normalized entropy is the entropy divided by *n*log2. Finally, the entropy ratio statistic equals 2(*n*log2)(1 - normalized entropy). The percent correct predicted values are discussed below.

The next set of results examines the success of the prediction rule

$$\text{Predict } y_i = 1 \text{ if } P_i > P^* \text{ and 0 otherwise}$$

where P^* is a defined threshold probability. The default value of P^* is 0.5, which makes the prediction rule equivalent to 'Predict $y_i = 1$ if the model says the predicted event $y_i = 1 \mid \mathbf{x}_i$ is more likely than the complement, $y_i = 0 \mid \mathbf{x}_i$.' You can change the threshold from 0.5 to some other value with

; Limit = your *P**

```
+----------------------------------------------------------------------+
| Information Statistics for Discrete Choice Model.                    |
|                            M=Model MC=Constants Only   M0=No Model |
| Criterion F (log L)   -17673.09788      -18019.55173  -18940.93986 |
| LR Statistic vs. MC      692.90772            .00000        .00000 |
| Degrees of Freedom         5.00000            .00000        .00000 |
| Prob. Value for LR          .00000            .00000        .00000 |
| Entropy for probs.     17673.09788       18019.55173   18940.93986 |
| Normalized Entropy          .93306            .95135       1.00000 |
| Entropy Ratio Stat.     2535.68395        1842.77624        .00000 |
| Bayes Info Criterion       1.29537           1.32072       1.38816 |
| BIC(no model) - BIC         .09270            .06744        .00000 |
| Pseudo R-squared            .01923            .00000        .00000 |
| Pct. Correct Pred.        62.85223            .00000      50.00000 |
| Means:       y=0    y=1    y=2    y=3    y=4    y=5     y=6   y>=7 |
| Outcome     .3709  .6291  .0000  .0000  .0000  .0000  .0000  .0000 |
| Pred.Pr     .3709  .6291  .0000  .0000  .0000  .0000  .0000  .0000 |
| Notes: Entropy computed as Sum(i)Sum(j)Pfit(i,j)*logPfit(i,j).     |
|        Normalized entropy is computed against M0.                  |
|        Entropy ratio statistic is computed against M0.             |
|        BIC = 2*criterion - log(N)*degrees of freedom.              |
|        If the model has only constants or if it has no constants,  |
|        the statistics reported here are not useable.               |
+----------------------------------------------------------------------+
```

A variety of fit measures for the model are listed.

```
+----------------------------------------+
| Fit Measures for Binomial Choice Model |
| Logit    model for variable DOCTOR     |
+----------------------------------------+
| Proportions P0= .370892   P1= .629108  |
| N =   27326 N0=   10135   N1=   17191  |
| LogL=   -17673.098 LogL0=  -18019.552  |
| Estrella = 1-(L/L0)^(-2L0/n) = .02528  |
+----------------------------------------+
|     Efron |  McFadden  |  Ben./Lerman  |
|    .02435 |    .01923  |       .54487  |
|    Cramer | Veall/Zim. |     Rsqrd_ML  |
|    .02470 |    .04348  |       .02504  |
+----------------------------------------+
| Information  Akaike I.C. Schwarz I.C.  |
| Criteria        1.29394      1.29574   |
+----------------------------------------+
+-----------------------------------------------------------+
|Predictions for Binary Choice Model.  Predicted value is |
|1 when probability is greater than  .500000, 0 otherwise.|
|Note, column or row total percentages may not sum to     |
|100% because of rounding. Percentages are of full sample.|
+------+---------------------------------+----------------+
|Actual|         Predicted Value         |                |
|Value |       0                1        | Total Actual   |
+------+----------------+----------------+----------------+
|  0   |    378 (  1.4%)|   9757 ( 35.7%)|  10135 ( 37.1%)|
|  1   |    394 (  1.4%)|  16797 ( 61.5%)|  17191 ( 62.9%)|
+------+----------------+----------------+----------------+
|Total |    772 (  2.8%)|  26554 ( 97.2%)|  27326 (100.0%)|
+------+----------------+----------------+----------------+
```

This table computes a variety of conditional and marginal proportions based on the results using the defined prediction rule. For examples, the 97.708% equals (16797/17191)100% while the 63.256% is (16797/26554)100%.

```
=======================================================================
Analysis of Binary Choice Model Predictions Based on Threshold =  .5000
-----------------------------------------------------------------------
Prediction Success
-----------------------------------------------------------------------
Sensitivity = actual 1s correctly predicted                     97.708%
Specificity = actual 0s correctly predicted                      3.730%
Positive predictive value = predicted 1s that were actual 1s    63.256%
Negative predictive value = predicted 0s that were actual 0s    48.964%
Correct prediction = actual 1s and 0s correctly predicted       62.852%
-----------------------------------------------------------------------
Prediction Failure
-----------------------------------------------------------------------
False pos. for true neg. = actual 0s predicted as 1s            96.270%
False neg. for true pos. = actual 1s predicted as 0s             2.292%
False pos. for predicted pos. = predicted 1s actual 0s          36.744%
False neg. for predicted neg. = predicted 0s actual 1s          51.036%
False predictions = actual 1s and 0s incorrectly predicted      37.148%
=======================================================================
```

Retained Results

The results saved by the binary choice models are:

Matrices:	*b*	=	estimate of β (also contains γ for the Burr model)
	varb	=	asymptotic covariance matrix
Scalars:	*kreg*	=	number of variables in Rhs
	nreg	=	number of observations
	logl	=	log likelihood function

5.2.3 Analysis of Marginal Effects

Marginal effects in a binary choice model may be obtained as

$$\frac{\partial E[y\,|\,\mathbf{x}]}{\partial \mathbf{x}} = \frac{\partial F(\beta'\mathbf{x})}{\partial \mathbf{x}} = \frac{dF(\beta'\mathbf{x})}{d(\beta'\mathbf{x})}\beta \;= F'(\beta'\mathbf{x})\beta \;=\; f(\beta'\mathbf{x})\beta$$

That is, the vector of marginal effects is a scalar multiple of the coefficient vector. The scale factor, $f(\beta'\text{x})$, is the density function, which is a function of $\mathbf{x}$. This function can be computed at any data vector desired. You can request the computation to be done automatically at the vector of means of the current sample by adding

; Marginal Effects

to your command.

Marginal Effects for Dummy Variables

When one of the variables in **x** is a dummy variable, the derivative approach to estimating the marginal effect is not appropriate. An alternative which is closer to the desired computation for a dummy variable which we denote *z*, is

$$\begin{aligned}\Delta F_z &= \text{Prob}[y = 1 \mid z = 1] - \text{Prob}[y = 1 \mid z = 0]\\ &= F(\boldsymbol{\beta}'\mathbf{x} + \alpha z \mid z = 1) - F(\boldsymbol{\beta}'\mathbf{x} + \alpha z \mid z = 0).\end{aligned}$$

For this type of variable, the asymptotic standard error must be changed as well. This is accomplished simply by changing the appropriate row of **G** to

$$\mathbf{G}_z = [f(\boldsymbol{\beta}'\mathbf{x}+ \alpha z)] \begin{pmatrix}\mathbf{x}\\ 1\end{pmatrix}' - [f(\boldsymbol{\beta}'\mathbf{x} + \alpha z)] \begin{pmatrix}\mathbf{x}\\ 0\end{pmatrix}'$$

NLOGIT examines the variables in the model and makes this adjustment automatically.

5.2.4 Robust Covariance Matrix Estimation

The preceding describes a covariance estimator that accounts for a specific, observed aspect of the data. The concept of the ‘robust’ covariance matrix is that it is meant to account for hypothetical, unobserved failures of the model assumptions. The intent is to produce an asymptotic covariance matrix that is appropriate even if some of the assumptions of the model are not met. (It is an important, but infrequently discussed issue whether the estimator, itself, remains consistent in the presence of these model failures – that is, whether the so called robust covariance matrix estimator is being computed for an inconsistent estimator.)

The Sandwich Estimator

It is becoming common in the literature to adjust the estimated asymptotic covariance matrix for possible misspecification in the model which leaves the MLE consistent but the estimated asymptotic covariance matrix incorrectly computed. One example would be a binary choice model with unspecified latent heterogeneity. A frequent adjustment for this case is the ‘sandwich estimator,’ which is the choice based sampling estimator suggested above with weights equal to one. (This suggests how it could be computed.) The desired matrix is

$$\text{Est.Asy.Var}\left[\hat{\beta}\right] = \left[\sum_{i=1}^{n}\left(\frac{\partial^2 \log F_i}{\partial\hat{\beta}\partial\hat{\beta}'}\right)\right]^{-1}\left[\sum_{i=1}^{n}\left(\frac{\partial \log F_i}{\partial\hat{\beta}}\right)\left(\frac{\partial \log F_i}{\partial\hat{\beta}'}\right)'\right]\left[\sum_{i=1}^{n}\left(\frac{\partial^2 \log F_i}{\partial\hat{\beta}\partial\hat{\beta}'}\right)\right]^{-1}$$

Three ways to obtain this matrix are

; Wts = one ; Choice Based sampling
or **; Robust**
or **; Cluster = 1**

The computation is identical in all cases. (As noted below, the last of them will be slightly larger, as it will be multiplied by *n*/(*n*-1).)

Clustering

A related calculation is used when observations occur in groups which may be correlated. This is rather like a panel; one might use this approach in a random effects kind of setting in which observations have a common latent heterogeneity. The parameter estimator is unchanged in this case, but an adjustment is made to the estimated asymptotic covariance matrix. The calculation is done as follows: Suppose the n observations are assembled in G clusters of observations, in which the number of observations in the ith cluster is n_i. Thus, $\sum_{i=1}^{G} n_i = n$. Let the observation specific gradients and Hessians be

$$\mathbf{g}_{ij} = \frac{\partial \log L_{ij}}{\partial \boldsymbol{\beta}} \quad \mathbf{H}_{ij} = \frac{\partial^2 \log L_{ij}}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}'}.$$

The uncorrected estimator of the asymptotic covariance matrix based on the Hessian is

$$\mathbf{V}_H = \mathbf{-H}^{-1} = \left(-\sum_{i=1}^{G} \sum_{j=1}^{n_i} \mathbf{H}_{ij}\right)^{-1}$$

Estimators for some models will use the BHHH estimator, instead;

$$\mathbf{V}_B = \left(\sum_{i=1}^{G} \sum_{j=1}^{n_i} \mathbf{g}_{ij}\mathbf{g}_{ij}'\right)^{-1}$$

Let $\mathbf{V}$ be the estimator chosen. Then, the corrected asymptotic covariance matrix is

$$\text{Est.Asy.Var}\left[\hat{\boldsymbol{\beta}}\right] = \mathbf{V}\,\frac{G}{G-1}\left[\sum_{i=1}^{G} \left(\sum_{j=1}^{n_i} \mathbf{g}_{ij}\right)\left(\sum_{j=1}^{n_i} \mathbf{g}_{ij}\right)'\right]\mathbf{V}$$

Note that if there is exactly one observation per cluster, then this is $G/(G$-1) times the sandwich estimator discussed above. Also, if you have fewer clusters than parameters, then this matrix is singular – it has rank equal to the minimum of G and K, the number of parameters.

To request the estimator, your command must include

; Cluster = specification

where the specification is either the fixed value if all the clusters are the same size, or the name of an identifying variable if the clusters vary in size. Note, this is not the same as the variable in the Pds function that is used to specify a panel. The cluster specification must be an identifying code that is specific to the cluster. For example, our health care data used in our examples is an unbalanced panel. The first variable is a family *id*, which we will use as follows

; Cluster = id

5.3 Ordered Choice Models

The basic ordered choice model is based on the following specification: There is a latent regression,

$$y_i^* \quad = \boldsymbol{\beta}'\mathbf{x}_i + \varepsilon_i,\ \ \varepsilon_i \sim F(\varepsilon_i\,|\boldsymbol{\theta}),\ E[\varepsilon_i|\mathbf{x}_i] = 0,\ \text{Var}[\varepsilon_i|\mathbf{x}_i] = 1,$$

The observation mechanism results from a complete censoring of the latent dependent variable as follows:

$$\begin{aligned} y_i \quad &= 0 \text{ if } y_i \le \mu_0, \\ &= 1 \text{ if } \mu_0 < y_i \le \mu_1, \\ &= 2 \text{ if } \mu_1 < y_i \le \mu_2, \\ &\dots \\ &= J \text{ if } y_i > \mu_{J-1}. \end{aligned}$$

The latent 'preference' variable, y_i^* is not observed. The observed counterpart to y_i^* is y_i. Four stochastic specifications are provided for the basic model shown above. The *ordered probit* model applies in applications such as surveys, in which the respondent expresses a preference with the above sort of ordinal ranking. The variance of ε_i is assumed to be one, since as long as y_i^*, $\boldsymbol{\beta}$, and ε_i are unobserved, no scaling of the underlying model can be deduced from the observed data. Since the μs are free parameters, there is no significance to the unit distance between the set of observed values of y. They merely provide the coding. Estimates are obtained by maximum likelihood. The probabilities which enter the log likelihood function are

$$\text{Prob}[y_i = j] = \text{Prob}[y_i^* \text{ is in the } j\text{th range}].$$

The model may be estimated either with individual data, with $y_i = 0, 1, 2, \dots$ or with grouped data, in which case each observation consists of a full set of J+1 proportions, $p_{0i},\dots,p_{Ji}$.

> **NOTE**: If your data are not coded correctly, this estimator will abort with one of several possible diagnostics – see below for discussion. Your dependent variable must be coded 0,1,...,J. We note that this differs from some other econometric packages which use a different coding convention.

There are numerous variants and extensions of this model which can be estimated: The underlying mathematical forms are shown below, where the CDF is denoted $F(z)$ and the density is $f(z)$. (Familiar synonyms are given as well.)

Probit: $$F(z) = \int_{-\infty}^{z} \frac{\exp(-t^2/2)}{\sqrt{2\pi}}dt = \Phi(z), \qquad f(z) = \phi(z)$$

Logit: $$F(z) = \frac{\exp(z)}{1+\exp(z)} = \Lambda(z), \qquad f(z) = \Lambda(z)[1 - \Lambda(z)]$$

The *ordered probit* model is an extension of the probit model for a binary outcome with normally distributed disturbances. The *ordered logit model* results from the assumption that ε has a standard logistic distribution instead of a standard normal.

5.3.1 Estimating Ordered Probability Models

The essential command for estimating ordered probability models is

ORDERED ; Lhs = y ; Rhs = regressors $

If you are using individual data, the Lhs variable must be coded 0,1,...,*J*. All the values must be present in the data. *LIMDEP* will look for empty cells. If there are any, estimation is halted. (If value '*j*' is not represented in the data, then the threshold parameter, μ_j is not estimable.) In this circumstance, you will receive a diagnostic such as

```
ORDE,Panel,BIVA PROBIT:A cell has (almost) no observations.
Empty cell: Y        never takes value   2
```

This diagnostic means exactly what it says. The ordered probability model cannot be estimated unless all cells are represented in the data. Users frequently overlook the coding requirement, y = 0,1,... If you have a dependent variable that is coded 1,2,..., you will see the following diagnostic:

```
Models - Insufficient variation in dependent variable.
```

The reason this particular diagnostic shows up is that *LIMDEP* creates a new variable from your dependent variable, say *y*, which equals zero when *y* equals zero, and one when *y* is greater than zero. It then tries to obtain starting values for the model by fitting a regression model to this new variable. If you have miscoded the Lhs variable, the transformed variable always equals one, which explains the diagnostic. In fact, there is no variation in the transformed dependent variable. If this is the case, you can simply use **CREATE** to subtract 1.0 from your dependent variable to use this estimator.

The probit model is the default specification. To estimate an ordered logit, add

; Model = Logit

to the command. The standardized logistic distribution (mean zero, standard deviation approximately 1.81) is used as the basis of the model instead of the standard normal.

5.3.2 Model Structure and Data

This model must include a constant term, *one*, as the first Rhs variable. Since the equation does include a constant term, one of the μs is not identified. We normalize μ_0 to zero. (Consider the special case of the binary probit model with something other than zero as its threshold value. If it contains a constant, this cannot be estimated.) Data may be grouped or individual. (Survey data might logically come in grouped form.) If you provide individual data, the dependent variable is coded 0, 1, 2, ..., *J*. There must be at least three values. Otherwise, the binary probit model applies. If the data are grouped, a full set of proportions, p_0, p_1, ..., p_J, which sum to one at every observation must be provided. In the individual data case, the data are examined to determine the value of *J*, which will be the largest observed value of *y* which appears in the sample. In the grouped data case, *J* is one less than the number of Lhs variables you provide. Once again, we note that other programs sometimes use different normalizations of the model. For example, if the constant term is forced to equal zero, then one will instead, add a nonzero threshold parameter, μ_0, which equals zero in the presence of a nonzero constant term.

5.3.3 Output from the Ordered Probability Estimators

All of the ordered probit/logit models begin with an initial set of least squares results of some sort. These are suppressed unless your command contains **; OLS**. The iterations are then followed by the maximum likelihood estimates in the usual tabular format. The final output includes a listing of the cell frequencies for the outcomes. When the data are stratified, this output will also include a table of the frequencies in the strata. The log likelihood function, and a log likelihood computed assuming all slopes are zero are computed. For the latter, the threshold parameters are still allowed to vary freely, so the model is simply one which assigns each cell a predicted probability equal to the sample proportion. This appropriately measures the contribution of the nonconstant regressors to the log likelihood function. As such, the chi squared statistic given is a valid test statistic for the hypothesis that all slopes on the nonconstant regressors are zero.

The sample below shows the standard output for a model with six outcomes. These are the German health care data described earlier. The dependent variable is the self reported health satisfaction rating. For the purpose of a convenient sample application, we have truncated the health satisfaction variable at five by discarding observations – in the original data set, it is coded 0,1,...,10.

```
+---------------------------------------------+
| Ordered Probability Model                   |
| Maximum Likelihood Estimates                |
| Dependent variable               NEWHSAT    |
| Weighting variable                  None    |
| Number of observations              8140    |
| Log likelihood function        -11284.69    |
| Number of parameters                   9    |
| Info. Criterion: AIC =           2.77486    |
| Info. Criterion: BIC =           2.78261    |
| Restricted log likelihood      -11308.02    |
| McFadden Pseudo R-squared       .0020635    |
| Chi squared                     46.66728    |
| Degrees of freedom                     4    |
| Prob[ChiSqd > value] =          .0000000    |
| Underlying probabilities based on Normal    |
|    Cell frequencies for outcomes            |
|  Y Count Freq  Y Count Freq  Y Count Freq   |
|  0   447 .054  1   255 .031  2   642 .078   |
|  3  1173 .144  4  1390 .170  5  4233 .520   |
+---------------------------------------------+
+---------+--------------+----------------+--------+--------+----------+
|Variable| Coefficient   | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|
+---------+--------------+----------------+--------+--------+----------+
---------+Index function for probability
 Constant|    1.32892012       .07275667    18.265   .0000
 FEMALE  |     .04525825       .02546350     1.777   .0755     .52936118
 HHNINC  |     .35589979       .07831928     4.544   .0000     .32998942
 HHKIDS  |     .10603682       .02664775     3.979   .0001     .33169533
 EDUC    |     .00927669       .00629721     1.473   .1407   10.8759203
---------+Threshold parameters for index
 Mu(1)   |     .23634786       .01236704    19.111   .0000
 Mu(2)   |     .62954428       .01439990    43.719   .0000
 Mu(3)   |    1.10763798       .01405938    78.783   .0000
 Mu(4)   |    1.55676227       .01527126   101.941   .0000
```

The model output is followed by a $(J+1)\times(J+1)$ frequency table of predicted versus actual values. (This table is not given when data are grouped or when there are more than 10 outcomes.) The predicted outcome for this tabulation is the one with the largest predicted probability.

```
+------------------------------------------------------------------------------+
|   Cross tabulation of predictions. Row is actual, column is predicted.       |
|   Model = Probit     .  Prediction is number of the most probable cell.      |
+-------+------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
| Actual|Row Sum|  0  |  1  |  2  |  3  |  4  |  5  |  6  |  7  |  8  |  9  |
+-------+------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
|      0|    447|    0|    0|    0|    0|    0|  447|
|      1|    255|    0|    0|    0|    0|    0|  255|
|      2|    642|    0|    0|    0|    0|    0|  642|
|      3|   1173|    0|    0|    0|    0|    0| 1173|
|      4|   1390|    0|    0|    0|    0|    0| 1390|
|      5|   4233|    0|    0|    0|    0|    0| 4233|
+-------+------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
|Col Sum|   8140|    0|    0|    0|    0|    0| 8140|    0|    0|    0|    0|
+-------+------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
```

Even though the model appears to be highly significant, the table of predictions has some large gaps in it. The estimation criterion for the ordered probability model is unrelated to its ability to predict those cells, and you will rarely see a predictions table that closely matches the actual outcomes. It often happens that even in a set of results with highly significant coefficients, only one or a few of the outcomes are predicted by the model.

Computation of predictions and ancillary variables is as follows: For each observation, the predicted probabilities for all J+1 outcomes are computed. Then if you request **; List**, the listing will contain

Predicted Y is the *Y* with the largest probability.

Residual is the largest of the J+1 probabilities (i.e., Prob[y = fitted *Y*]).

Var1 is the estimate of $E[y_i] = \sum_{i=0}^{J} i\times \text{Prob}[Y_i = i]$.

(Note that since the outcomes are only ordinal, this is not a true expected value.

Var2 is the probability estimated for the observed *Y*.

Estimation results kept by the estimator are as follows:

Matrices: *b* = estimate of $\boldsymbol{\beta}$,
varb = estimated asymptotic covariance,
mu = J-1 estimated μs.

Scalars: *kreg*, *nreg*, and *logl*.

Last Model: The labels are *b_variables*, *mu*1, ...

The specification **; Par** adds $\boldsymbol{\mu}$ (the set of estimated threshold values) to *b* and *varb*. The additional matrix, *mu* is kept regardless, but the estimated asymptotic covariance matrix is lost unless the command contains **; Par**.

5.3.4 Marginal Effects

Marginal effects in the ordered probability models are quite involved. Since there is no meaningful conditional mean function to manipulate, we consider, instead, the effects of changes in the covariates on the cell probabilities. These are:

$$\partial \text{Prob}[\text{cell } j]/\partial \mathbf{x}_i = [f(\mu_{j-1} - \boldsymbol{\beta}'\mathbf{x}_i) - f(\mu_j - \boldsymbol{\beta}'\mathbf{x}_i)] \times \boldsymbol{\beta},$$

where $f(.)$ is the appropriate density for the standard normal, $\phi(\bullet)$, logistic density, $\Lambda(\bullet)(1-\Lambda(\bullet))$, Weibull or Gompertz. Each vector is a multiple of the coefficient vector. But it is worth noting that the magnitudes are likely to be very different. In at least one case, Prob[cell 0], and probably more if there are more than three outcomes, the partial effects have exactly the opposite signs from the estimated coefficients. Thus, in this model, it is important to consider carefully the interpretation of the coefficient estimates. Marginal effects for all cells can be requested by including **; Marginal Effects** in the command. An example appears below.

NOTE: This estimator segregates dummy variables for separate computation in the marginal effects. The marginal effect for a dummy variable is the simple difference of the two probabilities, with and without the variable. See the application below for an illustration.

```
+-----------------------------------------------------+
| Marginal effects for ordered probability model      |
| M.E.s for dummy variables are Pr[y|x=1]-Pr[y|x=0]   |
| Names for dummy variables are marked by *.          |
+-----------------------------------------------------+
+---------+--------------+----------------+--------+--------+----------+
|Variable| Coefficient   | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|
+---------+--------------+----------------+--------+--------+----------+
---------+These are the effects on Prob[Y=00] at means.
 Constant|        .000000     ......(Fixed Parameter).......
 *FEMALE |     -.00498024        .00280960    -1.773   .0763     .52936118
 HHNINC  |     -.03907462        .00862973    -4.528   .0000     .32998942
 *HHKIDS |     -.01131976        .00277405    -4.081   .0000     .33169533
 EDUC    |     -.00101850        .00069179    -1.472   .1409   10.8759203
---------+These are the effects on Prob[Y=01] at means.
 Constant|        .000000     ......(Fixed Parameter).......
 *FEMALE |     -.00209668        .00118069    -1.776   .0758     .52936118
 HHNINC  |     -.01647123        .00362630    -4.542   .0000     .32998942
 *HHKIDS |     -.00483428        .00119623    -4.041   .0001     .33169533
 EDUC    |     -.00042933        .00029148    -1.473   .1408   10.8759203
Effects for Y=02, Y=03 and Y=04 are omitted.
---------+These are the effects on Prob[Y=05] at means.
 Constant|        .000000     ......(Fixed Parameter).......
 *FEMALE |      .01803285        .01014562     1.777   .0755     .52936118
 HHNINC  |      .14180876        .00073836   192.060   .0000     .32998942
 *HHKIDS |      .04218672        .00029837   141.390   .0000     .33169533
 EDUC    |      .00369631        .00250467     1.476   .1400   10.8759203
+--------------------------------------------------------------------------+
| Summary of Marginal Effects for Ordered Probability Model (probit)       |
+--------------------------------------------------------------------------+
Variable|    Y=00     Y=01     Y=02     Y=03     Y=04     Y=05     Y=06     Y=07 |
---------------------------------------------------------------------------+
ONE          .0000    .0000    .0000    .0000    .0000    .0000
*FEMALE     -.0050   -.0021   -.0041   -.0047   -.0021    .0180
HHNINC      -.0391   -.0165   -.0326   -.0373   -.0164    .1418
*HHKIDS     -.0113   -.0048   -.0096   -.0112   -.0052    .0422
EDUC        -.0010   -.0004   -.0008   -.0010   -.0004    .0037
```

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Chapter 6

The Multinomial Logit Model

6.1 Introduction

This chapter will describe a basic form of the 'multinomial logit' model. These models are also known variously as 'conditional logit,' 'discrete choice,' and 'universal logit' models, among other names. All of them can be viewed as special cases of a general model of utility maximization: An individual is assumed to have preferences defined over a set of alternatives (travel modes, occupations, food groups, etc.)

$$U_i(\text{alternative } 0) = \boldsymbol{\beta}_0'\mathbf{x}_{i0} + \varepsilon_{i0},$$

$$U_j(\text{alternative } 1) = \boldsymbol{\beta}_1'\mathbf{x}_{i1} + \varepsilon_{i1},$$

$$\ldots$$

$$U_i(\text{alternative } J) = \boldsymbol{\beta}_J'\mathbf{x}_{iJ} + \varepsilon_{iJ},$$

$$\text{Observed } Y_i = j \text{ if } U_i(\text{ alternative } j) > U_i(\text{ alternative } q) \ \forall\ q \neq j.$$

The 'disturbances' in this framework (individual heterogeneity terms) are assumed to be independently and identically distributed with identical type 1 extreme value distribution; the CDF is

$$F(\varepsilon_j) = \exp(-\exp(-\varepsilon_j)).$$

Based on this specification, the choice probabilities,

$$\text{Prob}(\text{choice } j) = \text{Prob}(U_j > U_q),\ \forall\ q \neq j$$

$$= \frac{\exp(\boldsymbol{\beta}_j'\mathbf{x}_{ji})}{\sum_{m=0}^{J}\exp(\boldsymbol{\beta}_m'\mathbf{x}_{mi})},\ j = 0,\ldots,J,$$

where 'i' indexes the observation, or individual, and 'j' and 'm' index the choices. The IID assumptions made about ε_j are quite stringent, and lead to the 'Independence from Irrelevant Alternatives' or IIA implications that characterize the model. Much (perhaps all) of the research on forms of this model consists of development of alternative functional forms and stochastic specifications that avoid this feature. The observed data consist of the Rhs vectors, $\mathbf{x}_{jt}$, and the outcome, or choice, y_t. (We also consider a number of variants.)

