Estimating Efficiency Spillovers with State Level Evidence for Manufacturing in the U.S.

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Abstract

Unit specific effects are often used to estimate non-spatial efficiency. We extend this efficiency estimator to the case where there is spatial autoregressive dependence and distinguish between own efficiency and spillover efficiency. We apply our estimator to a cost frontier model for state manufacturing in the U.S.

\textit{JEL Classification}: C23; C51; D24

\textit{Keywords}: Spatial Autoregression, Frontier Modeling, Panel Data, Efficiency Spillovers

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Preprint submitted to Economics Letters June 4, 2013
1. Introduction

The classic Schmidt & Sickles (1984) (SS from hereon) time-invariant efficiency estimator benchmarks the relative performance of the cross-sectional units using the fixed or random effects. The SS estimator was extended to the case of time-variant efficiency by Cornwell et al. (1990) (CSS from hereon). We extend the non-spatial CSS efficiency estimator to the case where there is spatial autoregressive dependence. This involves estimating direct efficiency (own efficiency accounting for feedback effects), indirect efficiency (spillover efficiency) and total (direct plus indirect) efficiency.

We apply our spatial time-variant efficiency estimator to a cost frontier model for manufacturing in the U.S. The model is estimated using annual data for the contiguous states from 1997 – 2008.

2. The Deterministic Spatial Autoregressive Cost Frontier Model

We develop a spatial CSS type efficiency estimator for deterministic frontier models. The estimator is applied to the deterministic spatial autoregressive cost frontier model in equation (1). We do not discuss spatial panel data models in detail here but for a comprehensive and up-to-date survey see Baltagi (2011).

\[
C_{it} = \kappa + \alpha_i + \tau_t + TL(h, q, t)_{it} + \lambda \sum_{j=1}^{N} w_{ij} C_{jt} + z_{it}\phi + \varepsilon_{it}, \quad (1)
\]

\[i = 1, \ldots, N; \quad t = 1, \ldots, T.
\]

$N$ is a cross-section of units; $T$ is the fixed time dimension; $C_{it}$ is the cost of the $i$th unit; $\alpha_i$ is a unit specific fixed effect; $\tau_t$ is a time period effect; $TL(h, q, t)_{it}$ represents the technology as the translog approximation of the log of the cost function where $h$ is a vector of normalized input prices and $q$ is a vector of outputs; $\lambda$ is the spatial autoregressive parameter; $w_{ij}$ is an element of the spatial weights matrix, $W$; $z_{it}$ is a vector of exogenous\footnote{In the application, $h$ is a vector of three input prices, price of production labor, price of non-production labor and price of energy, all of which are normalized by a fourth input price, the price of capital. Furthermore, in the application $q$ is a single output and not a vector of outputs.}.
characteristics and $\phi$ is the associated vector of parameters; $\varepsilon_{it}$ is an i.i.d. disturbance for $i$ and $t$ with zero mean and variance $\sigma^2$.

$W$ is a $(N \times N)$ matrix of known positive constants which describes the spatial arrangement of the cross-sectional units and also the strength of the spatial interaction between the units. All the elements on the main diagonal of $W$ are set to zero. $\lambda$ is assumed to lie in the interval $(1/r_{\text{min}}, 1)$, where $r_{\text{min}}$ is the most negative real characteristic root of $W$ and because $W$ is row-normalized in the application, 1 is the largest real characteristic root of $W$.\(^2\) We model the effects of time in equation (1) by, firstly, including a time trend, $t$, and the associated quadratic and cross terms in the translog function and secondly, via time period effects to account for common shocks to manufacturing costs across the cross-sectional units.

Equation (1) is estimated using maximum likelihood and we ensure that $\lambda$ lies in its parameter space, account for the endogeneity of the spatial autoregressive variable and also the fact that $\varepsilon_t$ is not observed by including the scaled logged determinant of the Jacobian transformation of $\varepsilon_t$ to $C_t$ (i.e. $T \log |I - \lambda W|$) as a term in the log-likelihood function. Details of the estimation of equation (1) by demeaning in the space dimension can be found in Elhorst (2009) with the following caveat. Lee & Yu (2010) show that demeaning in the space dimension to estimate a model such as that in equation (1) results in a biased estimate of $\sigma^2$ when $N$ is large and $T$ is fixed, which we denote $\sigma^2_B$. Following Lee & Yu (2010) we correct for this bias by replacing $\sigma^2_B$ in the variance matrix with the bias corrected estimate of $\sigma^2$, $\sigma^2_{BC} = T \sigma^2_B / (T - 1)$, which changes the t-values.