This chapter will examine what we call, for the present, the *multinomial logit* model. In this model, it is assumed that the Rhs variables consist of a set of individual specific characteristics, such as age, education, marital status, etc. These are the same for all choices, so the choice subscript on $\mathbf{x}$ in the formula above is dropped. The observation setting is the individual's choice among a set of alternatives, where it is assumed that the determinant of the choice is the *characteristics* of the individual. An example might be a model of choice of occupation. (This is the model originally devised by Nerlove and Press (1973).) The remaining chapters of this manual after this one will examine what we call (again only for convenience) the *discrete choice* model and, also, to differentiate the command, the *conditional logit* model. In this framework, we observe the *attributes* of the choices, rather than the characteristics of the individual. A well known example is travel mode choice. Samples of observations often consist of the attributes of the different modes and the choice actually made. Usually, no characteristics of the individuals are observed beyond their actual choice. Models may also contain mixtures of the two types of choice determinants. These are considered in the later chapters as well. (We emphasize, these naming distinctions are meaningless in the modeling framework – we just use them here only to organize the applicable parts of *NLOGIT*.

6.2 The Multinomial Logit Model

The general form of the *multinomial logit* model is

$$\text{Prob(choice } j) = \frac{\exp(\boldsymbol{\beta}_j'\mathbf{x}_t)}{\sum_{m=1}^{J}\exp(\boldsymbol{\beta}_m'\mathbf{x}_t)}, \; j = 0,...,J.$$

A possible J+1 *unordered* outcomes can occur. In order to identify the parameters of the model, we impose the normalization $\boldsymbol{\beta}_0 = \mathbf{0}$. This model is typically employed for individual or grouped data in which the '**x**' variables are characteristics of the observed individual(s), not the choices. The characteristics are the same across all outcomes. The study of occupational choice, by Schmidt and Strauss (1975) provides a well known application.

The data will appear as follows:

- Individual data: y_i coded 0, 1, ..., J,
- Grouped data: $y_{0i}, y_{1i},...,y_{Ji}$ give proportions or shares.

In the grouped data case, a weighting variable, n_t, may also be provided if the observations happen to be frequencies. The proportions variables must range from zero to one and sum to one at each observation. The full set must be provided, even though one is redundant. The data are inspected to determine which specification is appropriate. The number of Lhs variables given and the coding of the data provide the full set of information necessary to estimate the model, so no additional information about the dependent variable is needed.

This model proliferates parameters. There are $J \times K$ nonzero parameters in all, since there is a vector $\boldsymbol{\beta}_j$ for each probability except the first. Consequently, even moderately sized models quickly become very large ones if your outcome variable, y, takes many values. The maximum number of parameters which can be estimated in a model is 150 as usual with the standard configuration. However, if you are able to forego certain other optional features, the number of parameters can increase to 300. (This is the only model in *NLOGIT* that extends the 150 parameter limit.) The model size is detected internally. If your configuration contains more than 150 parameters, the following options and features become unavailable:

- marginal effects
- choice based sampling
- **; Rst** = list for imposing restrictions
- **; CML** = specification for imposing linear constraints
- **; Hold** for using the multinomial logit model as a sample selection equation

In addition, if your model size exceeds 150 parameters, the matrices *b* and *varb* cannot be retained. (But, see below for another way to retrieve large parameter matrices).

The choice set should be restricted to no more than 25 choices. If you have more than 25 choices, the number of characteristics that may be used becomes very small. Nonetheless, it is possible to fit models with up to 100 choices by using **CLOGIT**.

6.3 Model Command for the Multinomial Logit Model

The command for fitting this form of multinomial logit model is

MLOGIT ; Lhs = y or y0,y1,...yJ
; Rhs =regressors $

(The verb may also be **LOGIT**, which is what has always been used in previous versions of *LIMDEP and NLOGIT*.) All general options for controlling output and iterations are available except **;Keep=name**. (A program which can be used to obtain the fitted probabilities is listed below.) There are internally computed predictions for the multinomial logit model.

The **;Rst = list** form of restrictions is supported for imposing constraints on model parameters, either fixed value or equality. One possible application of the constrained model involves making the entire vector of coefficients in one probability equal that in another. You can do this as follows:

```
NAMELIST   ; x     = the entire set of Rhs variables $
CALC       ; k     = Col(x) $
LOGIT      ; Lhs   = y
           ; Rhs   = x
           ; Rst   = k_b, k_b, ... , k_b $
```

This would force the corresponding coefficients in all probabilities to be equal. You could also apply this to some, but not all of the outcomes, as in

; Rst = k_b, k_b, k_b2, k_b3

HINT: The coefficients in this model are not the marginal effects. But, forcing the coefficient on a characteristic in probability *j* to equal its counterpart in probability *m* also forces the two marginal effects to be equal.

6.4 Robust Covariance Matrix

It has become common in the literature to compute a ‘robust covariance matrix’ for the MLE. (The misspecification to which the matrix is robust is left unspecified in most cases.) The desired robust covariance matrix would result in the preceding computation if w_i equals one for all observations. This suggests a simple way to obtain it, just by specifying **;Choice Based;Wts=one**. Alternatively, just use

; Robust

which is equivalent.

A related calculation is used when observations occur in groups which may be correlated. This is rather like a panel; one might use this approach in a random effects kind of setting in which observations have a common latent heterogeneity. The parameter estimator is unchanged in this case, but an adjustment is made to the estimated asymptotic covariance matrix. The calculation is done as follows: Suppose the n observations are assembled in C clusters of observations, in which the number of observations in the cth cluster is n_c. Thus,

$$\sum_{c=1}^{C} n_c = n.$$

Denote by β the full set of model parameters, $[\beta_1', ..., \beta_J']'$. Let the observation specific gradients and Hessians for individual i in cluster c be

$$\mathbf{g}_{ic} = \frac{\partial \log L_{ic}}{\partial \beta},$$

$$\mathbf{H}_{ic} = \frac{\partial^2 \log L_{ic}}{\partial \beta \partial \beta'}.$$

The uncorrected estimator of the asymptotic covariance matrix based on the Hessian is

$$\mathbf{V}_H = -\mathbf{H}^{-1} = \left(-\sum_{c=1}^{C} \sum_{i=1}^{n_c} \mathbf{H}_{ic}\right)^{-1}.$$

The corrected asymptotic covariance matrix is

$$\text{Est.Asy.Var}\left[\hat{\beta}\right] = \mathbf{V}_H \frac{C}{C-1}\left[\sum_{c=1}^{C}\left(\sum_{i=1}^{n_c}\mathbf{g}_{ic}\right)\left(\sum_{i=1}^{n_c}\mathbf{g}_{ic}\right)'\right]\mathbf{V}_H.$$

Note that if there is exactly one observation per cluster, then this is $C/(C-1)$ times the sandwich (robust) estimator discussed above. Also, if you have fewer clusters than parameters, then this matrix is singular - it has rank equal to the minimum of C and JK, the number of parameters. This estimator is requested with

; Cluster = specification

where the specification is either a fixed number of observations per cluster, or an identifier that distinguishes clusters, such as an identification number. This estimator can also be extended to stratified as well as clustered data, using

; Stratum = specification

6.5 Output for the Multinomial Logit Model

Initial ordinary least squares results are used for the starting values for this model. For individual data, J binary variables are implied by the model. These are used in a least squares regression. For the grouped data case, a minimum chi squared, generalized least squares estimate is obtained by the weighted regression of

$$o_{ij} = \log(P_{ij} / P_{i0})$$

on the regressors, with weights $h_{ij} = (n_i P_{ij} P_{i0})^{1/2}$ (n_i may be 1.0). (Note that the dependent variables in these regressions are the 'odds ratios.') The OLS estimates based on the individual data are inconsistent, but the grouped data estimates are consistent (and, in the binomial case, efficient). The least squares estimates are included in the displayed results by including

; OLS

in the model command. The iterations are followed by the maximum likelihood estimates with the usual diagnostic statistics. An example is shown below.

NOTE: Minimum chi squared (MCS) is an estimator, not a model. Moreover, the MCS estimator has the same properties as, but is different from the maximum likelihood estimator. Since the MCS estimator in *NLOGIT* is not iterated, it should not be used as the final result of estimation. Without iteration, the MCS estimator is not a fixed point - the weights are functions only of the sample proportions, not the parameters. For current purposes, these are only useful as starting values.

Standard output for the logit model will begin with a table such as the following which results from estimation of a model in which the dependent variable takes values 0,1,2,3,5:

LOGIT; Lhs = newhsat ; Rhs = one,educ,hhninc,age,hhkids $

```
+---------------------------------------------+
| Multinomial Logit Model                     |
| Maximum Likelihood Estimates                |
| Model estimated: Mar 25, 2006 at 07:40:03PM.|
| Dependent variable             NEWHSAT      |
| Weighting variable                None      |
| Number of observations            8140      |
| Iterations completed                 5      |
| Log likelihood function      -11246.97      |
| Number of parameters                25      |
| Info. Criterion: AIC =         2.76953      |
|   Finite Sample: AIC =         2.76955      |
| Info. Criterion: BIC =         2.79104      |
| Info. Criterion:HQIC =         2.77688      |
| Restricted log likelihood    -11308.02      |
| McFadden Pseudo R-squared     .0053989      |
| Chi squared                   122.1013      |
| Degrees of freedom                  20      |
| Prob[ChiSqd > value] =        .0000000      |
+---------------------------------------------+
```

(This is based on the health satisfaction variable analyzed in the preceding chapter. We reduced the sample to those with *newhsat* reported zero to five. We would note, though these make for a fine numerical example, the multinomial logit model would be inappropriate for these ordered data.) The restricted log likelihood is computed for a model in which *one* is the only Rhs variable. In this case,

$$\log L_0 \;=\; \Sigma_j\, n_j \log P_j,$$

where n_j is the number of individuals who choose outcome j and $P_j = n_j/n$ = the jth sample proportion. The chi squared statistic is 2(log L - log L_0). If your model does not contain a constant term, this statistic need not be positive, in which case it is not reported. But, even if it is, the statistic is meaningless if your model does not contain a constant.

The diagnostic statistics are followed by the coefficient estimates: These are $\beta_1,...,\beta_J$. Recall β_0 is normalized to zero, and not reported.

Variable	Coefficient	Standard Error	b/St.Er.	P[|Z|>z]	Mean of X
Characteristics in numerator of Prob[Y = 1]					
Constant	-1.77566023	.69486152	-2.555	.0106	
EDUC	.07325707	.04476186	1.637	.1017	10.8759203
HHNINC	.28572052	.58129003	.492	.6231	.32998942
AGE	.00565832	.00838172	.675	.4996	46.9925061
HHKIDS	.27187563	.19642471	1.384	.1663	.33169533
Characteristics in numerator of Prob[Y = 2]					
Constant	-.54216913	.54865993	-.988	.3231	
EDUC	.06151644	.03616780	1.701	.0890	10.8759203
HHNINC	.85929376	.44943471	1.912	.0559	.32998942
AGE	-.00089766	.00650574	-.138	.8903	46.9925061
HHKIDS	.13920984	.15529658	.896	.3700	.33169533
Characteristics in numerator of Prob[Y = 3]					
Constant	-.25432932	.49206457	-.517	.6053	
EDUC	.10995580	.03246796	3.387	.0007	10.8759203
HHNINC	1.54516927	.40166793	3.847	.0001	.32998942
AGE	-.00955207	.00583708	-1.636	.1017	46.9925061
HHKIDS	.08177804	.14014086	.584	.5595	.33169533
Characteristics in numerator of Prob[Y = 4]					
Constant	.09378185	.48301274	.194	.8461	
EDUC	.10453491	.03201865	3.265	.0011	10.8759203
HHNINC	1.74362305	.39382043	4.427	.0000	.32998942
AGE	-.01430375	.00571476	-2.503	.0123	46.9925061
HHKIDS	.19548647	.13659829	1.431	.1524	.33169533
Characteristics in numerator of Prob[Y = 5]					
Constant	1.58458651	.45170179	3.508	.0005	
EDUC	.07526768	.03034831	2.480	.0131	10.8759203
HHNINC	1.64030015	.37209397	4.408	.0000	.32998942
AGE	-.01481141	.00525964	-2.816	.0049	46.9925061
HHKIDS	.19988328	.12654882	1.579	.1142	.33169533

The coefficient estimates are followed by an analysis of the multinomial logit criterion function, shown in the table below. Some of this table repeats part of the previous diagnostic information. Three log likelihoods are shown; M applies to the estimated model, MC is for the constants only model – these two values appear above – and M0 is the log likelihood for a model in which every predicted probability is $1/(J+1)$, that is, no model. Most of the remaining entries in this table relate to the information criteria and the and pseudo $R^2 = 1 - \text{MC/M}$. To underscore a point, we note that even though the model predicts more than half the observations correctly, the pseudo R^2 is only 0.0054. This is not a measure of fit. The sample means and average predicted probabilities match exactly because the model contains a constant term. This will always be the case. Finally, the table contains information about the entropy of the predicted probabilities.

```
+----------------------------------------------------------------------+
| Information Statistics for Discrete Choice Model.                    |
|                           M=Model MC=Constants Only   M0=No Model |
| Criterion F (log L)   -11246.96937      -11308.02002  -14584.92208 |
| LR Statistic vs. MC      122.10132            .00000        .00000 |
| Degrees of Freedom        20.00000            .00000        .00000 |
| Prob. Value for LR          .00000            .00000        .00000 |
| Entropy for probs.     11246.96937       11308.02002   14584.92208 |
| Normalized Entropy          .77114            .77532       1.00000 |
| Entropy Ratio Stat.     6675.90543        6553.80411        .00000 |
| Bayes Info Criterion       2.78551           2.80051       3.60564 |
| BIC(no model) - BIC         .82014            .80514        .00000 |
| Pseudo R-squared            .00540            .00000        .00000 |
| Pct. Correct Pred.        52.00246            .00000      16.66667 |
| Means:        y=0    y=1    y=2    y=3    y=4    y=5    y=6   y>=7 |
| Outcome     .0549  .0313  .0789  .1441  .1708  .5200  .0000  .0000 |
| Pred.Pr     .0549  .0313  .0789  .1441  .1708  .5200  .0000  .0000 |
| Notes: Entropy computed as Sum(i)Sum(j)Pfit(i,j)*logPfit(i,j).     |
|        Normalized entropy is computed against M0.                  |
|        Entropy ratio statistic is computed against M0.             |
|        BIC = 2*criterion - log(N)*degrees of freedom.              |
|        If the model has only constants or if it has no constants,  |
|        the statistics reported here are not useable.               |
+----------------------------------------------------------------------+
```

The statistical output for the coefficient estimates is followed by a table of predicted and actual frequencies, such as the following:

```
Frequencies of actual & predicted outcomes
Predicted outcome has maximum probability.

            Predicted
------  ------------------------------  +  -----
Actual      0    1    2    3    4    5  |  Total
------  ------------------------------  +  -----
  0         0    0    0    0    0  447  |    447
  1         0    0    0    0    0  255  |    255
  2         0    0    0    0    0  642  |    642
  3         0    0    0    0    0 1173  |   1173
  4         0    0    0    0    0 1390  |   1390
  5         0    0    0    0    0 4233  |   4233
------  ------------------------------  +  -----
Total       0    0    0    0    0 8140  |   8140
```

The prediction for any observation is the cell with the largest predicted probability for that observation.

NOTE: If you have more than three outcomes, it is very common, as occurred above, for the model to predict zero outcomes in one or more of the cells. Even in a model with very high t-ratios and great statistical significance, it takes a very well developed model to make predictions in all cells.

The **;List** specification produces a listing such as the following:

Observation	Observed Y	Predicted Y	Residual	MaxPr(i)	Prob[Y*=y]
1	2.0000	.00000	.0000	.2905	.1443
2	.00000	.00000	.0000	.2538	.2538
3	.00000	.00000	.0000	.2866	.2866
4	5.0000	3.0000	.0000	.2532	.1088
5	4.0000	3.0000	.0000	.2535	.2452
6	4.0000	3.0000	.0000	.2584	.2503
7	4.0000	4.0000	.0000	.2568	.2568
8	5.0000	.00000	.0000	.2354	.1440
9	.00000	4.0000	.0000	.2596	.2045
10	1.0000	.00000	.0000	.2554	.1027

In the listing, the MaxPR(i) is the probability attached to the outcome with the largest predicted probability; the outcome is shown as the Predicted Y. The last column shows the predicted probability for the observed outcome. Residuals are not computed - there is no significance to the reported zero.

The results kept for further use are:

Matrices: *b* and *varb*.

An additional matrix named *b_logit* is created which is $(J+1)\times K$. This matrix contains the parameters arranged so that β_j' is the jth row. The first row is zero. This matrix can be used to obtain fitted probabilities, as discussed below.

Scalars: *kreg*, *nreg*, *logl*, and *exitcode*.

Labels for **WALD** are constructed from the outcome and variable numbers. For example, if there are three outcomes and **Rhs=one,x1,x2**, the labels will be

Last Model: [*b*1_1,*b*1_2,*b*1_3,*b*2_1,*b*2_2,*b*2_3].

6.6 Marginal Effects

The marginal effects in this model are

$$\delta_j = \partial P_j / \partial \mathbf{x},\ j = 0,1,...,J.$$

For the present, ignore the normalization $\beta_0 = \mathbf{0}$. The notation P_j is used for Prob[y = j]. After some tedious algebra, we find

$$\delta_j = P_j(\beta_j - \bar{\beta}),$$

where

$$\bar{\beta} = \sum_{j=0}^{J} P_j \beta_j.$$

It follows that neither the sign nor the magnitude of δ_j need bear any relationship to those of β_j. (This is worth bearing in mind when reporting results.) The asymptotic covariance matrix for the estimator of δ_j would be computed using

$$\text{Asy.Var.}\left[\hat{\delta}_j\right] = \mathbf{G}_j \,\text{Asy.Var}\left[\hat{\beta}_j\right]\mathbf{G}_j',$$

where β is the full parameter vector. It can be shown that

$$\text{Asy.Var.}\left[\hat{\delta}_j\right] = \Sigma_l \Sigma_m \mathbf{V}_{jl}\,\text{Asy.Cov.}[\hat{\beta}_l, \hat{\beta}'_m]\mathbf{V}_{jm}', j\text{=}0,...,J,$$

where $$\mathbf{V}_{jl} = [\mathbf{1}(j = l) - P_l]\{P_j\mathbf{I} + \delta_j\mathbf{x}'\} - P_j\delta_l\mathbf{x}',$$

and $$\mathbf{1}(j = l) = 1 \text{ if } j = l, \text{ and } 0 \text{ otherwise.}$$

This full set of results is produced automatically when your **LOGIT** command includes

; Marginal Effects

NOTE: Marginal effects are computed at the sample averages of the Rhs variables in the model.

There is no conditional mean function in this model, so marginal effects are interpreted a bit differently from the usual case. What is reported is the derivatives of the probabilities. (Note this is the same as the ordered probability models.) These derivatives are saved in a matrix named *partials* which has J+1 rows and K columns. Each row is the vector of partial effects of the corresponding probability. Since the probabilities will always sum to one, the column sums in this matrix will always be zero. That is,

MATRIX ; list ; 1 ' partials $

will display a row matrix of zeros. The elasticities of the probabilities, $(\partial P_j/\partial x_k)\times(x_k/Pj)$ are placed in a $(J+1)\times K$ matrix named *elast_ml*. The format of the results is illustrated in the example below.

```
+-------------------------------------------+
| Partial derivatives of probabilities with |
| respect to the vector of characteristics. |
| They are computed at the means of the Xs. |
| Observations used for means are All Obs.  |
| A full set is given for the entire set of |
| outcomes, NEWHSAT  = 0 to NEWHSAT  =  5.  |
| Probabilities at the mean vector are      |
|  0= .052 1= .030 2= .078 3= .145 4= .171  |
|  5= .523                                  |
+-------------------------------------------+
+--------+--------------+----------------+--------+--------+----------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]|Elasticity|
+--------+--------------+----------------+--------+--------+----------+
---------+Marginal effects on Prob[Y = 0]
 Constant|    -.03681271       .02185753    -1.684   .0921
 EDUC    |    -.00415059       .00144841    -2.866   .0042   -.87310224
 HHNINC  |    -.07533229       .01759541    -4.281   .0000   -.48080659
 AGE     |     .00059378       .00025180     2.358   .0184    .53968780
 HHKIDS  |    -.00874507       .00608176    -1.438   .1505   -.05610378
---------+Marginal effects on Prob[Y = 1]
 Constant|    -.07581474       .01624087    -4.668   .0000
 EDUC    |    -.00021399       .00101558     -.211   .8331   -.07636415
 HHNINC  |    -.03569724       .01353007    -2.638   .0083   -.38652184
 AGE     |     .00052245       .00019922     2.622   .0087    .80558651
 HHKIDS  |     .00313091       .00463577      .675   .4994    .03407609
---------+Marginal effects on Prob[Y = 2]
 Constant|    -.09814200       .02502533    -3.922   .0001
 EDUC    |    -.00146816       .00158947     -.924   .3557   -.20405436
 HHNINC  |    -.04677448       .02027747    -2.307   .0211   -.19724874
 AGE     |     .00082844       .00031003     2.672   .0075    .49750446
 HHKIDS  |    -.00234229       .00728521     -.322   .7478   -.00992853
---------+Marginal effects on Prob[Y = 3]
 Constant|    -.13990259       .03064835    -4.565   .0000
 EDUC    |     .00429655       .00187257     2.294   .0218    .32276832
 HHNINC  |     .01275949       .02392200      .533   .5938    .02908292
 AGE     |     .00027978       .00039814      .703   .4822    .09081229
 HHKIDS  |    -.01264824       .00934649    -1.353   .1760   -.02897839
---------+Marginal effects on Prob[Y = 4]
 Constant|    -.10599103       .03277396    -3.234   .0012
 EDUC    |     .00415859       .00200931     2.070   .0385    .26381106
 HHNINC  |     .04913321       .02486677     1.976   .0482    .09457056
 AGE     |    -.00048333       .00042477    -1.138   .2552   -.13248126
 HHKIDS  |     .00451648       .00978660      .461   .6444    .00873817
---------+Marginal effects on Prob[Y = 5]
 Constant|     .45666308       .04483400    10.186   .0000
 EDUC    |    -.00262240       .00279117     -.940   .3475   -.05449699
 HHNINC  |     .09591130       .03450901     2.779   .0054    .06047510
 AGE     |    -.00174112       .00056626    -3.075   .0021   -.15633760
 HHKIDS  |     .01608821       .01313247     1.225   .2205    .01019657
 Marginal Effects Averaged Over Individuals
 --------+---------+---------+---------+---------+---------+---------+
 Variable|    Y=00 |    Y=01 |    Y=02 |    Y=03 |    Y=04 |    Y=05 |
 --------+---------+---------+---------+---------+---------+---------+
 ONE     |  -.0377 |  -.0772 |  -.0975 |  -.1380 |  -.1051 |   .4556 |
 EDUC    |  -.0044 |  -.0002 |  -.0014 |   .0043 |   .0042 |  -.0025 |
 HHNINC  |  -.0786 |  -.0361 |  -.0459 |   .0136 |   .0494 |   .0977 |
 AGE     |   .0006 |   .0005 |   .0008 |   .0003 |  -.0005 |  -.0018 |
 HHKIDS  |  -.0092 |   .0033 |  -.0023 |  -.0125 |   .0045 |   .0162 |
 --------+---------+---------+---------+---------+---------+---------+
 Averages of Individual Elasticities of Probabilities
```

Variable	Y=00	Y=01	Y=02	Y=03	Y=04	Y=05
ONE	-.7050	-2.4807	-1.2472	-.9593	-.6112	.8796
EDUC	-.8732	-.0764	-.2041	.3227	.2638	-.0545
HHNINC	-.4847	-.3904	-.2011	.0252	.0907	.0566
AGE	.5315	.7974	.4894	.0827	-.1406	-.1645
HHKIDS	-.0571	.0330	-.0110	-.0300	.0077	.0092

Matrix - B_LOGIT

[6, 5] Cell:

	1	2	3	4	5
1	0	0	0	0	0
2	-1.77566	0.0732571	0.285721	0.00565832	0.271876
3	-0.542169	0.0615164	0.859294	-0.000897661	0.13921
4	-0.254329	0.109956	1.54517	-0.00955207	0.081778
5	0.0937819	0.104535	1.74362	-0.0143038	0.195486
6	1.58459	0.0752677	1.6403	-0.0148114	0.199883

Matrix - ELAST_ML

[6, 5] Cell: -0.712012

	1	2	3	4	5
1	-0.712012	-0.873102	-0.480807	0.539688	-0.0561038
2	-2.48767	-0.0763641	-0.386522	0.805587	0.0340761
3	-1.25418	-0.204054	-0.197249	0.497504	-0.00992853
4	-0.966341	0.322768	0.0290829	0.0908123	-0.0289784
5	-0.61823	0.263811	0.0945706	-0.132481	0.00873817
6	0.872575	-0.054497	0.0604751	-0.156338	0.0101966

Matrix - PARTIALS

[6, 5] Cell: -0.0347176

	1	2	3	4	5
1	-0.0347176	-0.00354578	-0.065477	0.000527146	-0.00750032
2	-0.0512738	-0.000156155	-0.024176	0.000353446	0.00200267
3	-0.0843077	-0.00123466	-0.0402582	0.000704008	-0.00203835
4	-0.146084	0.00402383	0.00667303	0.00034874	-0.0125056
5	-0.123917	0.00415089	0.0436	-0.00039811	0.00424322
6	0.4403	-0.00323814	0.0796381	-0.00153523	0.0157984

Figure N9.1 Matrices Computed by MLOGIT

Marginal effects are computed by averaging the effects over individuals rather than computing them at the means. The difference between the two is likely to be quite small. Current practice favors the averaged individual effects, rather than the effects computed at the means. **MLOGIT** also reports elasticities with the marginal effects. An example appears below.

6.7 Computing Predicted Probabilities

Predicted probabilities can be computed automatically for the multinomial logit model. Since there are multiple outcomes, this must be handled a bit differently from other models. The procedure is as follows: Request the computation with

; Prob = name

as you would normally for a discrete choice model. However, for this model, *NLOGIT* does the following:

1. A namelist is created with name consisting of up to the first four letters of ‘name’ and *prob* is appended to it. Thus, if you use **;Prob=Pfit**, the namelist will be named *pfitprob*.
2. The set of variables, one for each outcome, are named with the same convention, with *prjj* instead of *prob*.

For example, in a five outcome model, the specification

; Prob = Job

produces a namelist

jpbprob = *jobpr*00, *jobpr*01, *jobpr*02, *jobpr*03, *jobpr*04.

The variables will then contain the respective probabilities. You may also use

; Fill

with this procedure to compute probabilities for observations that were not in the sample. Observations which contain missing data are bypassed as usual.

You can also compute a vector of probabilities for a specific observation, for example the sample means, by using the matrix *b_logit*. The following suggests how this might be done using the group means

```
NAMELIST   ; x      = the Rhs variables $
MATRIX     ; xb     = Mean(x)$
MATRIX     ; pvec   = b_logit*xb
           ; pvec   = Expn(pvec)
           ; pvec   = <1'pvec> * pvec $
```

Chapter 7

Data Setup for *NLOGIT*

7.1 Introduction

The preceding chapters of this manual described estimators for discrete choice models that are common to *NLOGIT* and *LIMDEP*, and use the data conventions and arrangements for the full set of models in *LIMDEP*. In this chapter and those to follow, we describe estimators that are specific to *NLOGIT*. In general, your data for these models will be arranged in a format that is set up to work well with this style of modeling. In almost all cases, the data used for all models that you fit with *NLOGIT* will be set up as if they were a panel. That is, each individual 'observation' will have a set of observations, with one 'line' of data for each choice in the choice set. Thus, in the analogy to a panel, the 'group' is a person and the group size would be the number of choices. You will use this arrangement in nearly all cases. This chapter will explain the various aspects of setting up the data for the NLOGIT models.

7.2 Basic Data Setup for *NLOGIT*

In the base case, the data are arranged as follows, where we use a specific set of values for the problem to illustrate. Suppose you observe 25 individuals. Each individual in the sample faces three choices and there are two attributes, q and w. For each observation, we also observe which choice was made. Suppose further that in the first three observations, the choices made were two, three, and one, respectively. The data matrix would consist of 75 rows, with 25 blocks of three rows. Within each block, there would be the set of attributes and a variable y, which, at each row, takes the value one if the alternative is chosen and zero if not. Thus, within each block of J rows, y will be one once and only once. For the hypothetical case, then, we have:

```
          Y          Q         W
i=1       0          q1,1      w1,1
       →  1          q2,1      w2,1
          0          q3,1      w3,1
          ---------------------------
i=2       0          q1,2      w1,2
          0          q2,2      w2,2
       →  1          q3,2      w3,2
          ---------------------------
i=3    →  1          q1,3      w1,3
          0          q2,3      w2,3
          0          q3,3      w3,3
```

and so on, continuing to $i = 25$, where → marks the row of the respondent's actual choice.

When you read these data, the data set is not treated any differently from any other panel. *Nobs* would be the total number of rows in the data set, in the hypothetical case, 75, not 25. The separation of the data set into the above groupings would be done at the time your particular model is estimated.

NOTE: Missing values are handled automatically by estimation programs in *NLOGIT*. You should not reset the sample or use **SKIP** with the *NLOGIT* models. Observations that have missing values are bypassed as a group.

Thus far, it is assumed that the observed outcome is an indicator of which choice was made among a fixed set of up to 100 choices. Numerous variations on this are possible:

- Data on the observed outcome may be in the form of frequencies, market shares, or ranks.
- The number of choices may differ across observations.
- The choice set may be extremely large.

The preceding described the base case model for a fixed number of choices using individual level data. There are several alternative formulations that might apply to the data set you are using.

7.3 Fixed and Variable Numbers of Choices

When every individual in the sample chooses from the same choice set, and all alternatives are available to all individuals, then the data set will appear as in the first example above, and will consist of *n* sets of *J* 'observations.' You indicate this case with a command such as:

	NLOGIT	**; Lhs = y**
or	**CLOGIT**	**; Choices = ... a list of *J* names for the choices**
or	**...**	**; ... the rest of the command $**

(Section N4.11 lists the eight different model commands that are used for estimation with *NLOGIT*. For convenience in what follows, where the same model format is used for all of them, we will use the generic model name **NLOGIT** in the command. The specific verbs, **CLOGIT**, **ECLOGIT**, **RLPOGIT**, etc. will be used in the specific chapters where the model itself is developed.) For example,

```
NLOGIT    ; Lhs = mode
          ; Choices = air,train,bus,car
          ; ... the rest of the command $
```

The list of choices is crucial, as it tells the program how many choices constitute an observation. (Otherwise, for example, there is no way to tell if 12 rows of data are three observations on a four choice setting or four observations on a three choice setting.)

We now consider the random utility model first in which the number of choices is not constant from one observation to the next. Two possible arrangements that might occur are as follows:

- There is a 'universal choice set,' from which individuals make their choices. But, not all choices are available to all individuals. Consider, for example, the choice of travel mode among (*air, train, bus, car*). If respondents are observed at many different locations, one or more of the choices, for example, *train*, might be unavailable to some of them, and those might vary from person to person.

- Individuals each choose among a set of J_i alternatives. However, there is no universal choice set defined as such. Consider, for example, the choice of which shopping center to shop at. If observations are taken in many different cities, we will observe numerous different choice sets, but there is no well defined universal choice set.

Either case can be accommodated. For both cases, you will provide a second **;Lhs** variable which gives the number of choices for each observation. The command is

NLOGIT ; Lhs = y,nij
; ... specification of the utility functions
; ... the rest of the command $

Note that the **;Choices=list** is not defined in this command, since in this case (the second one above), there is no clearly defined choice set. Nothing else need be changed. *NLOGIT* does all of the accounting internally. In this case, it is simply assumed that each individual has his or her own choice set. For example, one such data set might appear as follows.

	Y	*Q*	*W*	*Nij*
i=1	0	$q_{1,1}$	$w_{1,1}$	3
	1	$q_{2,1}$	$w_{2,1}$	3
	0	$q_{3,1}$	$w_{3,1}$	3
i=2	0	$q_{1,2}$	$w_{1,2}$	4
	0	$q_{2,2}$	$w_{2,2}$	4
	1	$q_{3,2}$	$w_{3,2}$	4
	0	$q_{4,2}$	$w_{4,2}$	4
i=3	1	$q_{1,3}$	$w_{1,3}$	2
	0	$q_{2,3}$	$w_{2,3}$	2

The model command might be

NLOGIT ; Lhs = y,nij
; Rhs = q,w $

Notice, once again, that the command does not contain a definition of the choice set, such as **;Choices=list** specification.

For the case of a universal choice set, suppose that the data set were, instead:

	Y	Q	W	Nij	Altij
i=1	0	$q_{1,1}$	$w_{1,1}$	3	1 (*Air*)
	→1	$q_{2,1}$	$w_{2,1}$	3	2 (*Train*)
	0	$q_{3,1}$	$w_{3,1}$	3	4 (*Car*)
i=2	0	$q_{1,2}$	$w_{1,2}$	4	1 (*Air*)
	0	$q_{2,2}$	$w_{2,2}$	4	2 (*Train*)
	→1	$q_{3,2}$	$w_{3,2}$	4	3 (*Bus*)
	0	$q_{4,2}$	$w_{4,2}$	4	4 (*Car*)
i=3	→1	$q_{1,3}$	$w_{1,3}$	2	3 (*Bus*)
	0	$q_{2,3}$	$w_{2,3}$	2	4 (*Car*)

The specific choice identifier, when it is needed, is provided as a *third* Lhs variable. For this case, the choice set would have to be defined. For example,

```
NLOGIT    ; Lhs       = y, nij, altij
          ; Choices   = air,train,bus,car $
          ; Rhs       = q,w $
```

Once again, in this setting, every individual is assumed to choose from a set of four alternatives, though the *altij* variable indicates that some of these choices are unavailable to some individuals.

7.4 Data for the Applications

The documentation of the *NLOGIT* program in the chapters to follow includes numerous applications based on the data set CLOGIT.DAT, that is distributed with *NLOGIT*. These data are a survey of the transport mode chosen by a sample of 210 travelers between Sydney and Melbourne (about 500 miles) and other points in nonmetropolitan New South Wales. As discussed in Section N10.7, data for *NLOGIT* will generally consist of a record (row of data) for each alternative in the choice set, for each individual. Thus, the data file contains 210 observations, or 840 records. The variables in the data set are as follows:

Original Data

mode = 0/1 for four alternatives: air, train, bus, car
(this variable equals one for the choice made, labeled *choice* below),
ttme = terminal waiting time,
invc = invehicle cost for all stages,
invt = invehicle time for all stages,
gc = generalized cost measure = Invc + Invt × value of time,
chair = dummy variable for chosen mode is air,
hinc = household income in thousands,
psize = traveling party size.

Transformed variables

aasc = choice specific dummy for air (generated internally),
tasc = choice specific dummy for train,
basc = choice specific dummy for bus,
casc = choice specific dummy for car,
hinca = *hinc* × *aasc*
psizea = *psize* × *aasc*,

The table below lists the first five observations in the data set. In the terms used here, each 'observation' is a block of four rows. The mode chosen in each block is boldfaced.

mode	*choice*	*ttme*	*invc*	*invt*	*gc*	*chair*	*hinc*	*psize*	*aasc*	*tasc*	*basc*	*casc*	*hinca*	*psizea*	*obs.*
Air	0	69	59	100	70	0	35	1	1	0	0	0	35	1	*i*=1
Train	0	34	31	372	71	0	35	1	0	1	0	0	0	0	
Bus	0	35	25	417	70	0	35	1	0	0	1	0	0	0	
Car	**1**	**0**	**10**	**180**	**30**	**0**	**35**	**1**	**0**	**0**	**0**	**1**	**0**	**0**	
Air	0	64	58	68	68	0	30	2	1	0	0	0	30	2	*i*=2
Train	0	44	31	354	84	0	30	2	0	1	0	0	0	0	
Bus	0	53	25	399	85	0	30	2	0	0	1	0	0	0	
Car	**1**	**0**	**11**	**255**	**50**	**0**	**30**	**2**	**0**	**0**	**0**	**1**	**0**	**0**	
Air	0	69	115	125	129	0	40	1	1	0	0	0	40	1	*i*=3
Train	0	34	98	892	195	0	40	1	0	1	0	0	0	0	
Bus	0	35	53	882	149	0	40	1	0	0	1	0	0	0	
Car	**1**	**0**	**23**	**720**	**101**	**0**	**40**	**1**	**0**	**0**	**0**	**1**	**0**	**0**	
Air	0	64	49	68	59	0	70	3	1	0	0	0	70	3	*i*=4
Train	0	44	26	354	79	0	70	3	0	1	0	0	0	0	
Bus	0	53	21	399	81	0	70	3	0	0	1	0	0	0	
Car	**1**	**0**	**5**	**180**	**32**	**0**	**0**	**3**	**0**	**0**	**0**	**1**	**0**	**0**	
Air	0	64	60	144	82	0	45	2	1	0	0	0	45	2	*i*=5
Train	0	44	32	404	93	0	45	2	0	1	0	0	0	0	
Bus	0	53	26	449	94	0	45	2	0	0	1	0	0	0	
Car	**1**	**0**	**8**	**600**	**99**	**0**	**45**	**2**	**0**	**0**	**0**	**1**	**0**	**0**	

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Chapter 8

NLOGIT Commands and Results

8.1 Introduction

NLOGIT is built around estimation of the parameters of the random utility model for discrete choice,

$$U(\text{ choice } j \text{ for individual } i\ ) = U_{ij} = \beta_{ij}'\mathbf{x}_{ij} + \varepsilon_{ij},\ j = 1,...,J_i,$$

in which individual i makes choice j if U_{ij} is the largest among the J_i utilities in the choice set. The parameters in the model are the weights in the utility functions and the deeper parameters of the distribution of the random terms. In some cases, the 'taste' parameters in the utility functions might vary across individuals and in most cases, they will vary across choices. The latter is simple to accommodate just by merging all parameters into one grand β and redefining $\mathbf{x}$ with some zeros in the appropriate places. But, for the former case, we will be interested in a lower level parameterization that involves what are sometimes labeled the 'hyperparameters.' Thus, it might be the extreme case (as in the random parameters logit model) that $\beta_{ij} = \mathbf{f}(\mathbf{z}_i, \Delta, \Gamma, \beta, \mathbf{v}_i)$ where Δ, Γ, β are lower level parameters, $\mathbf{z}_i$ is observed data, and $\mathbf{v}_i$ is a set of latent unobserved variables. The parameters of the random terms will generally be few in number, usually consisting of a small number of scaling parameters as in the heteroscedastic logit model, but they might be quite numerous, again in the random parameters model. In all cases, the main function of the routines is estimation of the structural parameters, then use of the estimated model for analysis of individual and aggregate behavior.

8.2 *NLOGIT* Commands

The essential command for the set of discrete choice models in *NLOGIT* is the same for all, with the exception of the model name:

```
Model    ; Lhs     = variable which indicates the choice made
         ; Choices = a set of J names for the set of choices
         ; Rhs     = choice varying attributes in the utility functions
         ; Rh2     = choice invariant variables, including one for ASCs $
```

The various models are as follows, where either of the two forms given may be used:

Model	Model Name	Alternative Command Form
Conditional Logit	**CLOGIT**	**NLOGIT**
Error Components Logit	**ECLOGIT**	**NLOGIT;ECM=...**
Heteroscedastic Extreme Value	**HLOGIT**	**NLOGIT;HET**
Nested Logit	**NLOGIT**	**NLOGIT;Tree=...**

Generalized Nested Logit	**GNLOGIT**	**NLOGIT;GNL**
Random Parameters Logit	**RPLOGIT**	**NLOGIT;RPL**
Latent Class Logit	**LCLOGIT**	**NLOGIT;LCM**
Multinomial Probit	**MNPROBIT**	**NLOGIT;MNP**

The description to follow in the rest of this chapter applies equally to all models. For convenience, we will use the generic **NLOGIT** command in most of the discussion, while you can use the specific model names in your estimation commands. The command builder for this model is found in Model:Discrete Choice/Discrete Choice. (Some features of the models, and the ECM model, are not provided by the command builders. Most of the features of these models are much easier to specify in the editor/command mode of entry.) The model and the choice set are set up on the Main page. The Rhs variables (attributes) and Rh2 variables (characteristics) are defined on the Options page. Note in the two windows on the options page, the Rhs of the model is defined in the left window and the Rh2 variables are specified in the right window.

A set of exactly J choice labels must be provided in the command. These are used to label the choices in the output. The number you provide is used to determine the number of choices there are in the model. Therefore, the set of the right number of labels is essential. Use any descriptor of eight or fewer characters desired - these do not have to be valid names, just a set of labels, separated in the list by commas.

The internal limit on J, the number of choices, is 100.

There are K attributes (Rhs variables) measured for the choices. The sections below will describe variations of this for different formulations and options. The total number of parameters in the utility functions will include K_1 for the Rhs variables and $(J\text{-}1)K_2$ for the Rh2 variables. The total number of utility function parameters is thus $K = K_1 + (J\text{-}1)K_2$.

The internal limit on K, the number of utility function parameters, is 100.

The random utility model specified by this setup is precisely of the form

$$U_{i,j} = \beta_1 x_{i,1} + \beta_2 x_{i,2} + ... + \beta_{K1} x_{i,K1} + \gamma_{1,j} z_{i,1} + ... + \gamma_{K2,j} z_{i,K2} + \varepsilon_{i,j},$$

where the x variables are given by the Rhs list and the z variables are in the Rh2 list. By this specification, the same attributes and the same characteristics appear in all equations, at the same position. The parameters, β_k appear in all equations, and so on. There are various ways to change this specification of the utility functions - i.e., the Rhs of the equations that underlie the model, and several different ways to specify the choice set. These will be discussed at several points below.

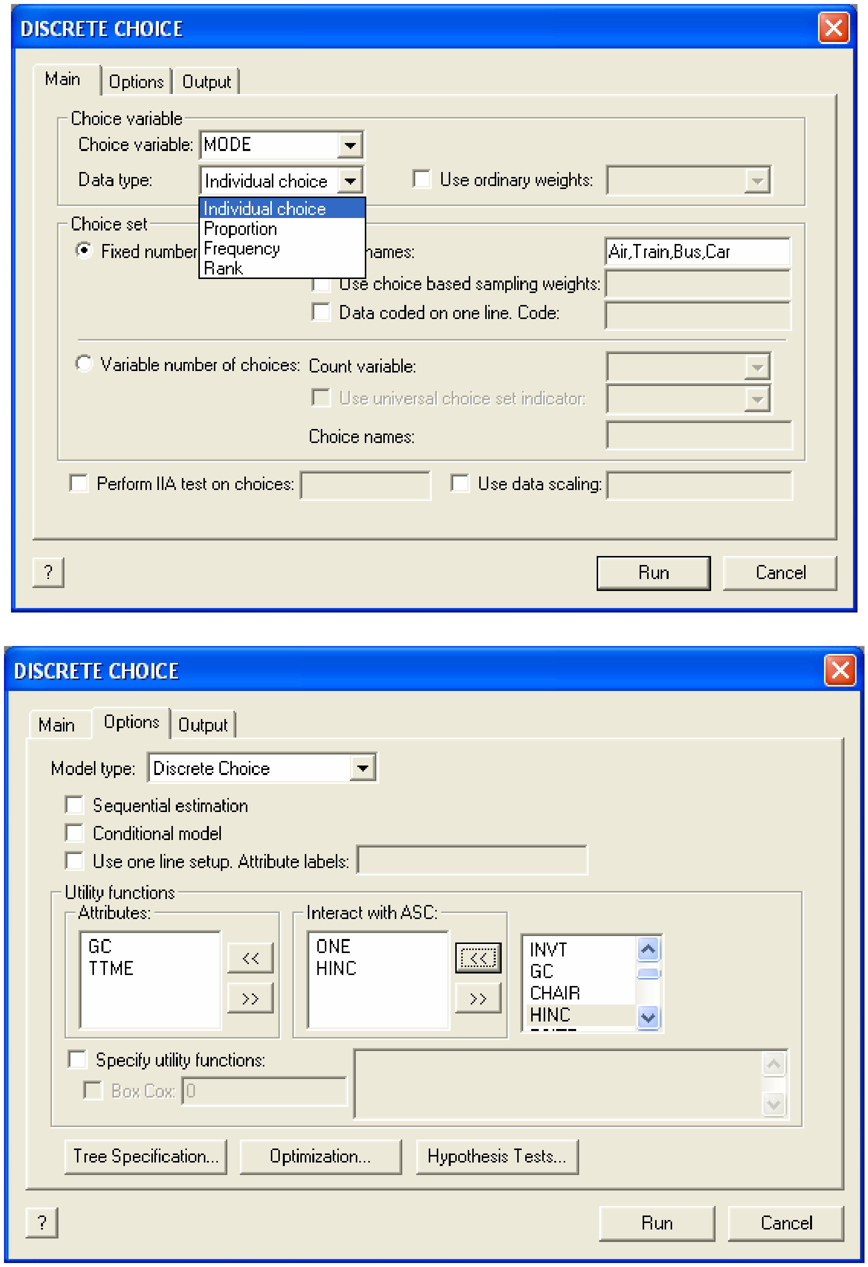


Figure 8.1 Command Builder for the Conditional Logit Model

8.2.1 Specifying the Choice Variable and the Choice Set

Every model fit by *NLOGIT* must include a specification for the choice variable and a definition of the choice set. The basic formulation would appear as

; Lhs = the dependent, or choice variable
; Choices = the names of the choices in the model

In general, your dependent variable is the name of a variable which indicates by a one or zero whether a particular alternative is selected, or it gives the proportion or frequency of individuals sampled that selected a particular alternative. When they are enumerated, the **;Choices** list gives names and possibly sampling weights for the set of alternatives.

All command builders begin with these two specifications. The discrete choice and nested logit models allow the full set of variants, while the other command builders expect the simple form with a fixed choice set. The Main page of the conditional logit command builder shown in Figure 8.2 illustrates. (A similar Main page is used for the nested logit command builder.) The command builder allows you to specify the choice variable and type of choice set in the three sections of this dialog box.

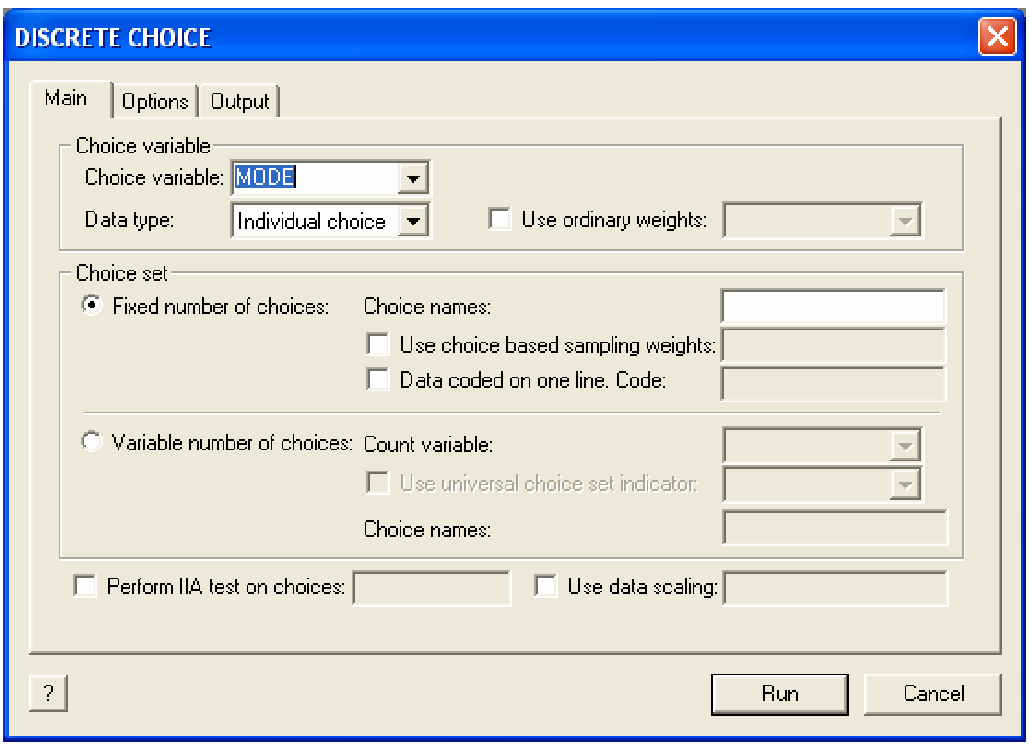


Figure 8.2. Main Page of Command Builder for Conditional Logit Model

NOTE: The command builder for the multinomial probit, HEV and RPL models requires you to provide a fixed sized choice set. This is a limitation of the command builder window, not the estimator. With the exception of the multinomial probit model, this is not a requirement of the models themselves. Only the multinomial probit model requires the number of choices to be fixed. For the HEV and RPL models, if you build your command in the text editor, rather than with the command builder, you may specify a variable choice set.

8.2.2 Specifying the Utility Functions with Rhs and Rh2

There are several ways to specify the utility functions in your **NLOGIT** command, in the text editor and in the command builder. In order to provide a simple explanation that covers the cases, we will develop the application that will be used in the chapters to follow to illustrate the models. The application is based on the data summarized in Section 7.4. We will model travel mode choice for trips between Sydney and Melbourne with utility functions for the four choices as follows:

		gc	***ttme***	***one***	***hinc***	***one***	***hinc***	***one***	***hinc***	***one***	***hinc***
U(*air*)	=	GC	TTME	A_AIR	AIR_HIN1	0	0	0	0	0	0
U(*train*)	=	GC	TTME	0	0	A_TRAIN	TRA_HIN2	0	0	0	0
U(*bus*)	=	GC	TTME	0	0	0	0	A_BUS	BUS_HIN3	0	0
U(*car*)	=	GC	TTME	0	0	0	0	0	0	0	0

The columns are headed by the names of variables, generalized cost (*gc*), terminal time (*ttme*) and household income (*hinc*). The entries in the body of the table are the names given to coefficients that will multiply the variables. Note that the generic coefficients in the first two columns are given the names of the variables they multiply while the interactions with the constants are given compound names. It is important to note the last two columns. The last one in a set of choice specific constants or variables that are interacted with them must be dropped to avoid a problem of collinearity in the model. In what follows, for brevity, we will omit these two columns. Before proceeding, we note the format of a set of parameter estimates for a model set up in exactly this fashion:

```
+--------+--------------+----------------+--------+--------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]|
+--------+--------------+----------------+--------+--------+
 GC      |    -.01092735       .00458775    -2.382   .0172
 TTME    |    -.09546055       .01047320    -9.115   .0000
 A_AIR   |    5.87481336       .80209034     7.324   .0000
 AIR_HIN1|    -.00537349       .01152940     -.466   .6412
 A_TRAIN |    5.54985728       .64042443     8.666   .0000
 TRA_HIN2|    -.05656186       .01397335    -4.048   .0001
 A_BUS   |    4.13028388       .67636278     6.107   .0000
 BUS_HIN3|    -.02858418       .01544418    -1.851   .0642
```

Note the construction of the compound names includes what might seem to be a redundant number at the end. This is necessary to avoid constructing identical names for different variables.

Utility Functions

A basic four choice model which contains *cost*, *time*, *one* and *income* will have utility functions

$$
\begin{aligned}
U_{i,air} &= \beta_{cost}\, cost_{i,air} + \beta_{time} time_{i,air} + \alpha_{air} + \gamma_{air} Income_i + \varepsilon_{i,air},\\
U_{i,train} &= \beta_{cost}\, cost_{i,train} + \beta_{time} time_{i,train} + \alpha_{train} + \gamma_{train} Income_i + \varepsilon_{i,train},\\
U_{i,bus} &= \beta_{cost}\, cost_{i,bus} + \beta_{time} time_{i,bus} + \alpha_{bus} + \gamma_{bus} Income_i + \varepsilon_{i,bus},\\
U_{i,car} &= \beta_{cost}\, cost_{i,car} + \beta_{time} time_{i,car} + \varepsilon_{i,bus}.
\end{aligned}
$$

The simple device you will use to construct utility functions in this fashion is

; Rhs = list of attributes that vary across choices

and

; Rh2 = list of variables that do not vary across choices.

The Rh2 variables are automatically expanded into a set of J-1 interactions with the choice specific constants, as they are in the matrix shown above. The implication is that, generally, you do not need to have these variables in your data set. They are automatically created by your command. (Note that our CLOGIT.DAT data set in Section 7.4 actually does contain the superfluous set of four choice specific constants, *aasc*, *tasc*, *basc* and *casc*.

NOTE: If you include *one* in your Rhs, it is automatically expanded to become a set of alternative specific constants. That is, *one* is automatically move to Rh2 if it is placed in the Rhs list.

The model specification for the four utility functions shown above would be

; Rhs = cost,time ; Rh2 = one,income

Note that the distinction between Rh2 and Rhs variables is that all variables in the first category are expanded by interacting with the choice specific binary variables. (The last term is dropped.)

Generic Coefficients

The simpler, but less flexible way to specify generic coefficients in a model is to use *NLOGIT*'s standard construction, by specifying a set of Rhs variables. The specification

; Rhs = gc,ttme

produces the utility functions in the first two columns in the table. Rhs variables are assumed to vary across the choices and will receive generic coefficients.

Alternative Specific Constants and Interactions with Constants

The logit model is homogeneous of degree zero in the attributes. Any attribute which does not vary across the choices, such as age, marital status, income etc., will simply fall out of the probability. Consider an example with a constant, one attribute and one characteristic,

$$\begin{aligned}\text{Prob}(\textit{choice } j) &= \frac{\exp(\alpha + \beta_1 cost_{ij} + \beta_2 income_i)}{\Sigma_{j=1}^{J} \exp(\alpha + \beta_1 cost_{ij} + \beta_2 income_i)} \\ &= \frac{\exp(\alpha + \beta_2 income_i)\exp(\beta_1 cost_{ij})}{\Sigma_{j=1}^{J} \exp(\alpha + \beta_2 income_i)\exp(\beta_1 cost_{ij})} \\ &= \frac{\exp(\alpha + \beta_2 income_i)\exp(\beta_1 cost_{ij})}{\exp(\alpha + \beta_2 income_i)\Sigma_{j=1}^{J} \exp(\beta_1 cost_{ij})} \\ &= \frac{\exp(\beta_1 cost_{ij})}{\Sigma_{j=1}^{J} \exp(\beta_1 cost_{ij})}.\end{aligned}$$

With a generic coefficient, the choice invariant characteristic falls out of the model. This includes the constant term, *one*. A model which contains such a characteristic with a generic coefficient is not estimable. This carries over to all of the more elaborate models such as the HEV, nested logit and MNP models as well. The solution to this complication is to create choice specific constant terms and, if need be, interact the invariant characteristic with the constant term. This is what appears in the last eight columns in the example above. (This is how the *MLOGIT* model in Chapter N9 arises – in that model, all variables are choice invariant.) Here, it produces a hybrid model, which can have both types of variables in the utility functions.

$$\text{Prob}(\textit{choice} = j) = \frac{\exp(\beta_1 cost_{i,j} + \alpha_j + \gamma_j Income_i)}{\sum_{j=1}^{J} \exp(\beta_1 cost_{i,j} + \alpha_j + \gamma_j Income_i)}.$$

There remains an indeterminacy in the model after it is expanded in this fashion. Suppose the same constant, say θ, is added to each γ_j. The resulting model is

$$\begin{aligned}\text{Prob}(\textit{choice} = j) &= \frac{\exp(\beta_1 cost_{i,j} + \alpha_j + (\gamma_j + \theta) Income_i)}{\sum_{j=1}^{J} \exp(\beta_1 cost_{i,j} + \alpha_j + (\gamma_j + \theta) Income_i)} \\ &= \frac{\exp(\beta_1 cost_{i,j} + \alpha_j + \gamma_j Income_i + \theta Income_i)}{\sum_{j=1}^{J} \exp(\beta_1 cost_{i,j} + \alpha_j + \gamma_j Income_i + \theta Income_i)} \\ &= \frac{\exp(\theta Income_i)\exp(\beta_1 cost_{i,j} + \alpha_j + \gamma_j Income_i)}{\exp(\theta Income_i)\sum_{j=1}^{J} \exp(\beta_1 cost_{i,j} + \alpha_j + \gamma_j Income_i)} \\ &= \frac{\exp(\beta_1 cost_{i,j} + \alpha_j + \gamma_j Income_i)}{\sum_{j=1}^{J} \exp(\beta_1 cost_{i,j} + \alpha_j + \gamma_j Income_i)}.\end{aligned}$$

So, the identical model arises for any θ. This means that the model still cannot be estimated in this form. The solution to this remaining issue is to normalize the coefficients so that one of the choice varying parameters is equal to zero. *NLOGIT* sets the last one to zero. The same result applies to the choice specific constant terms that you create with *one*. This produces the data matrix shown earlier, with the last two columns (in the dashed box) normalized to zeros.