We do not also demean in the time dimension even though we include time period dummies. This is because this would eliminate the time trend and the associated quadratic term which are significant in the application. Moreover, by retaining in equation (1) the time trend and the associated cross and quadratic terms, the fitted model could be used to conduct a spatial TFP growth decomposition with, among other things, direct and indirect technical change components. Glass et al. (2013) propose such a spatial TFP growth decomposition but include, among other things, own efficiency change rather than direct and indirect efficiency changes. Their spatial TFP growth decomposition but include, among other things, own efficiency change rather than direct and indirect efficiency changes. Their spatial TFP growth decomposition but include, among other things, own efficiency change rather than direct and indirect efficiency changes. Their spatial TFP growth decomposition but include, among other things, own efficiency change rather than direct and indirect efficiency changes. Their spatial TFP

\(^2\)Furthermore, $(I_N - \lambda W)$ is taken to be nonsingular for all values of $\lambda$ in the parameter space. It is also assumed that the row and column sums of $W$ and $(I_N - \lambda W)$ are bounded uniformly in absolute value. This limits the spatial correlation to a manageable degree.
growth decomposition could be extended to include direct and indirect efficiency changes by following the approach which we set out here to calculate direct and indirect efficiencies.\(^3\)

3. Marginal Effects and Direct, Indirect and Total Efficiencies

A key recent development in applied spatial econometrics is the demonstration by LeSage & Pace (2009) that the coefficients on the explanatory variables in a model with spatial autoregressive dependence such as that in equation (1), cannot be interpreted as elasticities. This is because the marginal effect of an explanatory variable is a function of the spatial autoregressive variable. In light of this, LeSage & Pace (2009) propose the following approach to calculate direct, indirect and total marginal effects for the explanatory variables. Stacking successive cross-sections we can rewrite equation (1) as:

\[
C_t = (I - \lambda W)^{-1} \kappa_t + (I - \lambda W)^{-1} \alpha + (I - \lambda W)^{-1} \tau_t + (I - \lambda W)^{-1} \Gamma_t \beta + (I - \lambda W)^{-1} \phi + (I - \lambda W)^{-1} \epsilon_t, \tag{2}
\]

where \(\epsilon_t\) is an \((N \times 1)\) vector of ones; \(\alpha\) is an \((N \times 1)\) vector of fixed effects; \(\Gamma_t\) is an \((N \times K)\) matrix of stacked observations for \(TL(h,q,t)_t\); and \(\beta\) is a vector of translog parameters.

Differentiating equation (2) with respect to the \(k\)-th variable in \(TL(h,q,t)_t\), \(\Gamma_{k,t}\), yields the following vector of partial derivatives:

\[
\left[ \frac{\partial C}{\partial \Gamma_{k,1}}, \ldots, \frac{\partial C}{\partial \Gamma_{k,N}} \right]_t = \left[ \frac{\partial C_1}{\partial \Gamma_{k,1}}, \ldots, \frac{\partial C_1}{\partial \Gamma_{k,N}} \right]_t = (I - \lambda W)^{-1} \left[ \begin{array}{ccc} \beta_k & 0 & \cdots & 0 \\ 0 & \beta_k & \cdots & \cdot \\ \cdot & \cdots & \cdots & \cdot \\ 0 & 0 & \cdots & \beta_k \end{array} \right], \tag{3}
\]

where the product of the matrices on the far right of equation (3) is independent of time. The main diagonal of this product consists of direct marginal

\(^3\)Not demeaning in the time dimension does not create an incidental parameter problem in the application as the sample only spans twelve years. In the application, when estimating the non-spatial model using standard software, a small number of time period dummies are dropped by the software for reasons of collinearity. We drop the same time period dummies when fitting the spatial frontier models.
effects and all the non-diagonal elements of this product are indirect marginal
effects. Equation (3) yields different direct and indirect marginal effects on
each unit so to facilitate interpretation LeSage & Pace (2009) advise report-
ing a mean direct effect (average of the diagonal elements of the product of
matrices on the far right of equation (3)) and a mean indirect effect (average
column sum or row sum of the non-diagonal elements from the same product
as the magnitude of these calculations is the same).