Finally, while it is necessary for choice invariant variables to appear in Rh2, it is not necessary that all variables in the Rh2 list actually be choice invariant. Indeed, one could specify the preceding model with choice specific coefficients on the *cost* variable; it would appear

$$
\begin{aligned}
U_{i,air} &= \gamma_{cost,air}\ cost_{i,air} + \beta_{time}\ time_{i,air} + \alpha_{air} + \gamma_{air} Income_i + \varepsilon_{i,air},\\
U_{i,train} &= \gamma_{cost,train}\ cost_{i,train} + \beta_{time}\ time_{i,train} + \alpha_{train} + \gamma_{train} Income_i + \varepsilon_{i,train},\\
U_{i,bus} &= \gamma_{cost,bus}\ cost_{i,bus} + \beta_{time}\ time_{i,bus} + \alpha_{bus} + \gamma_{bus} Income_i + \varepsilon_{i,bus},\\
U_{i,car} &= \gamma_{cost,car}\ cost_{i,car} + \beta_{time}\ time_{i,car} + \varepsilon_{i,bus}.
\end{aligned}
$$

Note also, that there is no need to drop one of the *cost* coefficients because the variable *cost* varies by choices. You *can* estimate a model with four separate coefficients for *cost*, one in each utility function. However, it is not possible to do it by including *cost* in the Rh2 list as described above, because this form will automatically drop the last term (the one in the *car* utility function). You could obtain this form, albeit a bit clumsily, by creating the four interaction terms yourself and including them on the Rhs. We already have the alternative specific constants, so the following would work

```
CREATE      ; cost_a = gc * aasc
            ; cost_t = gc * tasc
            ; cost_b = gc * basc
            ; cost_c = gc * casc $
NLOGIT      ; ...     ; Rhs = time,cost_a,cost_t,cost_b,cost_c
                      ; Rh2 = one,income $
```

Having to create the interaction variables is going to be inconvenient. The alternative method of specifying the model described in the next section will be much more convenient. This method also allows you much greater flexibility in specifying utility functions.

HINT: There are many different possible configurations of alternative specific constants (ASCs) and alternative specific variables. In estimating a model, it is not possible to determine a priori if a singularity will arise as a consequence of the specification. You will have to discern this from the estimation results for the particular model.

The constant term, *one* fits the hint above. Recognizing this, *NLOGIT* assumes that if your Rhs list includes *one*, you are requesting a set of alternative specific constants. As such, when the Rhs list includes *one*, *NLOGIT* will create a full set of *J*-1 choice specific constants. (One of them must be dropped to avoid what amounts to the dummy variable trap.)

HINT: You need not have choice specific dummy variables in your data set. The Rh2 setup described here allows you to produce these variables as part of the model specification.

The remaining columns of the utility functions in the example above are produced with

```
; Rh2 = one,hinc
```

You should note, in addition, how the variables are expanded, as a set, in constructing the utility functions.

Command Builders

You can specify utility functions in this format in any of the command builders, as shown in Figure 8.3. The two windows allow you to select variables from the list at the right and assemble the Rhs list at the left or the Rh2 list in the center.

8.2.3 Building the Utility Functions

The model specification thus far builds the utility functions from the common Rhs and Rh2 specification. For example, in a four outcome model which contains *cost*, *time*, *one* and *income*, the data for the choice variable and the utility functions are contained in

$$
\begin{array}{cccccccccc}
 & \text{choice} & \text{cost} & \text{time} & \text{constants} & & & & \text{income} & \\
\mathbf{Z}_i = & \left[\begin{array}{ccccccccc}
y_{air} & c_a & t_a & 1 & 0 & 0 & income & 0 & 0 \\
y_{train} & c_t & t_t & 0 & 1 & 0 & 0 & income & 0 \\
y_{bus} & c_b & t_b & 0 & 0 & 1 & 0 & 0 & income \\
y_{car} & c_c & t_c & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] . & & & & & & & &
\end{array}
$$

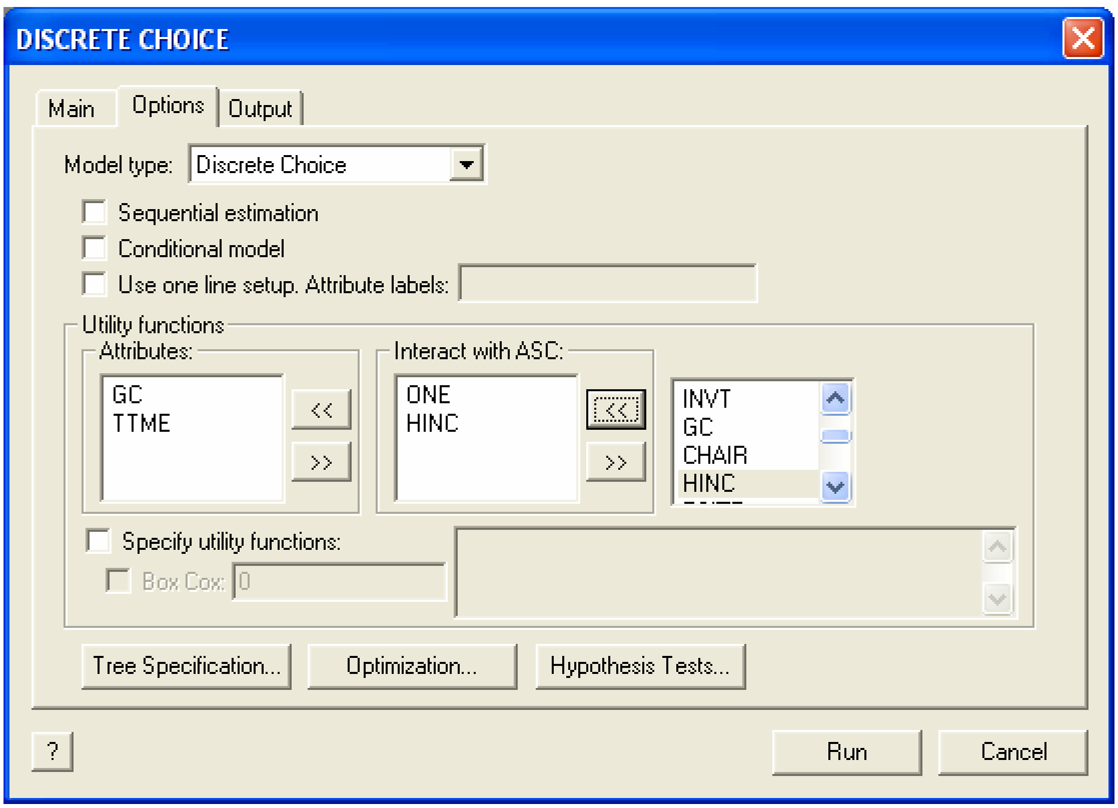


Figure 8.3. Specifying Utility Functions in the Command Builder

The utility functions are all the same;

$$U_{i,j} \;=\; \beta_1 cost_{i,j} + \beta_2 time_{i,j} + \alpha_j + \gamma_j income_i + \varepsilon_{i,j}.$$

One might want to have different attributes appear in the different utility functions, or impose other kinds of constraints on the parameters. This section will describe how to structure the utility functions individually, rather than generically with Rhs and Rh2.

The utility functions need not be the same for all choices. Different attributes may enter and the coefficients may be constrained in different ways. The following more flexible format can be used instead of the **;Rhs=list** and **Rh2=list** parts of the command described above. This format also provides way to provide starting values for parameters, so this can also replaces the **;Start=list** specification. Finally, you will also be able to use this format to fix coefficients, so it will be an easy way to replace the **;Rst = list** specification.

We begin with the case of a fixed (and named) set of choices, then turn to the cases of variable numbers of choices. We replace the Rhs/Rh2 setup with explicit definitions of the utility functions for the alternatives. Utility functions are built up from the format

```
; Model :    U ( choice 1 ) = linear equation  /
             U ( choice 2 ) = linear equation /
             ...
             U ( choice J ) = linear equation $
```

Though we have shown all *J* utility functions, for a given model specification, you could, in principle, not specify a utility function in the list. The implied specification would be $U_{ij} = \varepsilon_{ij}$.) The **:U(list)** is mandatory. *NLOGIT* scans for the '*U*' and the parentheses. For example:

```
; Model:  U ( air )  =  ba  +  bcost * gc
```

Note that the specification begins with '**;Model:**' - the colon ('**:**') is also mandatory. Parameters always come first, then variables. Constant terms need not multiply variables. Thus, *ba* in this model *could* be an '*Air specific constant*.' (It depends on whether *ba* appears elsewhere in the model.) Notice that the utility function defines both the variables and the parameters. Usually, you would give an equation for each choice in the model. For example:

```
NLOGIT    ; Lhs     = mode
          ; Choices = air,train,bus,car
          ; Model:  U( air )     = ba +   bcost * gc + btime * ttme /
                    U( car )     = bc +   bcost * gc   /
                    U( bus )     = bb +   bcost * gc   /
                    U(train)     =        bcost * gc +btime * ttme $
```

Utility functions are separated by slashes. Note also that the alternative specific constants stand alone without multiplying a variable. Your utility definitions now provide the names for the parameters. The estimates produced by this model command are as follows:

```
+--------+--------------+----------------+--------+--------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]|
+--------+--------------+----------------+--------+--------+
 BA      |    1.55491032       .37580063     4.138   .0000
 BCOST   |    -.02020918       .00434927    -4.647   .0000
 BTIME   |    -.08680295       .01122237    -7.735   .0000
 BC      |   -3.65316491       .46378035    -7.877   .0000
 BB      |   -3.91982604       .45611114    -8.594   .0000
```

One point that you might find useful to note. The order of the parameters in this list is determined by moving through the model definition from beginning to end. Each time a new parameter name is encountered, it is added to the list. Looking at the model command above, you can now see how the order in the displayed output arose.

The last example in the preceding subsection, which has four separate coefficients on a cost variable, *gc*, could be specified using

```
NLOGIT   ; Lhs  =  mode; Choices  =  air,train,bus,car
         ; Model :  U(air)   = bc*invc+bt*invt+aa+cha*hinc  + cga*gc /
                    U(train) = bc*invc+bt*invt+at+cht*hinc  + cgt*gc /
                    U(bus)   = bc*invc+bt*invt+ab+chb*hinc  + cgb*gc /
                    U(car)   = bc*invc+bt*invt              + cgc*gc $
```

The estimates are

Variable	Coefficient	Standard Error	b/St.Er.	P[|Z|>z]
BC	-.04386562	.01712959	-2.561	.0104
BT	-.00815115	.00241976	-3.369	.0008
AA	-1.37473591	.83837138	-1.640	.1011
CHA	.00703267	.01078793	.652	.5145
CGA	.03762100	.01676624	2.244	.0248
AT	2.53156832	.60800716	4.164	.0000
CHT	-.05096641	.01214303	-4.197	.0000
CGT	.03348741	.01506250	2.223	.0262
AB	1.17857565	.73948909	1.594	.1110
CHB	-.03339204	.01299642	-2.569	.0102
CGB	.03455919	.01516387	2.279	.0227
CGC	.03808057	.01523791	2.499	.0125

Alternative Specific Constants and Interactions

You can also specify alternative specific constants in this format, by using a special notation. When you have a **U(a1, a2, ..., aJ)** for *J* alternatives, then you may specify, instead of a single parameter, a list of parameters enclosed in pointed brackets, to signify interaction with choice specific constants. Thus, **<b1,b2,...,bL>** indicates *L* interactions with choice specific dummy variables. *L* may be any number up to the number of alternatives. Use a zero in any location in which the variable does not appear in the corresponding equation. For example,

```
; Model:   U( air )   = ba  +  bcost * gc   /
           U( car )   = bc  +  bcost * gc   /
           U( bus )   =        bcost * gc   /
           U(train)   = bt  +  bcost * gc   $
```

could be specified as

```
; Model:  U(air,car,bus,train) = <ba,bc,0,bt> + bcost * gc $
```

NOTE: Within a < ... > construction, the correspondence between positions in the list is with the U(... list ...) list, *not* with the original **;Choices** list.

Note the considerable savings in notation. The same device may also be used in interactions with attributes. For example:

```
; Model:   U( air )   = ba  +  bcprv * gc   /
           U( car )   = bc  +  bcprv * gc   /
           U( bus )   =        bcpub * gc   /
           U(train)   = bt  +  bcpub * gc   $
```

There are two cost coefficients, but the variable *gc* is common. This entire model can be collapsed into the single specification

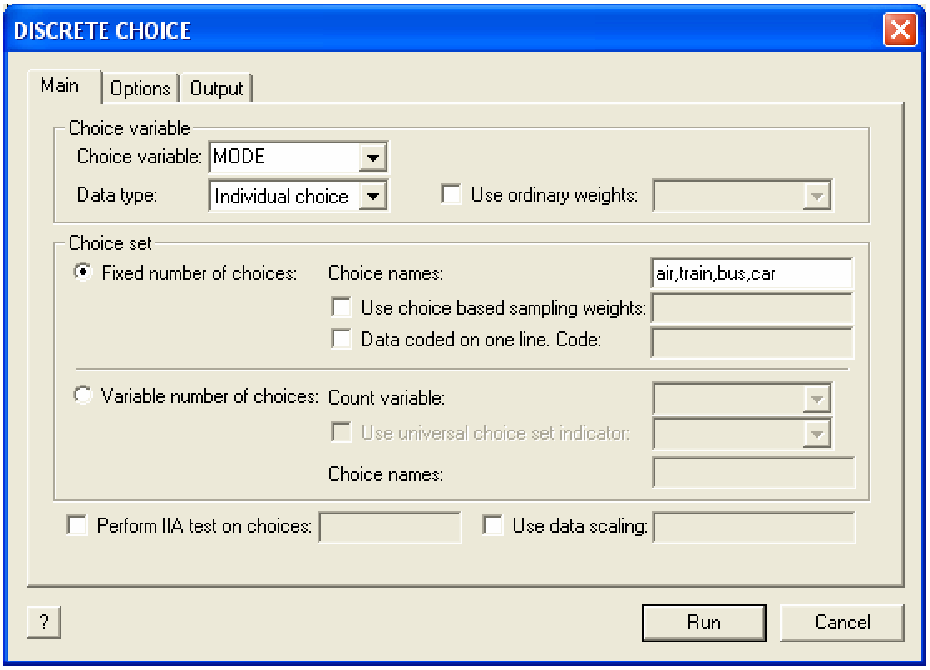
; Model: U(air,car,bus,train) = <ba,bc,0,bt> +
<bcprv,bcprv,bcpub,bcpub> * gc $

Parameters inside the brackets need not all be different if you wish to impose equality constraints.

Command Builders

The command builders provide space for you to build the utility functions in this fashion. See Figure 8.4. Since this is done by typing out the functions in the windows - there is no menu construction that would allow this - these will not save much effort.
Note that in the window, you must provide the entire specification for the utility functions, including the listing of which alternatives the definitions are to apply to. The model shown in the window in Figure 8.4 produces these results.

```
+---------------------------------------------+
| Discrete choice (multinomial logit) model   |
| Maximum Likelihood Estimates                |
| Dependent variable                 Choice   |
| Weighting variable                   None   |
| Iterations completed                    6   |
| Log likelihood function       -199.6825     |
| Number of parameters                    6   |
| Info. Criterion: AIC =          1.95888     |
|   Finite Sample: AIC =          1.96085     |
| Info. Criterion: BIC =          2.05451     |
| Info. Criterion:HQIC =          1.99754     |
| R2=1-LogL/LogL*  Log-L fncn  R-sqrd  RsqAdj |
| Constants only    -283.7588  .29630  .28953 |
| Chi-squared[ 3]          =    168.15262     |
| Prob [ chi squared > value ] =   .00000     |
| Response data are given as ind. choice.     |
| Number of obs.=   210, skipped   0 bad obs. |
+---------------------------------------------+
+---------------------------------------------+
| Notes No coefficients=> P(i,j)=1/J(i).      |
|       Constants only => P(i,j) uses ASCs    |
|         only. N(j)/N if fixed choice set.   |
|         N(j) = total sample frequency for j |
|         N    = total sample frequency.      |
|       These 2 models are simple MNL models. |
|       R-sqrd = 1 - LogL(model)/logL(other)  |
|       RsqAdj=1-[nJ/(nJ-nparm)]*(1-R-sqrd)   |
|         nJ   = sum over i, choice set sizes |
+---------------------------------------------+
+---------+--------------+----------------+--------+--------+
|Variable| Coefficient   | Standard Error |b/St.Er.|P[|Z|>z]|
+---------+--------------+----------------+--------+--------+
 AA      |    6.41353627      1.10452186      5.807   .0000
 AT      |    3.69564345       .52116476      7.091   .0000
 AB      |    2.96221779       .54485066      5.437   .0000
 BC      |    -.01702110       .00471351     -3.611   .0003
 BTA     |    -.10758045       .01791733     -6.004   .0000
 BTG     |    -.08939996       .01419339     -6.299   .0000
```



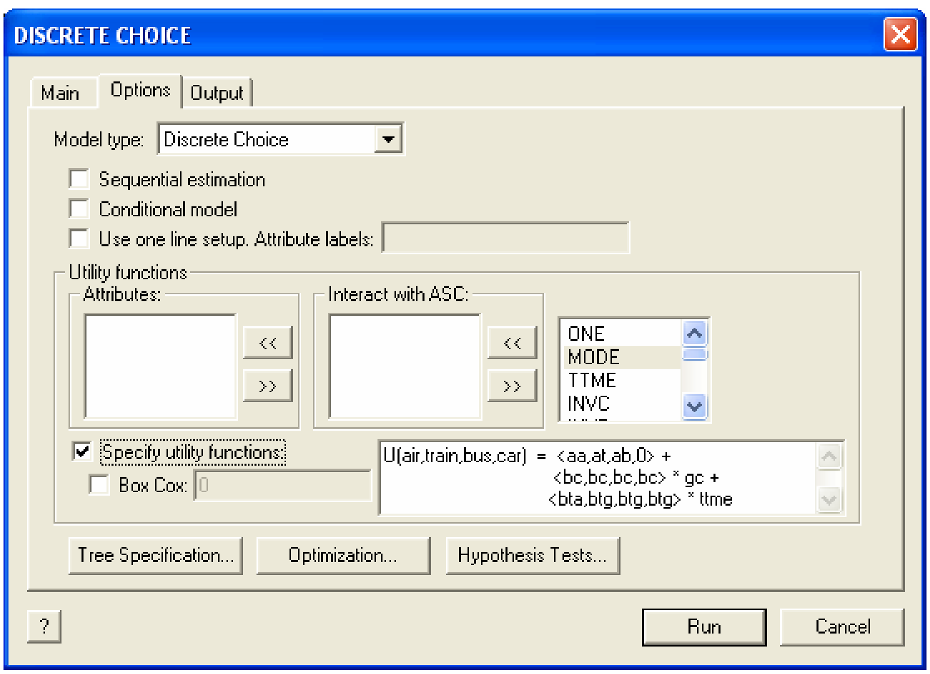


Figure 8.4 Utility Functions Assembled in Command Builder Window

8.3 Estimation Results

This section will detail the common results produced by the different models in *NLOGIT*.

8.3.1 Descriptive Headers for *NLOGIT* Models

The output for the **NLOGIT** estimators may contain a description of the model before the statistical results. The description consists of a table that shows the sample proportions and the tree structure if your fit a nested logit model, and a table that lists the components of the utility functions. You can request these listings by adding

; Show Model

to your **NLOGIT** command. (We used this device in several earlier examples.) Starting values for the iterations are either zeros or the values you provide with **;Start = list**. As such, there is no initial listing of OLS results. Output begins with the final results for the model. Here is a sample: The command is

```
NLOGIT    ; Lhs =  mode; Choices  =  air,train,bus,car
          ; Rhs    =  invc,invt,gc
          ; Rh2   =  one,hinc
          ; Show model $
```

```
Sample proportions are marginal, not conditional.
Choices marked with * are excluded for the IIA test.
+----------------+------+---+
|Choice    (prop.)|Weight|IIA|
+----------------+------+---+
|AIR        .27619| 1.000|   |
|TRAIN      .30000| 1.000|   |
|BUS        .14286| 1.000|   |
|CAR        .28095| 1.000|   |
+----------------+------+---+
+------------------------------------------------------------+
| Model Specification:  Table entry is the attribute that    |
| multiplies the indicated parameter.                        |
+--------+------+--------------------------------------------+
| Choice |******| Parameter                                  |
|        |Row  1| INVC      INVT      GC        A_AIR    AIR_HIN1 |
|        |Row  2| A_TRAIN   TRA_HIN2 A_BUS     BUS_HIN3         |
+--------+------+--------------------------------------------+
|AIR     |     1| INVC      INVT      GC        Constant HINC   |
|        |     2| none      none      none      none            |
|TRAIN   |     1| INVC      INVT      GC        none     none   |
|        |     2| Constant HINC      none      none            |
|BUS     |     1| INVC      INVT      GC        none     none   |
|        |     2| none      none      Constant HINC            |
|CAR     |     1| INVC      INVT      GC        none     none   |
|        |     2| none      none      none      none            |
+------------------------------------------------------------+
```

The initial header includes a display of the tree structure when you fit a nested logit model. For example, the command

```
NLOGIT   ; Lhs = mode; Choices = air,train,bus,car
         ; Rhs  = invc,invt,gc
         ; Rh2  = one,hinc
         ; Tree = Public[(air),(train,bus)],Private[(car)]
         ; Show model $
```

produces the header

```
Tree Structure Specified for the Nested Logit Model
Sample proportions are marginal, not conditional.
Choices marked with * are excluded for the IIA test.
----------------+----------------+----------------+----------------+------+---
Trunk    (prop.)|Limb     (prop.)|Branch   (prop.)|Choice   (prop.)|Weight|IIA
----------------+----------------+----------------+----------------+------+---
Trunk{1} 1.00000|PUBLIC    .71905|B(1|1,1)  .27619|AIR       .27619| 1.000|
                |                |B(2|1,1)  .44286|TRAIN     .30000| 1.000|
                |                |                |BUS       .14286| 1.000|
                |PRIVATE   .28095|B(1|2,1)  .28095|CAR       .28095| 1.000|
----------------+----------------+----------------+----------------+------+---
```

(Note, this particular model is not identified – we specified it only for purpose of illustrating the display of its tree structure.)

8.3.2 Standard Model Results

Estimation results for the model commands consist of the initial display of diagnostic followed by notes about the model, then the estimated coefficients. The preceding command, without the tree structure or the initial echo of the model specification,

```
NLOGIT   ; Lhs = mode; Choices = air,train,bus,car
         ; Rhs  = invc,invt,gc
         ; Rh2  = one,hinc $
```

produces the following results:

```
Normal exit from iterations. Exit status=0.
+---------------------------------------------+
| Discrete choice (multinomial logit) model   |
| Maximum Likelihood Estimates                |
| Dependent variable               Choice     |
| Weighting variable                 None     |
| Number of observations              210     |
| Iterations completed                  5     |
| Log likelihood function       -246.1098     |
| Number of parameters                  9     |
| Info. Criterion: AIC =          2.42962     |
|   Finite Sample: AIC =          2.43390     |
| Info. Criterion: BIC =          2.57306     |
| Info. Criterion:HQIC =          2.48761     |
| R2=1-LogL/LogL*  Log-L fncn  R-sqrd  RsqAdj |
| Constants only    -283.7588  .13268  .12011 |
| Chi-squared[ 6]          =     75.29796     |
| Prob [ chi squared > value ] =    .00000    |
| Response data are given as ind. choice.     |
| Number of obs.=   210, skipped   0 bad obs. |
+---------------------------------------------+
```

```
+--------------------------------------------+
| Notes No coefficients=> P(i,j)=1/J(i).      |
|       Constants only => P(i,j) uses ASCs    |
|         only. N(j)/N if fixed choice set.   |
|         N(j) = total sample frequency for j |
|         N    = total sample frequency.      |
|       These 2 models are simple MNL models. |
|       R-sqrd = 1 - LogL(model)/logL(other)  |
|       RsqAdj=1-[nJ/(nJ-nparm)]*(1-R-sqrd)   |
|         nJ   = sum over i, choice set sizes |
+--------------------------------------------+
+--------+--------------+----------------+--------+--------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]|
+--------+--------------+----------------+--------+--------+
 INVC    |    -.04612501       .01664864    -2.770   .0056
 INVT    |    -.00838543       .00214019    -3.918   .0001
 GC      |     .03633292       .01477727     2.459   .0139
 A_AIR   |   -1.31602481       .72323155    -1.820   .0688
 AIR_HIN1|     .00648950       .01079433      .601   .5477
 A_TRAIN |    2.10710471       .43179879     4.880   .0000
 TRA_HIN2|    -.05058498       .01206873    -4.191   .0000
 A_BUS   |     .86502331       .50318615     1.719   .0856
 BUS_HIN3|    -.03316081       .01299094    -2.553   .0107
```

NOTE: (This is one of our frequently asked questions.) The '*R*-squareds' shown in the output are R^2s in name only. They do not measure the fit of the model to the data. It has become common for researchers to report these with results as a measure of the improvement that the model gives over one that contains only a constant. But, users are cautioned not to interpret these measures as suggesting how well the model predicts the outcome variable. It is essentially unrelated to this.