The mean direct effect which includes feedback effects (i.e. effects which
pass through other units via the spatial multiplier matrix and back to the
unit which initiated the change) is the mean effect on a unit’s dependent
variable following a change in one of its independent variables. The mean
indirect effect can be interpreted in two ways: (i) the mean effect following a
change in an independent variable for one unit on the dependent variables of
all the other units (average column sum of the non-diagonal elements); (ii)
the mean change in the dependent variable for one particular unit following
a change in an independent variable for all the other units (average row sum
of the non-diagonal elements). The mean total effect is the sum of the mean
direct and indirect effects. We calculate the $t$-statistics for the mean effects
by obtaining the standard errors using the delta method.

The unit specific effects from a spatial model can be used to calculate
efficiencies by directly applying the non-spatial CSS ‘modifying’ estimation
procedure, where the efficiencies are directly comparable to those from a non-
spatial deterministic frontier model using the same procedure. This approach
has been applied using a spatial error cost frontier model with random effects
(Druska & Horrace (2004)). The steps involved are as follows. Firstly, solve
for the partial derivatives of the following log-likelihood function associated
with equations (1) and (2) with respect to $\alpha_i$ for $\alpha_i$ (see equation (5)).

$$
\log L = -\frac{NT}{2} \log (2\pi\sigma^2) + T \log |I - \lambda W| - \\
\frac{1}{2\sigma^2} \sum_{i=1}^{N} \sum_{t=1}^{T} \left( C_{it} - \lambda \sum_{j=1}^{N} w_{ij} C_{jt} - \kappa - \alpha_i - \tau_t - TL (h, q, t)_{it} - z_{it}\phi \right)^2 , \quad (4)
$$

$$
\alpha_i = \frac{1}{T} \sum_{t=1}^{T} \left( C_{it} - \lambda \sum_{j=1}^{N} w_{ij} C_{jt} - \kappa - \alpha_i - \tau_t - TL (h, q, t)_{it} - z_{it}\phi \right) . \quad (5)
$$
Secondly, the fixed effects can be used to estimate cost efficiency, \( CE_{it} \), as follows, where in each period it is assumed that the most efficient unit lies on the frontier.

\[
CE_{it} = \exp \left[ \min_i (\delta_{it}) - \delta_{it} \right],
\]

where \( \delta_{it} = \alpha_i + \theta_i t + \rho_i t^2 \) so \( \alpha_i \) is the time-invariant component of time-variant cost efficiency and the estimates of the parameters \( \theta_i \) and \( \rho_i \) are obtained by using the residuals from equation (1), \( \varepsilon_{it} \), to estimate \( \varepsilon_{it} = \theta_i t + \rho_i t^2 + e_{it} \), where \( e_{it} \) is an i.i.d. disturbance.

We extend the CSS methodology and set out a spatial ‘modifying’ estimation procedure by recognizing that in equation (2), \((I - \lambda W)^{-1} \alpha = \alpha^Tot\), where \( \alpha^Tot \) is a \((N \times 1)\) vector of total fixed effects. Equivalently using column vector notation:

\[
(I - \lambda W)^{-1} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \end{pmatrix} = \begin{pmatrix} \alpha_{11}^{Dir} + \alpha_{12}^{Ind} + \cdots + \alpha_{1N}^{Ind} \\ \alpha_{21}^{Ind} + \alpha_{22}^{Dir} + \cdots + \alpha_{2N}^{Ind} \\ \vdots \\ \alpha_{N1}^{Ind} + \alpha_{N2}^{Ind} + \cdots + \alpha_{NN}^{Dir} \end{pmatrix} = \begin{pmatrix} \alpha^Tot_1 \\ \alpha^Tot_2 \\ \vdots \\ \alpha^Tot_N \end{pmatrix},
\]

where \( \alpha_{ij}^{Dir} \) (i.e. where \( i = j \)) and \( \alpha_{ij}^{Ind} \) (i.e. where \( i \neq j \)) are direct and indirect fixed effects, respectively. In the same way as we obtain direct and indirect fixed effects as indicated in equation (7), we obtain direct and indirect residuals from \((I - \lambda W)^{-1} \varepsilon_t\) in equation (2), \( \varepsilon_{ijt}^{Dir} \) and \( \varepsilon_{ijt}^{Ind} \).

Direct cost efficiency, \( CE_{it}^{Dir} \), aggregate indirect cost efficiency, \( CE_{it}^{AggInd} \), and total cost efficiency, \( CE_{it}^{Tot} \), are calculated as follows.

\[
CE_{it}^{Dir} = \exp \left[ \min_i (\delta_{it}^{Dir}) - \delta_{it}^{Dir} \right],
\]

\[
CE_{it}^{AggInd} = \exp \left[ \min_i (\delta_{it}^{AggInd}) - \delta_{it}^{AggInd} \right],
\]

\[
CE_{it}^{Tot} = \exp \left[ \min_i (\delta_{it}^{Dir} + \delta_{it}^{AggInd}) - \delta_{it}^{Dir} - \delta_{it}^{AggInd} \right],
\]

where:

\[
\delta_{it}^{Dir} = \alpha_{ij}^{Dir} + \theta_i^{Dir} t + \rho_i^{Dir} t^2;
\]

\[
\delta_{it}^{AggInd} = \sum_{j=1}^{N} \alpha_{ij}^{Ind} + \theta_i^{AggInd} t + \rho_i^{AggInd} t^2.
\]
The $\theta^\text{Dir}_i$, $\rho^\text{Dir}_i$, $\theta^\text{AggInd}_i$ and $\rho^\text{AggInd}_i$ parameters needed to estimate $CE^\text{Dir}_i$ and $CE^\text{AggInd}_i$ can be obtained by regressing in turn $\varepsilon^\text{Dir}_{ijt}$ and $\sum_{j=1}^{N} \varepsilon^\text{Ind}_{ijt}$ on $t$ and $t^2$ for each unit. The corresponding parameters to estimate $CE^\text{Tot}_i$ are obtained by summing $\theta^\text{Dir}_i$ and $\theta^\text{AggInd}_i$, and $\rho^\text{Dir}_i$ and $\rho^\text{AggInd}_i$.

Interestingly, the aggregate indirect cost efficiency from equation (9), refers to the efficiency of cost spillovers to the $i$th unit from all the $j$th units, which is how indirect efficiency should be interpreted when incorporating change in efficiency spillovers into a spatial TFP growth decomposition for the $i$th unit. It is also valid to interpret aggregate indirect cost efficiency as the efficiency of cost spillovers to all the $i$th units from a particular $j$th unit. Since $\alpha^\text{Ind}_{ij} \neq \alpha^\text{Ind}_{ji}$ and $\varepsilon^\text{Ind}_{ijt} \neq \varepsilon^\text{Ind}_{jit}$, the efficiency of cost spillovers to the $i$th unit from all the $j$th units will not be equal to the efficiency of cost spillovers to all the $i$th units from a particular $j$th unit. We only consider efficiency spillovers to the $i$th unit here and leave examination of the asymmetry between efficiency spillovers for future research.

To calculate direct and aggregate indirect cost inefficiencies, $CIE^\text{Dir}_i$ and $CIE^\text{AggInd}_i$, as shares of total cost inefficiency, $CIE^\text{Tot}_i$, $SCIE^\text{Dir}_i$ and $SCIE^\text{AggInd}_i$, and $CIE^\text{Dir}_i$, $CIE^\text{AggInd}_i$ and $CIE^\text{Tot}_i$ must be calculated relative to the same unit where this unit is the best performing unit in the calculation of $CIE^\text{Tot}_i$. Formally, we recognize that $CIE^\text{Tot}_i$ can be disaggregated into its direct and aggregate indirect efficiency components:

$$CE^\text{Tot}_i = \exp \left[ \min_i CE^\text{Tot}_i (\delta^\text{Dir}_i) - \delta^\text{Dir}_i \right] \times \exp \left[ \min_i CE^\text{Tot}_i (\delta^\text{AggInd}_i) - \delta^\text{AggInd}_i \right].$$

(11)

Taking logs of equation (11) yields an expression for $CIE^\text{Tot}_i$:

$$CIE^\text{Tot}_i = \left[ \min_i CE^\text{Tot}_i (\delta^\text{Dir}_i) - \delta^\text{Dir}_i \right] + \left[ \min_i CE^\text{Tot}_i (\delta^\text{AggInd}_i) - \delta^\text{AggInd}_i \right],$$

(12)

from which $SCIE^\text{Dir}_i$ is:

$$SCIE^\text{Dir}_i = \left[ \min_i CE^\text{Tot}_i (\delta^\text{Dir}_i) - \delta^\text{Dir}_i \right] / CIE^\text{Tot}_i.$$  

(13)

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4We report $SCIE^\text{Dir}_i$ and $SCIE^\text{AggInd}_i$ but these shares are equal to the direct and aggregate indirect cost efficiency shares of $CE^\text{Tot}_i$. 