To underscore the point, we will examine in detail the computations in the diagnostic measures shown in the box that precedes the coefficient estimates. Consider the example below, which was produced by fitting a model with five coefficients subject to two restrictions, or three free coefficients - *npfree* = 3. (The effect is achieved by specifying **;Choices=air,(train),(bus),car**.

```
+---------------------------------------------------+
|WARNING:   Bad observations were found in the sample. |
|Found  93 bad observations among     210 individuals. |
|You can use ;CheckData to get a list of these points. |
+---------------------------------------------------+
Sample proportions are marginal, not conditional.
Choices marked with * are excluded for the IIA test.
+----------------+------+---
|Choice   (prop.)|Weight|IIA
+----------------+------+---
|AIR       .49573| 1.000|
|TRAIN     .00000| 1.000|*
|BUS       .00000| 1.000|*
|CAR       .50427| 1.000|
+----------------+------+---
```

```
+------------------------------------------------------------------+
| Model Specification:  Table entry is the attribute that          |
| multiplies the indicated parameter.                              |
+--------+------+--------------------------------------------------+
| Choice |******| Parameter                                        |
|        |Row  1| GC        TTME      A_AIR     A_TRAIN  A_BUS      |
+--------+------+--------------------------------------------------+
|AIR     |     1| GC        TTME      Constant none     none       |
|TRAIN   |     1| GC        TTME      none     Constant none       |
|BUS     |     1| GC        TTME      none     none     Constant   |
|CAR     |     1| GC        TTME      none     none     none       |
+------------------------------------------------------------------+
Normal exit from iterations. Exit status=0.
+----------------------------------------------+
| Discrete choice (multinomial logit) model    |
| Maximum Likelihood Estimates                 |
| Dependent variable               Choice      |
| Weighting variable                 None      |
| Number of observations              117      |
| Iterations completed                  6      |
| Log likelihood function       -62.58418      |
| Number of parameters                  3      |
| Info. Criterion: AIC =          1.12110      |
|   Finite Sample: AIC =          1.12291      |
| Info. Criterion: BIC =          1.19192      |
| Info. Criterion:HQIC =          1.14985      |
| R2=1-LogL/LogL*  Log-L fncn  R-sqrd  RsqAdj  |
| Constants only     -81.0939  .22825  .20794  |
| Chi-squared[ 2]          =     37.01953      |
| Prob [ chi squared > value ] =    .00000     |
| Response data are given as ind. choice.      |
| Number of obs.=   210, skipped  93 bad obs.  |
+----------------------------------------------+
| Restricted choice set. Excluded choices are  |
| TRAIN    BUS                                 |
+----------------------------------------------+
+--------+--------------+----------------+--------+--------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]|
+--------+--------------+----------------+--------+--------+
 GC      |     .01320101       .00694790     1.900   .0574
 TTME    |    -.07141256       .01604643    -4.450   .0000
 A_AIR   |    3.96116758       .98004184     4.042   .0001
 A_TRAIN |       .000000    ......(Fixed Parameter).......
 A_BUS   |       .000000    ......(Fixed Parameter).......
```

There are 210 individuals in the data set, but this model was fit to a restricted choice set which reduced the data set to $n = 210 - 93 = 117$ useable observations. The original choice set had $J_i = 4$ choices, but two were excluded, leaving $J_i = 2$ in the sample. The log likelihood is -62.58418. The 'constants only' log likelihood is obtained by setting each choice probability to the sample share for each outcome in the choice set. For this application, those are 0.49573 for air and 0.50427 for car. (This computation cannot be done if the choice set varies by person or if weights or frequencies are used.) Thus, the log likelihood for the restricted model is

$$\text{Log } L_0 = 117\,(\,0.49573 \times \log 0.49573 + 0.50427 \times \log 0.50427\,) = -81.09395.$$

The 'R^2' is 1 - (-62.54818/-81.0939) = 0.22869 (including some rounding error). The adjustment factor is

$$K = (\Sigma_i J_i - n) / [(\Sigma_i J_i - n) - npfree] = (234 - 117)/(234 - 117 - 3) = 1.02632.$$

and the '*Adjusted* R^2' is $1 - K(\log L / \text{Log}L_0)$;

$$\textit{Adjusted } R^2 = 1 - 1.02632\,(-62.54818/-81.0939) = 0.20794.$$

8.3.3 Retained Results

Results kept by this estimator are:

Matrices:	*b* and *varb*	= coefficient vector and asymptotic covariance matrix,
Scalars:	*logl*	= log likelihood function,
	nreg	= N, the number of observational units,
	kreg	= the number of Rhs variables.
Last Model:	*b_variable*	= the labels kept for the **WALD** command.

NOTE: In the *Last Model*, groups of coefficients for variables that that are interacted with constants get labels *choice_variable*, as in *trai_gco*. (Note that the names are truncated - up to four characters for the choice and three for the attribute.) The alternative specific constants are *a_choice***,** with names truncated to no more than 6 characters. For example, the sum of the three estimated choice specific constants could be analyzed as follows:

WALD ; Fn1 = a_air + a_train + a_bus $

```
+-----------------------------------------------+
| WALD procedure. Estimates and standard errors |
| for nonlinear functions and joint test of     |
| nonlinear restrictions.                       |
| Wald Statistic              =     57.91928    |
| Prob. from Chi-squared[ 1] =       .00000     |
+-----------------------------------------------+
+---------+--------------+----------------+--------+---------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] |
+---------+--------------+----------------+--------+---------+
 Fncn(1)      13.32858178       1.7513477    7.610   .0000
```

8.3.4 Robust Standard Errors.

The 'cluster' estimator described elsewhere in this document is available in *NLOGIT*. However, this routine does not support hierarchical samples. There may be only one level of clustering. Also, the cluster specification is defined with respect to the *NLOGIT* groups of data, not the data set. *NLOGIT* sorts out how many clusters there are and how they are delineated. But, since the row count of the data set is used in constructing the estimator, you must treat a group of NALT observations as one. For example, our sample data used in this section contain 210 groups of 4 rows of data. Each group of 4 is an observation. Suppose that these data were grouped in clusters of 3 choice situations. The estimation command with the cluster estimator would appear

NLOGIT ; ... (the model) ; Cluster = 12 $

The relevant part of the output would appear as follows:

```
+---------------------------------------------------------------------+
| Covariance matrix for the model is adjusted for data clustering.    |
| Sample of    210 observations contained     70 clusters defined by  |
|      3 observations (fixed number) in each cluster.                 |
+---------------------------------------------------------------------+
+--------+--------------+----------------+--------+--------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]|
+--------+--------------+----------------+--------+--------+
 GC      |    -.01578375       .00543575    -2.904   .0037
 TTME    |    -.09709052       .01366784    -7.104   .0000
 A_AIR   |    5.77635888       .74564933     7.747   .0000
 A_TRAIN |    3.92300124       .47890812     8.192   .0000
 A_BUS   |    3.21073471       .48991386     6.554   .0000
```

You may also request a cross tabulation of the model predictions against the actual choices. (The predictions are obtained as the integer part of $\Sigma_t\, \hat{P}_{jt}\, y_{jt}$.) Add

; Crosstab

to your model command. For the same model, this would produce

```
+----------------------------------------------------+
| Cross tabulation of actual vs. predicted choices.    |
| Row indicator is actual, column is predicted.        |
| Predicted total is F(k,j,i)=Sum(i=1,...,N) P(k,j,i). |
| Column totals may be subject to rounding error.      |
+----------------------------------------------------+
         AIR           TRAIN          BUS           CAR            Total
        +-----------------------------------------------------------------------
AIR     |   19.00000     13.00000      8.00000     18.00000     58.00000
TRAIN   |   12.00000     30.00000      9.00000     12.00000     63.00000
BUS     |   10.00000      8.00000      6.00000      6.00000     30.00000
CAR     |   17.00000     12.00000      7.00000     23.00000     59.00000
Total   |   58.00000     63.00000     30.00000     59.00000    210.00000
```

8.4 Marginal Effects and Elasticities

In the discrete choice model, the effect of a change in attribute '*k*' of alternative '*j*' on the probability that individual *i* would choose alternative '*m*' (where *m* may or may not equal *j*) is

$$\delta_{im}(k|j) \;=\; \partial \text{Prob}[y_i = m]/\partial x_i(k|j) \;=\; [\mathbf{1}(j = m) - P_{ij}]P_{im}\beta_k..$$

You can request a listing of the effects of a specific attribute on a specified set of outcomes with

; Effects : attribute [list of outcomes]

The outcomes listing defines the variables '*j*' in the definition above. The attribute is the '*k*th.' A calculated marginal effect is then listed for all alternatives (i.e., all '*m*') in the model. You can request additional tables by separating additional specifications with slashes. For example:

; Effects : gc [car, train] / ttme [bus,train]

> **HINT:** It may generate quite a lot of output if your model is large, but you can request an analysis of 'all' alternatives by using the wildcard, **attribute [*]**.

The tables below are produced by

```
NLOGIT    ; Lhs =  mode; Choices  =  air,train,bus,car
          ; Rhs    =  invc,invt,gc
          ; Rh2   =  one,hinc
          ; Effects:gc[*] $
```

```
+---------------------------------------------------+
| Derivative (times 100) averaged over observations.|
| Attribute is GC       in choice AIR               |
| Effects on probabilities of all choices in model: |
| * = Direct Derivative effect of the attribute.    |
|                               Mean    St.Dev      |
| *     Choice=AIR              .6042     .2397     |
|       Choice=TRAIN          -.2007     .1132     |
|       Choice=BUS            -.1237     .0798     |
|       Choice=CAR            -.2798     .2044     |
+---------------------------------------------------+
| Derivative (times 100) averaged over observations.|
| Attribute is GC       in choice TRAIN             |
| Effects on probabilities of all choices in model: |
| * = Direct Derivative effect of the attribute.    |
|                               Mean    St.Dev      |
|       Choice=AIR            -.2007     .1132     |
| *     Choice=TRAIN           .6180     .2612     |
|       Choice=BUS            -.1754     .1377     |
|       Choice=CAR            -.2420     .1305     |
+---------------------------------------------------+
| Derivative (times 100) averaged over observations.|
| Attribute is GC       in choice BUS               |
| Effects on probabilities of all choices in model: |
| * = Direct Derivative effect of the attribute.    |
|                               Mean    St.Dev      |
|       Choice=AIR            -.1237     .0798     |
|       Choice=TRAIN          -.1754     .1377     |
| *     Choice=BUS             .4332     .1431     |
|       Choice=CAR            -.1342     .0648     |
+---------------------------------------------------+
| Derivative (times 100) averaged over observations.|
| Attribute is GC       in choice CAR               |
| Effects on probabilities of all choices in model: |
| * = Direct Derivative effect of the attribute.    |
|                               Mean    St.Dev      |
|       Choice=AIR            -.2798     .2044     |
|       Choice=TRAIN          -.2420     .1305     |
|       Choice=BUS            -.1342     .0648     |
| *     Choice=CAR             .6559     .2159     |
+---------------------------------------------------+
```

These effects are always extremely small. They are multiplied by 100 in the output to make sure that some significant digits are shown in the tables. The effects are computed by averaging the individual specific results, so the report contains the average partial effects. Since the mean is computed over a sample of observations, we also report the standard deviation of the estimates.

NOTE: The standard deviations are not the asymptotic standard errors for the estimators of the marginal effects. In principle, that could be computed using the delta method. However, the estimates computed by *NLOGIT* are *average partial effects*. They are computed for each individual in the sample, then averaged. Computing an appropriate standard error for that statistic is difficult to impossible owing to its extreme nonlinearity and due to the fact that all observations in the average are correlated – they use the same estimated parameter vector. Nonetheless, it may be tempting to use the standard deviations for tests of hypotheses that the marginal effects are zero. We advise against this. There is no meaning that could be attached to an elasticity or marginal effect being zero – these are complicated of all parameters in the model. The hypothesis that a variable is not influential in the determination of the choices should be tested at the coefficient level.

8.5 Testing the Assumption of Independence from Irrelevant Alternatives (IIA)

Hausman and McFadden (1984) have proposed a specification test for this model to test the inherent assumption of the independence from irrelevant alternatives (IIA). (IIA is a consequence of the initial assumption that the stochastic terms in the utility functions are independent and extreme value distributed. Discussion may be found in standard texts on qualitative choice modeling, such as Hensher, Rose and Greene (2005) and Greene (2008).) The procedure is, first, to estimate the model with all choices. The alternative specification is the model with a smaller set of choices. Thus, the model is estimated with this restricted set of alternatives and the same model specification. The set of observations is reduced to those in which one of the smaller set of choices is made. The test statistic is

$$q = [\mathbf{b}_r - \mathbf{b}_u]'[\mathbf{V}_r - \mathbf{V}_u]^{-1}[\mathbf{b}_r - \mathbf{b}_u],$$

where '*u*' and '*r*' indicate unrestricted and restricted (smaller choice set) models and **V** is an estimated variance matrix for the estimates. To use *NLOGIT* to carry out this test, it is necessary to estimate both models. In the second, it is necessary to drop the outcomes indicated. This is done with the

; **Ias=list**

specification. The list gives the names of the outcomes to be dropped. This procedure is automated as shown in the following example:

```
CLOGIT    ; Lhs       = mode
          ; Choices   = air,train,bus,car
          ; Rhs       = invc, invt, gc, ttme $
CLOGIT    ; Lhs       = mode
          ; Choices   = air,train,bus,car
          ; Ias       = car
          ; Rhs       = invc, invt, gc, ttme $
```

```
+---------------------------------------------+
| Discrete choice (multinomial logit) model   |
| Dependent variable               Choice     |
| Number of observations              210     |
| Log likelihood function       -244.1342     |
| Number of parameters                  4     |
| R2=1-LogL/LogL*  Log-L fncn  R-sqrd  RsqAdj |
| Constants only    -283.7588  .13964  .13414 |
| Response data are given as ind. choice.     |
| Number of obs.=   210, skipped   0 bad obs. |
+---------------------------------------------+
+--------+--------------+----------------+--------+--------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]|
+--------+--------------+----------------+--------+--------+
 INVC    |    -.02242963       .01435409    -1.563   .1181
 INVT    |    -.00634473       .00184168    -3.445   .0006
 GC      |     .03182946       .01372856     2.318   .0204
 TTME    |    -.03480667       .00469397    -7.415   .0000
+------------------------------------------------------+
|WARNING:   Bad observations were found in the sample. |
|Found  59 bad observations among     210 individuals. |
|You can use ;CheckData to get a list of these points. |
+------------------------------------------------------+
Normal exit from iterations. Exit status=0.
+---------------------------------------------+
| Discrete choice (multinomial logit) model   |
| Dependent variable               Choice     |
| Number of observations              151     |
| Log likelihood function       -103.2012     |
| Number of parameters                  4     |
| R2=1-LogL/LogL*  Log-L fncn  R-sqrd  RsqAdj |
| Constants only    -159.0502  .35114  .34243 |
| Response data are given as ind. choice.     |
| Number of obs.=   210, skipped  59 bad obs. |
+---------------------------------------------+
| Hausman test for IIA. Excluded choices are  |
| CAR                                         |
| ChiSqrd[ 4] =  51.9631, Pr(C>c) =  .000000  |
+---------------------------------------------+
+--------+--------------+----------------+--------+--------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]|
+--------+--------------+----------------+--------+--------+
 INVC    |    -.04641792       .02108920    -2.201   .0277
 INVT    |    -.00963276       .00271137    -3.553   .0004
 GC      |     .04116251       .01984102     2.075   .0380
 TTME    |    -.07938809       .00991501    -8.007   .0000
```

In order to compute the coefficients in the restricted model, it is necessary to drop those observations that choose the omitted choice(s). In the example above, 59 observations were skipped. They are marked as bad data because with *car* excluded, no choice is made for those observations. As a consequence, the log likelihood functions are not comparable. The Hausman statistic is used to carry out the test. In the preceding example, the large value suggests that the IIA restriction should be rejected. Note that you can carry out several tests with different subsets of the choices without refitting the benchmark model. Thus, in the example above, you could follow with a third model in which **;Ias=bus** instead of **car**.

There is a possibility that restricting the choice set can lead to a singularity. It is possible that when you drop one or more alternatives, some attribute will be constant among the remaining choices. Thus, you might induce the case in which there is a 'regressor' which is constant across the choices. In this case, *NLOGIT* will issue a diagnostic about a singular Hessian (it is). Hausman and McFadden (1984) suggest estimating the model with the smaller number of choice sets *and* a smaller number of regressors. There is no question of consistency, or omission of a relevant attribute, since if the attribute is always constant among the choices, variation in it is obviously not affecting the choice. After estimation, the subvector of the larger parameter vector in the first model can be measured against the parameter vector from the second model using the Hausman statistic given earlier. This possibility arises in the model with alternative specific constants, so it is going to be a common case. The examples below suggest one way you might proceed in such as case.

The first step is to fit the original model using the entire sample and retrieve the results.

```
NLOGIT    ; Lhs        = mode
          ; Choices    = air,train,bus,car
          ; Rhs        = invc, invt, gc, ttme, one $
MATRIX    ; bu         = b(1:4) ; vu = varb(1:4,1:4) $
```

The variable choice takes values 1,2,3,4,1,2,3,4... indicating the indexing scheme for the choices

```
CREATE    ; choice     = Trn(-4,0)$
```

Chair is a dummy variable that equals one for all four rows when choice made is *air*. Now restrict the sample to the observations for choices train, bus, car.

```
REJECT    ; chair      = 1   |  choice    = 1 $
```

Fit the model with the restricted sample (choice set) and one less constant term.

```
NLOGIT    ; Lhs        = mode   ; Choices  =  train,bus,car
          ; Rhs        = invc, invt, gc, ttme,one $
```

Retrieve the restricted results and compute the Hausman statistic.

```
MATRIX    ; br         = b(1:4) ; vr = varb(1:4,1:4)
          ; db         = br - bu  ; vdb  = Nvsm(vr,-vu) $
CALC      ; List   ; q = Qfr(db,vdb)  ; 1 - chi(q,4) $
```

The results are:

```
Q       =  .33784450384775710D+02
Result  =  .82501941289780950D-06
```

NOTE: (We've been asked this one several times.) The difference matrix in this calculation, *vdb*, might be nonsingular (have an inverse), but not be positive definite. In such a case, the chi squared can be negative. If this happens, the right conclusion is probably that it should be zero.

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Chapter 9

The Conditional Logit Model

9.1 Introduction

In the multinomial logit model described in Chapter 6, there is a single vector of characteristics, which describes the individual, and a set of J parameter vectors. In the 'discrete choice' setting of this section, these are essentially reversed. The J alternatives are each characterized by a set of K 'attributes,' $\mathbf{x}_{ij}$. Respondent 'i' chooses among the J alternatives. There is a single parameter vector, $\boldsymbol{\beta}$. The model underlying the observed data is assumed to be the following random utility specification:

$$U(\text{ choice } j \text{ for individual } i\ ) \ = \ U_{ij} \ = \ \boldsymbol{\beta}'\mathbf{x}_{ij} \ + \ \varepsilon_{ij},\ j = 1,...,J_i.$$

The random, individual specific terms, $(\varepsilon_{i1},\varepsilon_{i2},...,\varepsilon_{iJ})$ are assumed to be independently distributed, each with an extreme value distribution. Under these assumptions, the probability that individual i chooses alternative j is

$$\text{Prob}(U_{ij} > U_{iq}) \text{ for all } q \neq j.$$

It has been shown that for independent extreme value distributions, as above, this probability is

$$\text{Prob}(y_i = j) = \frac{\exp\left(\boldsymbol{\beta}'\mathbf{x}_{ij}\right)}{\sum_{m=1}^{J_i}\exp\left(\boldsymbol{\beta}'\mathbf{x}_{im}\right)}$$

where y_i is the index of the choice made. Regardless of the number of choices, there is a single vector of K parameters to be estimated. This model does not suffer from the proliferation of parameters that appears in the logit model described in Section 6. It does, however, make the very strong 'Independence From Irrelevant Alternatives' assumption which will be discussed below.

NOTE: The distinction made here between 'discrete choice' and 'multinomial logit' is not hard and fast. It is made purely for convenience in the discussion. As noted in Chapters 6 and 8, by interacting the characteristics with the alternative specific constants, the discrete choice model of this chapter becomes the multinomial logit model of Chapter 6. From this point, in the remainder of this reference guide for *NLOGIT*, we will refer to the model described in this chapter, with mathematical formulation as given above, as the 'multinomial logit model,' or MNL model as is common in the literature.

The basic setup for this model consists of observations on n individuals, each of whom makes a single choice among J_i choices, or alternatives. There is a subscript on J_i because we do not restrict the choice sets to have the same number of choices for every individual. The data will typically consist of the choices and observations on K 'attributes' for each choice. The attributes that describe each choice, i.e., the arguments that enter the utility functions, may be the same for all choices, or may be defined differently for each utility function. The estimator described in this section allows a large number of variations of this basic model. In the discrete choice framework, the observed 'dependent variable' usually consists of an indicator of which among J_i alternatives was *most* preferred by the respondent. All that is known about the others is that they were judged inferior to the one chosen. But, there are cases in which information is more complete and consists of a subjective ranking of all J_i alternatives by the individual. *NLOGIT* allows specification of the model for estimation with 'ranks data.' In addition, in some settings, the sample data might consist of aggregates for the choices, such as proportions (market shares) or frequency counts.

9.2 Command for the Multinomial Logit Model

The simplest form of the command for the discrete choice models is

CLOGIT ; Lhs = variable which indicates the choice made
; Choices = a set of J names for the set of choices
; Rhs = choice varying attributes in the utility functions
; Rh2 = choice invariant characteristics $

(With no qualifiers to indicate a different model, such as RPL or MNP, **CLOGIT** and **NLOGIT** are the same.) There are various ways to specify the utility functions - i.e., the Rhs of the equations that underlie the model, and several different ways to specify the choice set.. These are discussed in Sections N11.2.4 and N11.2.5. The **;Rhs** specification may be replaced with an explicit definition of the utility functions, using **;Model...** This is described in Section N11.2.5.

A set of exactly J choice labels must be provided in the command. These are used to label the choices in the output. The number you provide is used to determine the number of choices there are in the model. Therefore, the set of the right number of labels is essential. Use any descriptor of eight or fewer characters desired - these do not have to be valid names, just a set of labels, separated in the list by commas.

The command builder for this model is found in Model:Discrete Choice/Discrete Choice. The Main and Options pages are both used to set up the model. The model and the choice set are defined in the Main page; the attributes are defined in the Options page.

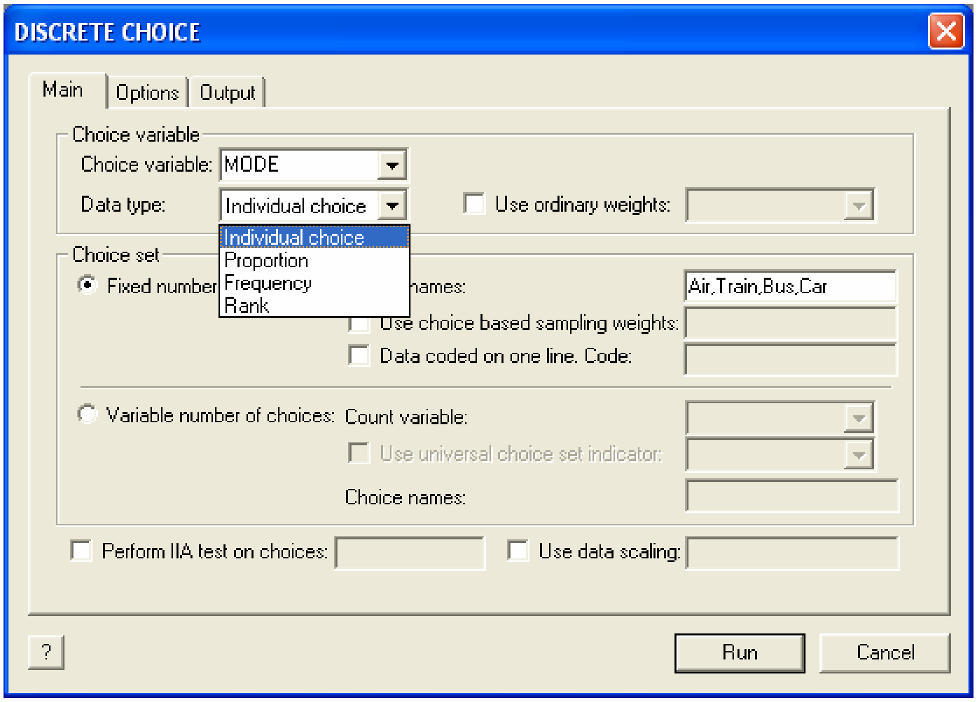


Figure 9.1 Main Page of Command Builder for Multinomial Logit Model

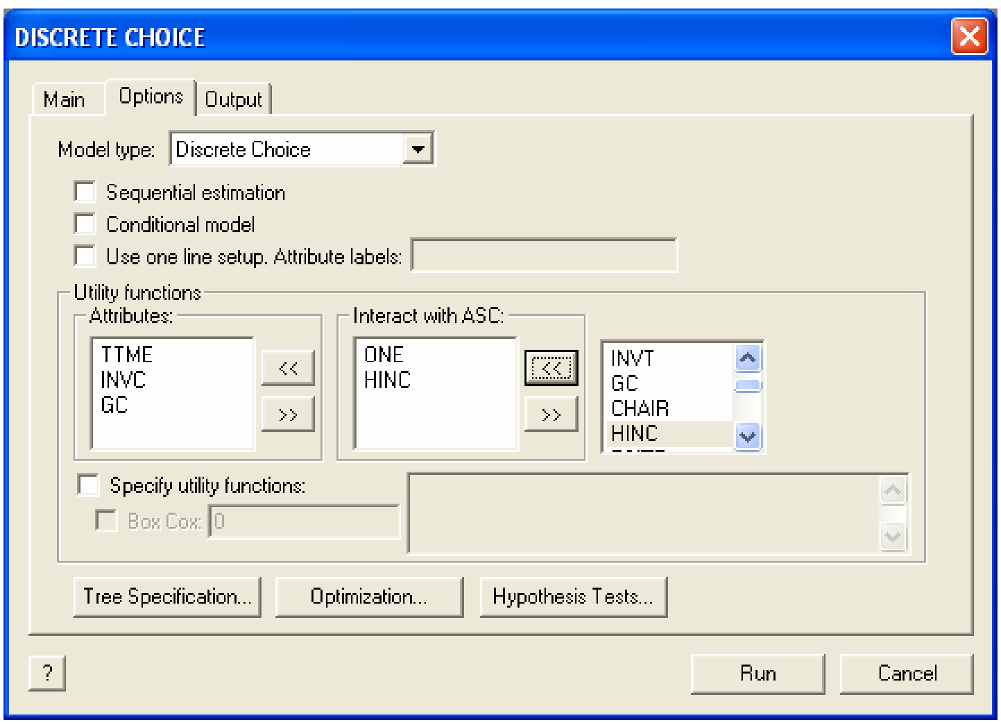


Figure 9.2 Options Page of Command Builder for Conditional Logit Model

9.3 Results for the Multinomial Logit Model

Results for the multinomial logit model will consist of the standard model results and any additional descriptive output you have requested. The application below will display the full set of available results. Results kept by this estimator are:

Matrices:	*b* and *varb*	= coefficient vector and asymptotic covariance matrix,
Scalars:	*logl*	= log likelihood function,
	nreg	= N, the number of observational units,
	kreg	= the number of Rhs variables,
Last Model:	*b_variable*	= the labels kept for the **WALD** command.