7
$SCIE_{it}^{AggInd}$ can be calculated in a similar manner.\(^5\)

4. Application: Cost Frontier for Manufacturing in U.S. States

4.1. Data

The starting point for the construction of our data set is the data used in the key study by Morrison & Schwartz (1996) (MS from hereon) to estimate a non-spatial cost function for state level manufacturing in the U.S. We focus, however, on estimation of efficiency spillovers whereas MS focus on the effect of investment in public capital. Summarizing, our data is for the period 1997-2008 for the contiguous states. We obtained all data from the Annual Survey of Manufactures (ASM) conducted by the U.S. Census Bureau unless otherwise stated and all monetary variables are expressed in 1997 prices using the CPI. The measure of output is value added ($q$), and the three input prices are average annual wages of a production worker ($h_1$) and a non-production worker ($h_2$), and the price of energy ($h_3$), where all three input prices are normalized by the price of capital.

Following MS, we assume a harmonized capital market and the price of capital is approximated by $TX_tP_t^I(r_t + \gamma)$. $TX_t$ is the corporate tax rate which we obtain for the U.S. from the OECD tax database, $P_t^I$ is the price deflator or more specifically, PPI for finished capital equipment, $r_t$ is the long-term lending rate for the manufacturing sector approximated by Moody’s Baa corporate bond yield and $\gamma$ is the depreciation rate, which following Hall (2005) we assume is 10%. The data for $h_3$ is from the U.S. Energy Administration and is the price paid by the industrial sector per million Btu. The data for total cost ($C$) is calculated by summing the wage bills for non-production and production workers, expenditure on new and used capital and expenditure on fuels and electricity. The ASM only contains manufacturing expenditure on fuels and electricity for the U.S. so this expenditure was

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\(^5\)We could proceed to estimate disaggregated indirect cost efficiencies for a given $i$. This would involve using the indirect residuals from equation (2), $\varepsilon_{ijt}^{Ind}$, to successively estimate $\varepsilon_{ijt}^{Ind} = \theta_t + \rho_t^2 + \epsilon_{ijt}$ for each $i$. The resulting disaggregated cost efficiencies refer to the efficiency of cost spillovers across the $j$th units to a given $i$th unit. The other valid estimator of disaggregated indirect cost efficiencies, which will yield a different estimate for the reasons given above, refers to the efficiency of cost spillovers across the $i$th units from a given $j$th unit. We leave the estimation of asymmetric disaggregate efficiency spillovers for future research.
allocated to the states using annual shares of U.S. energy expenditures by the industrial sector, where the state shares were calculated using data from the U.S. Energy Administration.

We extend the data set which MS use by including a number of $z$-variables which shift the cost frontier technology. To capture the effect of differences in tax conditions across states we include the ratio of personal current tax payments to personal income ($z_1$). Since the density of economic activity in a state is not meaningful because a lot of land is not productive, we follow Ciccone & Hall (1996) and control for agglomeration effects by including average county employment density within a state ($z_2$). We take account of urban roadway congestion effects by including the share of a state’s sampled urban national highway length with a volume-service flow (VSF) ratio: $< 0.21; 0.21-0.40; 0.71-0.79; 0.80-0.95; > 0.95$ ($z_3$-$z_7$, respectively, where we omit the 0.41-0.70 share). A VSF ratio $> 0.80$ indicates that congestion has set in.

Two states with small manufacturing sectors are highly efficient outliers (Rhode Island and Delaware) and were therefore omitted, which is in line with the high efficiency scores which we report for other small states in the North East region or just outside. Furthermore, we use two row-normalized specifications of $W$, a contiguity matrix, $W_1$, and a matrix weighted by average state real GDP for the manufacturing sector over the study period for contiguous states, $W_2$, which serves as a proxy for economic distance. With the exception of the data for $z_1$ and $z_3$-$z_7$, all the data is logged and normalized around the relevant sample mean so that the first order time trend, output and input prices can be interpreted as elasticities.

4.2. Estimation Results

To facilitate comparison, in Table 1 we present the non-spatial Within model as well as the models fitted using $W_1$ and $W_2$, where the time period effects are not reported for reasons of brevity. We get an indication of whether

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6The tax and income data to calculate $z_1$ and county employment data to calculate $z_2$ were obtained from the Bureau of Economic Analysis Regional Economic Accounts.

7$z_3$-$z_7$ were calculated using data from Highway Statistics published by the Federal Highway Administration. The availability of this data restricted the study period to 1997-2008.

8Descriptive statistics for the raw data are available from the corresponding author on request.
the $z$-variables are endogenous both individually and collectively from four Hausman-Wu tests by using the non-spatial Within and Hausman-Taylor estimators, where in the latter model $z_1$, $z_2$ and/or $z_3$-$z_7$ are assumed to be endogenous. All four tests fail to reject the null of no endogeneity bias at the 0.1% level. Moreover, at the 0.1% level an LR test rejects the null that the fixed effects are not jointly significant for both spatial models.

$\lambda$ cannot be interpreted as an elasticity but can be used to indicate how the spatial dependence of $C$ ia affected by the specification of $W$. The estimates of $\lambda$ are 0.21 from the $W_1$ model and 0.28 from the $W_2$ model, both of which are significant at the 0.1% level. This suggests that state manufacturing cost is about 33% more spatially dependent across our measure of economic distance than it is across state borders. In both models the direct $q$, $h_1$, $h_2$ and $h_3$ parameters are significant at the 5% level or lower. Also, in both models these four parameters are positive which suggests the monotonicity of the cost function is satisfied at the sample means.

The indirect $h_3$ parameter from the $W_2$ model is only significant at the 10% level, whereas all the other indirect input price and output parameters in the two models are significant at the 5% level or lower. The largest indirect input price or output parameter in both models is by some margin for $h_1$. This indicates that there are larger production wage spillovers than there are output, energy price or non-production wage spillovers. As expected we find evidence of technological progress from one year to the next because in both models the direct time trend parameters ($\beta_{15}$) are negative and significant at the 5% level or lower.

The parameter estimates for the $z$-variables are particularly interesting. To illustrate, we find that the direct $z_2$, $z_6$ and $z_7$ parameters are positive and significant at the 1% level or lower in both models. The implication is that state manufacturing cost will be higher in more urbanized states where employment density and urban roadway congestion are higher. The direct $z_3$ parameter is also positive and significant at the 5% level in both models. This suggests that state manufacturing cost is higher for the least urbanized states where low traffic levels on the state’s urban highways is a more frequently observed phenomenon.

4.3. Direct, Aggregate Indirect and Total Efficiencies

Efficiencies from the spatial models which are calculated using equations (8)-(10) are denoted by $CE_A$ in Table 2. To calculate the direct and aggregate indirect inefficiency shares, which are denoted by $SCIE$ in Table 2, we
Log-likelihood: $\lambda = -2 \sum_{i=1}^{N} w_i \log C_i$.

\begin{table}[h!]
\centering
\begin{tabular}{lllllllllll}
\hline
& \multicolumn{1}{c}{No SD} & \multicolumn{1}{c}{With SD: $W_1$} & \multicolumn{1}{c}{With SD: $W_2$} \\
\hline
Variable & Coef. & Direct Coef. & Indirect Coef. & Total Coef. & Direct Coef. & Indirect Coef. & Total Coef. \\
$\ln h_1$ & $\beta_1$ & 0.833*** & 0.815*** & 0.173*** & 0.988*** & 0.753*** & 0.246*** & 0.999*** \\
$\ln h_2$ & $\beta_2$ & 0.284*** & 0.294*** & 0.060*** & 0.344*** & 0.259*** & 0.084*** & 0.344*** \\
$\ln h_3$ & $\beta_3$ & 0.091*** & 0.079* & 0.017* & 0.096* & 0.059 & 0.019 & 0.078 \\
$\ln q$ & $\beta_4$ & 0.242*** & 0.233*** & 0.050*** & 0.283*** & 0.223*** & 0.073*** & 0.296*** \\
$(\ln h_1)^2$ & $\beta_5$ & 0.573 & 0.671 & 0.144 & 0.815 & 0.673 & 0.220 & 0.893 \\
$(\ln h_2)^2$ & $\beta_6$ & 0.932*** & 1.079*** & 0.231* & 1.310*** & 1.046*** & 0.342*** & 1.388*** \\
$(\ln h_3)^2$ & $\beta_7$ & -0.137** & -0.136** & -0.029* & -0.164** & -0.140** & -0.046* & -0.185** \\
$(\ln q)^2$ & $\beta_8$ & 0.02* & 0.019 & 0.004* & 0.023* & 0.014 & 0.005 & 0.019 \\
$(\ln h_1) \times t$ & $\beta_9$ & -1.188* & -1.325* & -0.282 & -1.608* & -1.287* & -0.420* & -1.707* \\
$(\ln h_2) \times t$ & $\beta_{10}$ & 0.290 & 0.206 & 0.143 & 0.249 & 0.191 & 0.062 & 0.253 \\
$(\ln h_3) \times t$ & $\beta_{11}$ & 0.121 & -0.086 & -0.018 & -0.103 & -0.054 & -0.018 & 0.072 \\
$(\ln q) \times t$ & $\beta_{12}$ & 0.127* & 0.110* & 0.023* & 0.133** & 0.113* & 0.037* & 0.150* \\
$t^2$ & $\beta_{13}$ & 0.179 & 0.109 & 0.138 & 0.217 & 0.179 & 0.097 & 0.163 \\
& $\beta_{14}$ & -0.032 & -0.034 & -0.007 & -0.041 & -0.042* & -0.014* & -0.056* \\
\hline
& \multicolumn{1}{c}{No SD} & \multicolumn{1}{c}{With SD: $W_1$} & \multicolumn{1}{c}{With SD: $W_2$} \\
\hline
$S = \sum_{i=1}^{N} w_i C_i$ & $\lambda = -2 \sum_{i=1}^{N} w_i \log C_i$ & 1.800*** & 0.258*** \\
Log-likelihood & & (4.52) & (6.80) \\
\hline
\end{tabular}
\caption{Results for selected deterministic cost frontier models}
\end{table}