In the *Last Model*, groups of coefficients for variables that are integrated with constants get labels *choice_variable*, as in *trai_gco*. (Note that the names are truncated - up to four characters for the choice and three for the attribute.) The alternative specific constants are *a_choice***,** with names truncated to no more than six characters. For example, the sum of the three estimated choice specific constants could be analyzed as follows:

WALD ; Fn1=a_air+a_train+a_bus$

```
+------------------------------------------------+
| WALD procedure. Estimates and standard errors |
| for nonlinear functions and joint test of     |
| nonlinear restrictions.                        |
| Wald Statistic             =     57.91928     |
| Prob. from Chi-squared[ 1] =       .00000     |
+------------------------------------------------+
+---------+--------------+----------------+--------+---------+
|Variable | Coefficient  | Standard Error |b/St.Er.|P[|Z|>z] |
+---------+--------------+----------------+--------+---------+
 Fncn(1)      13.32858178       1.7513477    7.610    .0000
```

9.4 Application

The MNL model based on the CLOGIT data is estimated with the command

```
CLOGIT    ; Lhs     = mode
          ; Choices = air,train,bus,car
          ; Rhs     = gc,ttme
          ; Rh2     = one,hinc
          ; Show model
          ; Describe
          ; Crosstab
          ; Effects: gc(*)
          ; Ivb     = incvlu
          ; Prob    = pmnl
          ; List $
```

This requests all the optional output from the model. The **;Describe** specification requests a set of descriptive statistics for the variables in the model, by choice. The leftmost set of results gives the coefficient estimates. Note that in this model, they are the same for the two generic coefficients, on *gc* and *ttme*, but they vary by choice for the alternative specific constant and its interaction with income. Also, since there is no ASC for *car* (it was dropped to avoid the dummy variable trap), there are no coefficients for the car grouping. The second set of values in the center section gives the mean and standard deviation for that attribute in that outcome for all observations in the sample. The third set of results gives the mean and variance for the particular attribute for the individuals that made that choice. The full set of results from the model is as follows.

```
+---------------------------------------------+
| Discrete choice (multinomial logit) model   |
+---------------------------------------------+
Sample proportions are marginal, not conditional.
Choices marked with * are excluded for the IIA test.
+----------------+------+---
|Choice   (prop.)|Weight|IIA
+----------------+------+---
|AIR       .27619| 1.000|
|TRAIN     .30000| 1.000|
|BUS       .14286| 1.000|
|CAR       .28095| 1.000|
+----------------+------+---
+------------------------------------------------------------+
| Model Specification:  Table entry is the attribute that    |
| multiplies the indicated parameter.                        |
+--------+------+--------------------------------------------+
| Choice |******| Parameter                                  |
|        |Row  1| GC       TTME     A_AIR    AIR_HIN1 A_TRAIN  |
|        |Row  2| TRA_HIN2 A_BUS    BUS_HIN3                 |
+--------+------+--------------------------------------------+
|AIR     |     1| GC       TTME     Constant HINC     none     |
|        |     2| none     none     none                     |
|TRAIN   |     1| GC       TTME     none     none     Constant |
|        |     2| HINC     none     none                     |
|BUS     |     1| GC       TTME     none     none     none     |
|        |     2| none     Constant HINC                     |
|CAR     |     1| GC       TTME     none     none     none     |
|        |     2| none     none     none                     |
+------------------------------------------------------------+
Normal exit from iterations. Exit status=0.
+---------------------------------------------+
| Discrete choice (multinomial logit) model   |
| Dependent variable               Choice     |
| Number of observations              210     |
| Log likelihood function       -189.5252     |
| Number of parameters                  8     |
| Info. Criterion: AIC =          1.88119     |
| R2=1-LogL/LogL*  Log-L fncn  R-sqrd  RsqAdj |
| Constants only    -283.7588  .33209  .32350 |
| Chi-squared[ 5]         =    188.46723      |
| Prob [ chi squared > value ] =   .00000     |
| Response data are given as ind. choice.     |
| Number of obs.=   210, skipped   0 bad obs. |
+---------------------------------------------+
```

```
+---------------------------------------------+
| Notes No coefficients=> P(i,j)=1/J(i).      |
|       Constants only => P(i,j) uses ASCs    |
|         only. N(j)/N if fixed choice set.   |
|         N(j) = total sample frequency for j |
|         N    = total sample frequency.      |
|       These 2 models are simple MNL models. |
|       R-sqrd = 1 - LogL(model)/logL(other)  |
|       RsqAdj=1-[nJ/(nJ-nparm)]*(1-R-sqrd)   |
|         nJ   = sum over i, choice set sizes |
+---------------------------------------------+
+--------+--------------+----------------+--------+--------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]|
+--------+--------------+----------------+--------+--------+
 GC      |    -.01092735       .00458775    -2.382   .0172
 TTME    |    -.09546055       .01047320    -9.115   .0000
 A_AIR   |    5.87481336       .80209034     7.324   .0000
 AIR_HIN1|    -.00537349       .01152940     -.466   .6412
 A_TRAIN |    5.54985728       .64042443     8.666   .0000
 TRA_HIN2|    -.05656186       .01397335    -4.048   .0001
 A_BUS   |    4.13028388       .67636278     6.107   .0000
 BUS_HIN3|    -.02858418       .01544418    -1.851   .0642

PREDICTED PROBABILITIES (* marks actual, + marks prediction.)
Indiv    AIR       TRAIN     BUS       CAR
    1   .0984     .3311     .1959     .3746*+
    2   .2566     .2262     .0530     .4641*+
    3   .1401     .1795     .1997     .4808*+
    4   .2732     .0297     .0211     .6759*+
    5   .3421     .1478     .0527     .4575*+
    6   .0831     .3962*+   .2673     .2534
    7   .6066*+   .0701     .0898     .2335
    8   .0626     .6059 +   .1925     .1390*
    9   .1125     .2932     .1995     .3947*+
   10   .1482     .0804     .1267     .6447*+
```

(Rows 11 – 210 are omitted.)

```
+-------------------------------------------------------+
| Cross tabulation of actual vs. predicted choices.     |
| Row indicator is actual, column is predicted.         |
| Predicted total is F(k,j,i)=Sum(i=1,...,N) P(k,j,i).  |
| Column totals may be subject to rounding error.       |
+-------------------------------------------------------+

Matrix Crosstab has  5 rows and  5 columns.
         AIR           TRAIN         BUS           CAR           Total
        +-----------------------------------------------------------------------
AIR     |   33.00000       7.00000       4.00000      14.00000      58.00000
TRAIN   |    7.00000      39.00000       5.00000      12.00000      63.00000
BUS     |    3.00000       6.00000      15.00000       6.00000      30.00000
CAR     |   15.00000      11.00000       6.00000      27.00000      59.00000
Total   |   58.00000      63.00000      30.00000      59.00000     210.00000
```

```
+---------------------------------------------------+
| Elasticity             averaged over observations.|
| Attribute is GC       in choice AIR                |
| Effects on probabilities of all choices in model: |
| * = Direct Elasticity effect of the attribute.    |
|                                  Mean    St.Dev   |
| *     Choice=AIR               -.8019     .3834   |
|       Choice=TRAIN              .3198     .3370   |
|       Choice=BUS                .3198     .3370   |
|       Choice=CAR                .3198     .3370   |
+---------------------------------------------------+
+---------------------------------------------------+
| Elasticity             averaged over observations.|
| Attribute is GC       in choice TRAIN              |
| Effects on probabilities of all choices in model: |
| * = Direct Elasticity effect of the attribute.    |
|                                  Mean    St.Dev   |
|       Choice=AIR                .3534     .3511   |
| *     Choice=TRAIN            -1.0693     .7134   |
|       Choice=BUS                .3534     .3511   |
|       Choice=CAR                .3534     .3511   |
+---------------------------------------------------+
+---------------------------------------------------+
| Elasticity             averaged over observations.|
| Attribute is GC       in choice BUS                |
| Effects on probabilities of all choices in model: |
| * = Direct Elasticity effect of the attribute.    |
|                                  Mean    St.Dev   |
|       Choice=AIR                .1679     .2308   |
|       Choice=TRAIN              .1679     .2308   |
| *     Choice=BUS              -1.0916     .5183   |
|       Choice=CAR                .1679     .2308   |
+---------------------------------------------------+
+---------------------------------------------------+
| Elasticity             averaged over observations.|
| Attribute is GC       in choice CAR                |
| Effects on probabilities of all choices in model: |
| * = Direct Elasticity effect of the attribute.    |
|                                  Mean    St.Dev   |
|       Choice=AIR                .2934     .2674   |
|       Choice=TRAIN              .2934     .2674   |
|       Choice=BUS                .2934     .2674   |
| *     Choice=CAR               -.7492     .4430   |
+---------------------------------------------------+
```

```
+----------------------------------------------------------------------------+
|               Descriptive Statistics for Alternative AIR                   :
|       Utility Function            |                     |     58.0 observs.  |
|       Coefficient                 | All      210.0 obs.|that chose AIR      |
| Name              Value  Variable | Mean      Std. Dev.|Mean      Std. Dev. |
| -------------------  -------- | -------------------+------------------- |
| GC             -.0109  GC       |  102.648     30.575|  113.552    33.198 |
| TTME           -.0955  TTME     |   61.010     15.719|   46.534    24.389 |
| A_AIR          5.8748  ONE      |    1.000       .000|    1.000      .000 |
| AIRxHIN1       -.0054  HINC     |   34.548     19.711|   41.724    19.115 |
+----------------------------------------------------------------------------+
+----------------------------------------------------------------------------+
|               Descriptive Statistics for Alternative TRAIN                 :
|       Utility Function            |                     |     63.0 observs.  |
|       Coefficient                 | All      210.0 obs.|that chose TRAIN    |
| Name              Value  Variable | Mean      Std. Dev.|Mean      Std. Dev. |
| -------------------  -------- | -------------------+------------------- |
| GC             -.0109  GC       |  130.200     58.235|  106.619    49.601 |
| TTME           -.0955  TTME     |   35.690     12.279|   28.524    19.354 |
| A_TRAIN        5.5499  ONE      |    1.000       .000|    1.000      .000 |
| TRAxHIN2       -.0566  HINC     |   34.548     19.711|   23.063    17.287 |
+----------------------------------------------------------------------------+
+----------------------------------------------------------------------------+
|               Descriptive Statistics for Alternative BUS                   :
|       Utility Function            |                     |     30.0 observs.  |
|       Coefficient                 | All      210.0 obs.|that chose BUS      |
| Name              Value  Variable | Mean      Std. Dev.|Mean      Std. Dev. |
| -------------------  -------- | -------------------+------------------- |
| GC             -.0109  GC       |  115.257     44.934|  108.133    43.244 |
| TTME           -.0955  TTME     |   41.657     12.077|   25.200    14.919 |
| A_BUS          4.1303  ONE      |    1.000       .000|    1.000      .000 |
| BUSxHIN3       -.0286  HINC     |   34.548     19.711|   29.700    16.851 |
+----------------------------------------------------------------------------+
+----------------------------------------------------------------------------+
|               Descriptive Statistics for Alternative CAR                   :
|       Utility Function            |                     |     59.0 observs.  |
|       Coefficient                 | All      210.0 obs.|that chose CAR      |
| Name              Value  Variable | Mean      Std. Dev.|Mean      Std. Dev. |
| -------------------  -------- | -------------------+------------------- |
| GC             -.0109  GC       |   95.414     46.827|   89.085    49.833 |
| TTME           -.0955  TTME     |     .000       .000|     .000      .000 |
+----------------------------------------------------------------------------+
```

9.5 Marginal Effects

We define the marginal effects in the multinomial logit model as the derivatives of the probability of choice *j* with respect to attribute *k* in alternative *m*. This is

$$\frac{\partial P_j}{\partial x_{km}} = \left[\mathbf{1}(j=m) - P_m\right] P_j \beta_k ,$$

where the function $\mathbf{1}(j = m)$ equals one if *j* equals *m* and zero otherwise. These are naturally scaled since the probability is bounded. They are usually very small, so **NLOGIT** reports 100 times the value obtained, as in the example below, which is produced by

;Effects:gc[air]

```
+---------------------------------------------------+
| Derivative (times 100) averaged over observations.|
| Attribute is GC       in choice AIR               |
| Effects on probabilities of all choices in model: |
| * = Direct Derivative effect of the attribute.    |
|                                  Mean     St.Dev  |
| *     Choice=AIR                -.1339     .0880  |
|       Choice=TRAIN               .0362     .0309  |
|       Choice=BUS                 .0204     .0204  |
|       Choice=CAR                 .0773     .0763  |
+---------------------------------------------------+
```

Derivatives and elasticities are obtained by averaging the observation specific values, rather than by computing them at the sample means. The listing reports the sample mean (average partial effect) and the sample standard deviation. Alternative approaches are discussed in Section N11.4.

It is common to report elasticities rather than the derivatives. These are

$$\frac{\partial \log P_j}{\partial \log x_{km}} = \left[\mathbf{1}(j=m) - P_m\right] x_{km} \beta_k .$$

The example below shows the counterpart to the preceding results produced by

; Effects: gc(air)

which requests a table of elasticities for the effect of changing *gc* in the *Air* alternative.

```
+---------------------------------------------------+
| Elasticity             averaged over observations.|
| Attribute is GC       in choice AIR               |
| Effects on probabilities of all choices in model: |
| * = Direct Elasticity effect of the attribute.    |
|                                  Mean     St.Dev  |
| *     Choice=AIR                -.8019     .3834  |
|       Choice=TRAIN               .3198     .3370  |
|       Choice=BUS                 .3198     .3370  |
|       Choice=CAR                 .3198     .3370  |
+---------------------------------------------------+
```

The difference between the two commands is the use of ‘[*air*]’ for derivatives and ‘(*air*)’ for elasticities. The full set of tables, one for each alternative, is requested with

alternative[*] or **alternative(*)**.

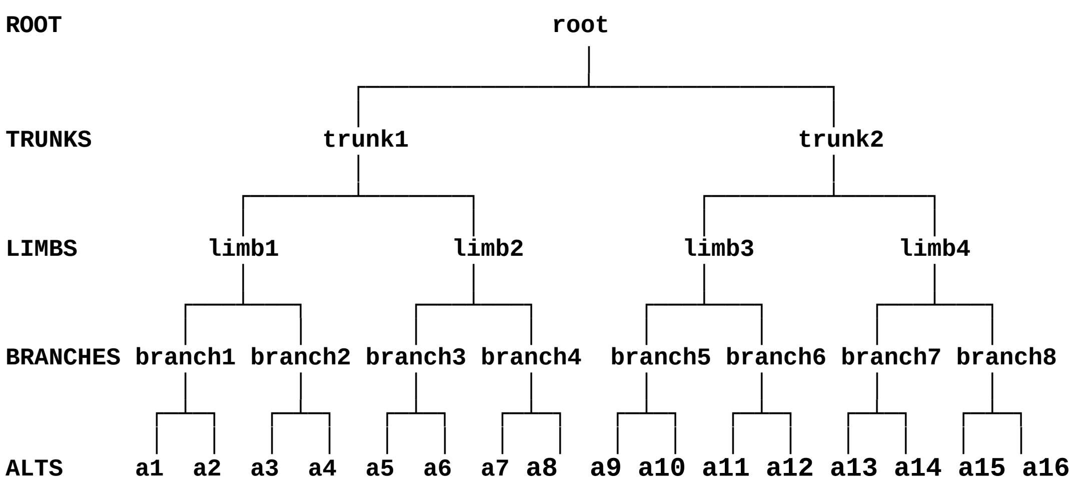
Note that for this model, the elasticities take only two values, the ‘own’ value when j equals m and the ‘cross’ elasticity when j is not equal to m. The fact that the cross elasticities are all the same is one of the undesirable consequences of the IIA property of this model.

Chapter 10

The Nested Logit Model

10.1. Introduction

The nested logit model is an extension of the multinomial model presented in Chapter 9 The models described here are based on variations of a four level tree structure such as the following:



Individuals are assumed to make a choice among NALT = *J* alternatives (alts) in a choice set. The ‘twigs’ in the tree are the elemental alternatives in the choice set. There may be up to 100 alternatives in the model, a total of 25 branches throughout the tree, 10 limbs, and 5 trunks. The model may contain one or more limbs. Each limb may contain one or more branches, and each branch may contain one or more twigs (choices). If there is only one trunk and one limb, the model is, by implication, a two level model. As for single level models, choice sets may vary by individual. However, in order to construct a tree for such a setting, a universal choice set, as described in Section N10.3, is necessary. The variable sized choice set is then indicated by setting up the full tree structure, and indicating that certain choices are unavailable for the particular individual.

The command for fitting nested logit models is the same as described in Chapter 9 for one level models, save for the addition of the tree definition in the command and, optionally, the specification of additional utility functions for choices made at higher levels in the tree. The nested logit model is limited to four level models for full information maximum likelihood (FIML) estimation. It also allows estimation of two and higher level models by sequential, or two step estimation.

Utility functions can be specified for trunks the same as for limbs and branches (though it is unlikely that there will be very many attributes at this level in a tree). All options are available, including logs, Box-Cox transformation, fixed values, starting values, trunk specific constants, interaction terms, and so on. Utility functions for the trunks may include up to 10 variables including the set of constant terms if used. Since the command structure and options for the nested logit model are the same as those for the one level model, we will present in this chapter only the parts of the command setup that are specific to nested models.

10.2 Mathematical Specification of the Model

Individuals are assumed to choose one of the alternatives at the lowest level of the tree. Thus, they also choose a branch, a limb and a trunk. We denote by '$j|b,l,r$' the choice of alternative j in branch b in limb l in trunk r. The number of alternatives in the branch/limb/trunk, $N_{b|l,r}$, can vary in every branch, limb, and trunk, and the number of branches in the l,rth limb/trunk, $N_{l|r}$ is likely to vary across limbs and trunks as well. No assumption of equal choice set sizes is made at any point in the following. (Note that for ease of presentation, we have dropped the observation subscript.)

The choice probability defined in the Chapter 9 is now redefined to be the conditional probability of alternative j in branch b, limb l, and trunk r, $j|b,l,r$:

$$P(j|b,l,r) = \frac{\exp(\boldsymbol{\beta}'\mathbf{x}_{j|b,l,r})}{\sum_{q|b,l,r}\exp(\boldsymbol{\beta}'\mathbf{x}_{q|b,l,r})} = \frac{\exp(\boldsymbol{\beta}'\mathbf{x}_{j|b,l,r})}{\exp(J_{b|l,r})},$$

where $J_{b|l,r}$ is the *inclusive value* for branch b in limb l, trunk r, $J_{b|l,r} = \log \Sigma_{q|b,l,r} \exp(\boldsymbol{\beta}'\mathbf{x}_{q|b,l,r})$. At the next level up the tree, we define the conditional probability of choosing a particular branch in limb l, trunk r,

$$P(b|l,r) = \frac{\exp(\boldsymbol{\alpha}'\mathbf{y}_{b|l,r} + \tau_{b|l,r}J_{b|l,r})}{\sum_{s|l,r}\exp(\boldsymbol{\alpha}'\mathbf{y}_{s|l,r} + \tau_{s|l,r}J_{s|l,r})} = \frac{\exp(\boldsymbol{\alpha}'\mathbf{y}_{b|l,r} + \tau_{b|l,r}J_{b|l,r})}{\exp(I_{l|r})},$$

where $I_{l|r}$ is the inclusive value for limb l in trunk r, $I_{l|r} = \log \Sigma_{s|l,r} \exp(\boldsymbol{\alpha}'\mathbf{y}_{s|l,r} + \tau_{s|l,r}J_{s|l,r})$. The probability of choosing limb l in trunk r is

$$P(l|r) = \frac{\exp(\boldsymbol{\delta}'\mathbf{z}_{l|r} + \sigma_{l|r}I_{l|r})}{\sum_{s|r}\exp(\boldsymbol{\delta}'\mathbf{z}_{s|r} + \sigma_{s|r}I_{s|r})} = \frac{\exp(\boldsymbol{\delta}'\mathbf{z}_{l|r} + \sigma_{l|r}I_{l|r})}{\exp(H_r)},$$

where H_r is the inclusive value for trunk r, $H_r = \log \Sigma_{s|r} \exp(\boldsymbol{\delta}'\mathbf{z}_{s|r} + \sigma_{s|r} I_{s|r})$. Finally, the probability of choosing a particular limb, r, is

$$P(r) = \frac{\exp(\boldsymbol{\theta}'\mathbf{h}_r + \phi_r H_r)}{\sum_s \exp(\boldsymbol{\theta}'\mathbf{h}_s + \phi_s H_s)}.$$

By the laws of probability, the unconditional probability of the observed choice made by an individual is

$$P(j,b,l,r) = P(j|b,l,r) \times P(b|l,r) \times P(l|r) \times P(r).$$

This is the contribution of an individual observation to the likelihood function for the sample.

The ‘nested logit’ aspect of the model arises when any of the $\tau_{j|i,l}$ or $\sigma_{i|l}$ or ϕ_l differ from 1.0. If all of these deep parameters are set equal to 1.0, the unconditional probability specializes to

$$P(j,bj,l,r) \quad = \quad \frac{\exp(\boldsymbol{\beta}'\mathbf{x}_{j|b,l,r} + \boldsymbol{\alpha}'\mathbf{y}_{b|l,r} + \boldsymbol{\delta}'\mathbf{z}_{l|r} + \boldsymbol{\theta}'\mathbf{h}_r)}{\sum_r \sum_l \sum_b \sum_j \exp(\boldsymbol{\beta}'\mathbf{x}_{jmbml,r} + \boldsymbol{\alpha}'\mathbf{y}_{b,l,r} + \boldsymbol{\delta}'\mathbf{z}_{l,r} + \boldsymbol{\theta}'\mathbf{h}_r)},$$

which is the probability for a one level model. The model is written in a very general form. The parameters of the model are, in exactly this order:

$$\beta_1,\beta_2,...,\beta_{nx},\alpha_1,\alpha_2,...,\alpha_{ny},\delta_1,\delta_2,...\delta_{nz},\theta_1,\theta_2,...,\theta_{nh},\tau_1...\tau_B,\sigma_1...,\sigma_L,\phi_1,...,\phi_R$$

where B is the total number of branches in the model, L is the number of limbs, and R is the number of trunks in the model. The **x**, **y**, **z**, and **h** vectors in the formulation above include all basic variables as well as all variables that interact with choice, branch, or limb specific dummy variables, etc. Once again, in this form, there may be different utility functions for each choice and, as described below, different utility functions defined for branches and limbs.

There is a vector of ‘shallow’ parameters, $[\boldsymbol{\beta},\boldsymbol{\alpha},\boldsymbol{\delta},\boldsymbol{\theta}]$ at each level, which multiplies the attributes (at the lowest level), or, e.g., demographics, at a higher level. There are also three vectors of ‘deep’ parameters, which multiply the inclusive values at the middle and high levels. In principle, there is one free inclusive value parameter for each branch in the model ($J_{b|l,r}$), one for each limb ($\sigma_{l|r}$), and one for each trunk (ϕ_r). But, some may have to be restricted to equal 1.0 for identification purposes. There are some degenerate cases:

- If the model has one trunk, then the one ϕ equals 1.0.
- If the model has one limb in a trunk, the one σ also equals 1.0.
- If a limb contains a single branch, the τ for that branch equals 1.0

The preceding describes a ‘nonnormalized’ model. The nested logit model also accommodates an explicit scaling factor at each level.

10.3 Commands for FIML Estimation

This section will describe how to set up a nested logit model. The default estimation technique is full information maximum likelihood (FIML). That is, the entire model is estimated in a single pass. In Section N16.9, we will describe how to obtain two step, limited information maximum likelihood (LIML) estimators for a two level model. In general, LIML has no advantage when FIML is available, and is generally inferior. Moreover, as will emerge below, the LIML estimator is not able to impose many of the parametric restrictions inherent in the model.

10.3.1 Data Setup

The arrangement of the data set for estimation of the nested logit model is exactly the same as shown in Chapter 7. There is no requirement that the choice sets be the same across individuals, but the nested logit model will require a definition of a universal choice set, so the command must contain the

; Choices = list of labels ...

specification. The nested model structure does mandate one special consideration if you are going to define utility functions for branches (*y*s), or limbs (*z*s). Since you have one line of data for each alternative, you will have more than one line of data for the variables in any branch or limb. In these cases, the values of '*y*' and '*z*' must be repeated for each alternative in the branch or limb. The following model and setup illustrate this for a three level model: (all in trunk 1)

			*x*1	*x*2	*y*1	*y*2	*z*1	*z*2
limb 1	branch 1|1	twig 1|1,1	.6	1	3	.02	104	.9
		twig 2|1,1	.1	2	3	.02	104	.9
	branch 2|1	twig 1|2,1	.8	2	7	.15	104	.9
		twig 2|2,1	.2	3	7	.15	104	.9
limb 2	branch 1|2	twig 1|1,2	.9	6	11	.08	96	.4
		twig 2|1,2	.3	1	11	.08	96	.4
		twig 3|1,2	.4	0	11	.08	96	.4

10.3.2 Tree Definition

The model command for estimating nested logit models is exactly as described in Chapter 8 for single level models, where the model name is now the generic **NLOGIT**;

NLOGIT ; Lhs = ... ; Choices = ... definition of choice set
; ... definition of utility functions for alternatives

All of the options described earlier are available. The nested logit model is requested by adding

; Tree = ... definition of the tree structure

to the command. In order to specify the tree, use these conventions:

{ } specifies a trunk,
[ ] specifies a limb within a trunk,
( ) specifies a branch within a limb in a trunk.

Entries in a list are separated by commas. Names for trunks, limbs and branches are optional before the opening '{' or '[' or '('. If you elect not to provide names, the defaults chosen will be 'Trunk{*l*}' 'Lmb[*i*|*l*]' and 'Br(*j*|*i,l*)' respectively, where the numbering is developed reading from left to right in your tree definition. Alternative names appear inside the parentheses. Some examples are as follows:

One limb:

; Tree = travel [fly (air), ground (train,bus,car)]

One limb: (With one limb, the [ ] is optional.)

; Tree = fly (air) , ground (train,bus,car)

One limb: (Branch names are optional. These would be Limb[1], Br(1|1) and Br(2|1).)

; Tree = (air) , (train,bus,car)

One limb, one branch, no nesting: (This would be unnecessary and could be omitted.)

```
; Tree  =  (air,train,bus,car)
```

Nested logit model - two limbs, one with one branch:

```
; Tree  =  private[ fly(air), ground(car_pas, car_drv) ],
           public [ (train, bus) ]
```

The fully nested 2×2×2×2 model shown in Section N16.1 could be specified with

```
; Choices = a1,a2,a3,a4,a5,a6,a7,a8,a9,a10,a11,a12,a13,a14,a15,a16
;Tree   = Trunk1 {   limb1 [ branch1 ( a1 , a2 ) ,    branch2 ( a3 , a4 )   ],
                     limb2 [ branch3 ( a5 , a6 ) ,    branch4 ( a7 , a8 )   ]  } ,
          Trunk2 {   limb3 [ branch5 ( a9 , a10 ) ,   branch6 ( a11 , a12 ) ] ,
                     limb4 [ branch7 ( a13 , a14 ) ,  branch8 ( a15 , a16 ) ]  }
```

10.3.3 Utility Functions

You may define the utility functions exactly as described in Chapter 8 for one level models. You may also define utility functions for branches and limbs and trunks, but note that in order to do so, you must use the explicit form described in Section 8.2.3. These are specified exactly the same as those for elemental alternatives. For example, in a two level model, you might put demographic characteristics, such as income or family size, at the top level. A complete model might appear as follows:

```
NLOGIT      ; Lhs       = mode ; Choices = air, train, bus, car
            ; Tree      = travel[public(bus,train), private(air, car)]
            ; Model:    U( air )       = ba  + bcost * gc + btime * ttme  /
                        U(train)       = bt  + bcost * gc + btime * ttme  /
                        U( car )       = bc  + bcost * gc + btime * ttme  /
                        U( bus )       =       bcost * gc + btime * ttme  /
                        U(public)      = ap  + apub * hinc /
                        U(private)     =       aprv  * hinc $
```

This model can be considerably collapsed;

```
; Model:  U(air, train, bus,car)  = <ba,bc,0,bt> +
                                    bcost * gc + btime * ttme /
          U(public,private)       = <ap,0>  +
                                    <apub, aprv> * Income $
```

Note that the same function specification U(...) is used for all three kinds of equations, for alternatives, branches, and limbs.

Finally, as noted earlier, you may impose equality constraints at any points in the model, just by using the same parameter name where you want the equality imposed. For example, if, for some reason, you desired to force the parameters *apub* and *bcost* to be equal, you could just change *apub* to *bcost* in the utility equation for *public*. That is, you can, if you wish, force equality of parameters at different levels of a model, once again, just by using the same parameter name in the model specification. (Given the impact of the scale parameters, this is probably inadvisable, but the program will allow you to do it nonetheless.)

The interaction of alternative specific constants, and branch and limb specific constants is complex, and it is difficult to draw generalities. As a general rule, models will usually become overdetermined, resulting in a singular Hessian, when there are more than NALT-1 constants, of all three types, in the entire model. Likewise, interactions of attributes and choice specific dummy variables can produce this effect as well. Users who encounter problems in which *NLOGIT* claims either that it is impossible to maximize the log likelihood function, or there is a singular Hessian, should examine the model for this pitfall.

10.3.4 Command Builder

The command builders can be used to specify the nested logit models. Select Model:Discrete Choice/Nested Logit to access the command builder. The choice variable is defined on the Main page, shown in Figure 10.1. The Options page, shown in Figure 19.2, may be used to specify the rest of the model.