\textbf{Note:} *, **, *** denote statistical significance at the 5%, 1% and 0.1% levels, respectively. SD refers to spatial dependence. $t$ statistics are in parentheses.
calculate direct, aggregate indirect and total efficiencies using equation (11) which are denoted by $CE_B$ in Table 2. The sample average own/direct $CE_A$ from the non-spatial model and the $W_2$ model is 0.40, which rises to 0.45 from the $W_1$ model. The sample average indirect (total) $CE_A$ is 0.87 (0.44) from the $W_1$ model and 0.74 (0.38) from the $W_2$ model. This suggests, firstly, that geographical contiguity ($W_1$) is a source of larger aggregate efficiency spillovers than economic contiguity ($W_2$) and, secondly, that direct efficiency is the principal component of total efficiency. Among other things, we can see from Figure 1 that average annual indirect $CE_A$ is greater than average annual direct $CE_A$ over the entire sample for both spatial models. We also observe that average annual indirect $CE_A$ from the $W_1$ model is always greater than that from the $W_2$ model.

Table 2 indicates that from both spatial models, average direct and/or indirect $CE_A$ is relatively high for several small states in the North East region or just outside (Maryland; Connecticut; New Jersey; Maine; Massachusetts; New Hampshire; Vermont). Interestingly, the two states with the largest average real GDP and average state manufacturing real GDP over the study period (California and Texas) have the lowest average direct $CE_A$ for both spatial models. The state with the third largest average real GDP and average state manufacturing real GDP (New York), however, has a respectable average direct $CE_A$ of the order 0.60 (14) and 0.63 (9), where the corresponding rankings are in parentheses. In terms of average indirect $CE_A$, California and Texas fair much better and are much closer to New York (0.96 (5) and 0.86 (6)). A comparison of average direct, indirect and total $CE_A$ for California and Texas indicate that average direct $CE_A$ is the reason for their low average total $CE_A$.

Some of the estimates of average direct (aggregate indirect) $CE_B$ are greater than 1 and when this is the case average direct (aggregate indirect) $SCIE$ is negative. This is because Connecticut is the best performing state in each period for the calculation of total cost efficiency but this is not the case for the calculation of direct and indirect cost efficiencies. To illustrate, consider the one of the more complex cases- the estimates of average direct and indirect $SCIE$ of $-5.30$ and $6.30$ for Maryland from the $W_1$ model. These estimates indicate that, on average, Maryland operates beyond the direct reference level but below the indirect reference level. The net indirect $SCIE$ is 1, indicating that Maryland’s relative total inefficiency is all due to its relative indirect inefficiency.
Table 2: Selected average cost efficiencies and efficiency shares

<table>
<thead>
<tr>
<th>State</th>
<th>CE_A</th>
<th>CE_B</th>
<th>SCIE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Direct</td>
<td>Agg Indirect</td>
<td>Total</td>
</tr>
<tr>
<td>Maryland</td>
<td>1.00(1)</td>
<td>0.90(16)</td>
<td>0.99(1)</td>
</tr>
<tr>
<td></td>
<td>1.05(1)</td>
<td>0.93(15)</td>
<td>0.98(2)</td>
</tr>
<tr>
<td></td>
<td>-5.30</td>
<td>0.30</td>
<td>-3.77</td>
</tr>
<tr>
<td>Connecticut</td>
<td>0.86(2)</td>
<td>0.96(2)</td>
<td>1.00(1)</td>
</tr>
<tr>
<td></td>
<td>1.00(2)</td>
<td>1.00(5)</td>
<td>1.00(1)</td>
</tr>
<tr>
<td></td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>New Jersey</td>
<td>0.84(3)</td>
<td>0.88(19)</td>
<td>0.86(3)</td>
</tr>
<tr>
<td></td>
<td>0.96(3)</td>
<td>0.91(19)</td>
<td>0.95(2)</td>
</tr>
<tr>
<td></td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Illinois</td>
<td>0.18(44)</td>
<td>0.82(35)</td>
<td>0.19(44)</td>
</tr>
<tr>
<td></td>
<td>0.23(44)</td>
<td>0.85(35)</td>
<td>0.19(44)</td>
</tr>
<tr>
<td></td>
<td>0.90</td>
<td>0.30</td>
<td>0.84</td>
</tr>
<tr>
<td>California</td>
<td>0.17(45)</td>
<td>0.91(11)</td>
<td>0.15(45)</td>
</tr>
<tr>
<td></td>
<td>0.16(45)</td>
<td>0.94(11)</td>
<td>0.15(45)</td>
</tr>
<tr>
<td></td>
<td>0.07</td>
<td>0.03</td>
<td>0.95</td>
</tr>
<tr>
<td>Texas</td>
<td>0.08(46)</td>
<td>0.88(18)</td>
<td>0.08(46)</td>
</tr>
<tr>
<td></td>
<td>0.09(46)</td>
<td>0.92(17)</td>
<td>0.09(46)</td>
</tr>
<tr>
<td></td>
<td>0.96</td>
<td>0.04</td>
<td>0.93</td>
</tr>
<tr>
<td>Maine</td>
<td>0.63(10)</td>
<td>1.00(1)</td>
<td>0.68(9)</td>
</tr>
<tr>
<td></td>
<td>0.66(12)</td>
<td>1.04(1)</td>
<td>0.68(9)</td>
</tr>
<tr>
<td></td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Massachusetts</td>
<td>0.81(4)</td>
<td>0.74(8)</td>
<td>0.73(4)</td>
</tr>
<tr>
<td></td>
<td>0.72(9)</td>
<td>1.03(2)</td>
<td>0.74(8)</td>
</tr>
<tr>
<td></td>
<td>1.10</td>
<td>-0.10</td>
<td>1.00</td>
</tr>
<tr>
<td>New Hampshire</td>
<td>0.75(5)</td>
<td>0.82(7)</td>
<td>0.76(7)</td>
</tr>
<tr>
<td></td>
<td>0.81(7)</td>
<td>1.01(3)</td>
<td>0.82(7)</td>
</tr>
<tr>
<td></td>
<td>1.07</td>
<td>-0.07</td>
<td>1.13</td>
</tr>
<tr>
<td>Vermont</td>
<td>0.63(9)</td>
<td>0.89(4)</td>
<td>0.89(5)</td>
</tr>
<tr>
<td></td>
<td>0.84(6)</td>
<td>1.01(3)</td>
<td>0.85(5)</td>
</tr>
<tr>
<td></td>
<td>1.08</td>
<td>-0.08</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Note: The corresponding ranking is given in parenthesis where the rankings are all in descending order.
We have extended the CSS estimator of non-spatial time-variant efficiency to the case where there is spatial autoregressive dependence. This involved deriving estimators of direct efficiency (own efficiency accounting for feedback effects), indirect (spillover) efficiency and total (direct plus indirect) efficiency. We then estimated a state level cost frontier model for U.S. manufacturing to apply our spatial efficiency estimators. There are a number of possibilities for further analysis of the estimators some of which we have alluded to here such as asymmetric indirect efficiencies.

Acknowledgments

The application to state manufacturing was inspired by the participants in the special session dedicated to the memory of Catherine Morrison Paul at the 2012 North American Productivity Workshop.