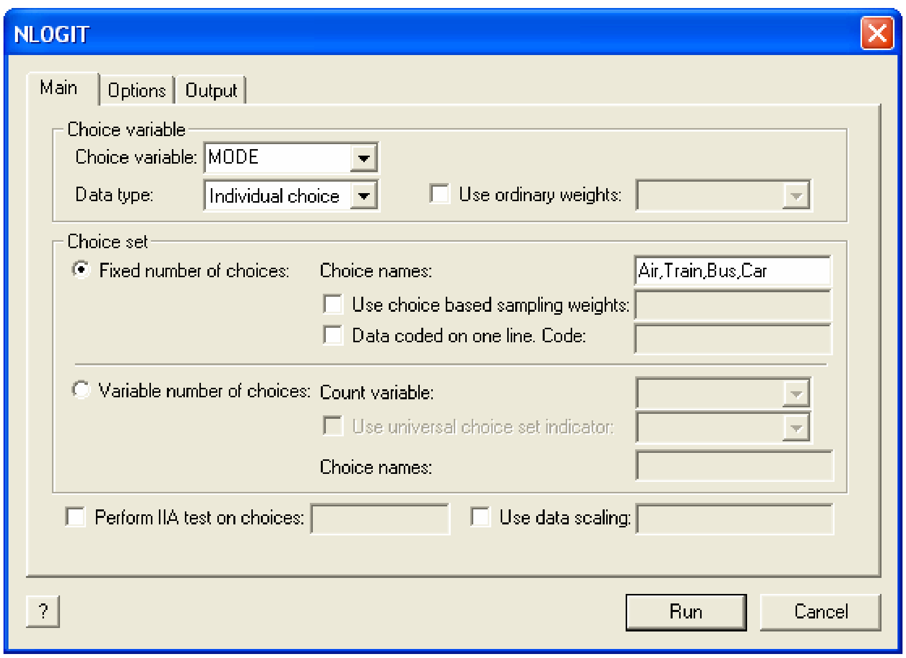


Figure 10. 1 Main Page of Command Builder for Nested Logit Models

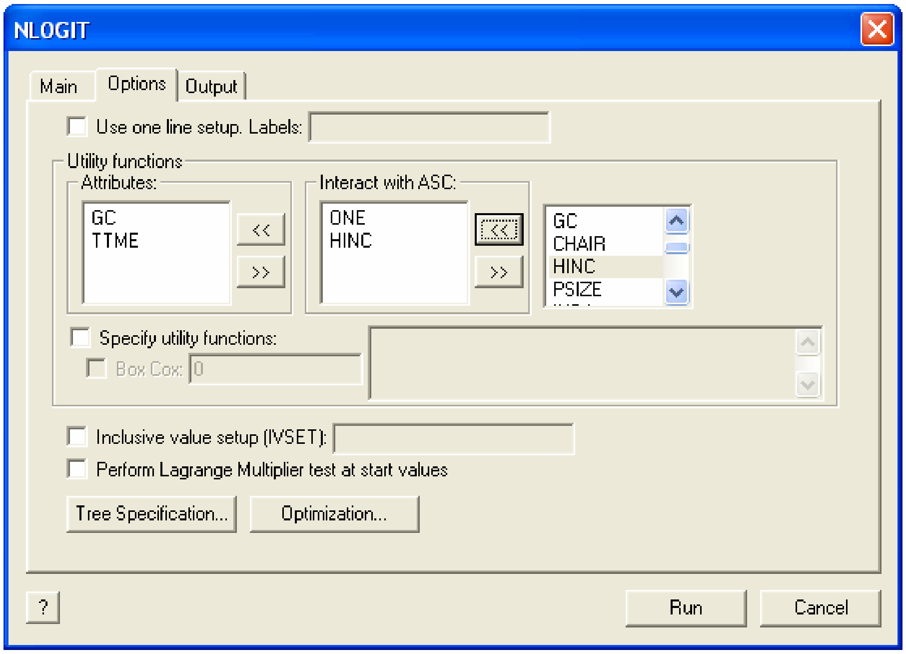


Figure 10.2 Options Page of Command Builder for Nested Logit Models

The tree is specified in a subsidiary dialog box by selecting Tree Specification at the bottom of the Options page. The dialog box, shown in Figure 10.3, allows you to define the tree graphically. Note in the dialog shown, *public* and *private* are siblings while *bus* is a child node of *public*.

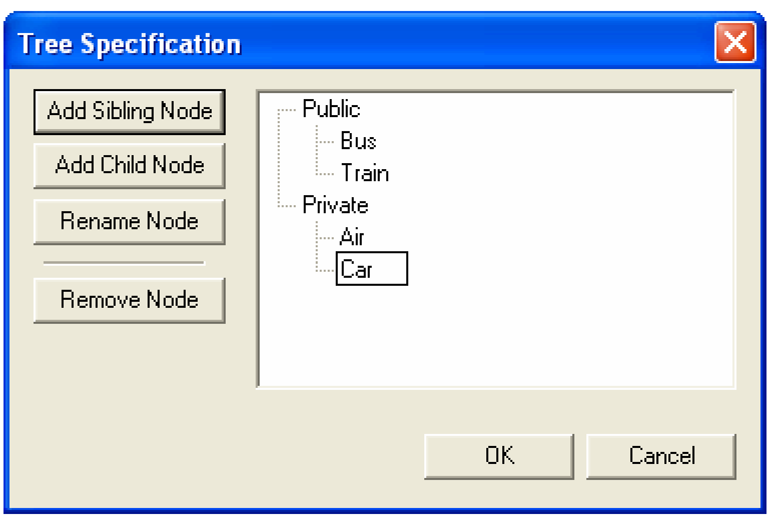


Figure 10.3Tree Specification Dialog Box for Defining the Tree Structure

The remaining options for output and results to be saved are defined in the Output page as shown in Figure 10.4.

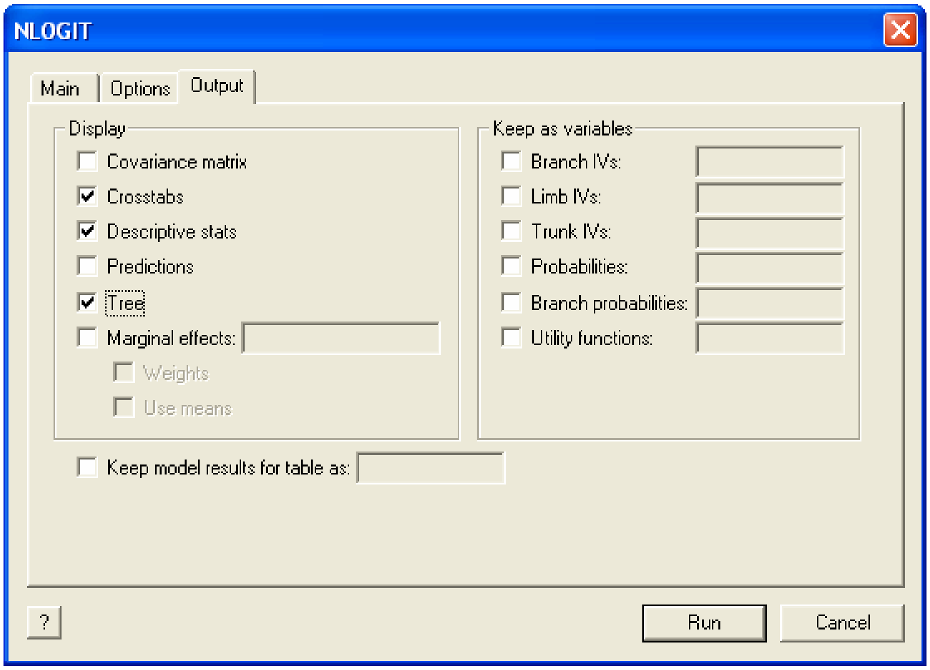


Figure 10.4 Output Page of Command Builder for Nested Logit Models

10.4 Marginal Effects and Elasticities

In the nested logit model with $P(j,b,l,r) = P(j|b,l,r) \times P(b|l,r) \times P(l|r) \times P(r)$, the marginal effect of a change in attribute 'k' in the utility function for alternative 'J' in branch 'B' of limb 'L' of trunk 'R' on the probability of choice 'j' in branch 'b' of limb 'l' of trunk 'r' is computed using the following result: Lower case letters indicate the twig, branch, limb and trunk of the outcome upon which the effect is being exerted. Upper case letters indicate the twig, branch, limb and trunk which contain the outcome whose attribute is being changed:

$$\frac{\partial \log P(alt=j, limb=l, branch=b, trunk=r)}{\partial x(k) \mid alt=J, limb=L, branch=B, trunk=r)} = D(k \mid J,B,L,R) = \Delta(k) \times F\ ,$$

where $\Delta(k)$ = coefficient on $x(k)$ in $U(J|B,L,R)$

and F = $\mathbf{1}(r=R) \times \mathbf{1}(l=L) \times \mathbf{1}(b=B) \times [\mathbf{1}(j=J) - P(J|BLR)]$ (trunk effect),
$\mathbf{1}(r=R) \times \mathbf{1}(l=L) \times [\mathbf{1}(b=B) - P(B|LR)] \times P(J|BLR)] \times \tau_{B|LR}$ (limb effect),
$\mathbf{1}(r=R) \times [\mathbf{1}(l=L) - P(L|R)] \times P(B|LR) \times P(J|BLR) \times \tau_{B|LR} \times \sigma_{L|R}$ (branch effect),
$[\mathbf{1}(r=R) - P(R)] \times P(L|R) \times P(B|LR) \times P(J|BLR) \times \tau_{B|LR} \times \sigma_{L|R} \times \phi_R$ (twig effect).

(Note, in this expression, J, B, L and R are being used generically to indicate a particular choice, branch, limb and trunk, not the total numbers of twigs, branches, limbs and trunks.) The marginal effect is

$$\partial P(j,b,l,r)/\partial x(k)|J,B,L,R = P(j,b,l,r)\ \Delta(k)\ F.$$

A marginal effect has four components, an effect on the probability of the particular trunk, one on the probability for the limb, one for the branch, and one for the probability for the twig.

(Note that with one trunk, $P(l) = P(1) = 1$, and likewise for limbs and branches.) For continuous variables, such as cost, you might be interested, instead, in the

$$Elasticity = x(k)|J,B,L,R \times \Delta(k|J,B,L,R) \times F.$$

NLOGIT will provide either. As in the case of nonnested models, marginal effects are requested with

;Effects: attribute [list of outcomes] / ...

or

attributes (list) / ... for elasticities

This generates a table of results for each of the outcomes listed. For example,

```
NLOGIT   ; Lhs        = mode
         ; Choices    = air,train,bus,car
         ; Tree       = travel[public(bus,train), private(air,car)]
         ; Model:  U( air )   = ba + bcost  * gc + btime * ttme /
                   U(train)   = bc + bcost  * gc + btime * ttme /
                   U(bus)     =      bcost  * gc + btime * ttme /
                   U(car)     = bc + bcost  * gc
         ; Effects:  gc ( car ) $
```

This lists the effects on all four probabilities of changes in attribute generalized cost (*gc*) of choice *car*.

```
+------------------------------------------------------------+
| Partial effects = average over observations                |
|                                                            |
| dlnP[alt=j,br=b,lmb=l,tr=r]                                |
| ---------------------------- = D(k:J,B,L,R) = delta(k)*F   |
| dx(k):alt=J,br=B,lmb=L,tr=R]                               |
| delta(k) = coefficient on x(k) in U(J|B,L,R)               |
| F = (r=R)  (l=L) (b=B) [(j=J)-P(J|BLR)]                    |
|  +  (r=R)  (l=L) [(b=B) -P(B|LR)]P(J|BLR)t(B|LR)           |
|  +  (r=R) [(l=L)-P(L|R)] P(B|LR) P(J|BLR)t(B|LR)s(L|R)     |
|  + [(r=R) -P(R)] P(L|R)  P(B|IR) P(J|BIR)t(B|LR)s(L|R)f(R) |
|                                                            |
| P(J|BLR)=Prob[choice=J |branch=B,limb=L,trunk=R]           |
| P(B|LR), P(L|R), P(R) defined likewise.                    |
| (n=N) = 1 if n=N, 0 else, for n=j,b,l,r and N=J,B,L,R.     |
| Elasticity = x(k) * D(j|B,L,R)                             |
| Marginal effect = P(JBLR)*D = P(J|BLR)P(B|LR)P(L|R)P(R)D   |
| F is decomposed into the 4 parts in the tables.            |
+------------------------------------------------------------+
+-----------------------------------------------------------------------+
| Elasticity             averaged over observations.                    |
| Attribute is GC       in choice CAR                                   |
| Effects on probabilities of all choices in the model:                 |
| * indicates direct Elasticity effect of the attribute.                |
|                        Decomposition of Effect if Nest    Total Effect|
|                        Trunk   Limb   Branch   Choice     Mean  St.Dev|
| Trunk=Trunk{1}                                                        |
| Limb=TRAVEL                                                           |
|    Branch=PUBLIC                                                      |
|       Choice=BUS        .000   .000    .857    .000        .857   .532 |
|       Choice=TRAIN      .000   .000    .857    .000        .857   .532 |
|    Branch=PRIVATE                                                     |
|       Choice=AIR        .000   .000  -1.015    .571       -.444   .746 |
| *     Choice=CAR        .000   .000  -1.015   -.338     -1.353  1.059 |
+-----------------------------------------------------------------------+
```

Note that across a row, the effects sum to the total effect given. The default method of computing the elasticities is to average the observation specific results. The results show the mean and the sample standard deviations. If you use the **;Means** specification, then the elasticities are computed once, and the results reflect the change, as shown below. (The differences are noticeably large.)

```
+-----------------------------------------------------------------------+
| Elasticity             computed at sample means.                      |
| Attribute is GC       in choice CAR                                   |
| Effects on probabilities of all choices in the model:                 |
| * indicates direct Elasticity effect of the attribute.                |
|                        Decomposition of Effect if Nest    Total Effect|
|                        Trunk   Limb   Branch   Choice     Mean  St.Dev|
| Trunk=Trunk{1}                                                        |
| Limb=TRAVEL                                                           |
|    Branch=PUBLIC                                                      |
|       Choice=BUS        .000   .000    .584    .000        .584   .000 |
|       Choice=TRAIN      .000   .000    .584    .000        .584   .000 |
|    Branch=PRIVATE                                                     |
|       Choice=AIR        .000   .000   -.411    .303       -.107   .000 |
| *     Choice=CAR        .000   .000   -.411   -.605     -1.016   .000 |
+-----------------------------------------------------------------------+
```

Chapter 11

The Random Parameters Logit Model

11.1 Introduction

The random parameters (RP) logit model, also referred to as the mixed logit model is the most general model form in *NLOGIT* in terms of the variety of model specifications it can accommodate and in terms of the range of behavior that it can model. (On this latter point, see McFadden and Train (2000).) This chapter will develop the numerous different specifications of the model that can be accommodated.

NLOGIT offers an extensive set of specifications within the mixed logit structure. This model is gaining great popularity in applications. Capabilities provided by the estimator include (i) choosing from among a large number of analytical distributions for each random parameter, (ii) accounting for the non-independence between observations associated with the same respondent (a theme of importance in stated choice studies), (iii) decomposing the mean and standard deviation of one or more random parameters to reveal sources of systematic taste heterogeneity, (iv) accounting for correlation of random parameters, (v) imposing priors based on known choices in model estimation, (vi) imposing constraints on distributions (e.g. constraining the triangular or normal to ensure that it does not change sign over its range), (vii) selecting subsets of pre-specified variables to interact with the mean and standard deviation of random parameterized attributes, and (viii) deriving willingness to pay estimates when both the numerator and denominator are random parameter estimates.

11.2 Random Parameters (Mixed) Logit Models

This model is somewhat similar to the random coefficients model for linear regressions. (See Bhat (1996), Jain, Vilcassim, and Chintagunta (1994), Revelt and Train (1998), Train, Revelt, and Ruud (1996), and Berry, Levinsohn, and Pakes (1995).) The model formulation is a one level multinomial logit model, for individuals $i = 1,\ldots,N$ in choice setting t. Neglecting for the moment the error components aspect of the model, we begin with the basic form of the multinomial logit model, with (optional) alternative specific constants α_{ji} and attributes $\mathbf{x}_{ji}$,

$$\text{Prob}(y_{it} = j) \quad = \frac{\exp\left(\alpha_{ji} + \boldsymbol{\beta}_i'\mathbf{x}_{ji}\right)}{\sum_{q=1}^{J_i}\exp\left(\alpha_{qi} + \boldsymbol{\beta}_i'\mathbf{x}_{qi}\right)}.$$

The RP model emerges as the form of the individual specific parameter vector, $\boldsymbol{\beta}_i$ is developed. The most familiar, simplest version of the model specifies

$$\beta_{ki} \quad = \beta_k + \sigma_k v_{ik},$$

and

$$\alpha_{ji} \quad = \alpha_j + \sigma_j v_{ji},$$

where β_k is the population mean, v_{ik} is the individual specific heterogeneity, with mean zero and standard deviation one, and σ_k is the standard deviation of the distribution of β_{ik}s around β_k. The term 'mixed logit' is often used in the literature (e.g., Revelt and Train (1998)) for this model. The choice specific constants, α_{ji} and the elements of $\boldsymbol{\beta}_i$ are distributed randomly across individuals with fixed means. A refinement of the model is to allow the means of the parameter distributions to be heterogeneous with observed data, $\mathbf{z}_i$, (which does not include *one*). This would be a set of choice invariant characteristics that produce individual heterogeneity in the means of the randomly distributed coefficients so that

$$\beta_{ki} \quad = \beta_k + \boldsymbol{\delta}_k'\mathbf{z}_i + \sigma_k v_{ki},$$

and likewise for the constants. The model is not limited to the normal distribution. We consider several alternatives below. One important variation is the lognormal model,

$$\beta_{ki} \quad = \exp(\rho_k + \boldsymbol{\delta}_k'\mathbf{z}_i + \sigma_k v_{ki}).$$

The v_{jki}s are individual and choice specific, unobserved random disturbances - the source of the heterogeneity. Thus, as stated above, in the population, if the random terms are normally distributed,

$$\alpha_{ji} \text{ or } \beta_{ki} \sim \text{Normal or Lognormal } [\rho_{j\ or\ k} + \boldsymbol{\delta}_{j\ or\ k}'\mathbf{z}_i, \sigma_{j\ or\ k}^2].$$

(Other distributions may be specified.) For the full vector of K random coefficients in the model, we may write the full set of random parameters as

$$\boldsymbol{\rho}_i = \boldsymbol{\rho} + \boldsymbol{\Delta}\mathbf{z}_i + \boldsymbol{\Gamma}\mathbf{v}_i.$$

where Γ is a diagonal matrix which contains σ_k on its diagonal. For convenience at this point, we will simply gather the parameters, choice specific or not, under the subscript 'k.' (The notation is a bit more cumbersome for the lognormally distributed parameters. We will return to that in the technical details.)

We can go a step further and allow the random parameters to be correlated. All that is needed to obtain this additional generality is to allow Γ to be a triangular matrix with nonzero elements below the main diagonal. Then, the full covariance matrix of the random coefficients is $\Sigma = \Gamma\Gamma'$. The standard case of uncorrelated coefficients has $\Gamma = \text{diag}(\sigma_1,\sigma_2\ ,\ldots,\sigma_k)$. If the coefficients are freely correlated, Γ is a full, unrestricted, lower triangular matrix and Σ will have nonzero off diagonal elements. (It will be convenient to aggregate this one step further. We may gather the entire parameter vector for the model in this formulation simply by specifying that for the nonrandom parameters in the model, the corresponding rows in Δ and Γ are zero.) We will also define the data and parameter vector so that any choice specific aspects are handled by appropriate placements of zeros in the applicable parameter vector.

An additional extension of the model allows the distribution of the random parameters to be heteroscedastic. As stated above, the variance of v_{ik} is taken to be a constant. The model is made heteroscedastic by assuming, instead, that

$$\text{Var}[v_{ik}] = \sigma_{jk}^2\ [\exp(\omega_k'\mathbf{hr}_i)]^2$$

A convenient way to parameterize this is to write the full model as

$$\rho_\text{i} = \rho + \Delta\mathbf{z}_i + \Gamma\Omega_i\mathbf{v}_i$$

where $\boldsymbol{\Omega}_i$ is the diagonal matrix of individual specific variance terms; $\omega_{ik} = \exp(\omega_k'\mathbf{hr}_i)$.

The list of variations above produces an extremely flexible, general model. Typically, you would use only some of them, though in principle, all could appear in the model at once. We will develop them in parts in the sections to follow. A convenient form of the full random parameters logit model to begin with is

$$\text{Prob}(y_{it} = j) = \frac{\exp(\alpha_{ji} + \boldsymbol{\beta}_i'\mathbf{x}_{jit})}{\sum_{q=1}^{J_{it}} \exp(\alpha_{qi} + \boldsymbol{\beta}_i'\mathbf{x}_{qit})},$$

Finally, an additional layer of individual heterogeneity may be added to the model in the form of the error components detailed in Chapter N14. The full model with all components is

$$\text{Prob}(y_{it} = j) = \frac{\exp\left[\alpha_{ji} + \boldsymbol{\beta}_i'\mathbf{x}_{jit} + \Sigma_{m=1}^{M} d_{jm}\theta_m \exp(\boldsymbol{\gamma}_m'\mathbf{he}_i)E_{im}\right]}{\sum_{q=1}^{J_i} \exp\left[\alpha_{qi} + \boldsymbol{\beta}_i'\mathbf{x}_{qit} + \Sigma_{m=1}^{M} d_{qm}\theta_m \exp(\boldsymbol{\gamma}_m'\mathbf{he}_i)E_{im}\right]},$$

where the components of the model are as follows:

Random Alternative Specific Constants and Taste Parameters:

$(\alpha_{ji},\beta_i) = (\alpha_j,\beta) + \Delta\mathbf{z}_i + \Gamma\Omega_i\mathbf{v}_i$, $\mathbf{\Omega}_i =$ diag(ω_{i1}, ω_{i2}, ...) or $\mathbf{\Omega}_i$ = diag(σ_1,...,σ_k),

β,α_{ji} = constant terms in the distributions of the random taste parameters,

Uncorrelated Parameters with Homogeneous Means and Variances

β_{ik} = $\beta_k + \sigma_k v_{ik}$ when $\Delta = \mathbf{0}$, $\Gamma = \mathbf{I}$, $\mathbf{\Omega}_i$ = diag(σ_1,...,σ_k),

$\mathbf{x}_{jit}$ = all observed choice attributes and individual characteristics,

$\mathbf{v}_i$ = random unobserved taste variation, with mean vector $\mathbf{0}$ and covariance matrix $\mathbf{I}$

Uncorrelated Parameters with Heterogeneous Means and Variances

β_{ik} = $\beta_k + \delta_k'\mathbf{z}_i + \sigma_k \exp(\omega_k'\mathbf{hr}_i)v_{ik}$ when $\Gamma = \mathbf{I}$, $\mathbf{\Omega}_i =$ diag(ω_{i1}, ω_{i2}, ...)

Δ = parameters that enter the heterogeneous means of the distributions of the random parameters; $\beta + \Delta\mathbf{z}_i$ = the heterogeneous means,

ω_{ik} = $\exp(\omega_k'\mathbf{hr}_i)$ = heterogeneity in the variances of the distributions of the random parameters,

ω_k = parameters in the variance heterogeneity of the random parameters,

σ_{ik} = $\sigma_k\omega_{ik}$ = heterogeneous standard deviations in the distributions of the random parameters; $\sigma_{ik} = \sigma_k$ in a homoscedastic model,

$\mathbf{z}_i$ = observed variables that measure the heterogeneity in the means of the random parameters,

$\mathbf{hr}_i$ = observed variables that measure the heterogeneity in the variances of the random parameters,

Correlated Parameters with Heterogeneous Means

β_{ik} = $\beta_k + \delta_k'\mathbf{z}_i + \Sigma_{s=1}^{k}\Gamma_{ks}\, v_{is}$ when $\Gamma \neq \mathbf{I}$, and $\mathbf{\Omega}_i$ = diag(σ_1,...,σ_k),

Γ = lower triangular matrix with ones on the diagonal that allows correlation across random parameters when $\Gamma \neq \mathbf{I}$,

Individual Error Components

E_{im} = the individual specific underlying random error components, $m = 1,...,M$, $E_{im} \sim$ N[0,1].

d_{jm} = 1 if E_{im} appears in utility for alternative j and 0 otherwise.

θ_m = scale factor for error component m,

γ_{im} = $\exp(\gamma_m'\mathbf{he}_i)$ = heterogeneity in the variances of the error components

λ_{im} = $\theta_m\gamma_{im}$ = standard deviations of random error components,

γ_m = parameters in the heteroscedastic variances of the error components,

$\mathbf{he}_i$ = individual choice invariant characteristics that produce heterogeneity in the variances of the error components,

The model specification will dictate which parameters are random and which are not, how the heteroscedasticity if any is parameterized, the distributions of the random terms, and how the error components enter the model.

The probabilities defined above are conditioned on the random terms, $\mathbf{v}_i$ and the error components, $\mathbf{E}_i$. The unconditional probabilities are obtained by integrating v_{ik} and E_{im} out of the conditional probabilities: $P_j = E_{\mathbf{v},\mathbf{E}}[P(j|\mathbf{v}_i,\mathbf{E}_i)]$. This is a multiple integral which does not exist in closed form. The integral is approximated by sampling *nrep* draws from the assumed populations and averaging. (See Bhat (1996) and Revelt and Train (1998) and Greene (2003) for discussion.) Parameters are estimated by maximizing the simulated log likelihood,

$$\log L_s = \sum_{i=1}^{N} \log \frac{1}{R} \sum_{r=1}^{R} \prod_{t=1}^{T_i} \frac{\exp\left[\alpha_{ji} + \boldsymbol{\beta}_{ir}'\mathbf{x}_{jit} + \Sigma_{m=1}^{M} d_{jm}\theta_m \exp(\boldsymbol{\gamma}_m'\mathbf{he}_i)E_{im,r}\right]}{\sum_{q=1}^{J_i} \exp\left[\alpha_{qi} + \boldsymbol{\beta}_{ir}'\mathbf{x}_{qit} + \Sigma_{m=1}^{M} d_{qm}\theta_m \exp(\boldsymbol{\gamma}_m'\mathbf{he}_i)E_{im,r}\right]},$$

with respect to ($\boldsymbol{\beta}$, $\boldsymbol{\Delta}$, $\boldsymbol{\Gamma}$, $\boldsymbol{\Omega}$, θ, γ), where

R = the number of replications,

$\boldsymbol{\beta}_{ir}$ = $\boldsymbol{\beta} + \boldsymbol{\Delta}\mathbf{z}_i + \boldsymbol{\Gamma}\boldsymbol{\Omega}_i\mathbf{v}_{ir}$ = the rth draw on $\boldsymbol{\beta}_i$,

$\mathbf{v}_{ir}$ = the rth multivariate draw for individual i,

$E_{im,r}$ = the rth univariate normal draw on the underlying effect for individual i.

(Note that the multivariate draw, $\mathbf{v}_{ir}$ is actually K independent draws. The heteroscedasticity is induced first by multiplying by $\boldsymbol{\Omega}_i$, then the correlation is induced by multiplying $\boldsymbol{\Omega}_i\mathbf{v}_{ir}$ by $\boldsymbol{\Gamma}$.)

The model components may be restricted and varied in several ways.

- A variety of distributions may be chosen for the random parameters, and they need need not be the same for all parameters.
- The observed heterogeneity, $\boldsymbol{\Delta}\mathbf{z}_i$, is optional. You may specify that a coefficient is randomly distributed around a fixed mean. Thus, $\boldsymbol{\delta}_k$ may be set to a zero vector for some or all random coefficients.
- σ_k may be set equal to zero for some coefficients. This may change the way a coefficient enters the model. If $\sigma_k = 0$ and $\boldsymbol{\delta}_k = \mathbf{0}$, then the coefficient is a nonrandom fixed parameter. But, including it in $\boldsymbol{\beta}$ allows you to force a coefficient to be positive. This device also allows you to form a hierarchical model with nonrandom coefficients.
- Any coefficient in the model may be fixed at a specific value.
- The heteroscedasticity may apply to some or all (or none) of the random parameters.
- Different variables may be placed in the heterogeneous means ($\boldsymbol{\Delta}\mathbf{z}_i$) or the heteroscedastic variances ($\boldsymbol{\Omega}_i$) of any of the random parameters.
- The variables that enter the heteroscedasticity of the error components may be different.

11.3 Command for the Random Parameters Logit Models

The command for the mixed logit model is as follows:

```
RPLOGIT    ; Lhs    = ... as usual
           ; Choices = ...
           ; ... Utility function specification using
               ; Rhs  = ... ;Rh2=...  or
               ; Model: U(...) = ... to specify utilities
           ; Fcn    = specification of random parameters $
```

(The model command **NLOGIT;RPL** is equivalent.) The last specification is used to define the random parameters. There are many variants. We begin with the simplest, and add features as we proceed. The ;Fcn specification takes the basic form

; Fcn = parameter label (type)

where **parameter label** is defined either by a variable name that you use in your **;Rhs** specification or by the name you give in your **;Model:**... definitions and the **type** is one of the distributions defined in the next section. Alternative specific constants are a special case. You will generally not want to specify the parameters that multiply Rh2 variables as random. These two cases are considered specifically below. For example, the following specifies two normally distributed random parameters:

```
RPLOGIT    ; Lhs = mode ; Choices = air,train,bus,car
           ; Rhs = gc,ttme,invc  ;  Rh2 = hinc
           ; Fcn = gc(n), ttme(n) $
```

(The 'type' in the example is n, indicating normally distributed parameters. Several other specifications would probably be added.) Alternatively, you might use the following to specify a model with two random parameters:

```
RPLOGIT    ; Lhs = mode ; Choices = air,train,bus,car
           ; Model:
             U(air) =a_air+bgc*gc + btt*ttme + binvc*invc + ghinc*hinc /
             U(train,bus,car) = a_ground + bgc*gc
           ; Fcn = a_ground(n), btt(n) $
```

Note that the specifications of the random parameters are separated by commas, not semicolons. The next several subsections will describe the various parts of the specifications of the random parameters. The last part of this section describes the command builder for this model. Because so much of this model is custom made for the particular application, the command builder is somewhat limited compared to the command form indicated above.

11.3.1 Distributions of Random Parameters in the Model

There are many distributions that can be used for the random parameters. The most common will be the normal, which is used in the example above. Many alternatives are supported, however. A few of these are listed below. The basic distributions are specified with the following:

; Fcn = parameter name (type), ...

The types are

n	normal	$\beta_i = \beta + \sigma v_i$, $v_i \sim$ N[0,1],
l	lognormal	$\beta_i = \exp(\beta + \sigma v_i)$, $v_i \sim$ N[0.1],
u	uniform	$\beta_i = \beta + \sigma v_i$, $v_i \sim$ U[-1.1,]
t	triangular	$\beta_i = \beta + \sigma v_i$, $v_i \sim$ Triangle[-1.1],
d	dome	$\beta_i = \beta + \sigma v_i$, $v_i \sim 2\times$Beta(2,2) - 1
e	Erlang	$\beta_i = \beta + \sigma v_i$, $v_i \sim$ Gamma(1,4) - 4,
w	Weibull	$\beta_i = \beta + \sigma\, v_i$, $v_i = 2(-\log u_i)^{\sqrt{.5}}$, $u_i \sim$ U[0,1],
p	exponential	$\beta_i = \beta + \sigma v_i$, $v_i \sim$ exponential - 1
c	nonstochastic	$\beta_i = \beta$.

In the list above, we have denoted the constant in the distribution as 'β.' However, the parameter definition may involve heterogeneity in the mean so, what appears there may be of the form $\theta_i = \beta + \boldsymbol{\delta}'\mathbf{z}_i$. We have also written the scaling parameter in each form as 'σ,' however, you may also specify heterogeneity in the variances so what appears there may be of the form $\sigma_i = \sigma \exp(\boldsymbol{\omega}'\mathbf{h}_i)$. The list above suggests the variety of different distributions that may be used.

Any distribution may be used for any parameter. The normal distribution will the the usual choice. However, you may wish to restrict a particular coefficient in the model to be positive. The lognormal distribution is the obvious choice, though there are several other possibilities. The normal, lognormal, exponential, Erlang and Weibull distributions all have infinite ranges. If you wish to restrict the range of variation of a parameter, then the triangular, dome or uniform can be used. The lognormal distribution has an infinite tail in the positive direction and is anchored at zero while the Erlang and Weibull models as specified have infinite range from $\beta - \sigma E[v_i]$ to $+\infty$.

It is important to note that the means and variances of the distributions are not always simple functions when the parameters are not linear functions of the underlying random variables. For all but the Weibull distributions shown above, the mean of v_i is zero, which centers the distributions at β. For the lognormal and Weibull models, the mean depends on the parameters. This is also true of the modified distributions shown below. This means that one must be careful in interpretating the estimated coefficients, even in simple cases in which there is no heterogeneity in the means or variances. It is possible to learn about these empirically, as described in Section N17.8, however, it is often not possible to state a priori what the population means are for most of the distributions. The problem becomes yet more complicated as additional features such as heterogeneity in the means and heteroscedasticity are added to the model.

Some practical aspects of the specifications are as follows:

- If you will be mixing distributions, the specification of correlated parameters, while allowable, produces ambiguous results. The nature of the correlation is difficult to define. However, the program will have no unusual difficulty estimating a model in which correlated parameters have different distributions. One particular case worth noting is a mixture of normal and lognormal parameters. In such a model, the reported correlation will be between the normally distributed parameter and the log of the lognormally distributed parameter. This is probably not a useful result.
- Researchers often find that the long, thick tail of the lognormal distribution produces an implausible distribution of parameters. The restricted triangular distribution as well as several alternatives may be preferable.
- Type '*c*' is the same as not including the parameter in the Fcn list, which is how this usually should be done. But sometimes, for convenience, this might be preferred. Variablename(*c*) specifies a free mean and zero variance of the parameter.

Model results for these distributions will display the structural parameters, not necessarily the means and variances of the parameter distributions. Note, for example, that the means of the lognormal and the Weibull distributions are not equal to β; for the lognormal it is $\exp(\beta+\sigma^2/2)$ while for the Weibull it is $\beta+2\sigma\Gamma(1+1/\sqrt{2})$. Consider an example. The following estimates a model with two random parameters. We will use the normal, Weibull and exponentiated Weibull (our 'Rayleigh') distributions. Since the exponentiated Weibull estimator forces the coefficient to be positive, and the coefficients on the two variables would naturally be negative, we reverse the signs on the data before estimation.

```
CREATE    ; mgc=-gc ; mttme=-ttme$
RPLOGIT   ; lhs=mode
          ; choices=air,train,bus,car
          ; rhs=mgc,mttme
          ; rh2=one
          ; fcn=mgc(n),mttme(n) ? Normally distributed parameters
          ; maxit=50;pts=25;halton; pds=3 $
RPLOGIT   ; lhs=mode
          ; choices=air,train,bus,car
          ; rhs=mgc,mttme
          ; rh2=one
          ; fcn=mgc(w),mttme(w) ? Weibull distributed parameters
          ; maxit=50;pts=25;halton; pds=3 $
RPLOGIT   ; lhs=mode
          ; choices=air,train,bus,car
          ; rhs=mgc,mttme
          ; rh2=one
          ; fcn=mgc(r),mttme(r) ? Modified Weibull distributed parameters
          ; maxit=50;pts=25;halton; pds=3 $
```

11.3.2 Alternative Specific Constants

If you have used the **;Rhs=list** specification with choices specific constants, then the constants will be labeled **a_name**. For example, if you have used

; Choices = bus, train, car
; Rhs = one, cost

then to specify the model for random ASCs, you might use

; Fcn = a_bus(n), a_train(n)

If you are using the **;Model:** form, then you will have supplied your own names for the ASCs.

Random choice specific constants in the random utility model with cross section data produce a random term that is a convolution of the original extreme value random variable and the one specified in your model command. Suppose, for example, that you specify a normally distributed random constant for '*car*.' Then, the utility function for *car* will be

$$U(car) = \alpha_{car} + \text{(the rest of the utility function)} + \sigma_{car}v_{car} + \varepsilon_{car}$$

$$= \alpha_{car} + \text{(the rest of the utility function)} + u_{car}.$$

The random term in this equation is the sum of a normally distributed variable and one with an extreme value distribution. This produces a different stochastic model, but probably not a useful extension of the model in general. For this reason, unless you are using panel data it is generally not useful to specify random constant terms in the random parameters logit model. That said, however, there is an exception which might prove useful. Random constant terms that are correlated will produce correlation across the alternatives, which is one of the oft cited virtues of the multinomial probit model. In addition, the error components logit specification produces a useful extension that serves much the same function as a random constant term.

11.3.3 Heterogeneity in the Means of the Random Parameters

The **RPLOGIT** command requests the random parameters model generally, with the parameters specified in the **;Fcn** list varying around a mean that is the same for all individuals. The variables in $\mathbf{z}_i$ provide the variation of the mean across individuals. To specify the variables in $\mathbf{z}_i$, use

; RPL = list of variables in $\mathbf{z}_i$

If you desire to specify that $\mathbf{z}_i$ enter the means of some of the coefficients but not all, you can change the specification of the random coefficients in the **;Fcn** specification as follows:

name (type) implies $\mathbf{z}_i$ enters the mean
name [type] implies that $\mathbf{z}_i$ does not enter the mean.

The difference here is the parentheses in the first as opposed to the brackets in the second. The second of these forces the applicable row of Δ to contain zeros instead of free parameters. There are also some variations on this that allow some flexibility in the construction of Δ. First, an alternative, equivalent form of **name[type]** is

name (type | #)

This requests that if there are RPL variables (**;RPL = list**) that these not appear in the mean for this parameter. This puts a row of zeros in the **Δ** matrix. For example,

; RPL = Income
; Fcn = gc(n), ttme (n|#)

specifies that *income* does not appear in the mean of the *ttme* parameter. This form may be extended to exclude and include specific variables from the RPL list in the mean of a particular parameter. The specification is

name(type | # pattern)

where the pattern consists of 1s and 0s which indicate which variables in the list are included (1s) and excluded (zeros). There must be the same number of items in the pattern as there in the list. For example, the specification

; RPL = age,sex,income
; Fcn = gc(n),
ttme(n|#101)
invt (n|# 011)
invc (n|#000)

includes all three variables in the mean of *gc*, excludes *sex* from the mean of *ttme*, excludes *age* from the men of *invt*, and excludes all three variables from the mean of *invc*. All parameters may be specified independently, and there is no restriction on how this feature is used. Do note, however, if you exclude an RPL variable from all parameters, the model becomes inestimable.

11.3.4 Correlated Parameters

The model specified thus far assumes that the random parameters are uncorrelated. Use

; Correlation

to allow free correlation among the parameters. In this case, estimates of the below diagonal elements of Γ will be obtained with the other parameters of the model. No restrictions may be imposed on these new parameters. After these are presented, the elements of $\Sigma = \Gamma\Gamma'$ are given. An example appears below. The second note in Section N17.3.1 gives some cautions about this specification. In particular, some ambiguity in the results will be unavoidable when this feature is used with other modifications of the model, such as mixed distributions and heteroscedasticity. The most favorable case for use of this feature would be a sparse model,

$$\beta_i = \beta + \Gamma \mathbf{v}_i.$$

We would note, many, perhaps most of the received applications of the mixed logit model are of this form – it is much less restrictive than its bare appearance would suggest.

In the model developed thus far, the covariance matrix for the random components for the simple distributions (normal, uniform, triangle) is

$$\text{Var}[\boldsymbol{\beta}_i|\mathbf{x}_i,\mathbf{z}_i] \quad = \boldsymbol{\Sigma} = \boldsymbol{\Gamma}\boldsymbol{\Gamma}'.$$

In the uncorrelated case, $\boldsymbol{\Gamma}$ is a diagonal matrix, and the variance of β_{ik} is simply σ_k^2. When the parameters are correlated, then the diagonal element of $\boldsymbol{\Sigma}$ is $\gamma_k'\gamma_k$ where γ_k is the kth row of $\boldsymbol{\Gamma}$. The model results will show the elements of $\boldsymbol{\Gamma}$ and the implied standard deviations. The following demonstrates the computations. The command below specifies two correlated random parameters.

```
RPLOGIT   ; lhs=mode ; choices=air,train,bus,car
          ; rhs=gc,ttme
          ; rh2=one
          ; fcn=gc(n),ttme(n) ; Correlated
          ; maxit=50;pts=25;halton;output=3; pds=3 $
```

The relevant results from estimation are as follows. The coefficients reported are, first, β from the random parameter distributions, then the nonstochastic β from the distributions of the nonrandom alternative specific constants. The next results display the elements of the 2×2 lower triangular matrix, Γ. The diagonal elements appear first, then the below diagonal element(s). The matrix Γ is shown again, in natural form at the end of the results, labeled 'Cholesky matrix.' The 'Standard deviations of parameter distributions' are derived from $\boldsymbol{\Gamma}$. The first is $(.011001342)^{1/2} = .001100134$. The second is $((-.07458)^2 + .03678^2)^{1/2} = .08315251$. The standard errors for these estimators are computed using the delta method. Hensher, Rose and Greene (2005) discuss the Cholesky decomposition in detail with numerous examples.

```
+---------+--------------+----------------+--------+--------+
|Variable| Coefficient  | Standard Error |b/St.Er.|P[|Z|>z]|
+---------+--------------+----------------+--------+--------+
---------+Random parameters in utility functions
 GC      |    -.02260684       .00724332    -3.121   .0018
 TTME    |    -.14522848       .02205029    -6.586   .0000
---------+Nonrandom parameters in utility functions
 A_AIR   |    8.70238058      1.22465947     7.106   .0000
 A_TRAIN |    6.95973395      1.03548341     6.721   .0000
 A_BUS   |    6.12199207      1.13357506     5.401   .0000
---------+Diagonal values in Cholesky matrix, L.
 NsGC    |     .01100134       .01124017      .979   .3277
 NsTTME  |     .03678160       .03024421     1.216   .2239
---------+Below diagonal values in L matrix. V = L*Lt
 TTME:GC |    -.07457516       .02353048    -3.169   .0015
---------+Standard deviations of parameter distributions
 sdGC    |     .01100134       .01124017      .979   .3277
 sdTTME  |     .08315251       .01967123     4.227   .0000
Correlation Matrix for Random Parameters
Matrix COR.MAT. has  2 rows and  2 columns.
         GC            TTME
        +----------------------------
GC      |    1.00000      -.89685
TTME    |    -.89685      1.00000

Covariance Matrix for Random Parameters
Matrix COV.MAT. has  2 rows and  2 columns.
         GC            TTME
        +----------------------------
GC      |     .00012      -.00082
TTME    |    -.00082       .00691

Cholesky Matrix for Random Parameters
Matrix Cholesky has  2 rows and  2 columns.
         GC            TTME
        +----------------------------
GC      |     .01100    .0000000D+00
TTME    |    -.07458       .03678
```

We emphasize, these results apply to the linear functions of the underlying random variables, not necessarily to the implied distributions of the random parameters themselves. In most of the specifications, the parameters involve nonlinear transformations of these variables.

Chapter 12

The Multinomial Probit Model

12.1 Introduction

In this model, the individual's choice among J alternatives is the one with maximum utility, where the utility functions are

$$U_{ji} = \boldsymbol{\beta}'\mathbf{x}_{ji} + \varepsilon_{ji},$$

where U_{ji} = utility of alternative j to individual i,
$\mathbf{x}_{ji}$ = union of all attributes that appear in all utility functions. For some alternatives, $x_{i,tk}$ may be zero by construction for some attribute k which does not enter their utility function for alternative j,
ε_{ji} = unobserved heterogeneity for individual i and alternative j.

The multinomial logit model specifies that ε_{ji} are draws from independent extreme value distributions (which induces the IIA condition). In the multinomial probit model, we assume that ε_{ji} are normally distributed with standard deviations Sdv[ε_{ji}] = σ_j and correlations Cor[ε_{ji}, ε_{mi}] = ρ_{jm} (the same for all individuals). Observations are independent, so Cor[ε_{ji},ε_{ms}] = 0 if i is not equal to s, for all j and m. A variation of the model allows the standard deviations and covariances to be scaled by a function of the data, which allows some heteroscedasticity across individuals.

The correlations ρ_{jm} are restricted to $-1 < \rho_{jm} < 1$, but they are otherwise unrestricted save for a necessarily normalization. The correlations in the last row of the correlation matrix must be fixed at zero. The standard deviations are unrestricted with the exception of a normalization - two standard deviations are fixed at 1.0 - *NLOGIT* fixes the last two. In principle, up to 20 alternatives may be in the model, but our experience thus far is that this model is extremely difficult to estimate, and will usually not be estimable with a completely free correlation matrix even with only five alternatives. The difficulty increases greatly with the number of alternatives. (Imposition of constraints which may improve this situation is discussed below.)

This model may also be fit with panel data. In this case, the utility function is modified as follows:

$$U_{ji,t} = \boldsymbol{\beta}'\mathbf{x}_{jt,t} + \varepsilon_{ji,t} + v_{ji,t},$$

where 't' indexes the periods or replications. There are two formulations for $v_{ji,t}$,

Random effects	$v_{ji,t}$	$= v_{ji,s}$ (the same in all periods),
First order autoregressive	$v_{ji,t}$	$= \alpha_j v_{ji,t-1} + a_{ji,t}$.

12.2 Model Command

This is a one level (nonnested) model. To request it, use

MNPROBIT ; Lhs = ... ; Choices = ...
; Rhs = ... or ; Model: U(...)=... / U(...) = ... all as usual
; ... any other options $

(The alternative model command used in earlier versions of *NLOGIT*, **NLOGIT;MNP** is equivalent and may be used instead.) Other options include

; Prob = name to use for estimated probabilities
; Utility = name for estimated utilities

and the usual other options for output, technical output, elasticities, descriptive statistics, etc. There are some special cases for this estimator:

- The number of alternatives must be fixed - it may not vary across observations.
- The choice set must be fixed.
- Choice based sampling is not supported, though you can use ordinary weights.
- Data may be individual, proportions, or frequencies.

(The second derivatives matrix is not computed for this model, so it is not possible to compute a robust covariance matrix estimator.) An additional option is

; Pts = number of replications to compute multivariate normal probabilities

The command builder may also be used for this model by selecting Model/Discrete Choice/Multinomial Probit, HEV, RPL. The choice set and utility functions for the model are defined on the Main page and the MNP format of the model is selected on the Options page.

The following features of **NLOGIT** are not available for this model:

; Tree ...	This is not a nested logit model
; Ivb	**= name, ; Ivl=name, ; Ivt=name.** No inclusive values are computed.
; IIA	**= list**. IIA is not testable here, since it is not imposed.
; Cprob	**= name**. Conditional and unconditional probabilities are the same.
; Ranks	This estimator may not be based on ranks data.
; Scale ...	Data scaling is only for the nested logit model.

The remainder of the command setup is identical to the multinomial logit model with one level. All other options are available, including

; Probs = name to retain the predicted probabilities
; Utility = name to retain the predicted systematic utilities

and so on.

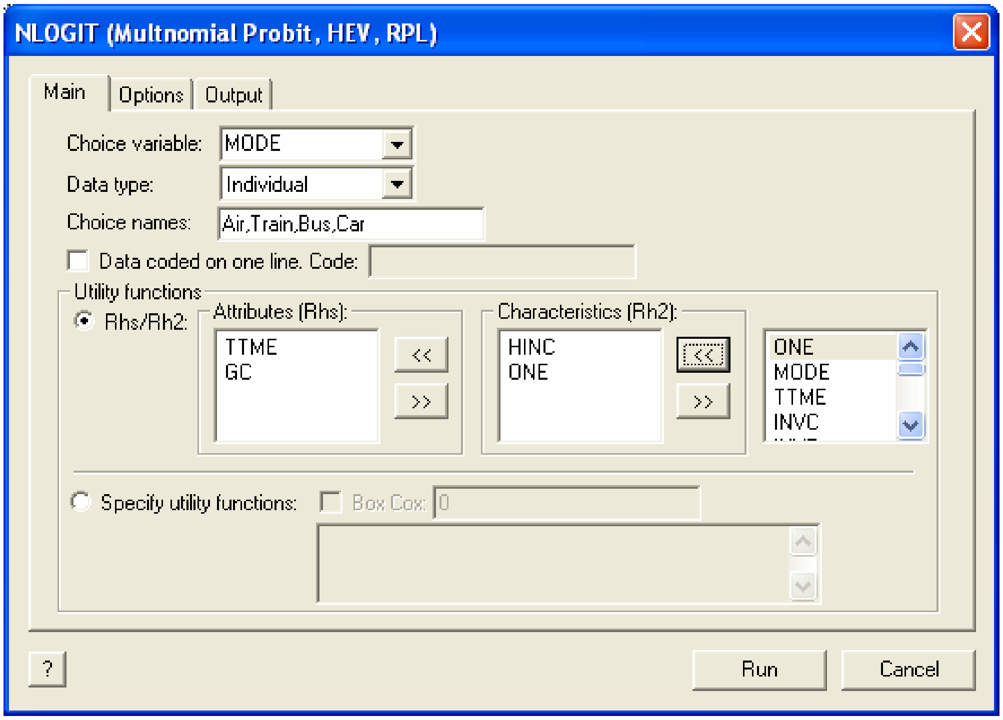


Figure 12.1 Main Page of Command Builder for the MNP model

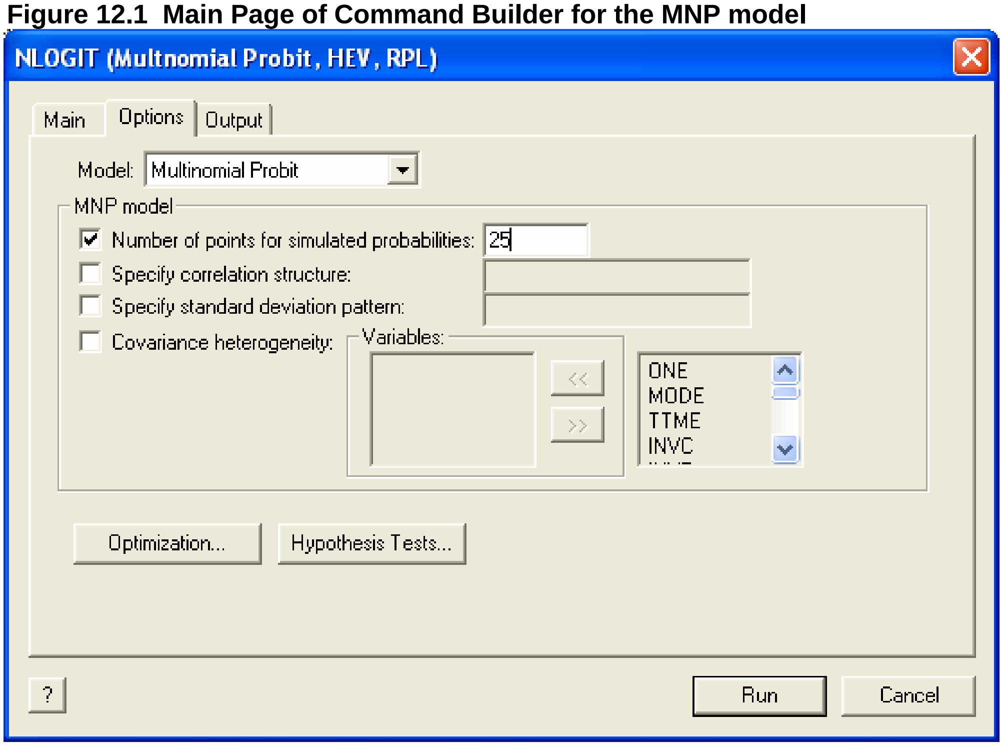


Figure 12.2 Options Page of Command Builder for the MNP model

12.3 An Application

The multinomial probit (MNP) model based on the CLOGIT data is estimated with the command

MNPROBIT ; Lhs = mode
; Choices = air,train,bus,car
; Rhs = gc,ttme
; Rh2 = one,hinc
; Effects: gc(air)
; Pts = 20 $

This is the model that was fit as an MNL model in Chapter 9. We have now relaxed the equal variances assumption and replaced the extreme value distribution with a multivariate normal distribution. The probabilities are computed with 20 replications, which is fairly small; we do this for purposes of a simple illustration. Results are shown below. The MNL model is fit first to obtain the starting values for the iterations. The results for the MNP model are given next. The two sets of results are merged in the display below.

```
+---------------------------------------------+
| Discrete choice (multinomial logit) model   |
| Dependent variable                    MODE  |
| Log likelihood function       -189.5252     |
| Info. Criterion: AIC =          1.88119     |
|   Finite Sample: AIC =          1.88460     |
| Info. Criterion: BIC =          2.00870     |
| Info. Criterion:HQIC =          1.93274     |
| R2=1-LogL/LogL*  Log-L fncn  R-sqrd  RsqAdj |
| Constants only    -283.7588  .33209  .31802 |
| Chi-squared[ 5]          =    188.46723     |
| Prob [ chi squared > value ] =    .00000    |
| Response data are given as ind. choice.     |
| Number of obs.=   210, skipped   0 bad obs. |
+---------------------------------------------+

+---------------------------------------------+
| Multinomial Probit Model                    |
| Log likelihood function       -189.8452     |
| Info. Criterion: AIC =          1.93186     |
|   Finite Sample: AIC =          1.94070     |
| Info. Criterion: BIC =          2.13906     |
| Info. Criterion:HQIC =          2.01562     |
| Restricted log likelihood     -291.1218     |
| McFadden Pseudo R-squared      .3478840     |
| Chi squared                    202.5532     |
| Degrees of freedom                   13     |
| Prob[ChiSqd > value] =         .0000000     |
| R2=1-LogL/LogL*  Log-L fncn  R-sqrd  RsqAdj |
| No coefficients   -291.1218  .34788  .33414 |
| Constants only    -283.7588  .33096  .31687 |
| At start values   -216.5343  .12326  .10478 |
+---------------------------------------------+
```

These are the estimates for the multinomial logit model

Variable	Coefficient	Standard Error	b/St.Er.	P[|Z|>z]
GC	-.01092735	.00458775	-2.382	.0172
TTME	-.09546055	.01047320	-9.115	.0000
A_AIR	5.87481336	.80209034	7.324	.0000
AIR_HIN1	-.00537349	.01152940	-.466	.6412
A_TRAIN	5.54985728	.64042443	8.666	.0000
TRA_HIN2	-.05656186	.01397335	-4.048	.0001
A_BUS	4.13028388	.67636278	6.107	.0000
BUS_HIN3	-.02858418	.01544418	-1.851	.0642

These are the estimates for the multinomial probit model

Variable	Coefficient	Standard Error	b/St.Er.	P[|Z|>z]
Attributes in the Utility Functions (beta)				
GC	-.02333086	.00896463	-2.603	.0093
TTME	-.09131236	.03629673	-2.516	.0119
A_AIR	4.68057508	1.91530359	2.444	.0145
AIR_HIN1	.00832932	.02520384	.330	.7410
A_TRAIN	5.90782858	1.92699048	3.066	.0022
TRA_HIN2	-.06016958	.02223662	-2.706	.0068
A_BUS	4.40097868	1.27339698	3.456	.0005
BUS_HIN3	-.01884772	.01615587	-1.167	.2434
Std. Devs. of the Normal Distribution.				
s[AIR]	2.85536857	1.29978748	2.197	.0280
s[TRAIN]	1.96198515	.91344112	2.148	.0317
s[BUS]	1.00000000	......(Fixed Parameter).......		
s[CAR]	1.00000000	......(Fixed Parameter).......		
Correlations in the Normal Distribution				
rAIR,TRA	.12923578	.74351679	.174	.8620
rAIR,BUS	.11759913	.92452141	.127	.8988
rTRA,BUS	.61859572	.38300577	1.615	.1063
rAIR,CAR	.000000	......(Fixed Parameter).......		
rTRA,CAR	.000000	......(Fixed Parameter).......		
rBUS,CAR	.000000	......(Fixed Parameter).......		

The table below compares the elasticities from the MNP model to the MNL model. The MNL results appear first. The are clearly similar, but the specification does make a difference.

Elasticity averaged over observations.			
Attribute is GC in choice AIR			
* = Direct Elasticity effect of the attribute.			
		Mean	St.Dev
*	Choice=AIR	-.8019	.3834
	Choice=TRAIN	.3198	.3370
	Choice=BUS	.3198	.3370
	Choice=CAR	.3198	.3370
Effects on probabilities of all choices in model:			
*	Choice=AIR	-1.0453	.4797
	Choice=TRAIN	.3796	.3184
	Choice=BUS	.5557	.3826
	Choice=CAR	.4221	.2957

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Index